Lecture 8

Red-Black Trees, Height Bound, Insertion

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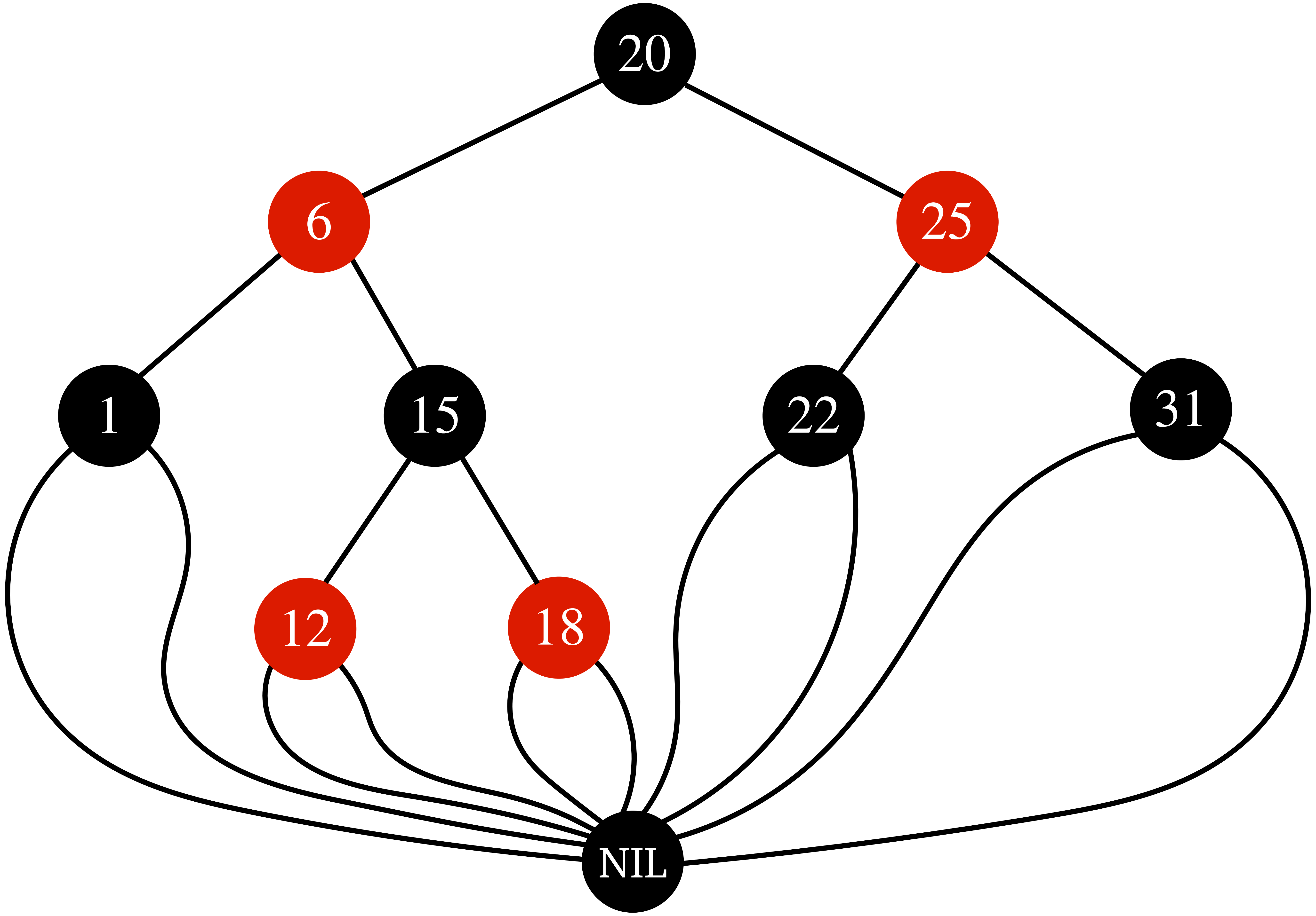

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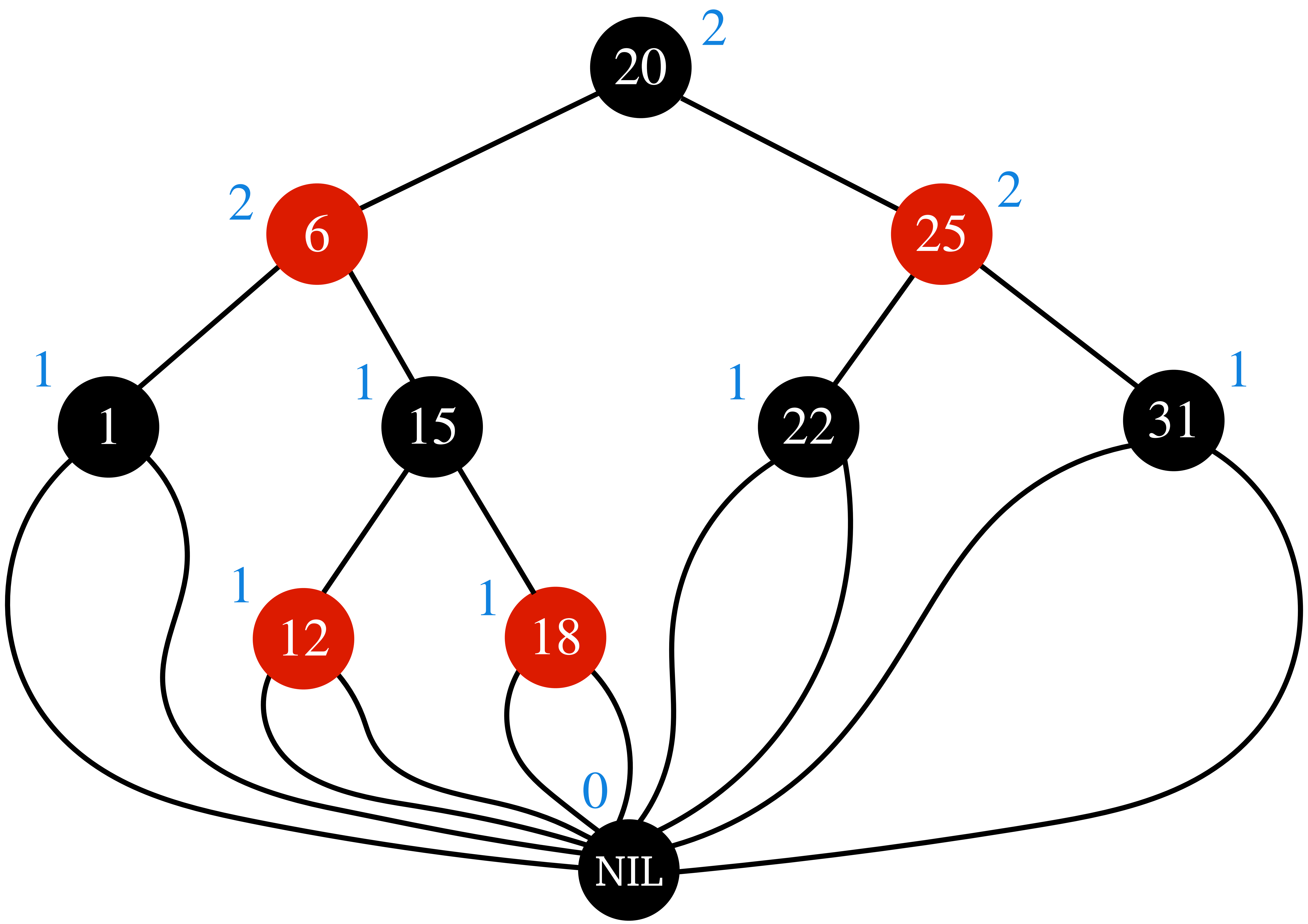

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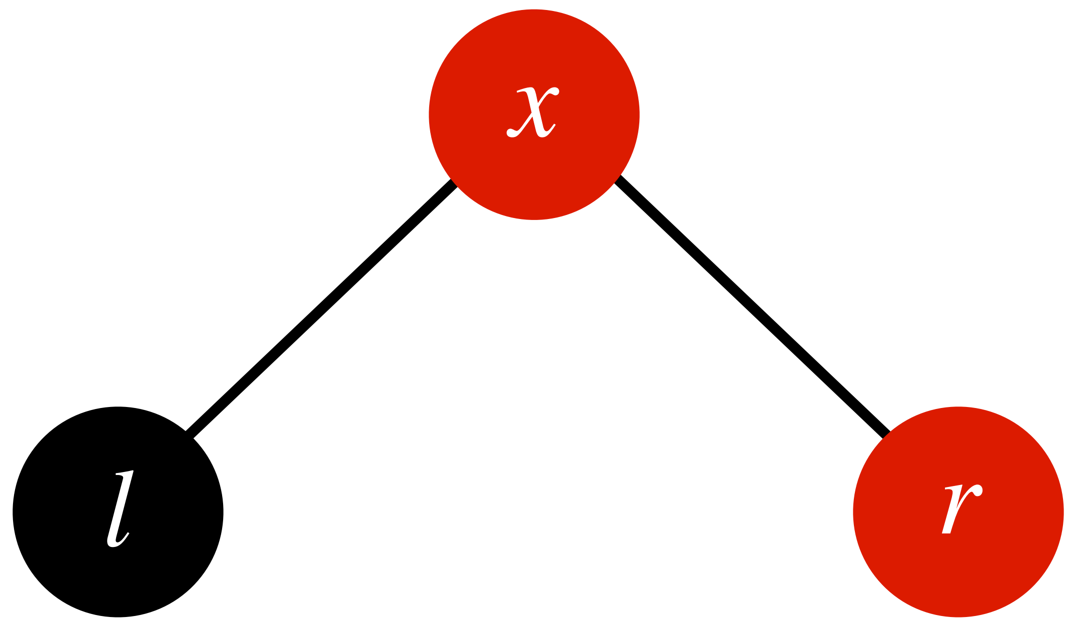

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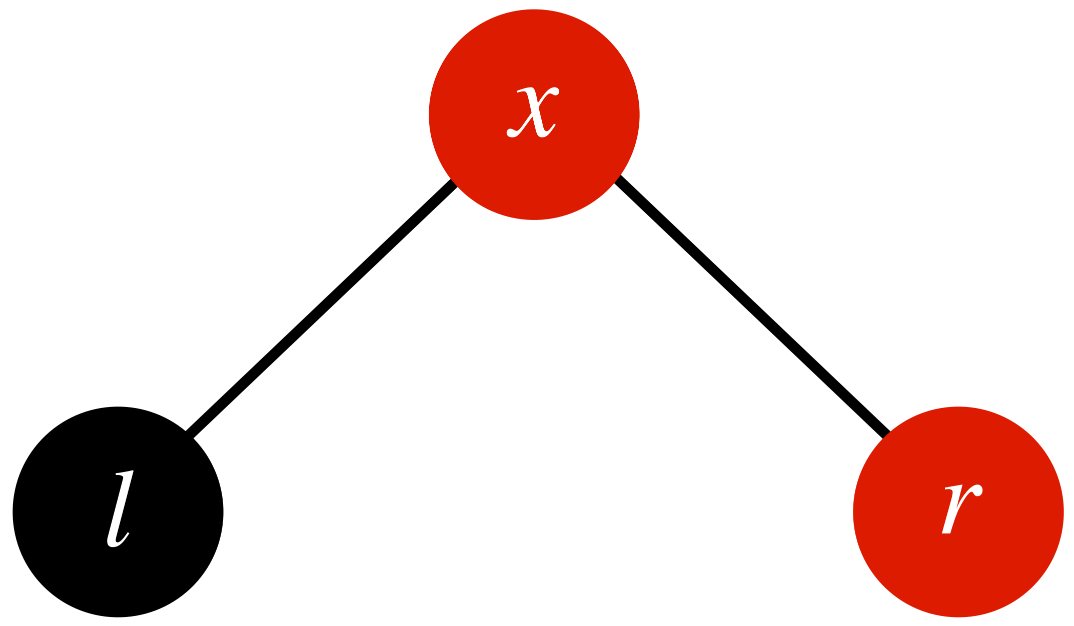

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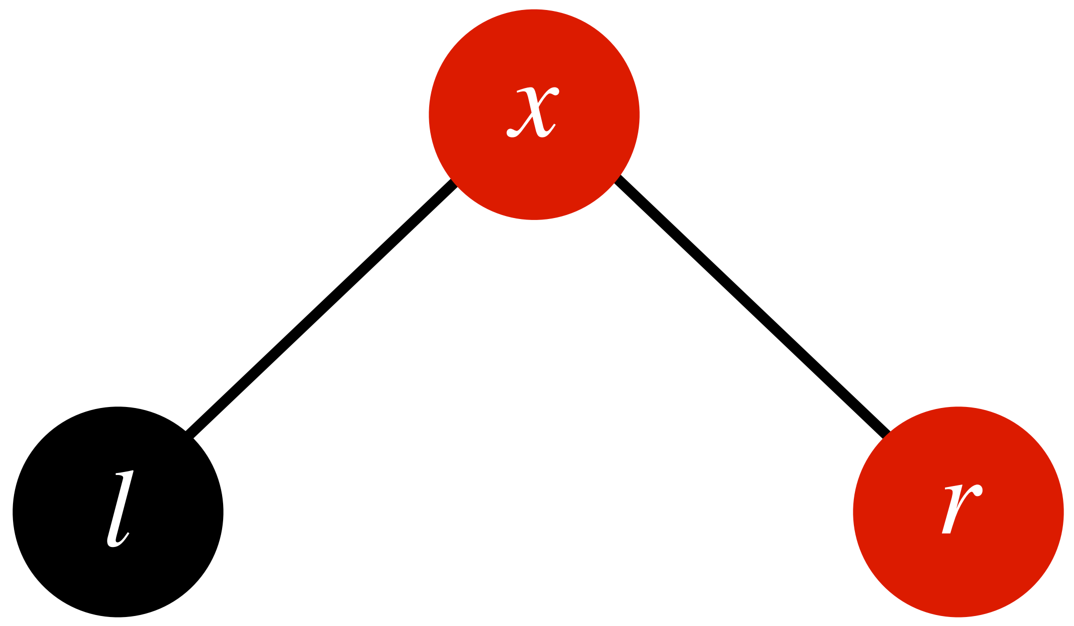

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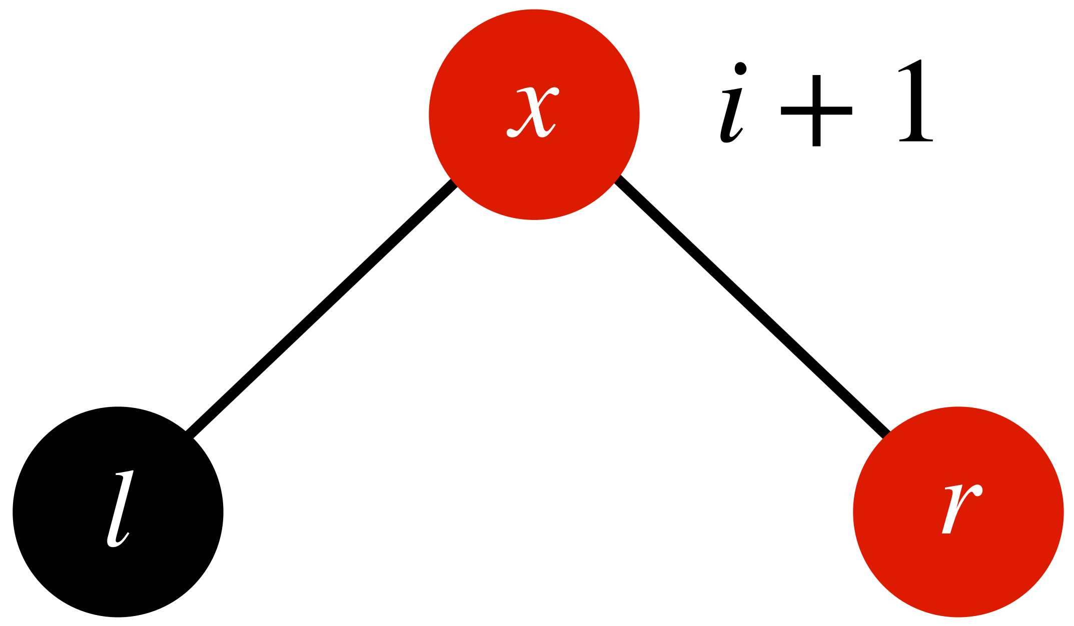

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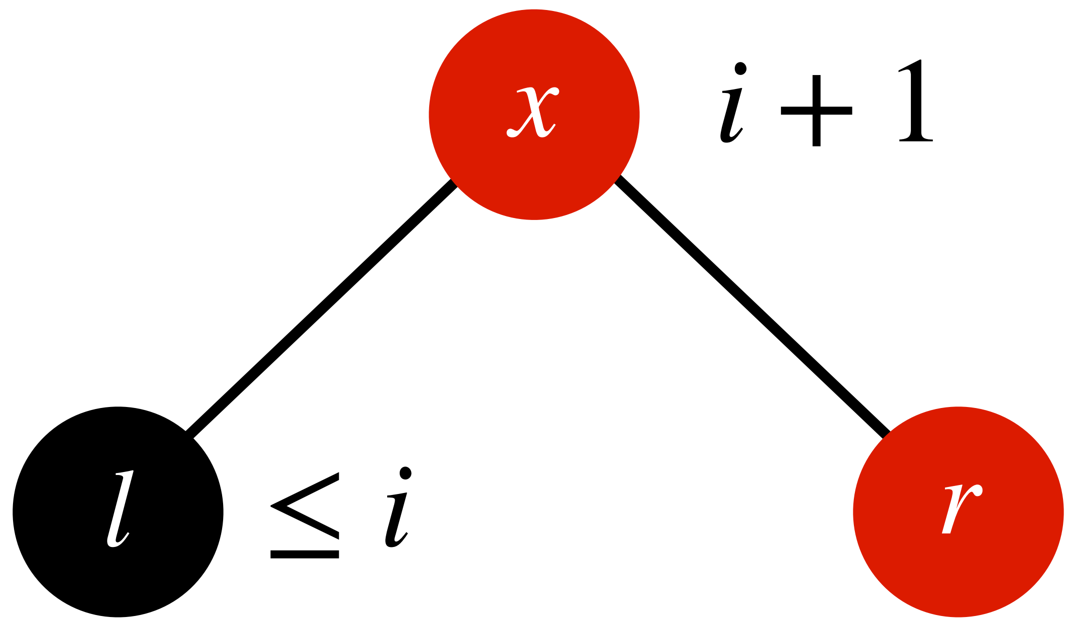

