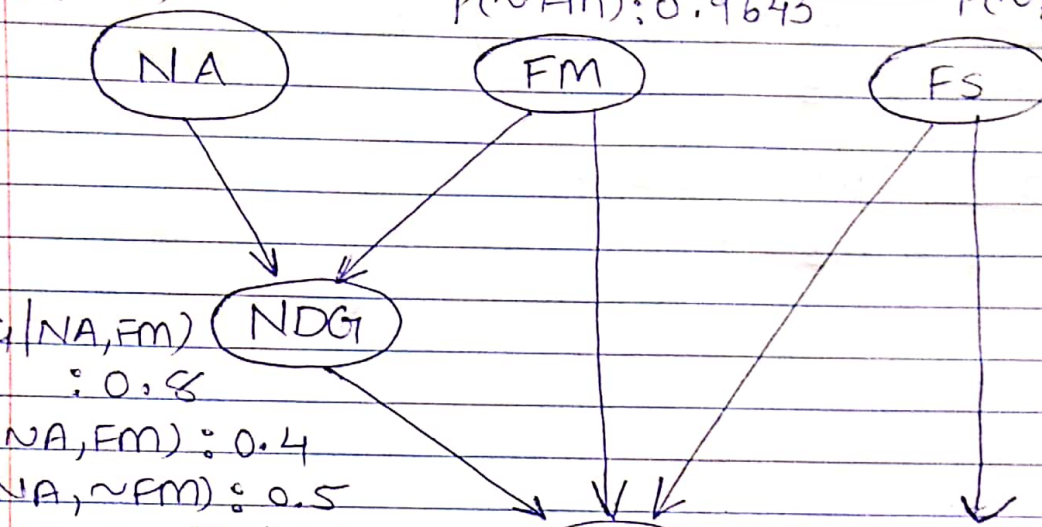
Bayes Net

(1)



P(NA) : 0.3
P(~NA) : 0.7

P(FM) : 0.0357
P(~FM) : 0.9643

P(FS) : 0.05
P(~FS) : 0.95

P(NDG|NA,FM) : 0.8
P(NDG|~NA,FM) : 0.4
P(NDG|NA,~FM) : 0.5
P(NDG|~NA,~FM) : 0
P(~NDG|NA,FM) : 0.2
P(~NDG|~NA,FM) : 0.6
P(~NDG|NA,~FM) : 0.5
P(~NDG|~NA,~FM) : 1

P(FH|FM,NDG,FS) : 0.99
P(FH|~FM,NDG,FS) : 0.75
P(FH|FM,~NDG,FS) : 0.9
P(FH|FM,NDG,~FS) : 0.65
P(FH|~FM,~NDG,FS) : 0.5
P(FH|~FM,NDG,~FS) : 0.2
P(FH|FM,~NDG,~FS) : 0.4
P(FH|~FM,~NDG,~FS) : 0.

P(FB|FS) : 0.6
P(FB|~FS) : 0.1
P(~FB|FS) : 0.4
P(~FB|~FS) : 0.9

(2) $P(FH)$

$$= \sum_{FS,NDG,NA,FM,FB} P(FH, FS, FB, FM, NA, NDG)$$

$$= \sum_{FS,NDG,NA,FM,FB} P(FH \mid NDG, FM, FS) \cdot P(FB \mid FS) \cdot P(NDG \mid NA, FM) \cdot P(NA) \cdot P(FS) \cdot P(FM)$$

$$= \sum_{FS} f_1(FS) \sum_{NDG} \sum_{NA} f_2(NA) \sum_{FM} f_3(NDG, NA, FM) \cdot f_4(FM)$$

$$f_5(FH, NDG, FM, FS) \sum_{FB} f_6(FB, FS)$$

Initially factors are

$f_1(FS)$ = FS : 0.05
~FS : 0.95

$f_2(NA)$ = NA : 0.3
~NA : 0.7

$f_3(NDG, NA, FM)$ = NDG, NA, FM : 0.8
~NDG, NA, FM : 0.2
NDG, ~NA, FM : 0.4
NDG, NA, ~FM : 0.5
~NDG, ~NA, FM : 0.6
~NDG, NA, ~FM : 0.5
NDG, ~NA, ~FM : 0
~NDG, ~NA, ~FM : 1

