time-series

December 5, 2020

1 IBEX35 Time Series

In this Jupyter Notebook we will analyse the time series of the closing value of the Spanish index IBEX35.

Data has been found in https://es.finance.yahoo.com/quote/%5EIBEX/history/

The study will be centered in the absolute value of the difference in the closing proce between two consecutive days.

1.0.1 Read and clean the data

We are going to read the data from the file downloaded and clean it to keep just the values we are interested in.

Read the file

[1]:
```
import pandas as pd
# read the data
data = pd.read_csv('../data/ibex.csv')
data
```

[1]:

	Date	Open	High	Low	Close	Adj Close	Volume
0	1993-02-15	9834.599609	9913.400391	9764.099609	9865.299805	9865.290039	10256100.0
1	1993-02-16	NaN	NaN	NaN	NaN	NaN	NaN
2	1993-02-17	NaN	NaN	NaN	NaN	NaN	NaN
3	1993-02-18	NaN	NaN	NaN	NaN	NaN	NaN
4	1993-02-19	NaN	NaN	NaN	NaN	NaN	NaN
…	…	…	…	…	…		
7169	2020-11-27	8069.100098	8190.700195	8054.399902	8190.700195		
7170	2020-11-30	8130.600098	8165.200195	8075.399902	8076.899902		
7171	2020-12-01	8112.700195	8176.399902	8100.200195	8140.799805		
7172	2020-12-02	8111.100098	8226.200195	8104.200195	8220.799805		
7173	2020-12-03	8208.099609	8223.400391	8154.799805	8200.700195		

```
4              NaN          NaN
…              …            …
7169  8190.700195  415126100.0
7170  8076.899902  544905000.0
7171  8140.799805  284946100.0
7172  8220.799805  310837500.0
7173  8200.700195  233469300.0

[7174 rows x 7 columns]
```

Clean the data

[2]:
```
# I will use only the close value so ill drop the rest of the colums and ill␣
↪keep just the date and the close.
data = data.loc[:, ['Date', 'Close']]
# and we want the date to be a date and to be the index of our dataframe
data['Date'] = pd.to_datetime(data['Date'])
data
```

[2]:
```
           Date        Close
0    1993-02-15  9865.299805
1    1993-02-16          NaN
2    1993-02-17          NaN
3    1993-02-18          NaN
4    1993-02-19          NaN
…           …           …
7169 2020-11-27  8190.700195
7170 2020-11-30  8076.899902
7171 2020-12-01  8140.799805
7172 2020-12-02  8220.799805
7173 2020-12-03  8200.700195

[7174 rows x 2 columns]
```

[3]:
```
# now ill check where the NaNs are
nan_index = data['Close'].isnull().to_numpy().nonzero()[0]
nan_index
```

[3]:
```
array([   1,    2,    3,    4,    5,    6,    7,    8,    9,   10,   11,
         12,   13,   14,   15,   16,   17,   18,   19,   20,   21,   22,
         23,   25,   26,   27,   28,   29,   30,   31,   32,   33,   34,
         35,   38,   39,  171,  185,  210,  212,  224,  233,  294,  295,
        315,  432,  446,  452,  471,  473,  485,  494,  545,  563,  564,
        575,  576,  585,  651,  693,  707,  732,  734,  745,  750,  818,
        819,  837,  838,  847,  913,  969,  994, 1006, 1007, 1012, 1015,
       1073, 1074, 1098, 1099, 1108, 1159, 1174, 1255, 1267, 1268, 1273,
       1276, 1328, 1343, 1344, 1345, 1359, 1380, 1475, 1516, 1528, 1529,
```

```
            1533, 1534, 1537, 1598, 1599, 1600, 1736, 1750, 1775, 1777, 1789,
            1794, 1798, 1874, 1875, 1880, 1998, 2012, 2037, 2039, 2050, 2051,
            2055, 2129, 2130, 2141, 2217, 2236, 2259, 2298, 2310, 2311, 2312,
            2315, 2316, 2379, 2380, 2402, 2478, 2534, 2559, 2571, 2572, 2573,
            2576, 2577, 2580, 2654, 2655, 2663, 2739, 2820, 2832, 2833, 2834,
            2837, 2838, 2841, 2909, 2910, 3000, 3041, 3055, 3080, 3082, 3094,
            3099, 3103, 3159, 3160, 3355, 3615, 3867, 6170, 6932])
```