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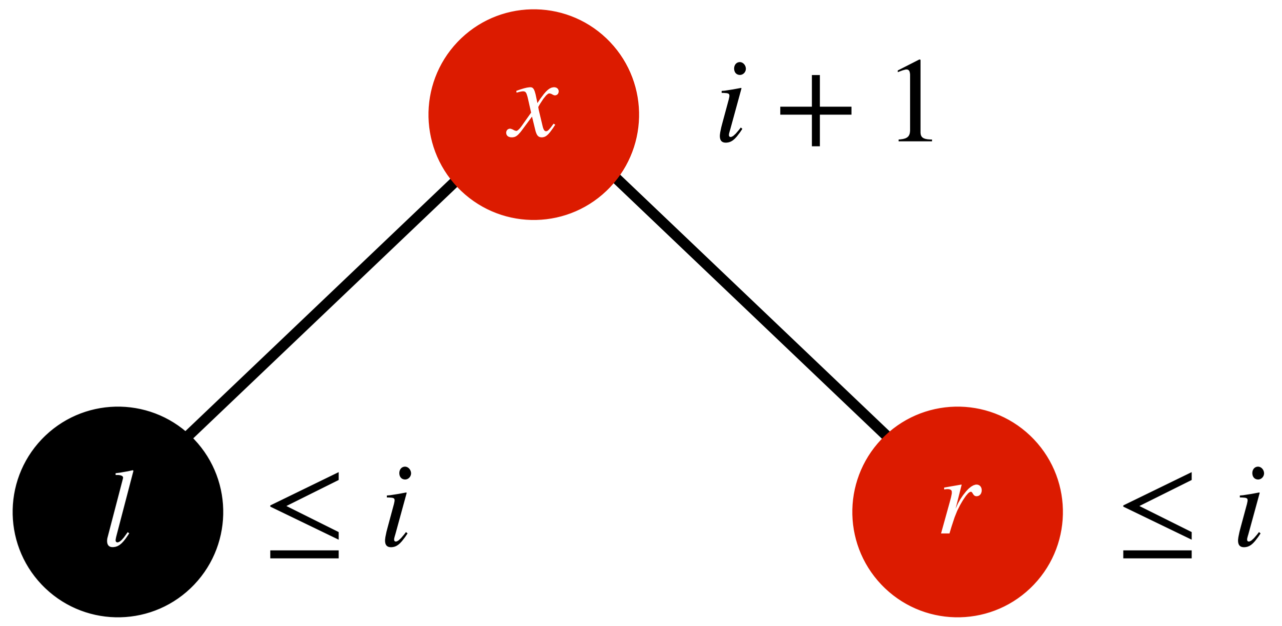

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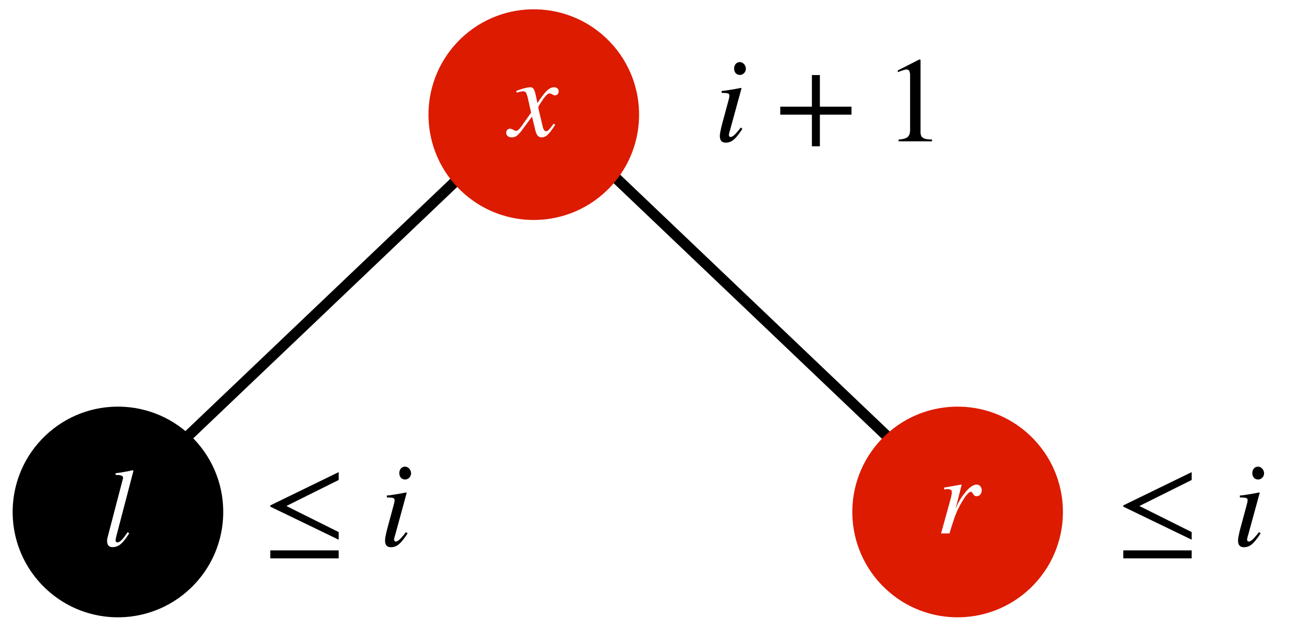

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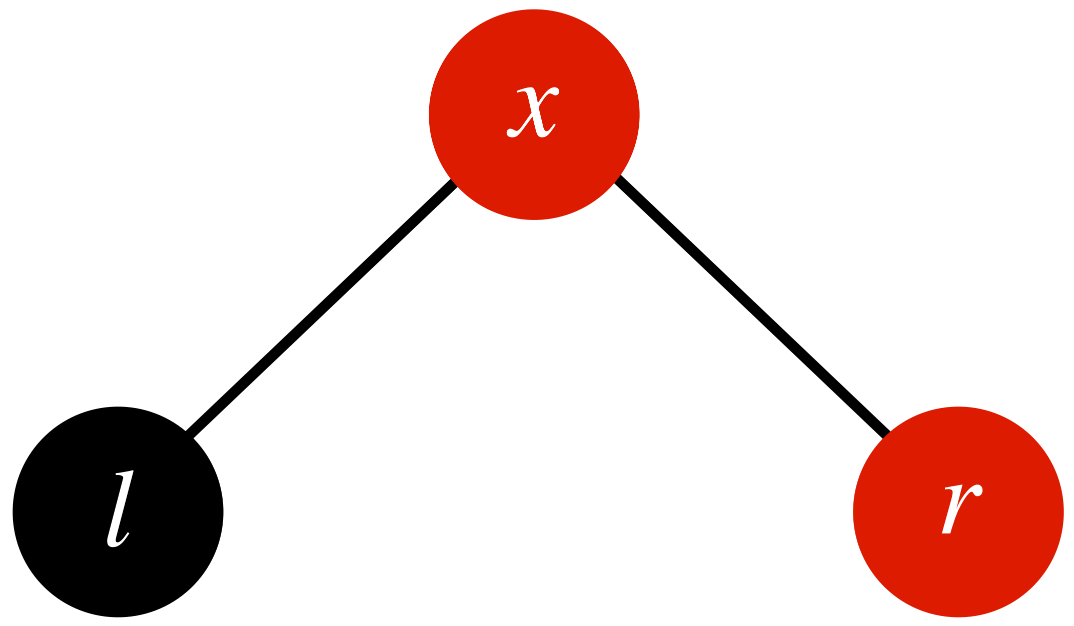

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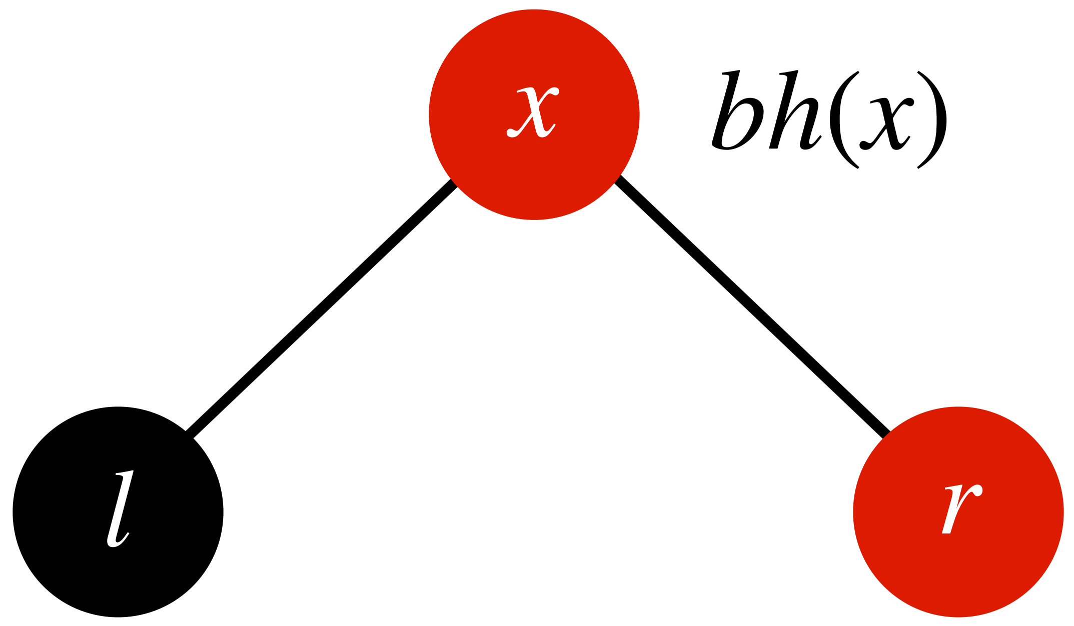

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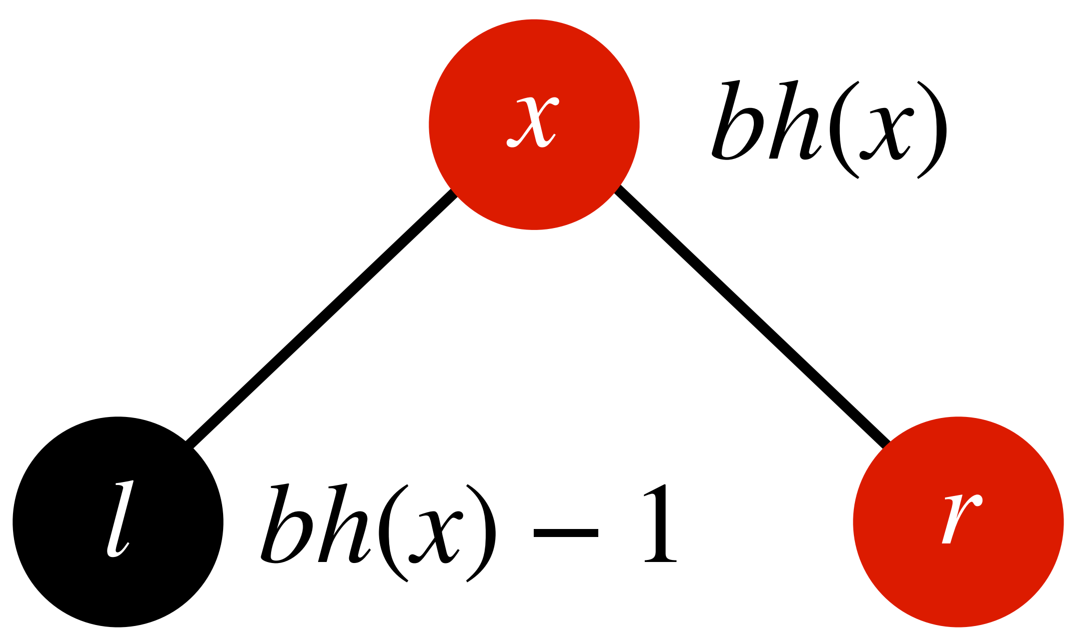

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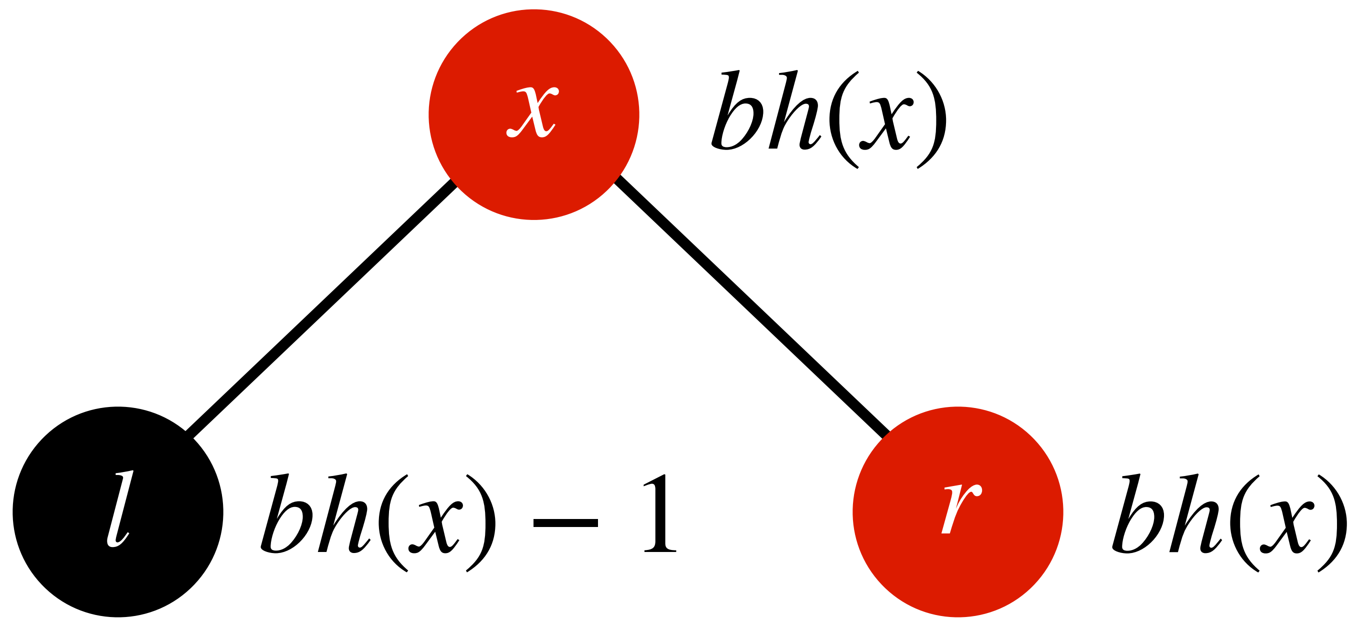

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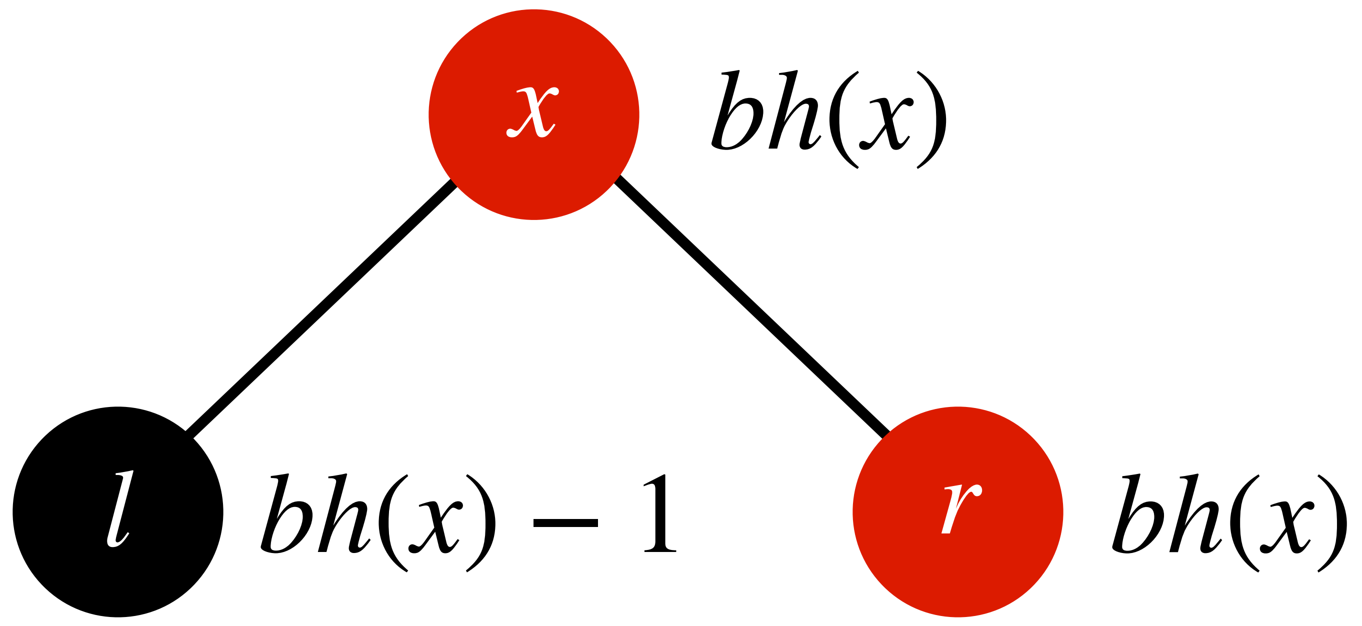

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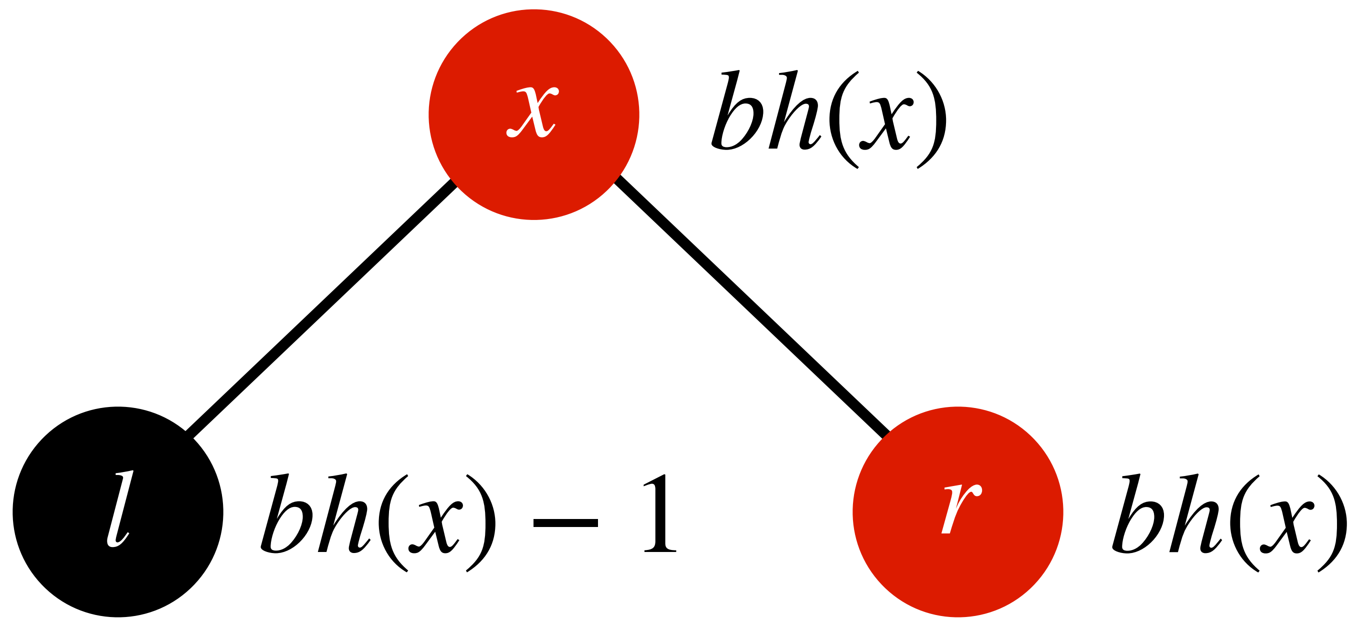