$f_4(FM)$ = FM : 0.0357
~FM : 0.9643

f_5(FH, NDG, FM, FS) =

= FH, NDG, FM, FS : 0.99
~FH, NDG, FM, FS : 0.01
FH, ~NDG, FM, FS : 0.9
FH, NDG, ~FM, FS : 0.75
FH, NDG, FM, ~FS : 0.65
~FH, ~NDG, FM, FS : 0.1
~FH, NDG, ~FM, FS : 0.25
~FH, NDG, FM, ~FS : 0.35
FH, ~NDG, ~FM, FS : 0.5
FH, ~NDG, FM, ~FS : 0.4
FH, NDG, ~FM, ~FS : 0.2
~FH, ~NDG, ~FM, FS : 0.5
~FH, ~NDG, FM, ~FS : 0.6
~FH, NDG, ~FM, ~FS : 0.8
FH, ~NDG, ~FM, ~FS : 0
~FH, ~NDG, ~FM, ~FS : 1

f_6(FB, FS) = FB, FS : 0.6
~FB, FS : 0.4
FB, ~FS : 0.1
~FB, ~FS : 0.9

(a) Multiply and sum out FB.

$f_7(FS)$ = FS : 1
~FS : 1

Factors are: $f_1(FS)$, $f_2(NA)$,
$f_3(NDG, NA, FM)$, $f_4(FM)$,
$f_5(FH, NDG, FM, FS)$, $f_7(FS)$

(b) Multiply and sum out FM.

i.e. $\sum_{FM} f_3(NDG, NA, FM)\, f_4(FM)\, f_5(FH, NDG, FM, FS)$

$= f_8(FH, NDG, NA, FS)$

= NDG, NA, FH, FS : 0.3898
~NDG, NA, FH, FS : 0.2475
NDG, ~NA, FH, FS : 0.01414
NDG, NA, ~FH, FS : 0.12082
NDG, NA, FH, ~FS : 0.11499
~NDG, ~NA, FH, FS : 0.501428
~NDG, NA, ~FH, FS : 0.241789
~NDG, NA, FH, ~FS : 0.002856
NDG, ~NA, ~FH, ~FS : 0.0001428
NDG, ~NA, FH, ~FS : 0.009282
NDG, NA, ~FH, ~FS : 0.39571
~NDG, ~NA, ~FH, FS : 0.484292
~NDG, ~NA, FH, ~FS : 0.008568
~NDG, NA, ~FH, ~FS : 0.48643
NDG, ~NA, ~FH, ~FS : 0.004998
~NDG, ~NA, ~FH, ~FS : 0.977152

Factors are: $f_1(FS), f_2(NA), f_7(FS),$
$f_8(FH, NDG, NA, FS)$

(C) Multiply and sum out NA.

$$\sum_{NA} f_2(NA)\, f_8(FH, NDG, NA, FS)$$
$$= f_9(FH, NDG, FS)$$

= FH, NDG, FS : 0.1268
~FH, NDG, FS : 0.036346
FH, ~NDG, FS : 0.42525
FH, NDG, ~FS : 0.040994.
~FH, ~NDG, FS : 0.41154
~FH, NDG, ~FS: 0.12221
FH, ~NDG, ~FS: 0.00685
~FH, ~NDG, ~FS: 0.82993

Factors are: $f_1(FS), f_7(FS),$
$f_9(FH, NDG, FS.$

(d) Multiply and sum out NDG:
i.e $\sum_{NDG} f_9(FH, NDG, FS)$
$= f_{10}(FH, FS)$ = FH, FS: 0.5521
~FH, FS: 0.447884.
FH, ~FS: 0.04784.
~FH, ~FS: 0.952142.