[4]:
```
# lets check what dates are those
data['Date'][nan_index].tail(20)
```

[4]:
```
2837   2003-12-31
2838   2004-01-01
2841   2004-01-06
2909   2004-04-09
2910   2004-04-12
3000   2004-08-16
3041   2004-10-12
3055   2004-11-01
3080   2004-12-06
3082   2004-12-08
3094   2004-12-24
3099   2004-12-31
3103   2005-01-06
3159   2005-03-25
3160   2005-03-28
3355   2005-12-26
3615   2007-01-01
3867   2007-12-24
6170   2017-01-02
6932   2019-12-25
Name: Date, dtype: datetime64[ns]
```

[5]:
```
# we can see that there are a bunch of rows at the begginig without data and␣
 ↪the rest are holiday days
# for example: 2005-12-26 and 2017-01-02 are bank holidays because 2005-12-25␣
 ↪and 2017-01-01 were Sundays
# so I will just not use those rows because they the markets were not open␣
 ↪those days
# also I will just use data from 1994 onwards
data = data.dropna()
data = data.set_index('Date')
data = data.loc[pd.datetime(1994,1,1):]
data
```

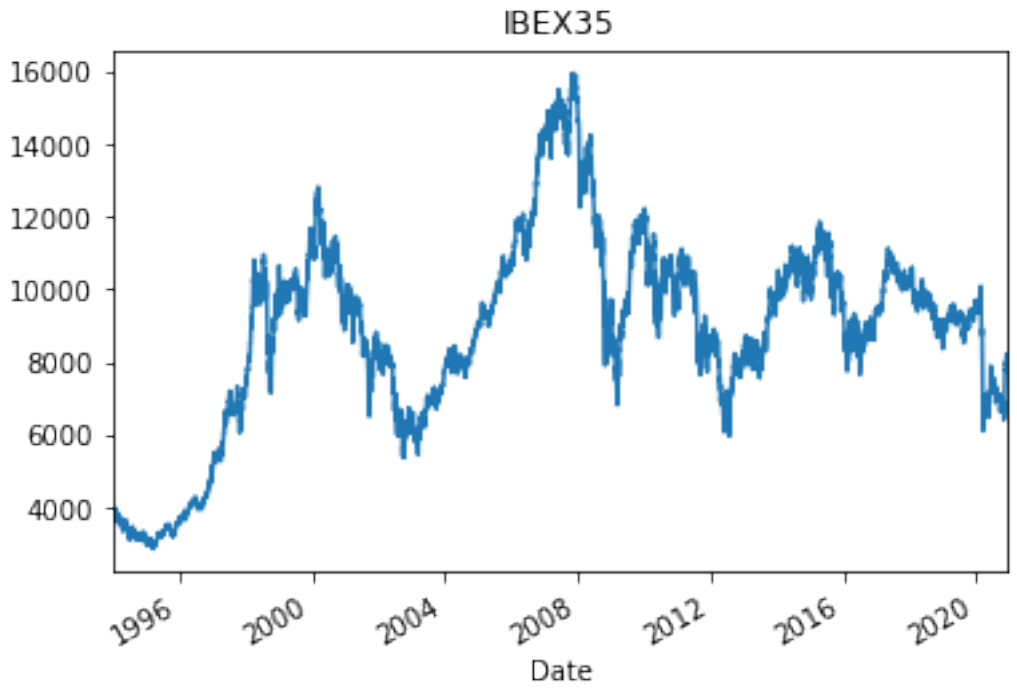
[5]:
```
                Close
Date
```

```
1994-01-03  3654.500000
1994-01-04  3630.300049
1994-01-05  3621.199951
1994-01-07  3636.399902
1994-01-10  3660.600098
…                 …
2020-11-27  8190.700195
2020-11-30  8076.899902
2020-12-01  8140.799805
2020-12-02  8220.799805
2020-12-03  8200.700195

[6811 rows x 1 columns]
```