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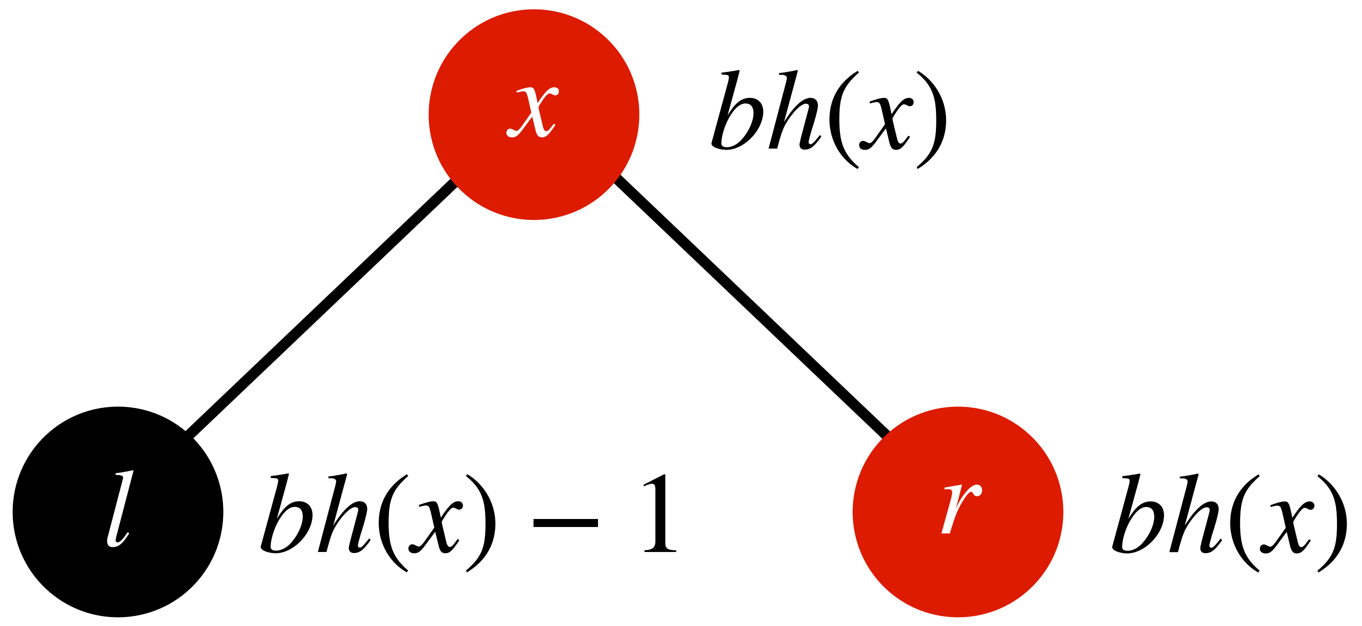

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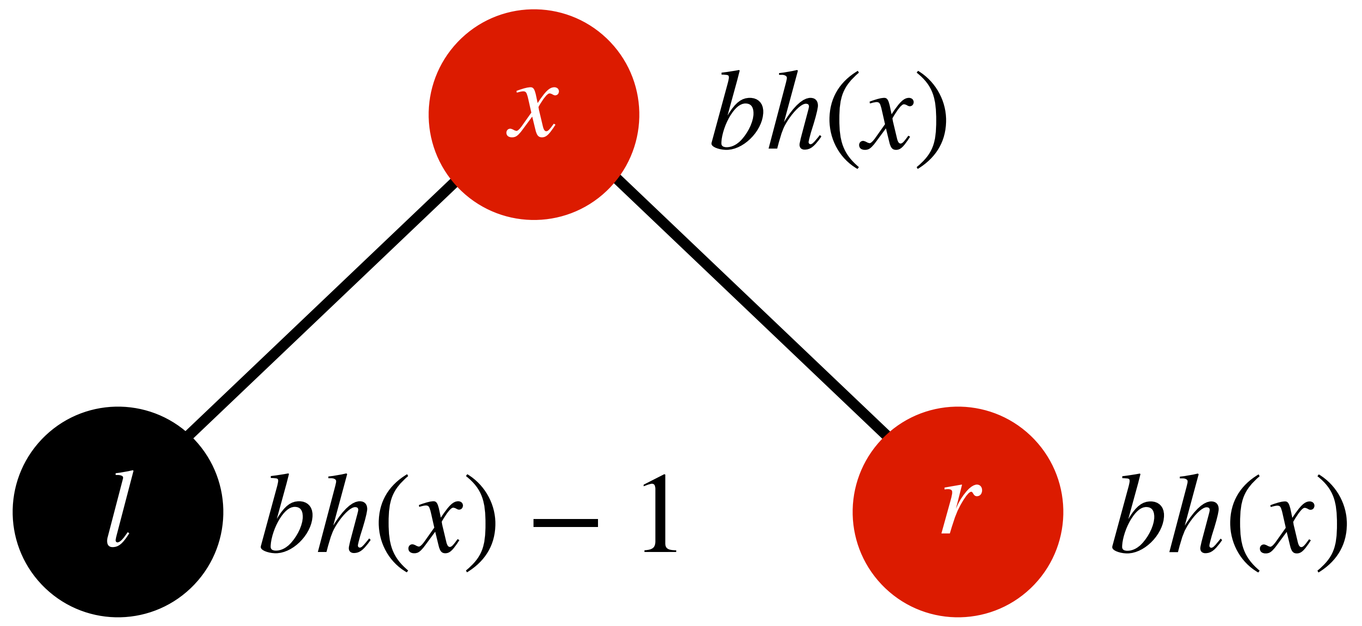

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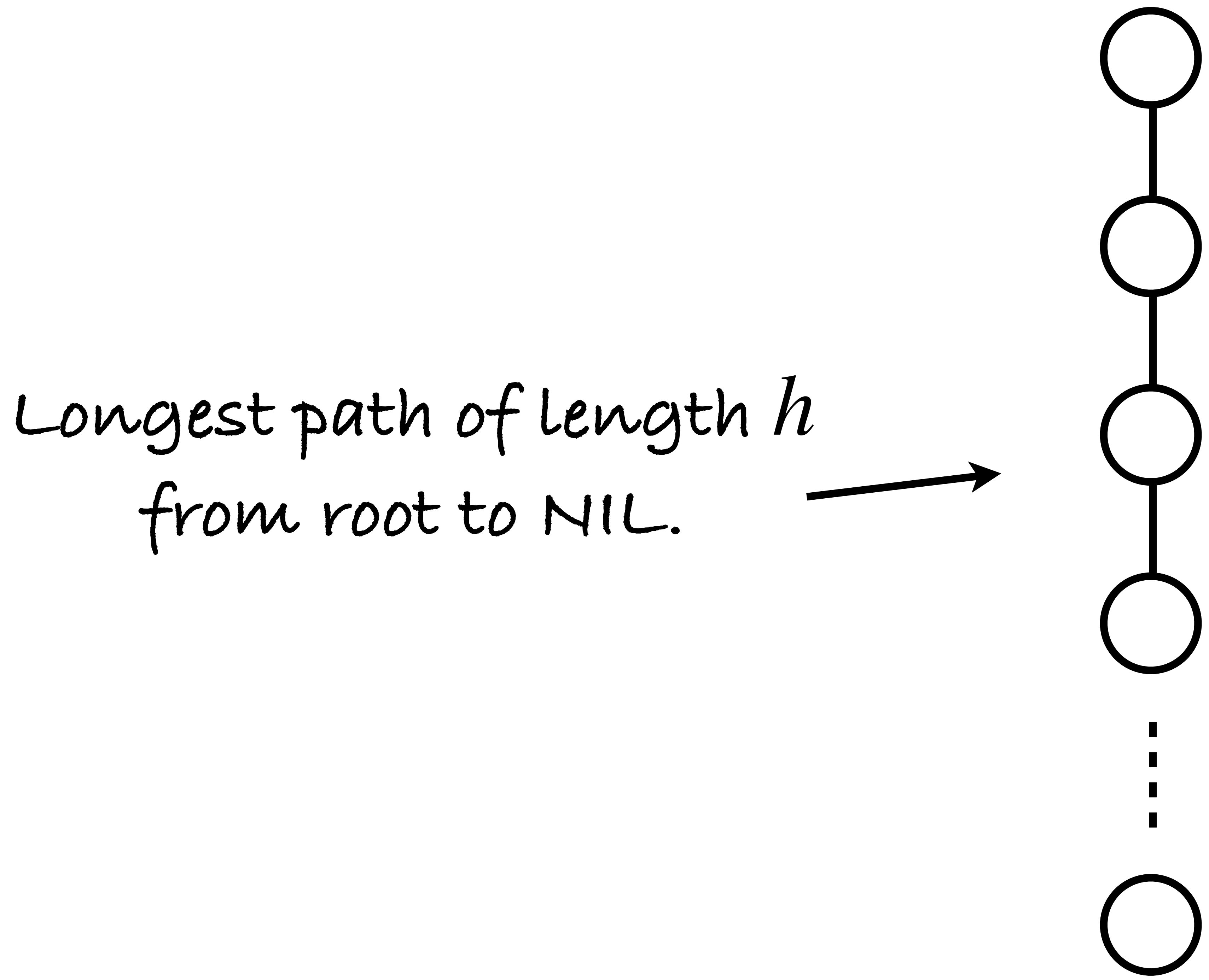

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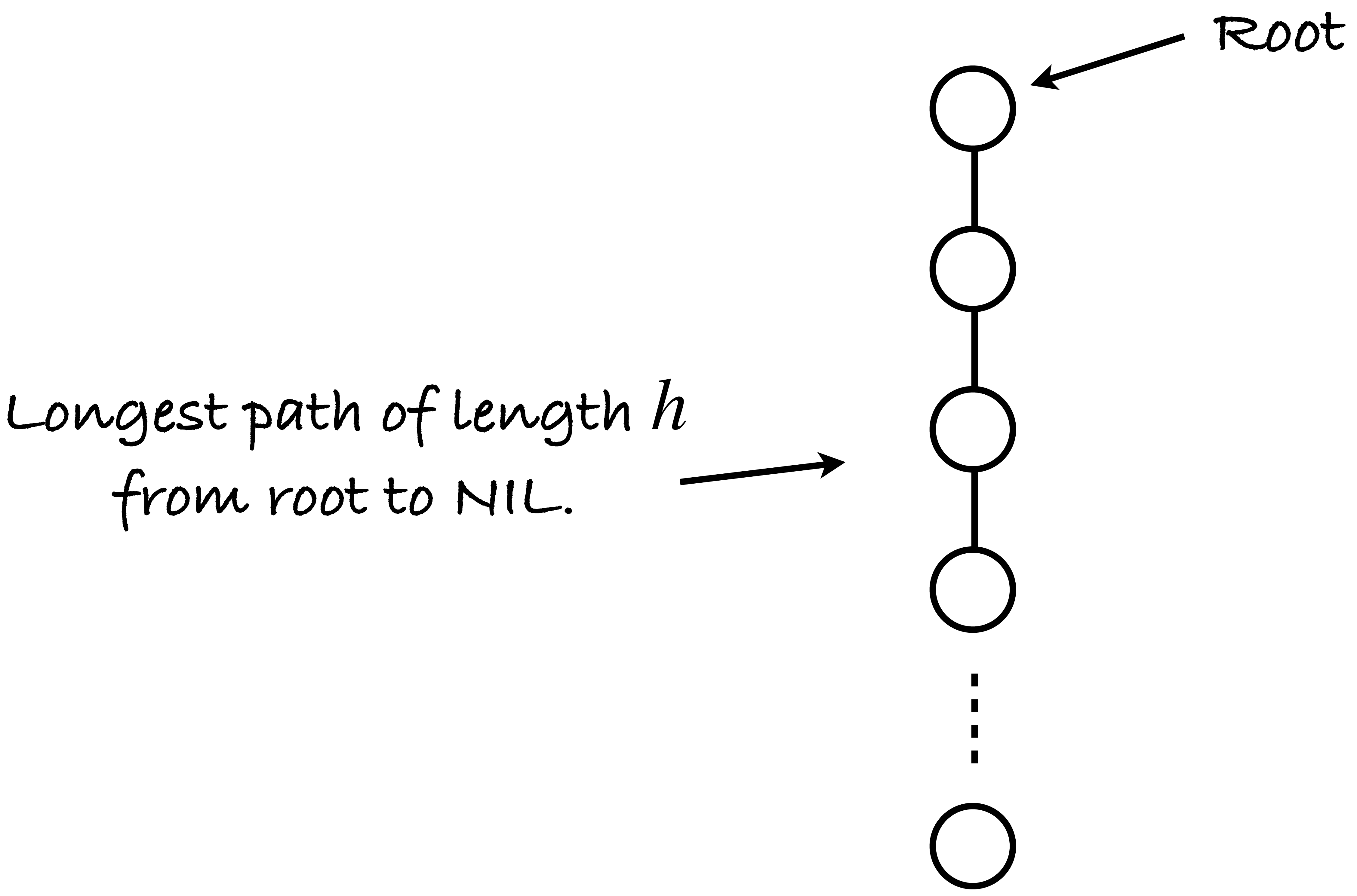

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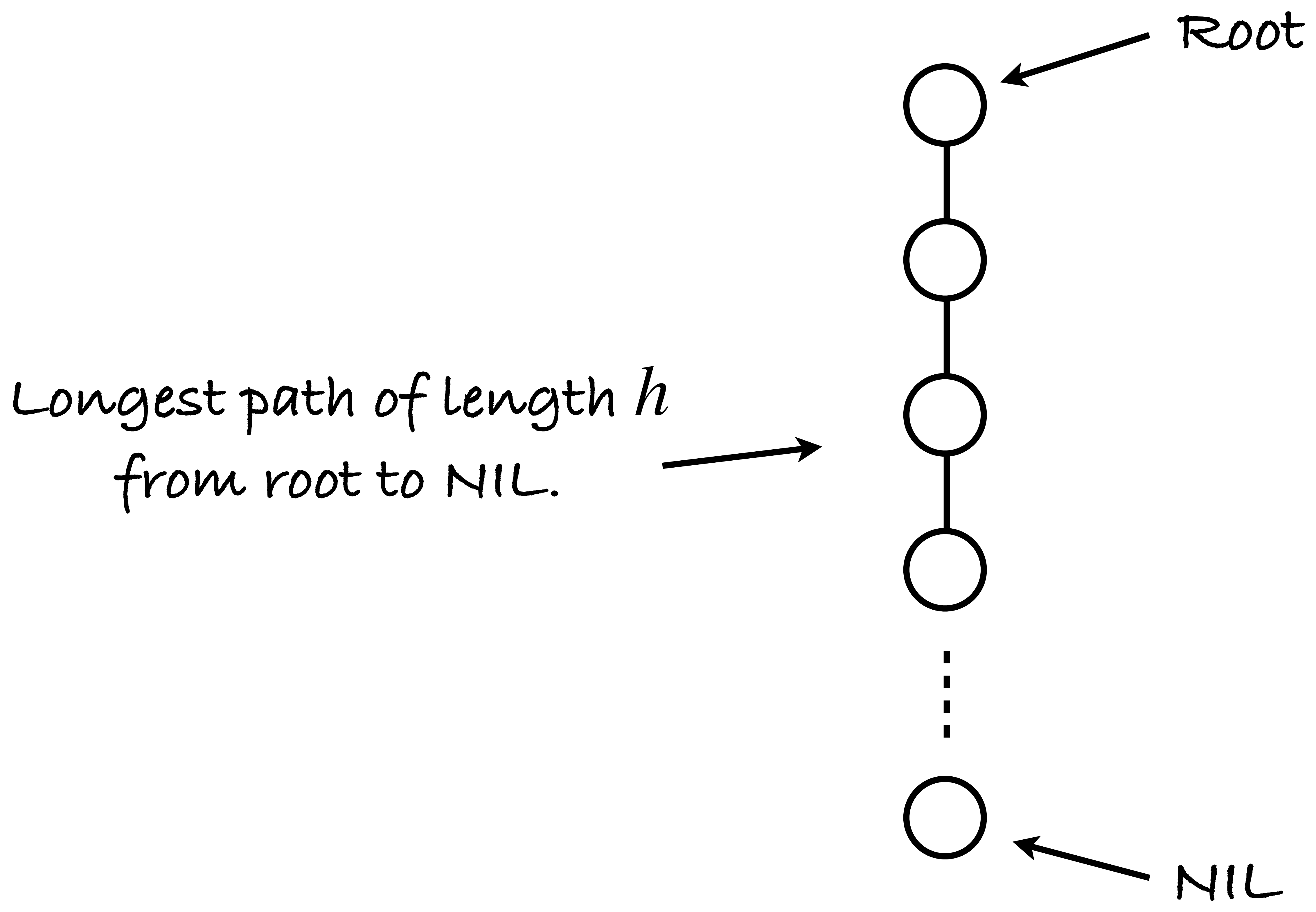

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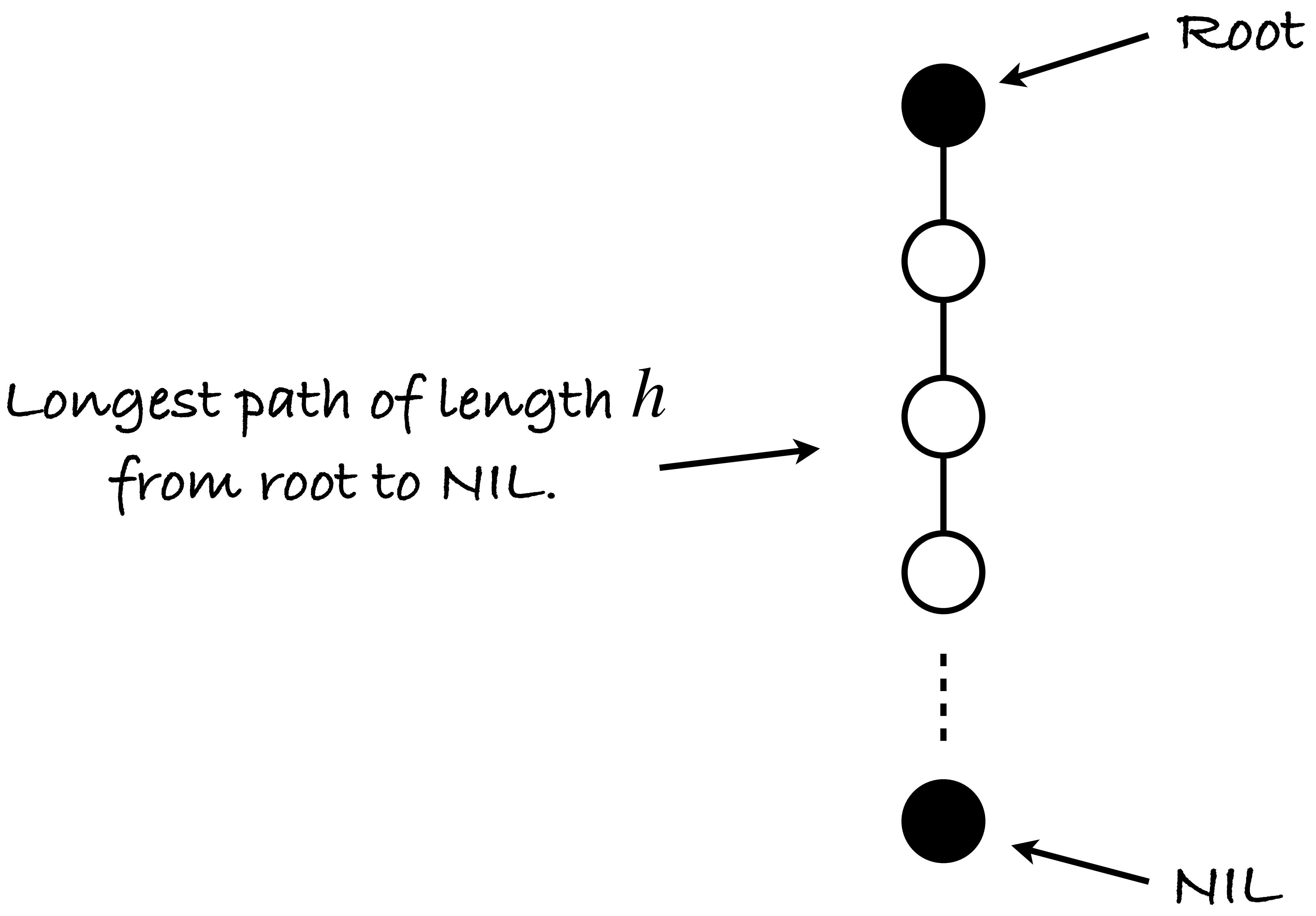