Factors are: $f_1(FS) \cdot f_7(FS),$
$f_{10}(FH, FS)$

(e) Multiply and sum out FS.

i.e. $\sum_{FS} f_1(FS) \cdot F_{10}(FH, FS) \cdot f_7(FS)$

$f(FH) = FH : 0.0730625$

$\sim FH : 0.92693$

$\therefore P(FH) = 0.07306$

(3) $P(FS \mid FM, FH)$

$$= \frac{P(FS, FM, FH)}{P(FM, FH)}$$

Numerator

$$P(FS, FM, FH)$$

$$= \sum_{NA, NDG, FB} P(FS, FM, FH, NA, NDG, FB)$$

$$= \sum_{NA, NDG, FB} P(FH \mid NDG, FM, FS) \cdot P(FB \mid FS) \cdot P(NDG \mid NA, FM) \cdot P(NA) \cdot P(FM) \cdot P(FS)$$

$$= P(FM) \cdot P(FS) \sum_{NDG} P(FH \mid NDG, FM, FS) \cdot \sum_{NA} P(NDG \mid NA, FM) \cdot P(NA) \sum_{FB} P(FB \mid FS)$$

$$= P(FM)\, f_1(FS) \sum_{NDG} f_2(NDG, FS) \cdot \sum_{NA} f_3(NA, NDG) \sum_{FB} f_4(FB, FS)$$

Restricting factors given $E = \{FM, FH\}$, we get the following factors:

$f_1(FS) = FS : 0.05$
$\sim FS : 0.95$

$f_2(NDG, FS) = NDG, FS : 0.99$
$\sim NDG, FS : 0.9$
$NDG, \sim FS : 0.65$
$\sim NDG, \sim FS : 0.4$

$f_3(NA, NDG) = NA, NDG : 0.8$
$\sim NA, NDG : 0.4$
$NA, \sim NDG : 0.2$
$\sim NA, \sim NDG : 0.6$

$f_4(NA) = NA : 0.3$
$\sim NA : 0.7$

$f_5(FB, FS) = FB, FS : 0.6$
$\sim FB, FS : 0.4$
$FB, \sim FS : 0.1$
$\sim FB, \sim FS : 0.9$

(a) Multiply and sum out B

$f_6(FS) = FS : 1$
$\sim FS : 1$

Factors are: $f_1(FS), f_2(NDG, FS), f_3(NA, NDG), f_4(NA), f_6(FS)$

(b) Multiply and sum out NA.

i.e. $\sum_{NA} f_3(NA, NDG) \cdot f_4(NA)$

$= f_7(NDG) = NDG : 0.52$
$\sim NDG : 0.48$

factors are: $f_1(FS), f_2(NDG, FS), f_6(FS), f_7(NDG)$

(C) Multiply and sum out NDG

i.e. $\sum_{NDG} f_2(FS, NDG) \cdot f_7(NDG)$

$= f_8(FS) = FS : 0.9468$
$\sim FS : 0.53$

Factors are: $f_1(FS), f_6(FS), f_8(FS)$

$\therefore P(FS, FM, FH) = P(FM) \cdot f_1(FS) \cdot f_6(FS) \cdot f_8(FS)$

Multiplying them, we get

$f(FS) = FS : 0.001689$
$\sim FS : 0.017974$

Denominator

$P(FM, FH) = P(FH|FM) \cdot P(FM)$

~~$P(FH|FM) = P(FH, FM)$~~

$\therefore$ Evidence = FM.

$$P(FH, FM) = \sum_{NA, NDG, FS, FB} P(FH, FM, NA, NDG, FS, FB)$$

$$= \sum_{NA, NDG, FS, FB} P(FH|NDG, FM, FB) P(FB|FS) \cdot P(NDG|NA, FM) \cdot P(NA) \cdot P(FM) \cdot P(FS)$$

$$= P(FM) \cdot \sum_{FS} f_1(FS) \sum_{NDG} f_2(FH, NDG, FS) \cdot \sum_{NA} f_3(NDG, NA) f_4(NA) \sum_{FB} f_7(FB, FS)$$

Restricting factors given evidence fm, we get the following :

$f_1(FS) = FS : 0.05$
$\sim FS : 0.95$

$f_2(FH, NDG, FS) = FH, NDG, FS : 0.99$
$\sim FH, NDG, FS : 0.01$
$FH, \sim NDG, FS : 0.9$
$FH, NDG, \sim FS : 0.65$
$\sim FH, \sim NDG, FS : 0.1$
$\sim FH, NDG, \sim FS : 0.35$
$FH, \sim NDG, \sim FS : 0.4$
$\sim FH, \sim NDG, \sim FS : 0.6.$