[6]:
```
# now lets plot the time serie
data['Close'].plot(title='IBEX35')
```

[6]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x7fdbf6ea9d00>
```



1.0.2 Transform the data

Wee are not going to study the serie like it is, we will study the daily change in the absolute value of the closing price, so we need to calculate that change.

[7]:
```
# calculate the diference with the previous day
data = data.diff().apply(abs)
# the firs line will be a nan so we are just going to delete it
data = data.dropna()
data=data.rename(columns = {'Close':'abs_diff'})
data
```

[7]:
```
              abs_diff
Date
1994-01-04   24.199951
1994-01-05    9.100098
1994-01-07   15.199951
1994-01-10   24.200196
1994-01-11   51.799804
…                  …
2020-11-27   86.100097
2020-11-30  113.800293
2020-12-01   63.899903
2020-12-02   80.000000
2020-12-03   20.099610

[6810 rows x 1 columns]
```

1.0.3 Discretize the data

I am going to discretize the data with intervals of 10, so:

[0, 10) -> 0

[10, 20) -> 10

…

[8]:
```
# lets discretize it with intervals of 10
data = data.apply(lambda x: x/10).astype('int16')*10
data
```

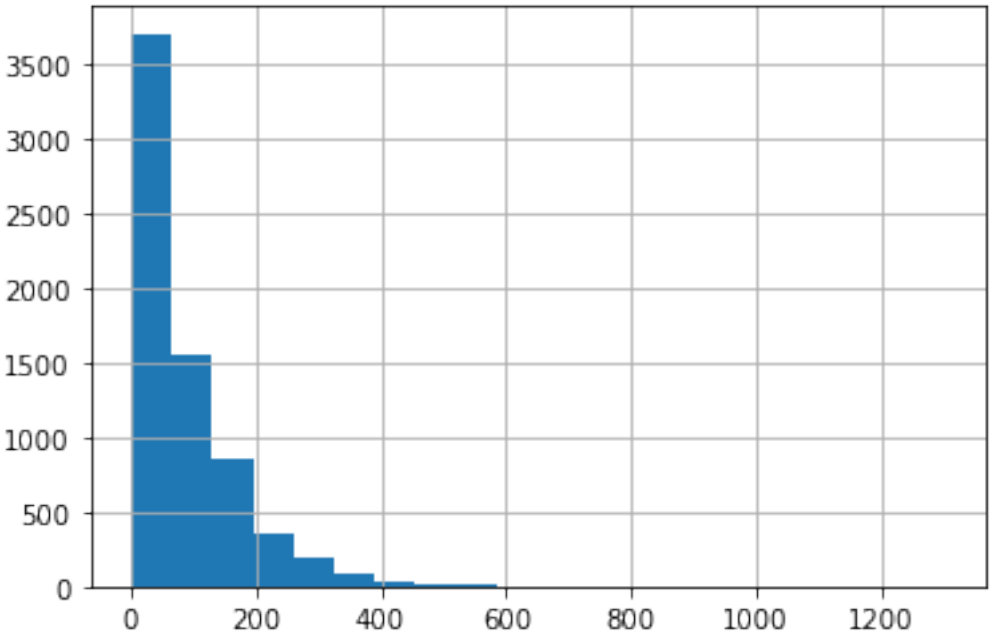
[8]:
```
            abs_diff
Date
1994-01-04        20
1994-01-05         0
1994-01-07        10
1994-01-10        20
1994-01-11        50
…                  …
2020-11-27        80
2020-11-30       110
2020-12-01        60
2020-12-02        80
```

```
2020-12-03        20

[6810 rows x 1 columns]
```

[9]:
```
# lets do a histogram to see wich values are the most common
data['abs_diff'].hist(bins=20)
```

[9]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x7fdbf6c8e7c0>
```



We can see that most of the days the change is small, close to 0.