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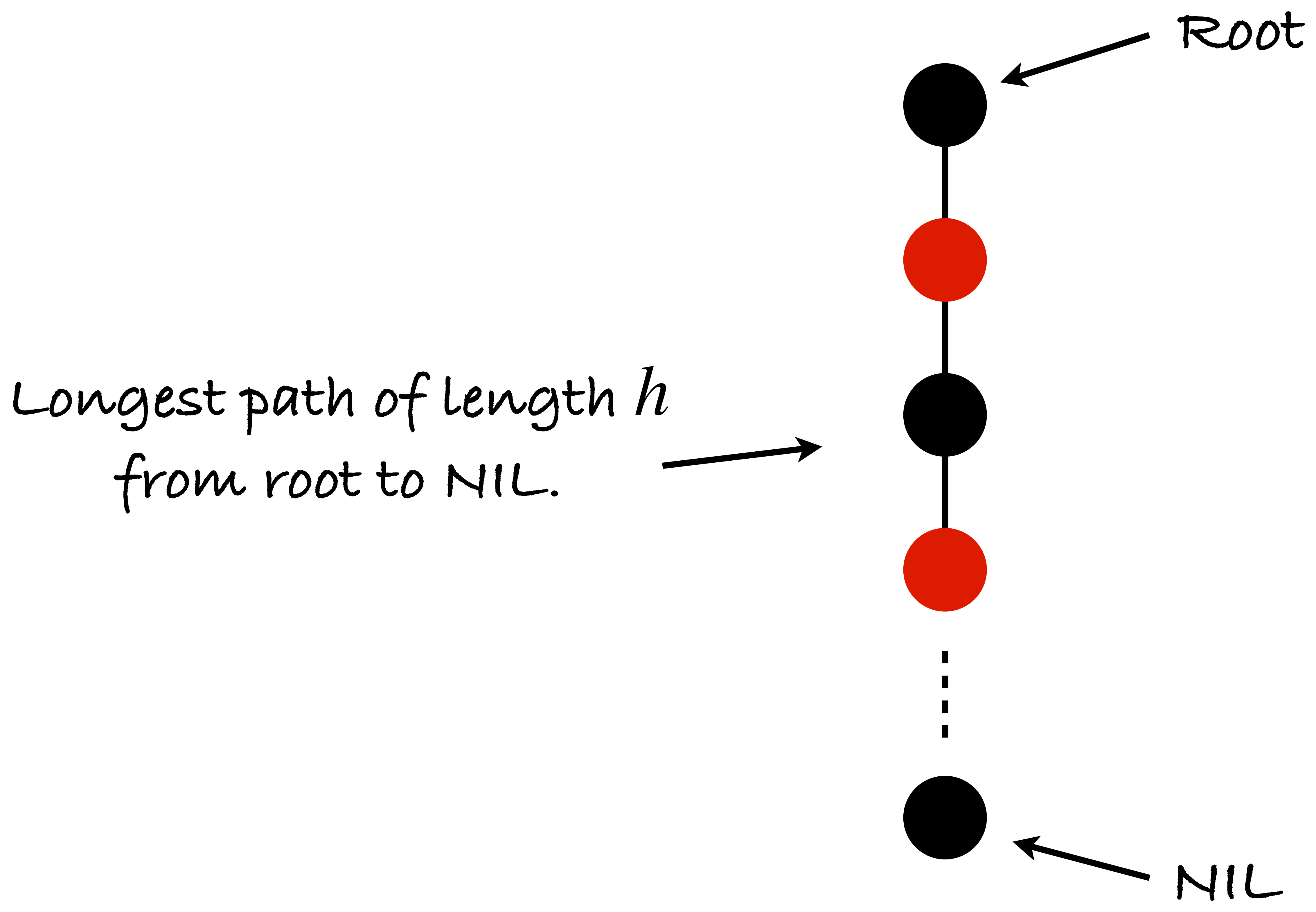

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$\blacksquare$

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Require care

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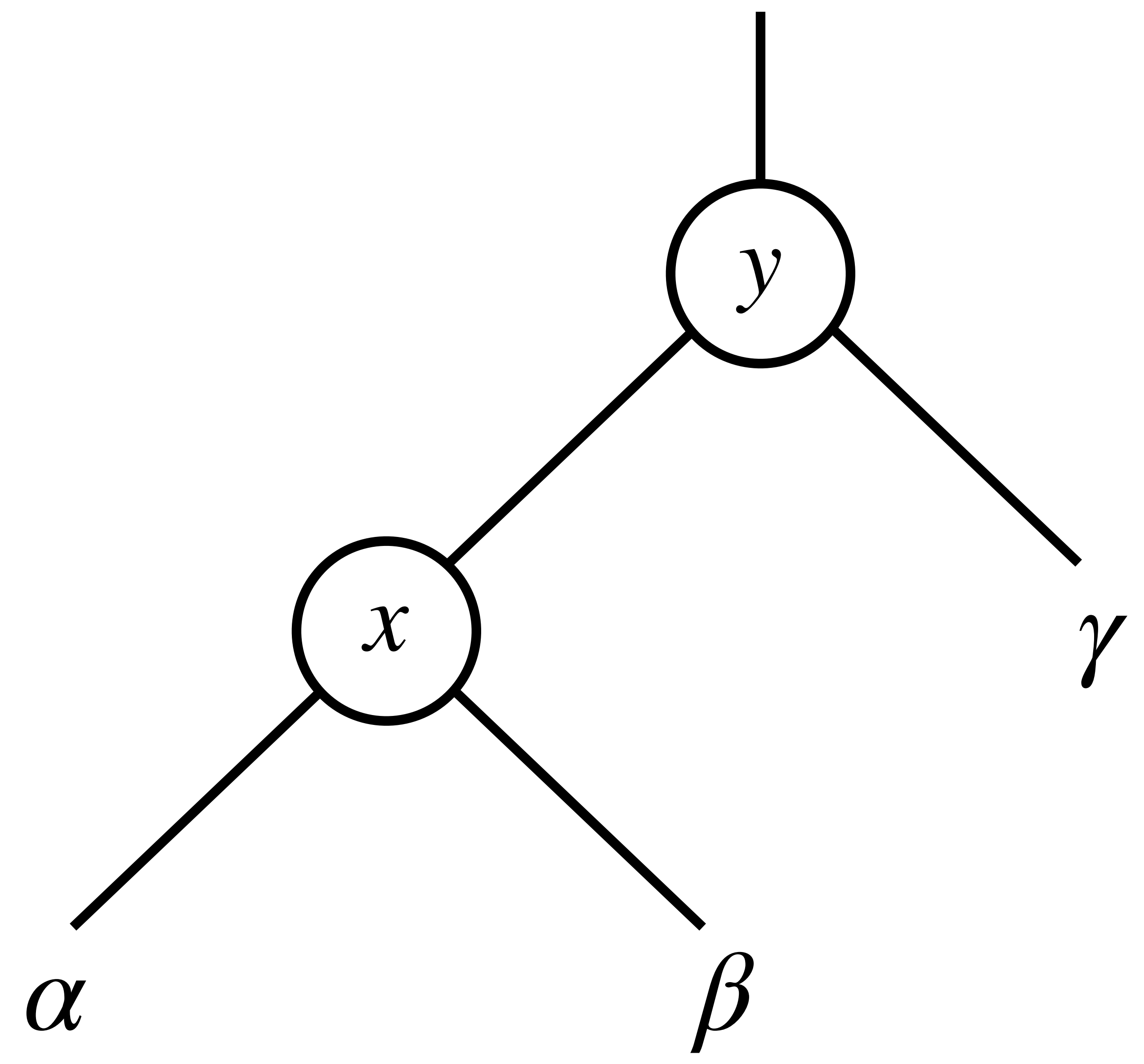

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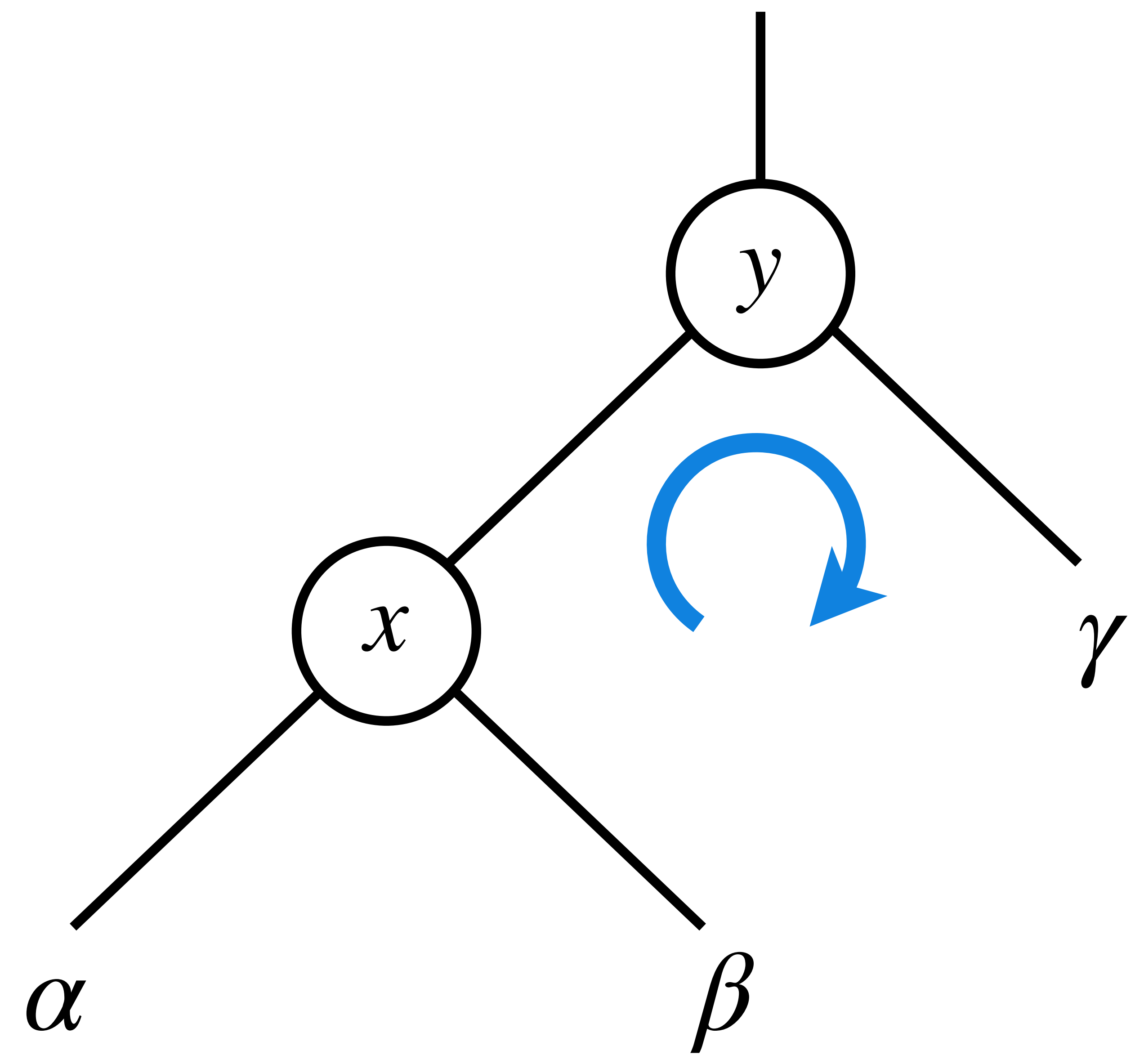

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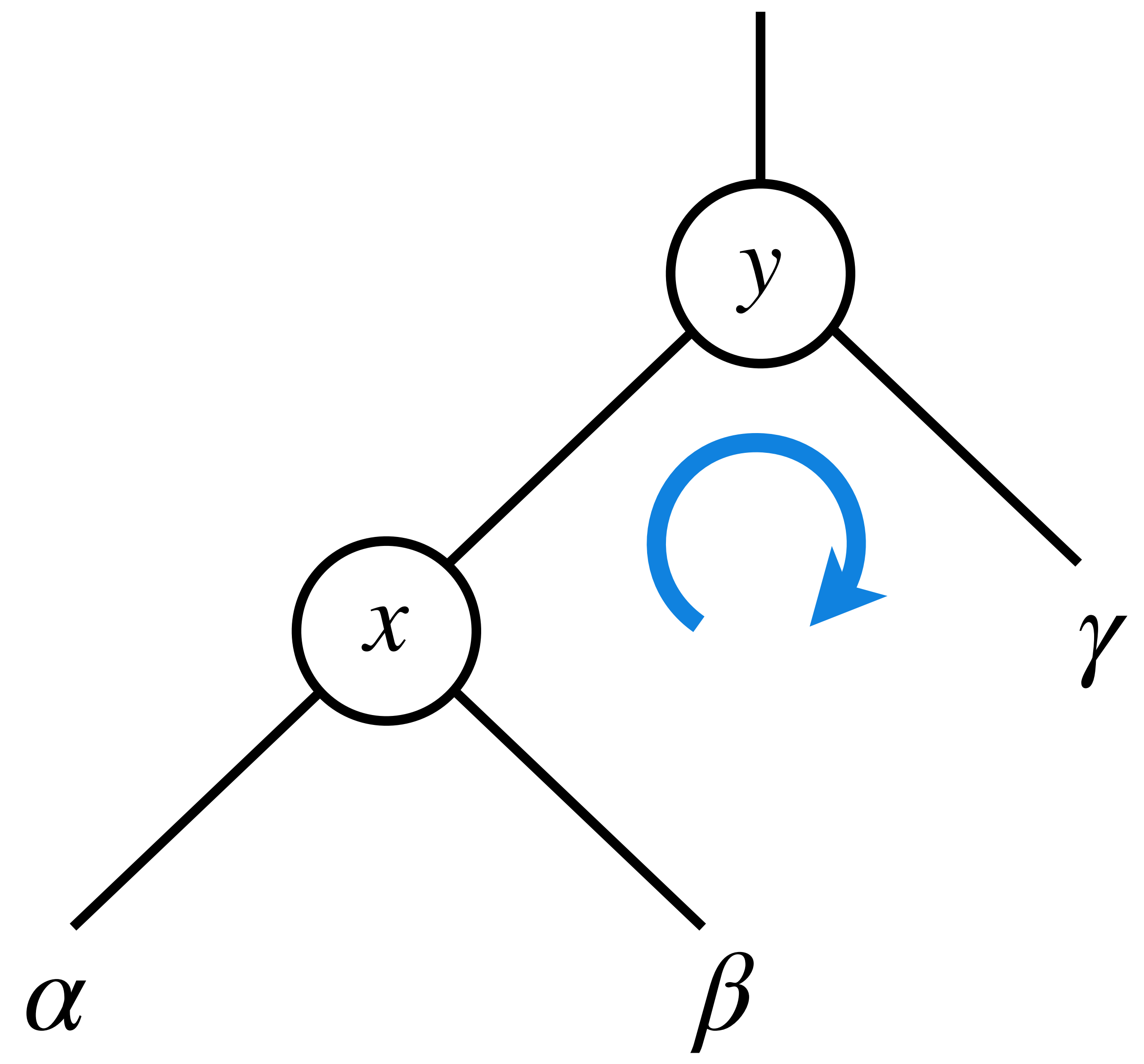

Right-rotate(T, y)

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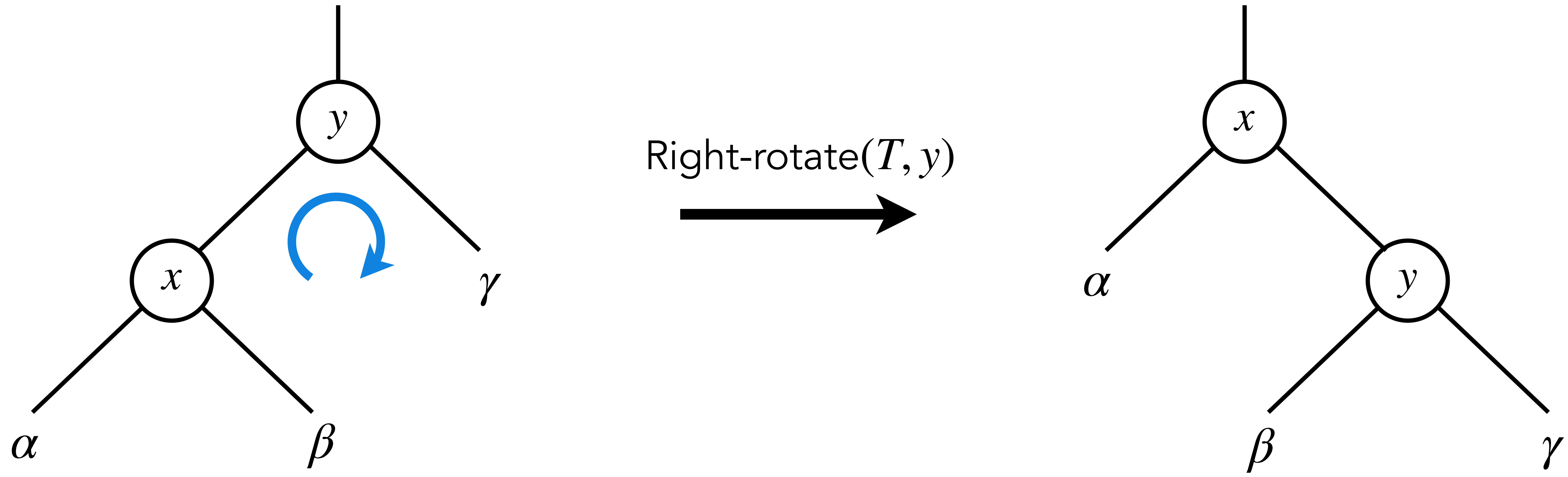