$f_3(NDG, NA) = NDG, NA : 0.8$
$\sim NDG, NA : 0.2$
$NDG, \sim NA : 0.4$
$\sim NDG, \sim NA : 0.6.$

$f_4(NA) = NA : 0.3$
$\sim NA : 0.7$

$f_5(FB, FS) = FB, FS : 0.6$
$\sim FB, FS : 0.4$
$FB, \sim FS : 0.1$
$\sim FB, \sim FS : 0.9.$

(a) Multiply and sum out FB.

$f_6(FS) = FS: 1$
$\sim FS: 1.$

Factors are: $f_1(FS), f_2(FH, NDG, FS),$
$f_3(NDG, NA), f_4(NA), f_6(FS)$

(b) Multiply and sum out NA.

i.e. $\sum_{NA} f_3(NDG, NA) \cdot f_4(NA)$

$= f_7(NDG) = NDG: 0.52$
$\sim NDG: 0.48.$

Factors are: $f_1(FS), f_2(FH, NDG, FS),$
$f_6(FS), f_7(NDG)$

(C) Multiply and sum out NDG.

i.e. $\sum_{NDG} f_2(FH, NDG, FS) \cdot f_7(NDG)$

$= f_8(FH, FS) = FH, FS: 0.9468$
$\sim FH, FS: 0.0532$
$FH, \sim FS: 0.53$
$\sim FH, \sim FS: 0.47.$

Factors are: $f_1(FS) \cdot f_6(FS) \cdot f_8(FH, FS)$

(d) Multiply and sum out FS

i.e. $\sum_{FS} f_1(FS)\, f_8(FH, FS) \cdot f_6(FS)$

$= f_9(FH) = FH : 0.55084$

$\sim FH : 0.44916$

$\therefore P(FM, FH) = P(fm) \cdot f_9(FH)$

Multiplying them, we get

$f(FH) = FH : 0.019665$

$\sim FH : 0.016035$

$P(FS \mid FM, FH)$

$= \frac{P(FS, FM, FH)}{P(FM, FH)} = \frac{0.001689}{0.019665}$

$= 0.0859$

(4) $P(FS \mid FM, FH, FB)$

$$= \frac{P(FS, FM, FH, FB)}{P(FM, FH, FB)}$$

Numerator $P(FS, FM, FH, FB)$

$$= \sum_{NA, NDG} P(FH \mid NDG, FM, FS) \cdot P(FB \mid FS) \cdot P(NDG \mid NA, FM) \cdot P(NA) \cdot P(FM) \cdot P(FS)$$

$$= P(FM) \cdot P(FS) \cdot P(FB \mid FS) \sum_{NDG} P(FH \mid NDG, FM, FS) \cdot \sum_{NA} P(NDG \mid NA; FM) \cdot P(NA)$$

$$= P(FM) \cdot f_1(FS) \cdot f_2(FS) \sum_{NDG} f_3(NDG, FS) \sum_{NA} f_4(NDG, NA) \cdot f_5(NA)$$

Restricting factors given $E = \{FM, FH, FB\}$, we get the following factors:

$f_1(FS) = FS : 0.05$
$\sim FS : 0.95$

$f_2(FS) = FS : 0.6$
$\sim FS : 0.1$

$f_3(NDG, FS) = NDG, FS : 0.99$
$\sim NDG, FS : 0.9$
$NDG, \sim FS : 0.65$
$\sim NDG, \sim FS : 0.4$

$f_4(NDG, NA) = NDG, NA : 0.8$
$\sim NDG, NA : 0.2$
$NDG, \sim NA : 0.4$
$\sim NDG, \sim NA : 0.6$