1.0.4 PST Vocabulary

The vocabulary of the PST are the different states the time serie can have.

We had a continuous time serie and we discretized it with intervals of 10 so the vocabulary of our PST is {0, 10, 20, ..., 900)

[10]:
```
# lets see what are the most common states
data['abs_diff'].value_counts()
```

[10]:
```
0      653
10     650
20     621
30     521
40     464
      ...
```

```
740      1
700      1
680      1
600      1
900      1
Name: abs_diff, Length: 74, dtype: int64
```

We can see that we have 74 states and the most common one are 0 and 10 with 650 or more occurencies.

1.0.5 PST States

The states are subsequeces of of the time serie that appears an enough number of times.

Order 1 states are the same than the vocabulary (0, 10, 20, ...), lets study orde2 and 3.

The order 2 states and the order 3 states are:

[11]:
```
# The different states in different orders of the series will be formed for␣
↪columns.
# Order 2 will be colum 1 and 2, order 3 will be column 1, 2 and 3

data['abs_diff+1'] = data['abs_diff']
data['abs_diff+2'] = data['abs_diff']
data['abs_diff+1'][:-1] = data['abs_diff'][1:]
data['abs_diff+2'][:-1] = data['abs_diff+1'][1:]
data
```

[11]:

Date	abs_diff	abs_diff+1	abs_diff+2
1994-01-04	20	0	10
1994-01-05	0	10	20
1994-01-07	10	20	50
1994-01-10	20	50	30
1994-01-11	50	30	0
…	…	…	…
2020-11-27	80	110	60
2020-11-30	110	60	80
2020-12-01	60	80	20
2020-12-02	80	20	20
2020-12-03	20	20	20

[6810 rows x 3 columns]

Now lets see how many of these states of order 2 and 3 we have and which ones are the mos common ones.

[12]:
```
from collections import Counter
```

```
# we will get the states combining the column abs_diff and abs_diff+1 in a␣
 ↪string and grouping by this column
data.groupby(['abs_diff', 'abs_diff+1'])['abs_diff'].count().
 ↪sort_values(ascending=False).head(20)
```

[12]:
```
abs_diff  abs_diff+1
0         20            88
          0             82
20        0             82
10        0             81
0         10            79
10        10            78
          20            76
30        10            72
20        20            71
          10            69
0         30            67
30        0             66
10        30            63
40        10            58
20        30            55
0         40            50
30        20            47
10        50            46
30        30            45
20        40            45
Name: abs_diff, dtype: int64
```

[13]:
```
# we will get the states combining the column abs_diff, abs_diff+1 and␣
 ↪abs_diff+2 in a string and grouping by this column
data.groupby(['abs_diff', 'abs_diff+1', 'abs_diff+2'])['abs_diff'].count().
 ↪sort_values(ascending=False).head(40)
```

[13]:
```
abs_diff  abs_diff+1  abs_diff+2
10        0           20            18
20        0           0             18
          20          10            17
0         20          0             15
10        20          20            15
0         10          20            15
          30          10            14
20        10          0             14
0         0           20            14
          20          10            14
10        10          0             14
          20          0             14
          0           0             13
```

```
                    10              13
0           10      10              13
20          20      30              12
0           20      20              12
10          0       30              12
            10      10              12
30          20      0               12
            10      20              12
20          0       10              12
                    20              12
            20      0               12
0           10      0               12
20          10      20              12
10          10      20              11
30          0       30              11
            10      0               11
0           0       10              11
30          0       20              11
20          30      0               10
            10      30              10
10          30      10              10
0           30      0               10
30          20      20              10
            30      10              10
40          0       0               10
20          10      10              10
0           0       0               10
Name: abs_diff, dtype: int64
```

We can see from the results that both for those of order 2 and for those of order 3 the lower values of our states (0, 10, 20, 30) are the most common one. That is not strange because as we saw before they are the most common values of our time serie.