RB-Trees: Rotations

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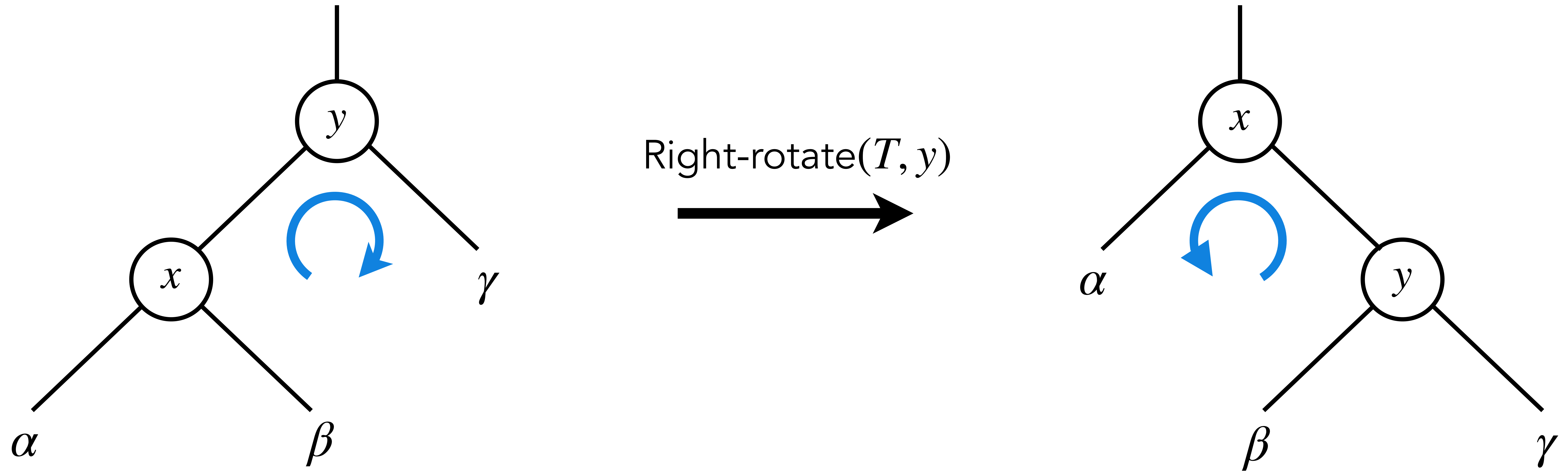

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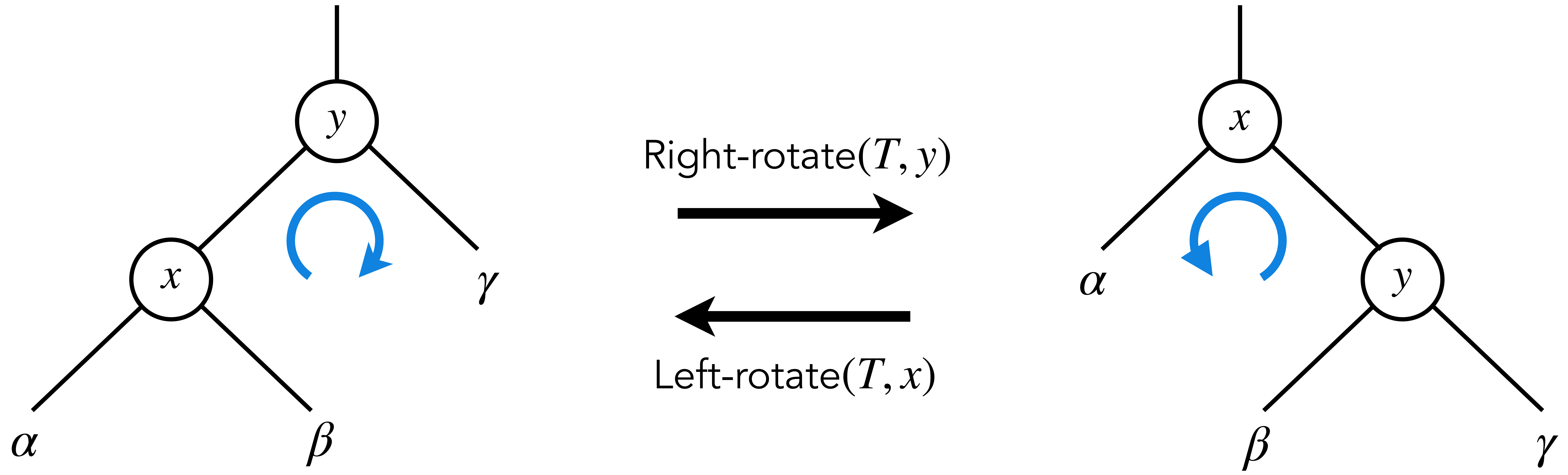

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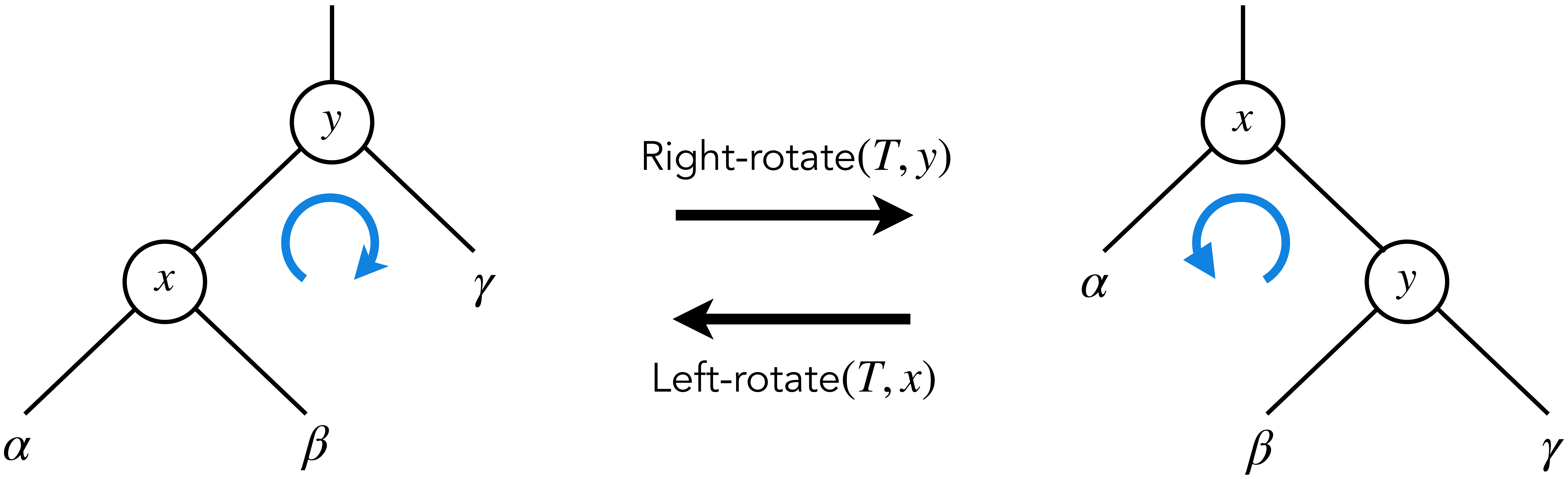

Note: Rotations do not disturb BST property and can be performed in constant time.

RB-Trees: Insertion

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Suppose we want to insert 19 in the following RB-tree:

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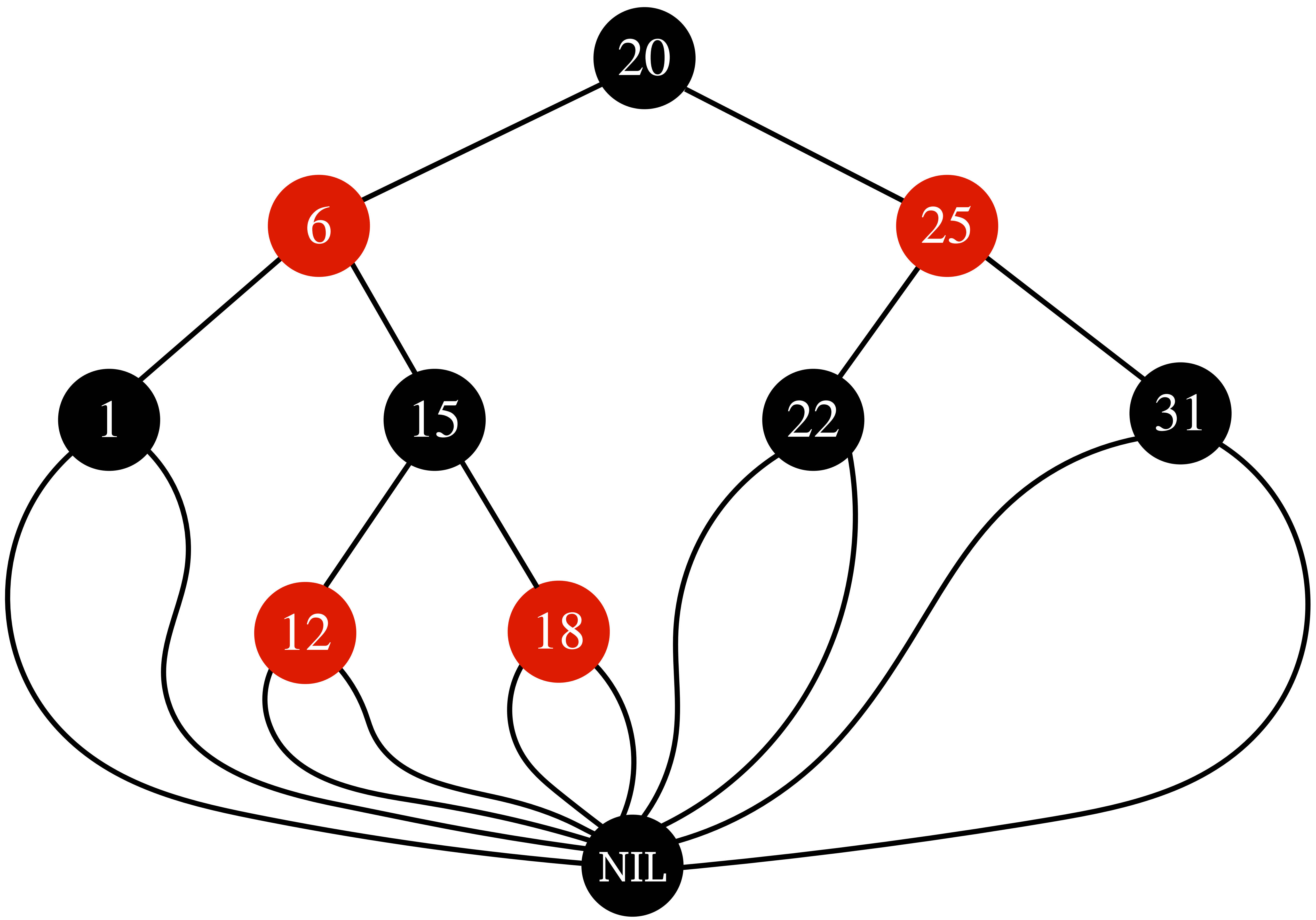

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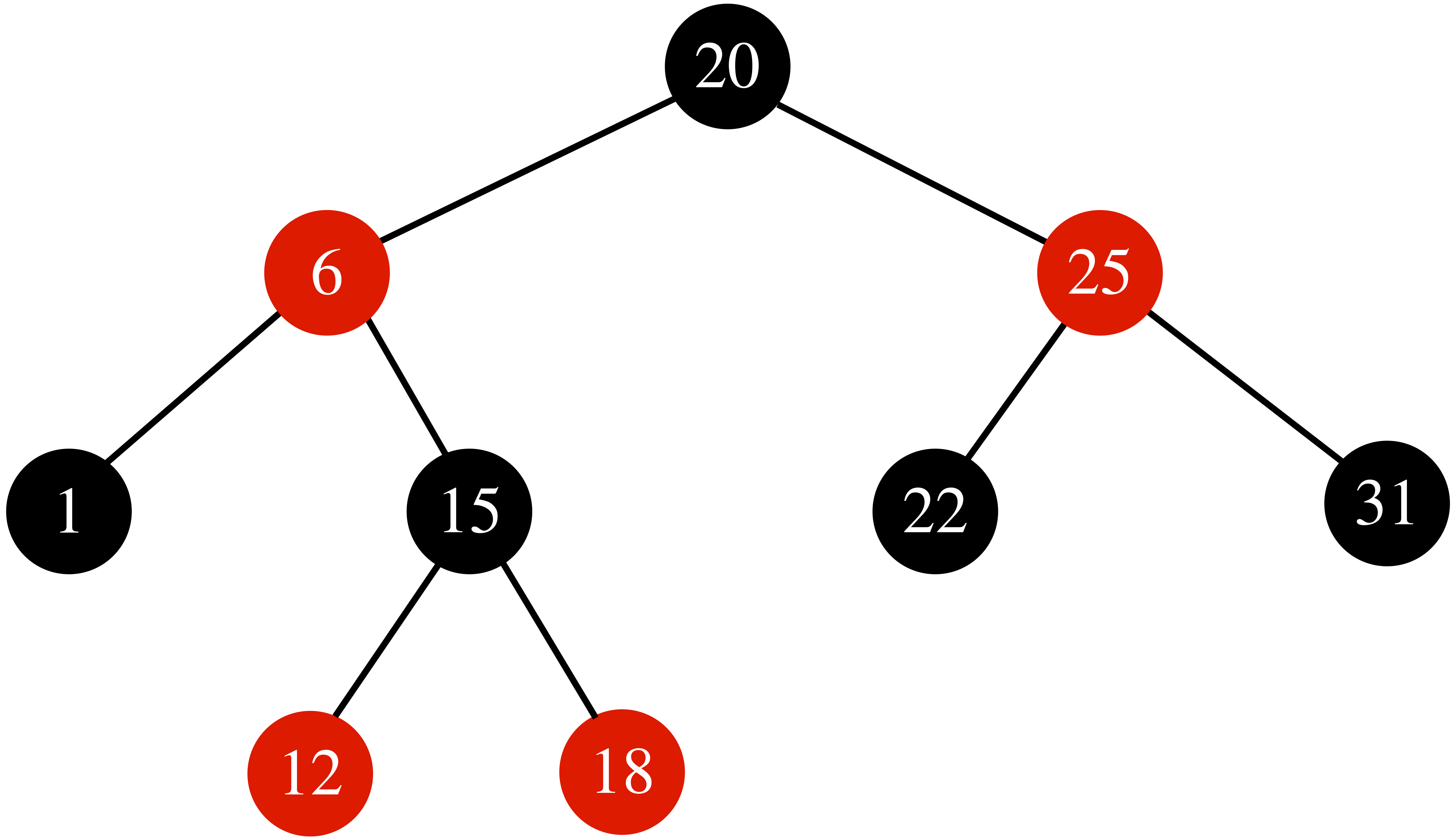

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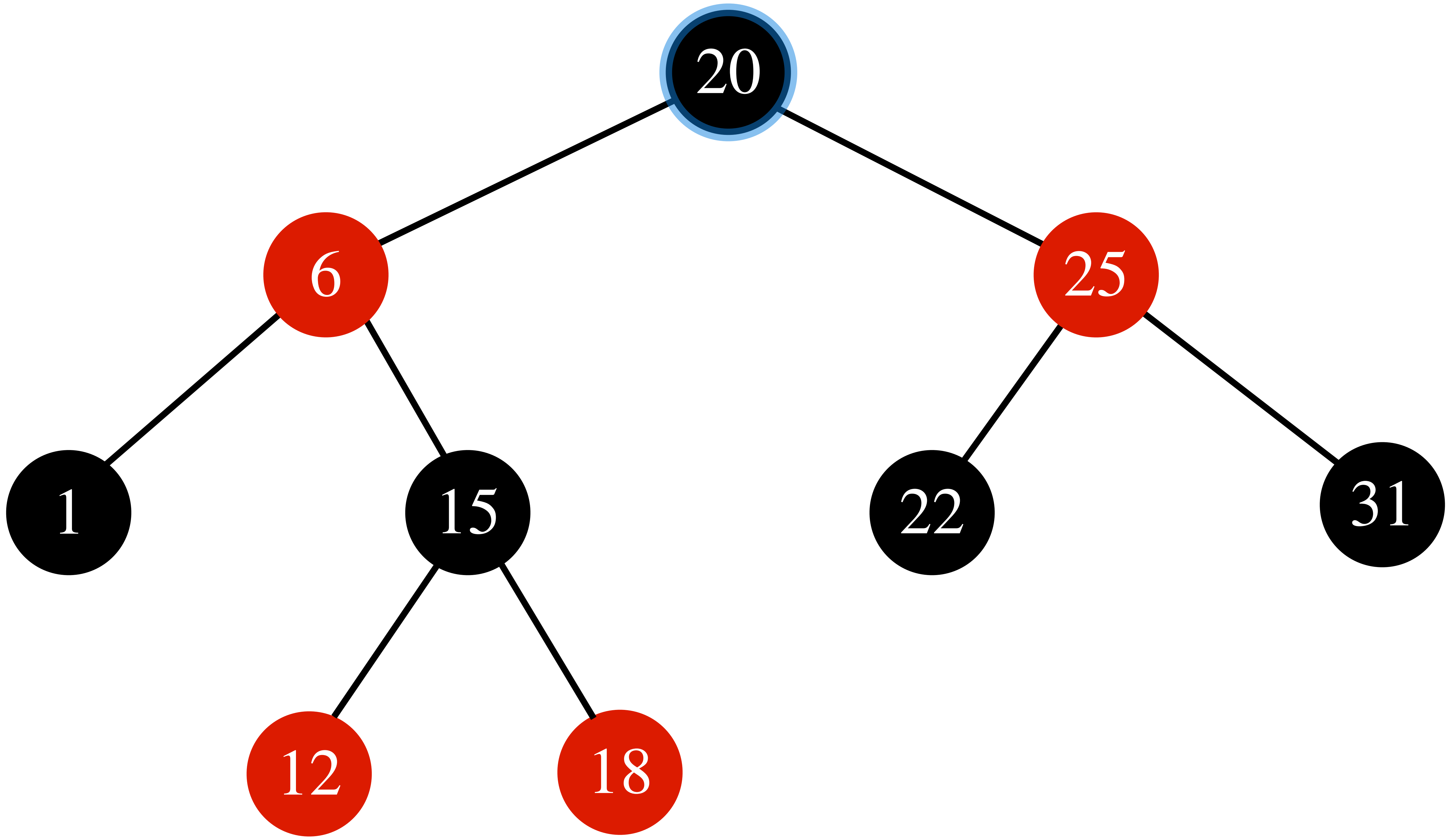