$f_5(NA) = NA : 0.3$
$\sim NA : 0.7$

(a) Multiply and sum out NA

i.e. $\sum_{NA} f_4(NDG, NA) \cdot f_5(NA)$

$= f_6(NDG) = NDG : 0.52$
$\sim NDG : 0.48$

Factors are: $f_1(FS), f_2(FS), f_3(NDG, FS), f_6(NDG)$

(b) Multiply and sum out NDG

i.e. $\sum_{NDG} f_3(NDG, FS) \cdot f_6(NDG)$

$= f_7(FS) = FS : 0.9468$
$\sim FS : 0.53$

Factors are: $f_1(FS), f_2(FS), f_7(FS)$

$P(FS, FM, FH, FB) = P(FM) \cdot f_1(FS) \cdot f_2(FS) \cdot f_7(FS)$

Multiplying them, we get

$f(FS)$ $FS : 0.001014$
$\sim FS : 0.001798.$

Denominator $P(FB, FM, FH)$

$= P(FB \mid FM, FH) \times P(FM, FH)$

$\therefore$ Evidence $= FM, FH)$

$$P(FB, FM, FH) = \sum_{NA, NDG, FS} P(FH \mid NDG, FM, FS) \cdot P(FB \mid FS) \cdot P(NDG \mid NA, FM) \cdot P(NA) \cdot P(FM) \cdot P(FS)$$

$$= P(FM) \sum_{FS} P(FB \mid FS) \cdot P(FS) \sum_{NDG} P(FH \mid NDG, FM, FS) \cdot \sum_{NA} P(NDG \mid NA, FM) \cdot P(NA)$$

$$= P(FM) \sum_{S} f_1(FB, FS)\, f_2(FS) \sum_{NDG} f_3(NDG, FS) \sum_{NA} f_4(NDG, NA)\, f_5(NA)$$

Restricting factors given $E = \{FM, FH\}$, we get the following factors:

$f_1(FB, FS) =$ FB, FS : 0.6
~FB, FS : 0.4
FB, ~FS : 0.1
~FB, ~FS 0.9

$f_2(FS) =$ FS : 0.05
~FS : 0.95

$f_3(NDG, FS) =$ NDG, FS : 0.99
~NDG, FS : 0.9
NDG, ~FS : 0.65
~NDG, ~FS : 0.4

$f_4(NDG, NA) =$ NDG, NA : 0.8
~NDG, NA : 0.2
NDG, ~NA : 0.4
~NDG, ~NA : 0.6

$f_5(NA) =$ NA : 0.3
~NA : 0.7

(a) Multiply and Sum out NA

$\sum_{NA} f_4(NDG, NA) \cdot f_5(NA)$

$= f_6(NDG)$ = NDG: 0.52

~NDG: 0.48.

Factors are: $f_1(FB, FS)$, $f_2(FS)$, $f_3(NDG, FS)$, $f_6(NDG)$

(b) Multiply and sum out NDG

i.e. $\sum_{NDG} f_3(NDG, FS) \cdot f_6(NDG)$

~~$= f_7(NDF$~~ $= f_7(FS)$ = FS: 0.9468

~FS: 0.53

Factors are: $f_1(FB, FS)$, $f_2(FS)$, $f_7(FS)$

(C) Multiply and Sum out FS

i.e. $\sum_{FS} f_1(FB, FS) \cdot f_2(FS) \cdot f_7(FS)$

$= f_8(FB)$ = FB: 0.07875

~FB: 0.472086.

$\therefore P(FM, FH, FB) = P(FM) \cdot f_8(FB)$

Multiplying them, we get f(FB) = FB: 0.002811

~FB: 0.016853

$\therefore P(FS|FM, FH, FB)$

$$= \frac{P(FS, FM, FH, FB)}{P(FM, FH, FB)} = \frac{0.001014}{0.002811}$$

$$= 0.3607$$

(5) $P(FS \mid FH, FM, FB, NA)$

$$= \frac{P(FS, FH, FM, FB, NA)}{P(FH, FM, FB, NA)}$$

Numerator

$$P(FS, FH, FM, FB, NA)$$

$$= \sum_{NDG} P(FS, FH, FM, FB, NA, NDG)$$

$$= \sum_{NDG} P(FH \mid NDG, FM, FS) \cdot P(FB \mid FS) \cdot P(NDG \mid NA, FM) \cdot P(NA) \cdot P(FM) \cdot P(FS)$$

$$= P(NA) \cdot P(FM) \, f_1(FS) \cdot f_2(FS) \sum_{NDG} f_3(NDG, FS) \, f_4(NDG)$$

where $f_1(FS) = FS : 0.6$ (restriction)
$\sim FS : 0.1$

$f_2(FS) = FS : 0.05$
$\sim FS = 0.95$

$f_3(NDG, FS) : NDG, FS : 0.99$
$\sim NDG, FS : 0.9$
$NDG, \sim FS : 0.65$
$\sim NDG, \sim FS : 0.4$