1.0.6 Transition matrix

The transition matrix is a square matrix used to describe the transitions of a Markov chain. Each of its entries is a nonnegative real number representing a probability.

If the probability of been in a state s_i after a state s_j is p(s_i|s_j) then the item in the transition matrix aij is described as p(s_i|s_j). If we have the states a, b and c the transition matrix would be:

	a	b	c
a	p(a|a)	p(b|a)	p(c|a)
b	p(a|b)	p(b|b)	p(c|b)
c	p(a|c)	p(b|c)	p(c|c)

Now we are going to see an example for the subsequence (0, 0, 0).

[14]:
```
# the data this group contain is
data.groupby(['abs_diff', 'abs_diff+1', 'abs_diff+2']).get_group((0,0,0))
```

[14]:

Date	abs_diff	abs_diff+1	abs_diff+2
1995-05-16	0	0	0
1995-06-02	0	0	0
1995-06-14	0	0	0
1996-02-16	0	0	0
1996-05-13	0	0	0
1996-11-07	0	0	0
2005-05-13	0	0	0
2010-10-19	0	0	0
2017-01-18	0	0	0
2017-05-30	0	0	0

[15]:
```
# Now lest see what is the next value for those subsequences
data['abs_diff+3'] = data['abs_diff']
data['abs_diff+3'][:-1] = data['abs_diff+2'][1:]
data000 = data.groupby(['abs_diff', 'abs_diff+1', 'abs_diff+2']).
 ↪get_group((0,0,0))
data000
```

[15]:

Date	abs_diff	abs_diff+1	abs_diff+2	abs_diff+3
1995-05-16	0	0	0	10
1995-06-02	0	0	0	10
1995-06-14	0	0	0	10
1996-02-16	0	0	0	50
1996-05-13	0	0	0	40
1996-11-07	0	0	0	30
2005-05-13	0	0	0	150
2010-10-19	0	0	0	30
2017-01-18	0	0	0	70
2017-05-30	0	0	0	20

[16]:
```
# calculate the frequecy for each value
data000.groupby(['abs_diff+3'])['abs_diff'].count().
 ↪sort_values(ascending=False).apply(lambda x: x/data000.shape[0])
```

[16]:

abs_diff+3	
10	0.3
30	0.2
150	0.1
70	0.1

```
50     0.1
40     0.1
20     0.1
Name: abs_diff, dtype: float64
```

The transition matrix for the state (0, 0, 0) would be:

State	Count	Next state	10	30	150	70	50	40	20
(0,0,0)	10	Probability	0.3	0.2	0.1	0.1	0.1	0.1	0.1

The probabilities that do not appear in the table are 0.

1.0.7 Conclusions

On the one hand, PST may not be the best method to "learn" from the data of this example because the vocabulary on this example is very big and sparse.

Most of the useful states will be of length 1 or 2 with some of length 3 (and I doubt there is any with lenght 4 given the frequecy outpputed by the subsequences of order 3).

On the other hand, the frequency of the states with low numbers like 0, 10 and 20 are very frequent so we could mainly those. This would have a drawback and it is that with this approach we would not be able to predict peaks beacuse we excluded them from our vocabulary.

Maybe one solution for this would be a non linear discretization, something like:

[0, 3) -> 1

[3, 10) -> 2

[10, 20) -> 3

…

[90, 100) -> 11

[100, 150) -> 12

[150, 200) -> 13

…

With this kind of discretization we would be able to detect peaks but no exactly their size. The prediction of the size would depend on the resolution of the discretization in those values.