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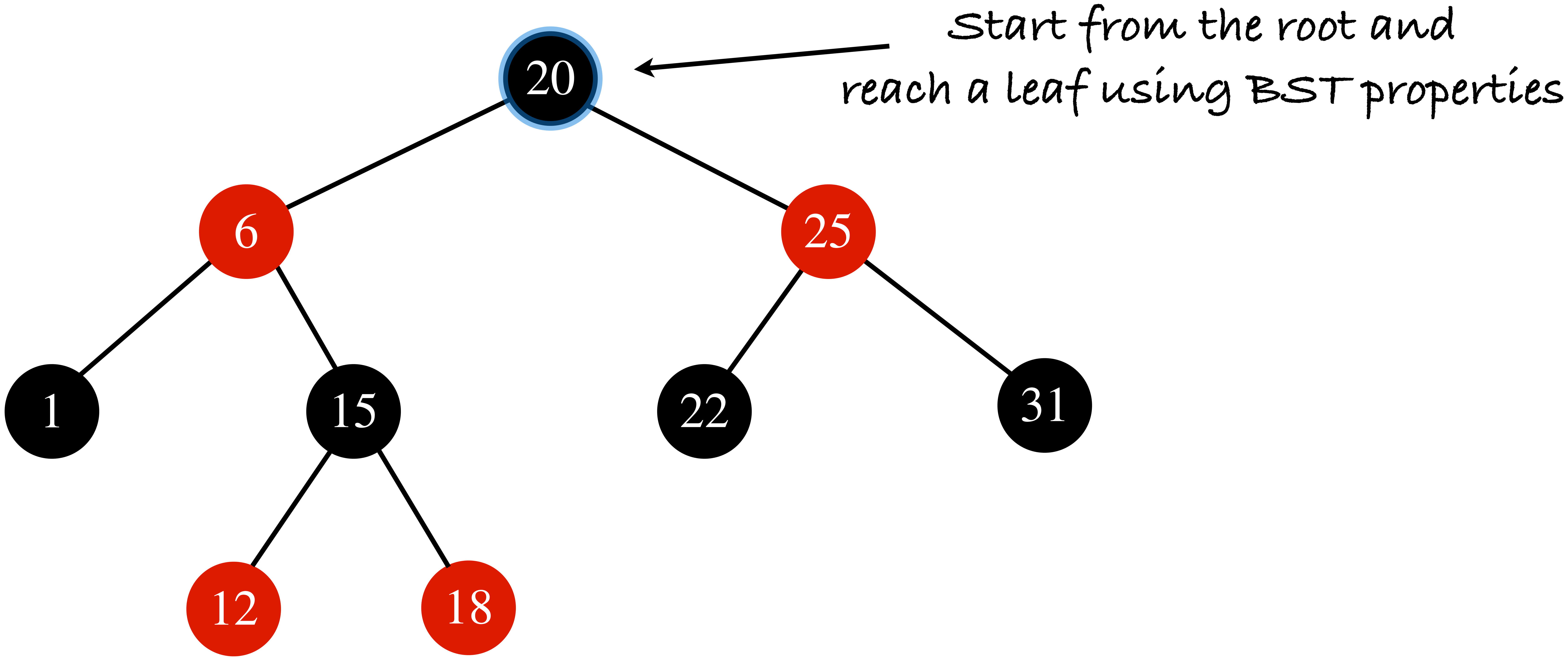

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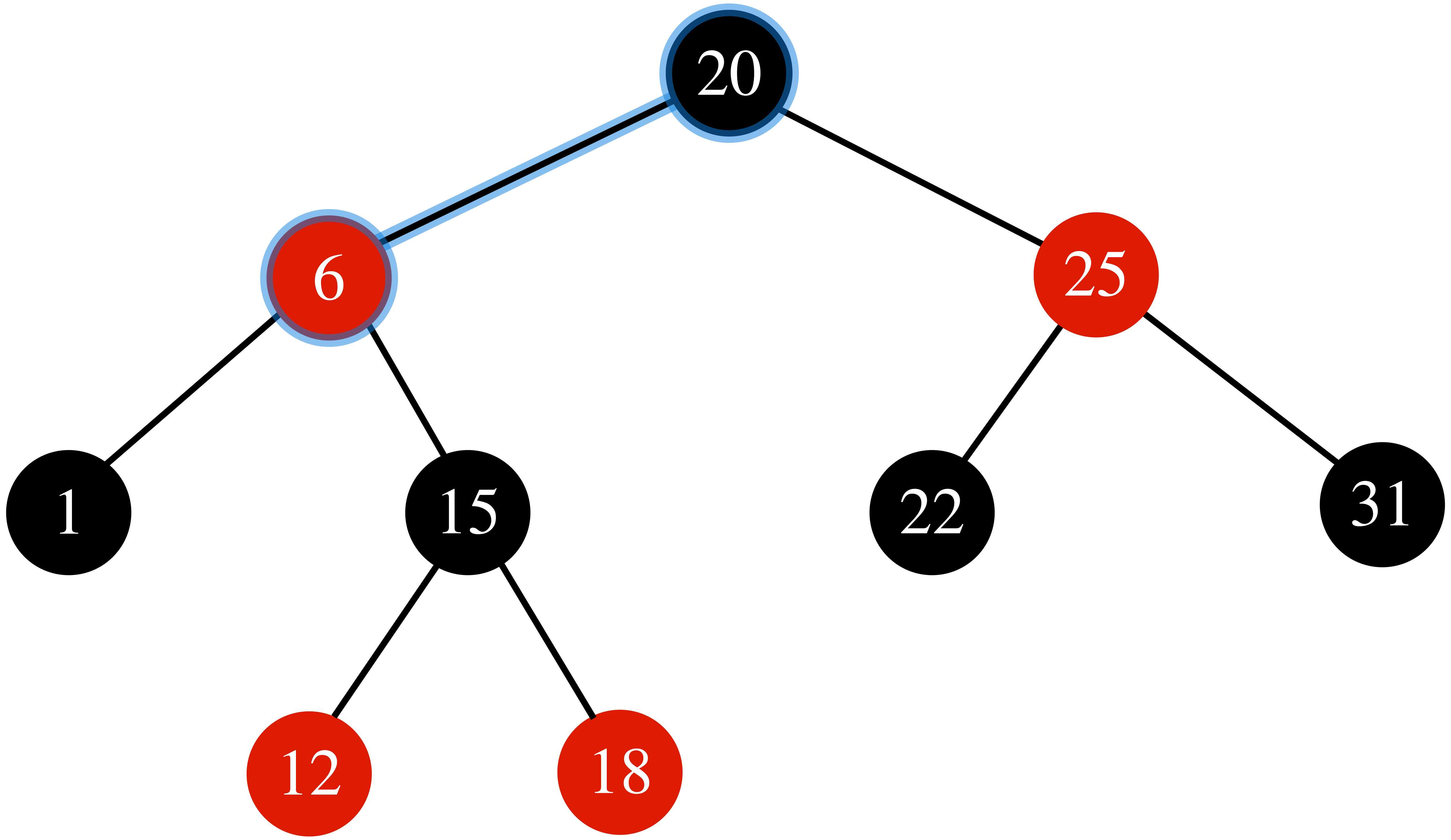

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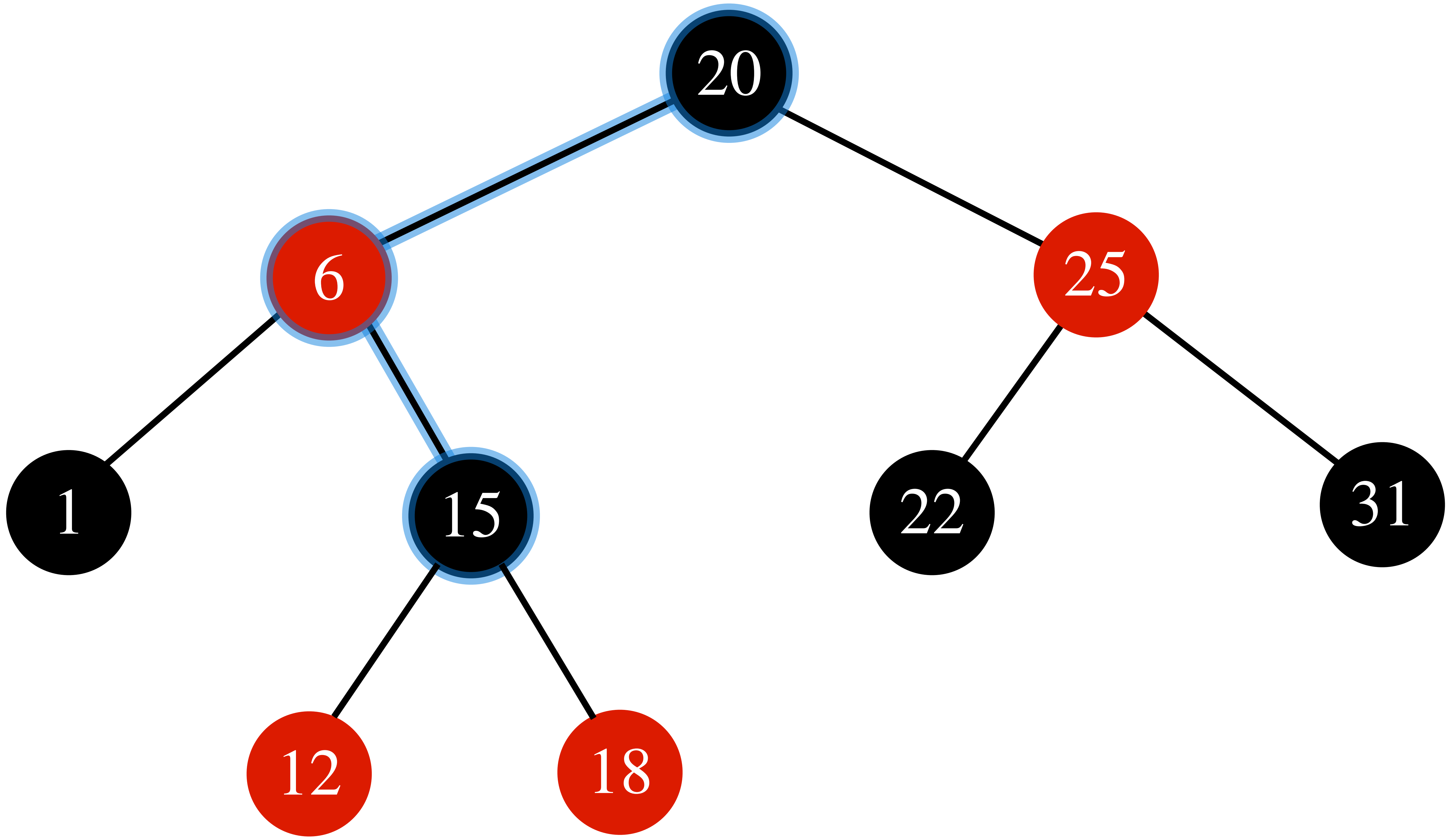

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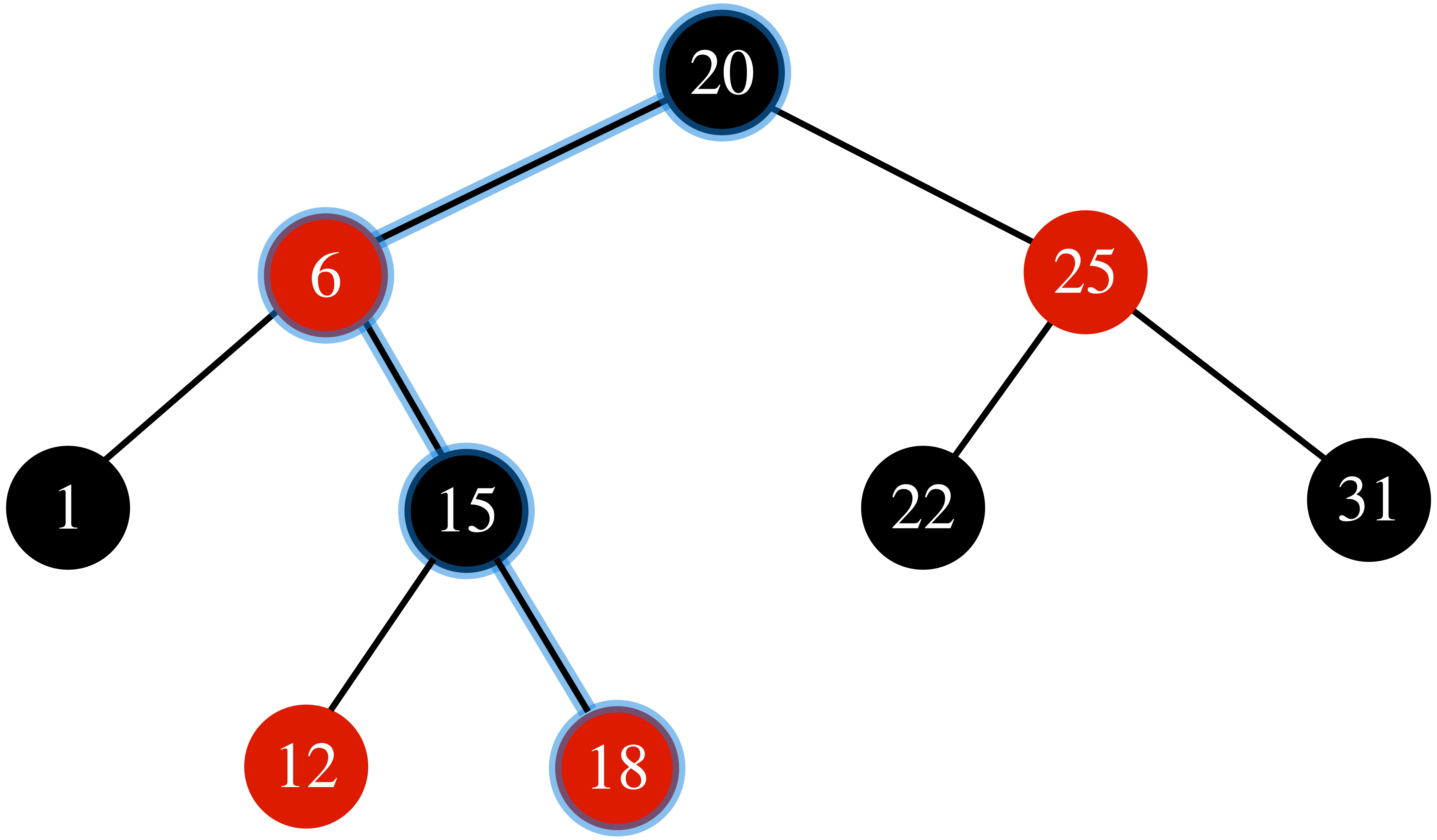

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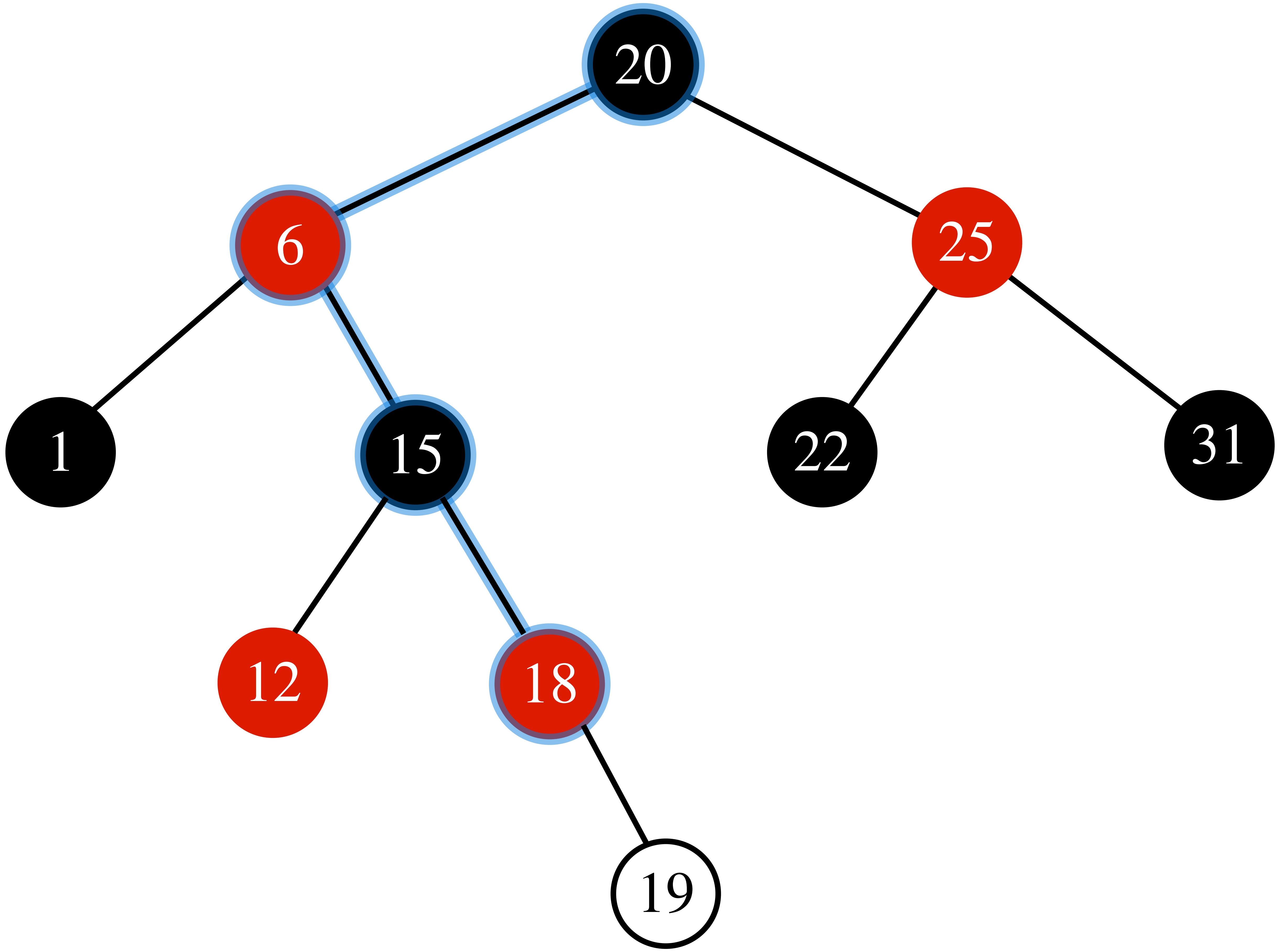