$f_4(NDG) = NDG : 0.8$
$\sim NDG : 0.2$

→ multiply and sum out NDG.

i.e. $\sum_{NDG} f_3(NDG, FS)\, f_4(NDG)$

$= f_5(FS) = FS : 0.972$

$\sim FS : 0.6.$

Factors are: $f_1(FS)$, $f_2(FS)$, $f_5(FS)$

$\therefore P(FS, FH, FM, EB, NA)$

$= P(NA) \cdot P(FM) \cdot f_1(FS) \cdot f_2(FS) \cdot f_5(FS)$

where $f_1(FS) = FS : 0.6$

$\sim FS : 0.1$

$f_2(FS) = FS : 0.05$

$\sim FS : 0.95$

$f_3(FS) = FS : 0.972.$

$\sim FS : 0.6.$

Multiplying them, we get

$f(FS) = FS : 0.0003123.$

$\sim FS = 0.00061047.$

Denominator : $P(FH, FM, FB, NA)$

$= P(NA \mid FH, FM, FB) \cdot P(FH, FM, FB)$

$\therefore$ Evidence = FH, FM, FB.

$P(FH, FM, FB, NA)$

$= \sum_{NDG, FS} P(FH \mid NDG, FM, FS) \cdot P(FB \mid FS) \cdot P(NDG \mid NA, FM) \cdot P(NA) \cdot P(FM) \cdot P(FS)$

$= P(FM) \cdot P(NA) \sum_{FS} P(FS) P(FB \mid FS) \cdot \sum_{NDG} P(FH \mid NDG, FM, FS) \cdot P(NDG \mid NA, FM)$

$= P(FM) \cdot f_1(NA) \sum_{FS} f_2(FS) f_3(FS) \cdot \sum_{NDG} f_4(NDG, FS) f_5(NDG, NA)$

After restricting, we get the following factors:

$f_1(NA)$ = NA : 0.3
~NA : 0.7

$f_2(FS)$ = FS : 0.05
~FS : 0.95

$f_3(FS)$ = FS : 0.6
~FS : 0.1

$f_4(NDG, FS)$ = NDG, FS : 0.99
~NDG, FS : 0.9
NDG, ~FS : 0.65
~NDG, ~FS : 0.4

$f_5(NDG, NA)$: NDG, NA : 0.8
~NDG, NA : 0.2
NDG, ~NA : 0.4
~NDG, ~NA : 0.6

(a) Summing out NDG:

$$\sum_{NDG} f_4(NDG, FS)\, f_5(NDG, NA)$$

$= f_6(FS, NA)$ = FS, NA : 0.972
~FS, NA : 0.6
FS ~NA : 0.936
~FS, ~NA : 0.5

Factors: $f_1(NA)$, $f_2(FS)$, $f_3(FS)$, $f_6(FS, NA)$

(b) Summing out FS:

$$\sum_{FS} f_2(FS)\, f_3(FS)\, f_6(FS, NA)$$

$= f_7(NA)$ = NA : 0.08616
~NA : 0.07558

Factors: $f_1(NA)$, $f_7(NA)$

$\therefore P(FH, FM, FB, NA) = P(FM) \cdot f_1(NA) \cdot f_7(NA)$

Multiplying them, we get
$f(NA)$ = NA : 0.00092277
~NA : 0.001888

$$\therefore P(FS \mid FH, FM, FB, NA)$$

$$= \frac{P(FS, FH, FM, FB, NA)}{P(FH, FM, FB, NA)}$$

$$= \frac{0.0003123}{0.0009277} = \boxed{0.33644}$$