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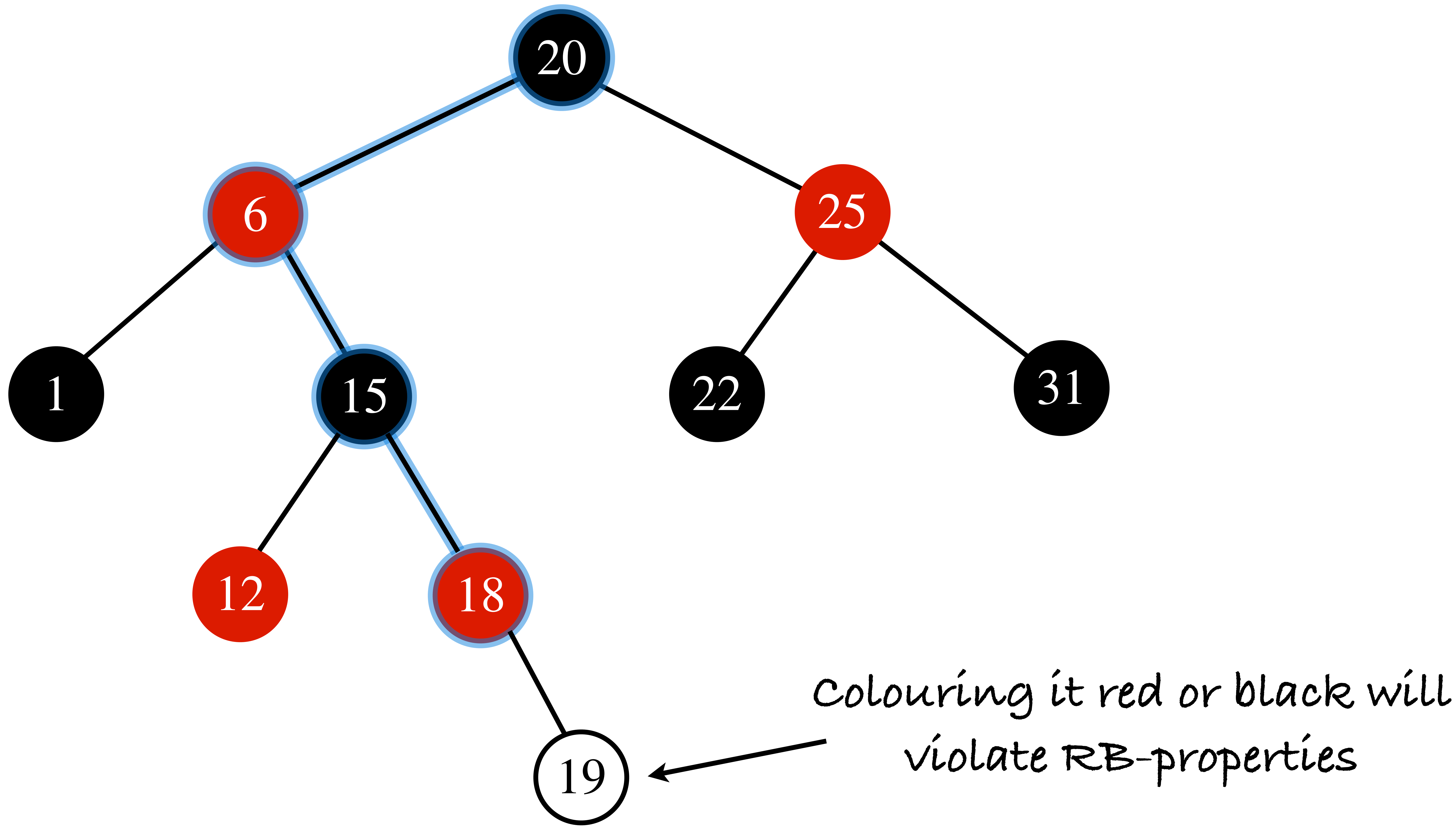

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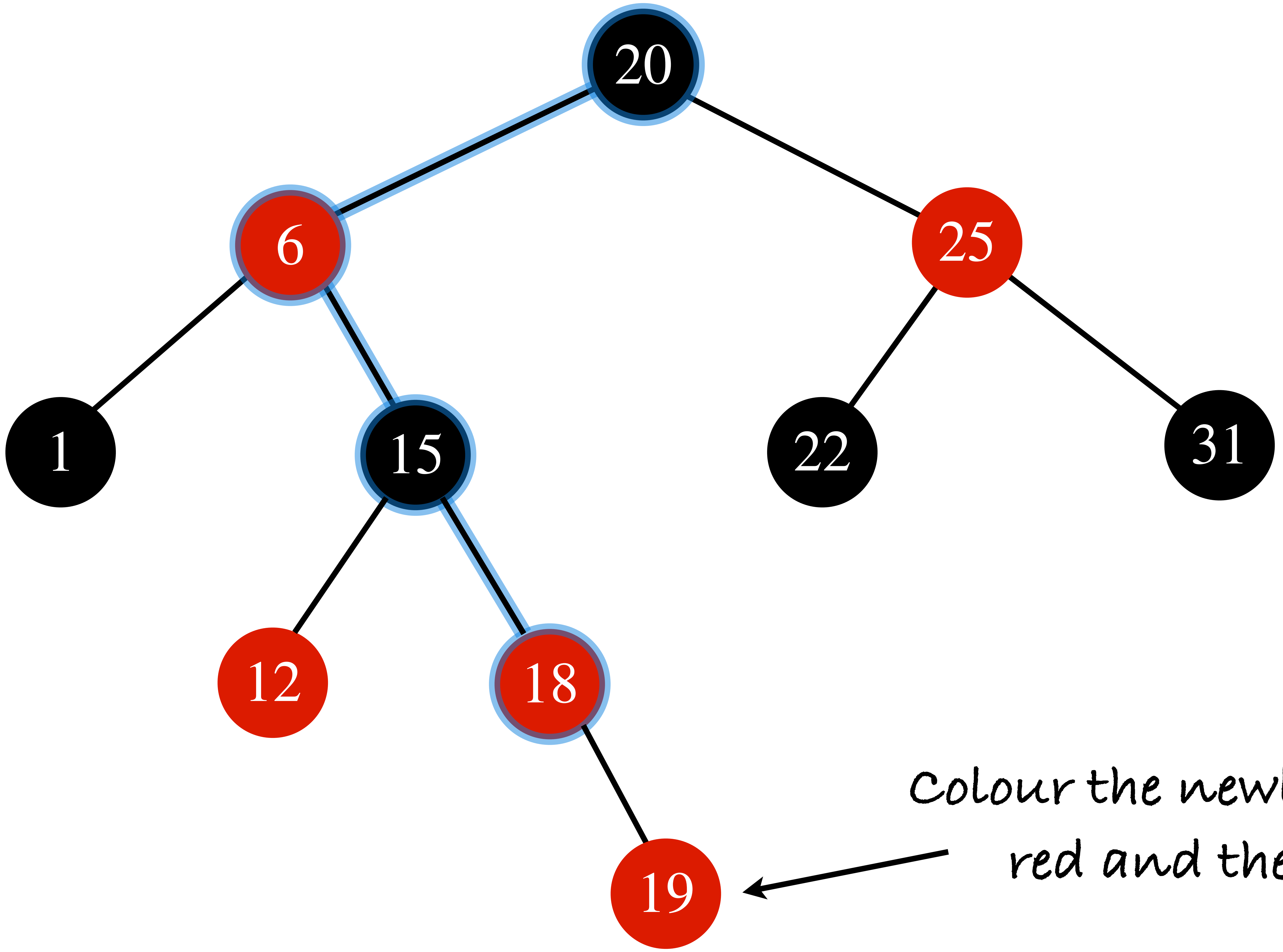

Colour the newly inserted node red and then do fix-up.

RB-Trees: Insertion

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Two stages of insertion:

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- Insert the new node as it is done in a BST and colour the new node red.

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- Insert the new node as it is done in a BST and colour the new node red.
- Do fix-ups as parent of the new node may also be a red node.

RB-Trees: Insertion Cases

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Let z be the newly inserted node with colour red.

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Let z be the newly inserted node with colour red.

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After doing local fix-up, z will set to its parent's parent.

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Gets converted to Case 3

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Gets converted to Case 3

Fix-up will be enough to terminate the process

RB-Trees: Insertion Case 1

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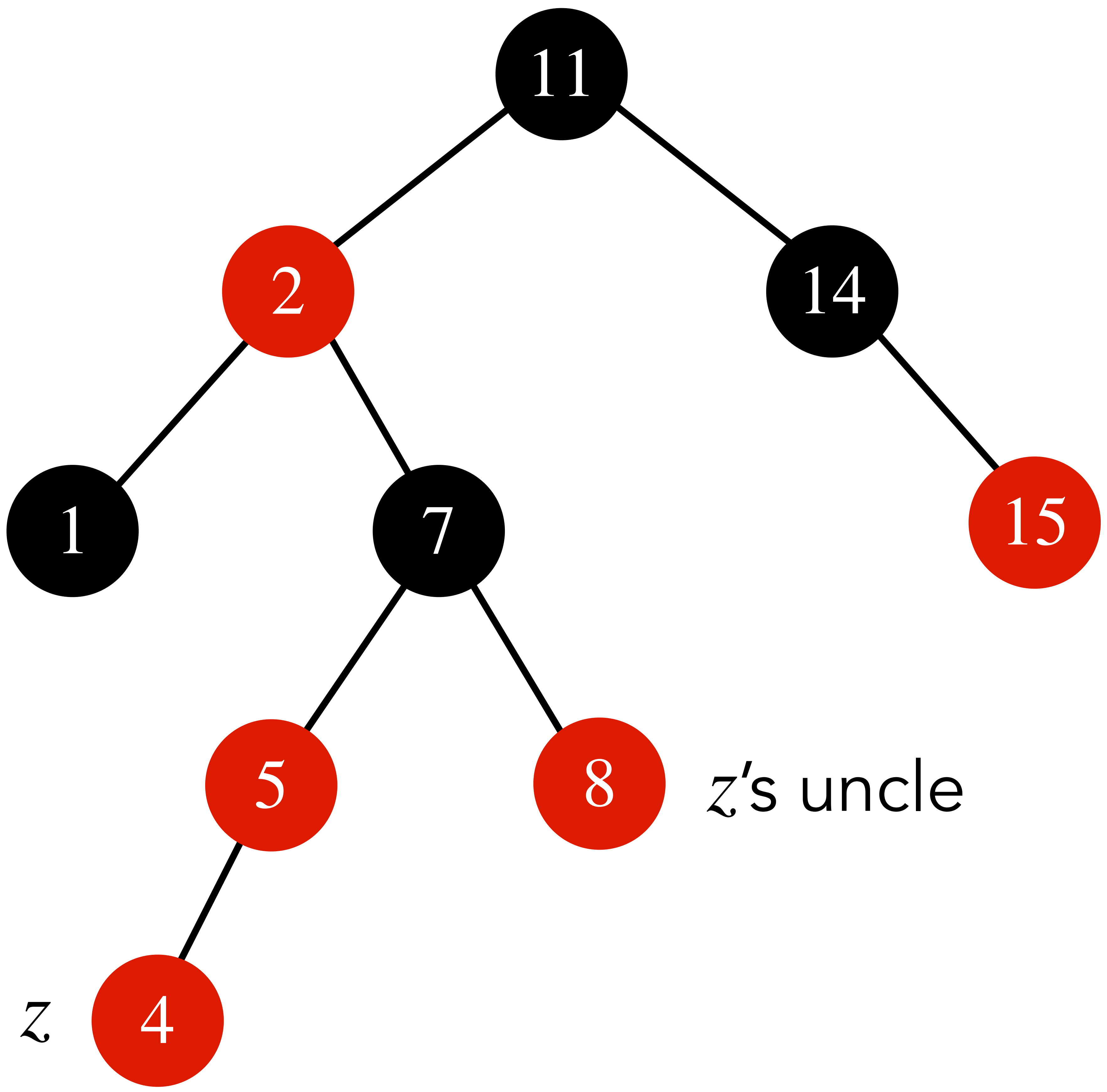

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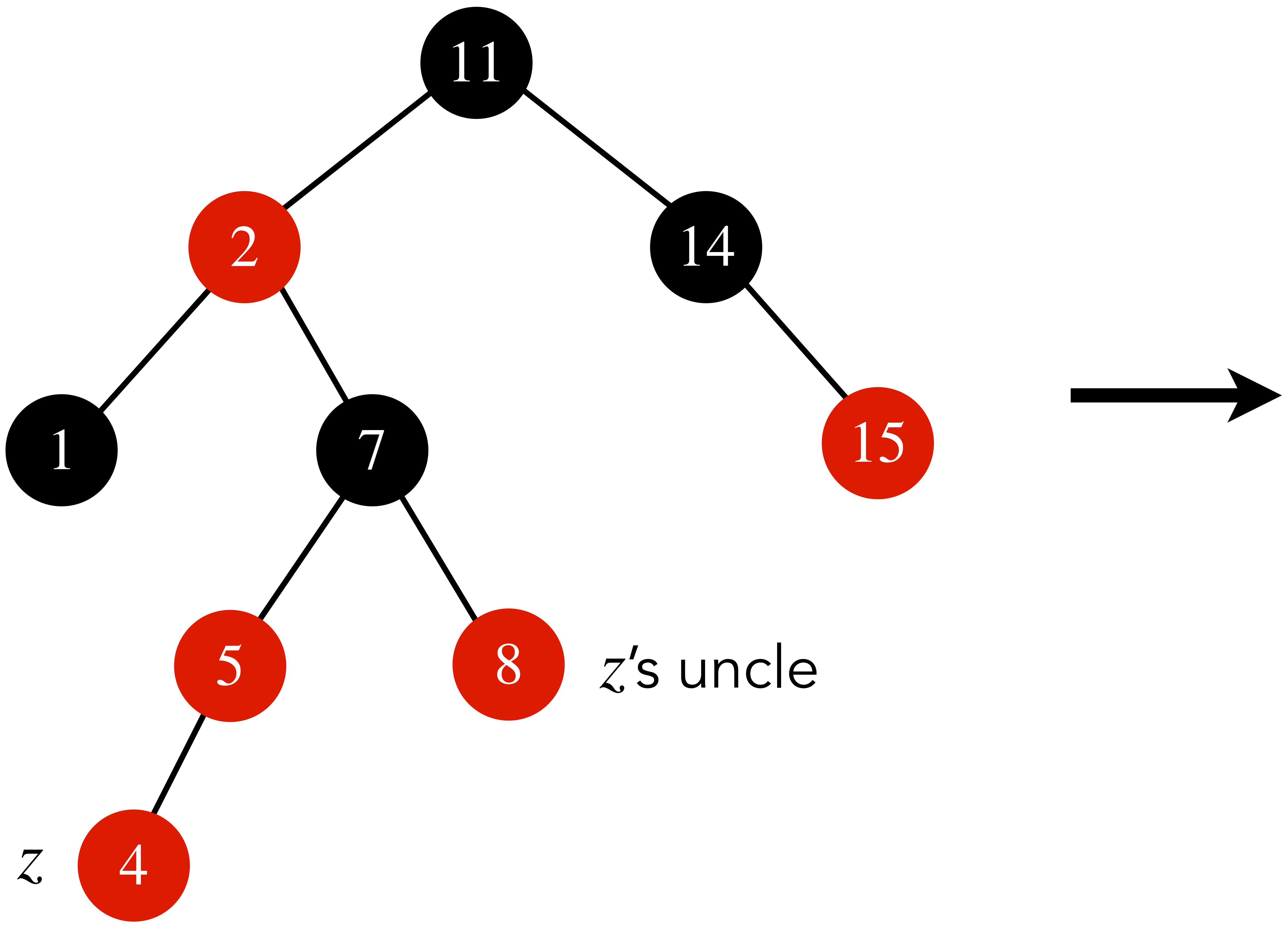

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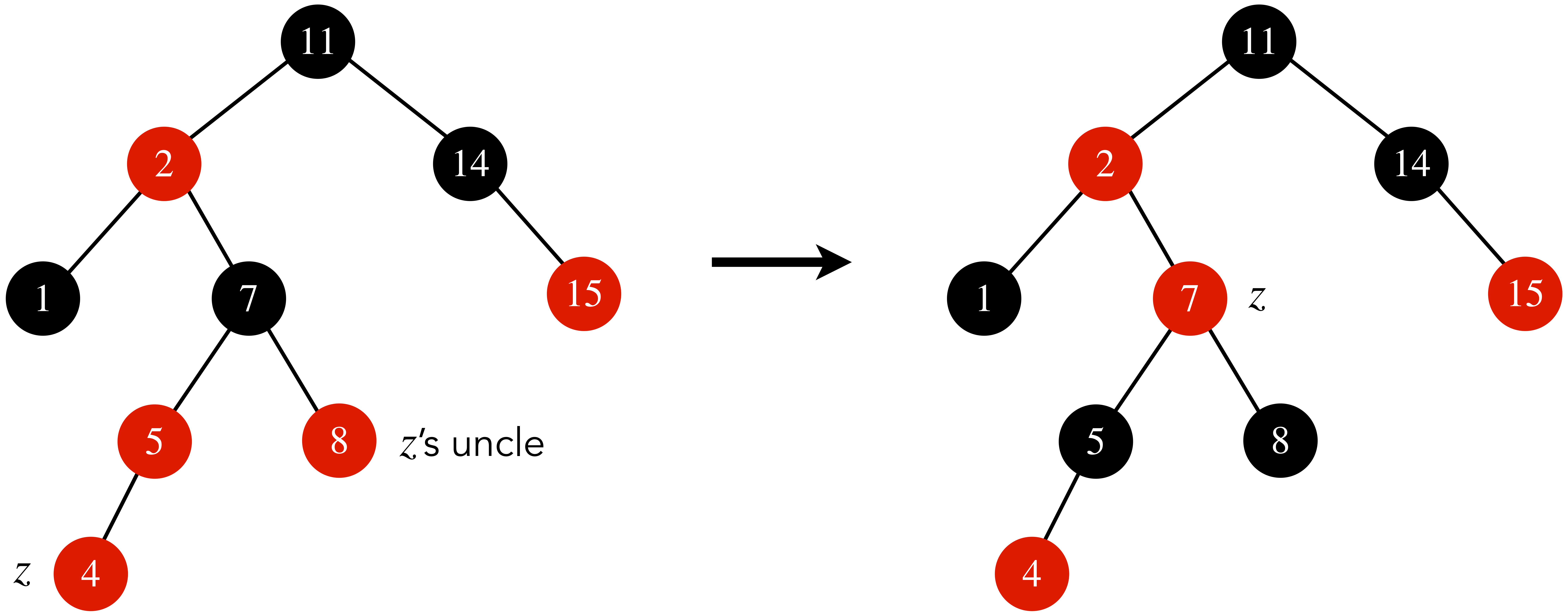

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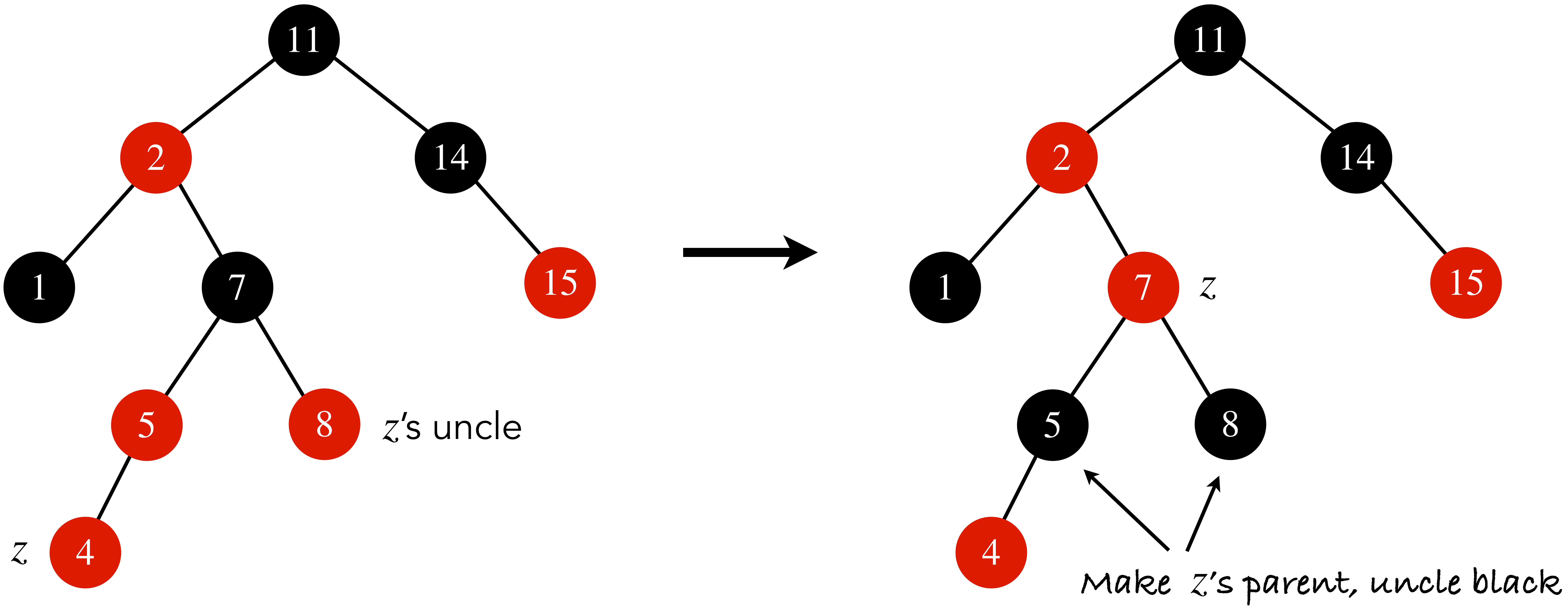

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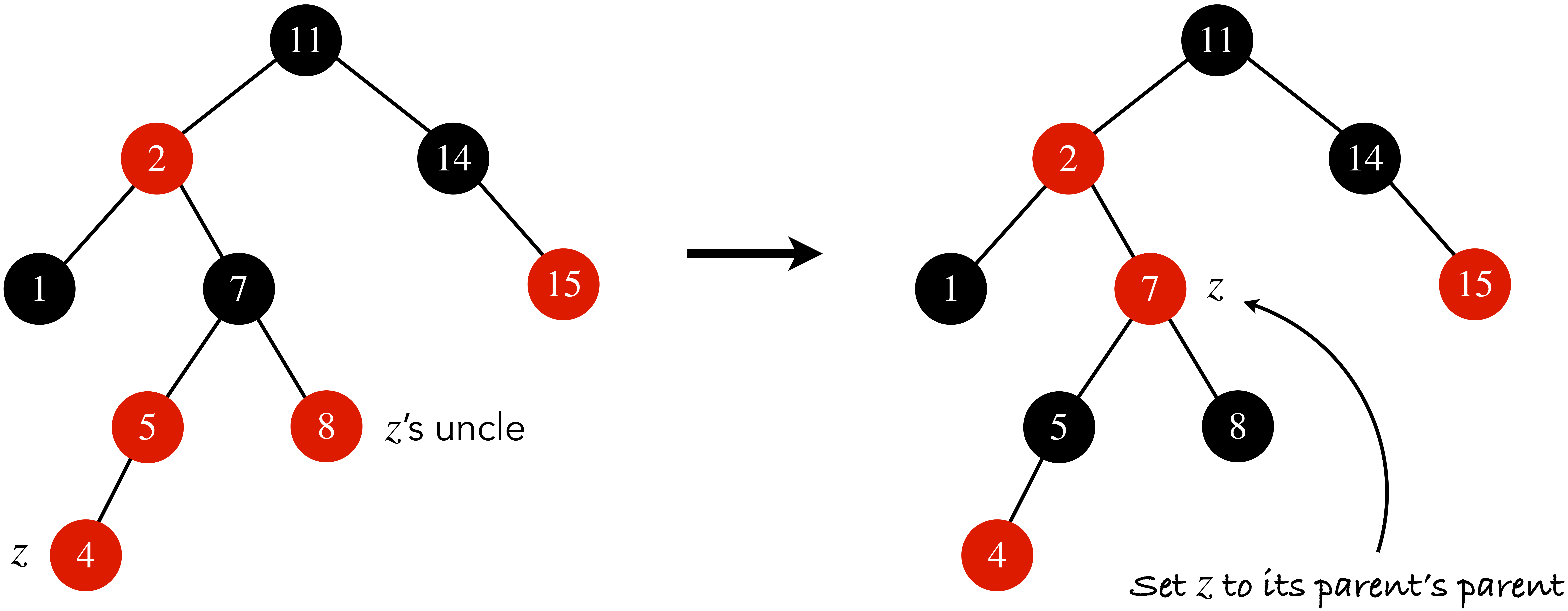
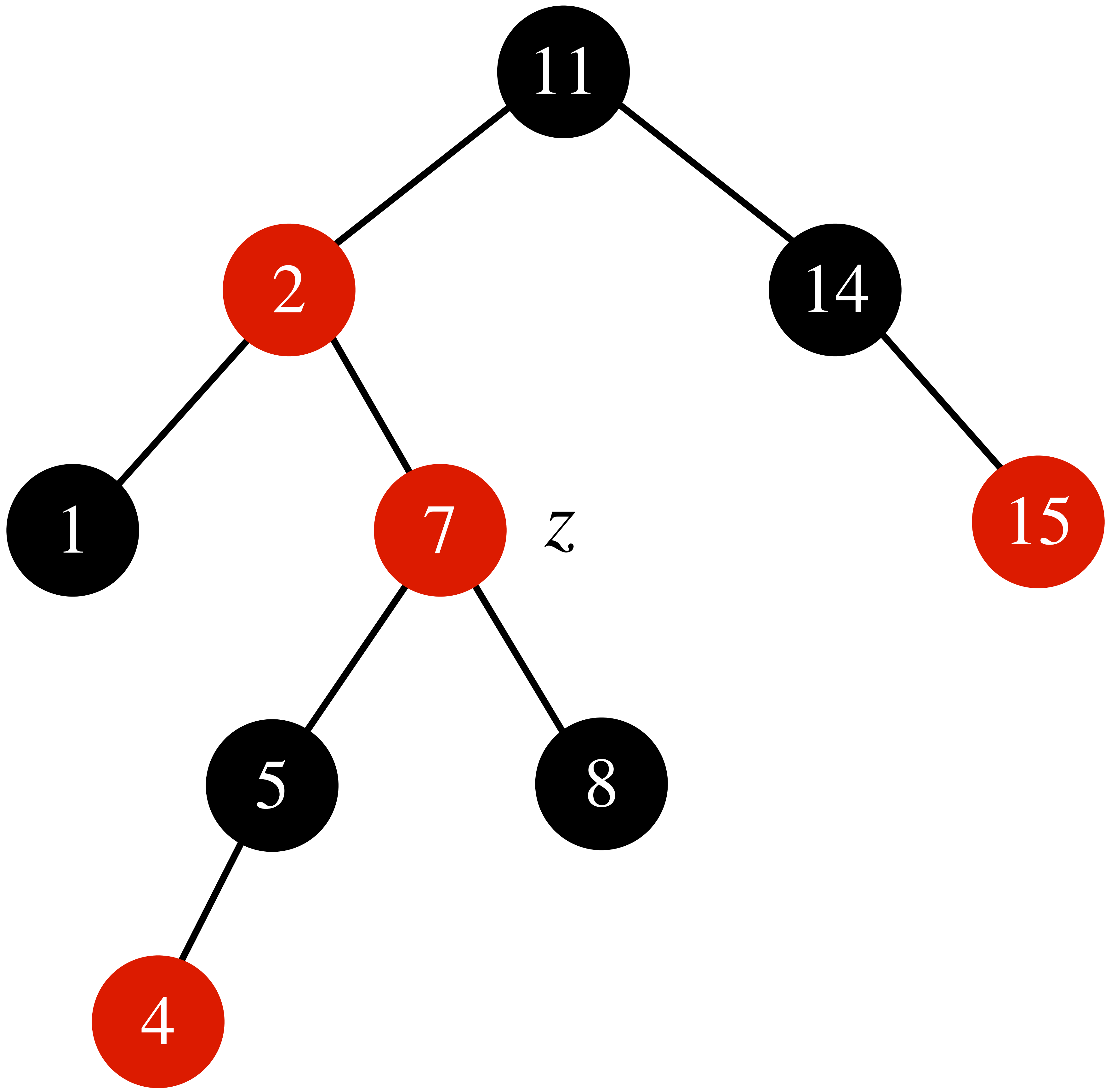


RB-Trees: Insertion Case 2

Case 2: z's uncle is black and z is a right child.



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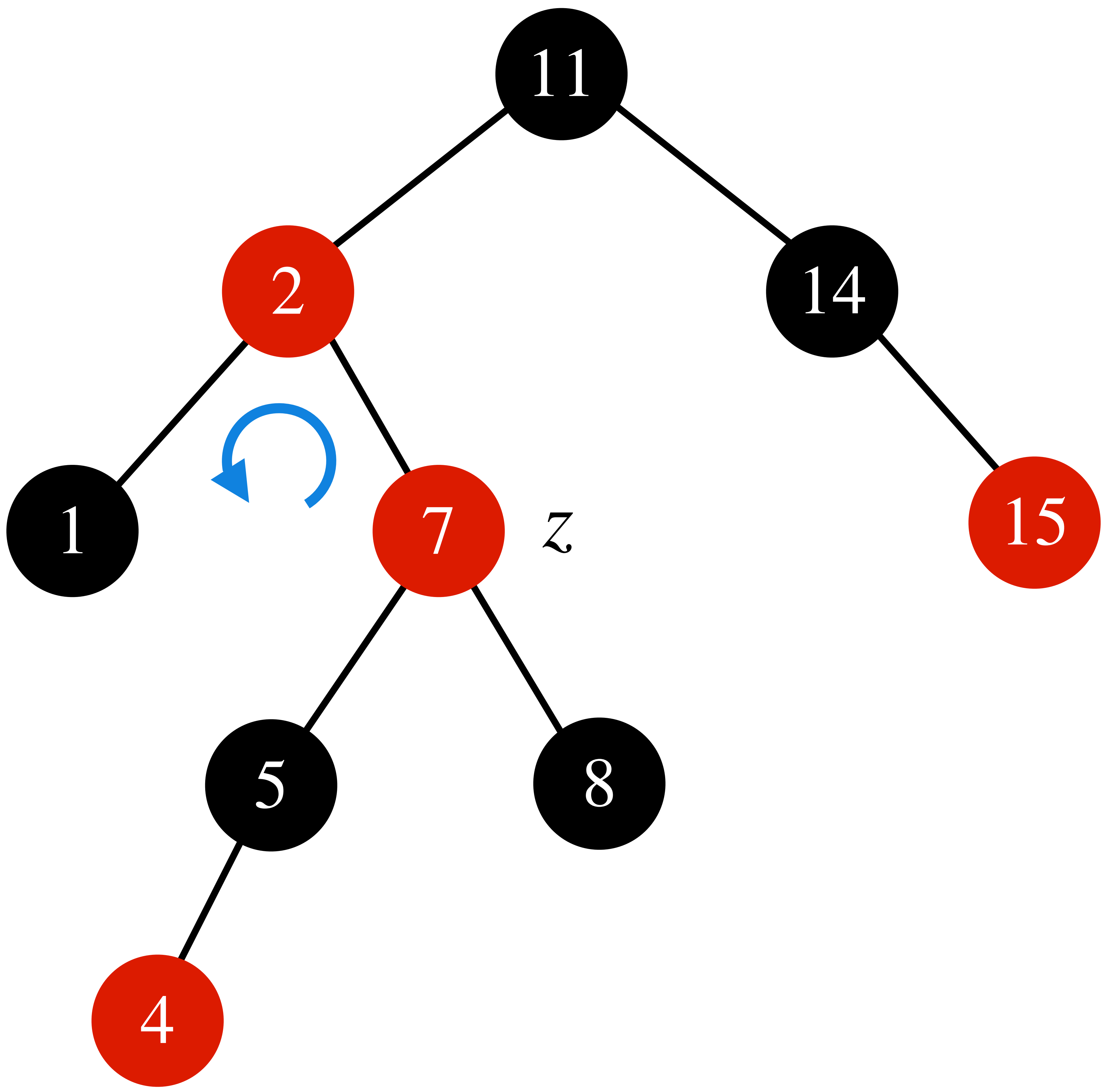

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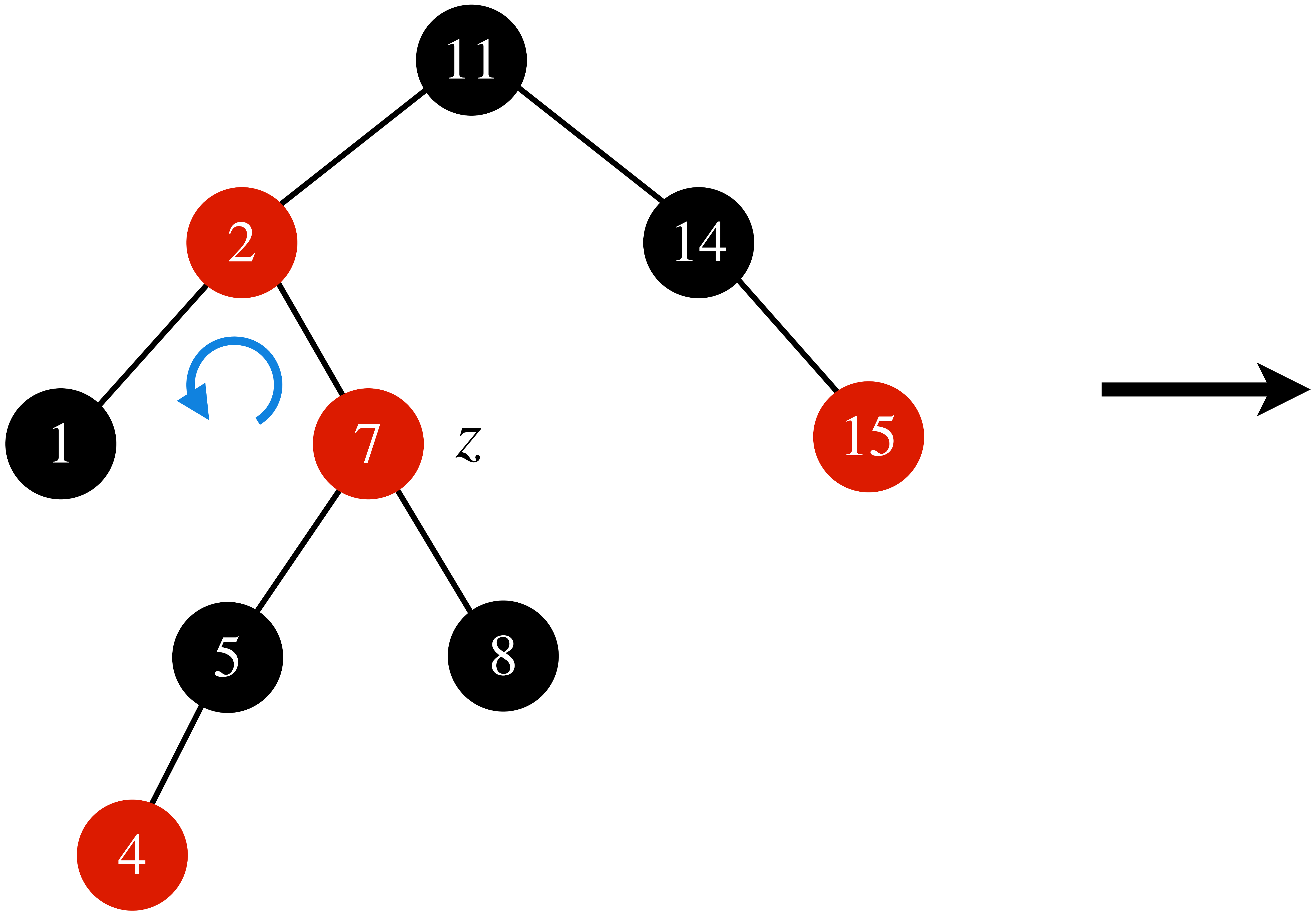

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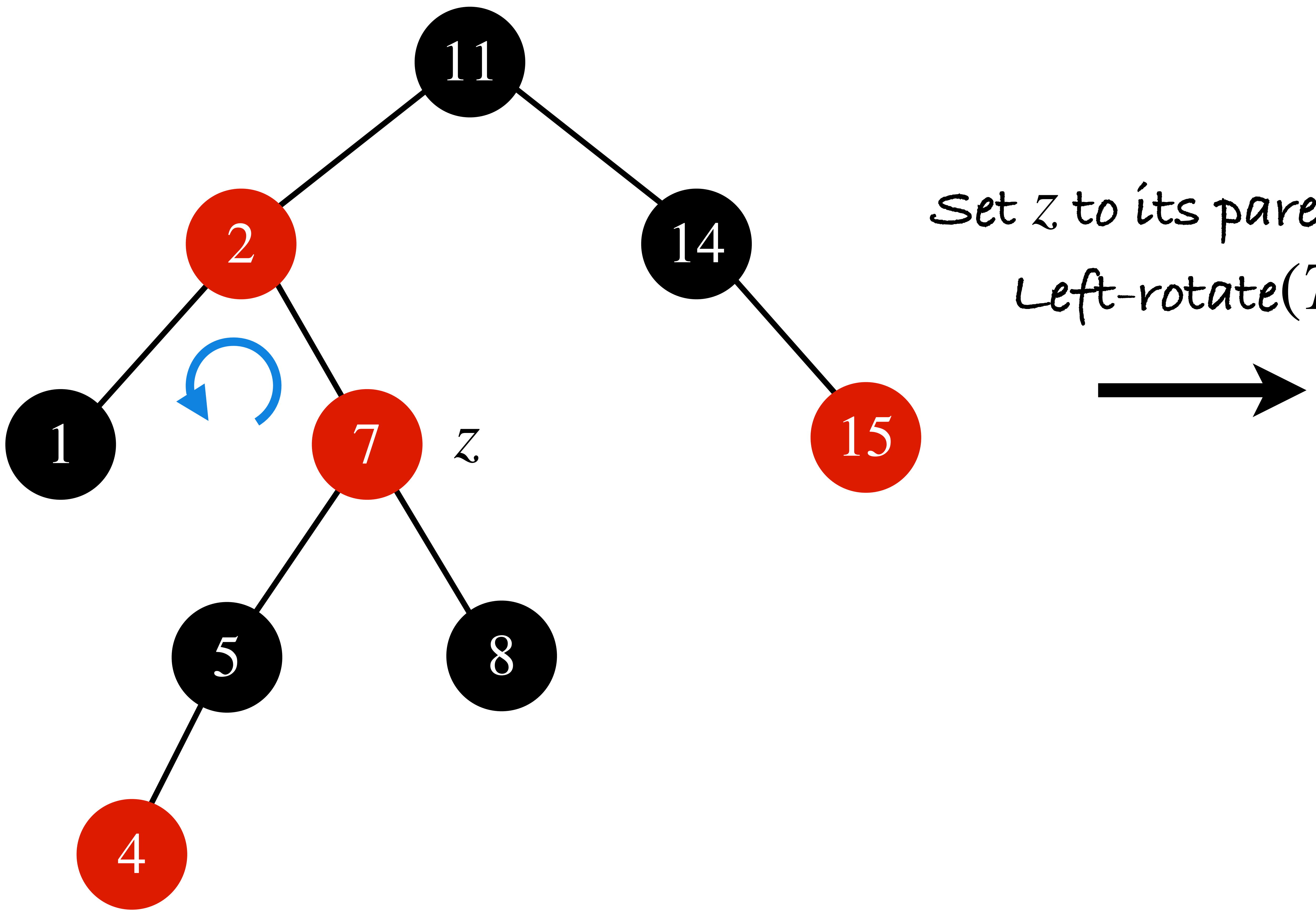

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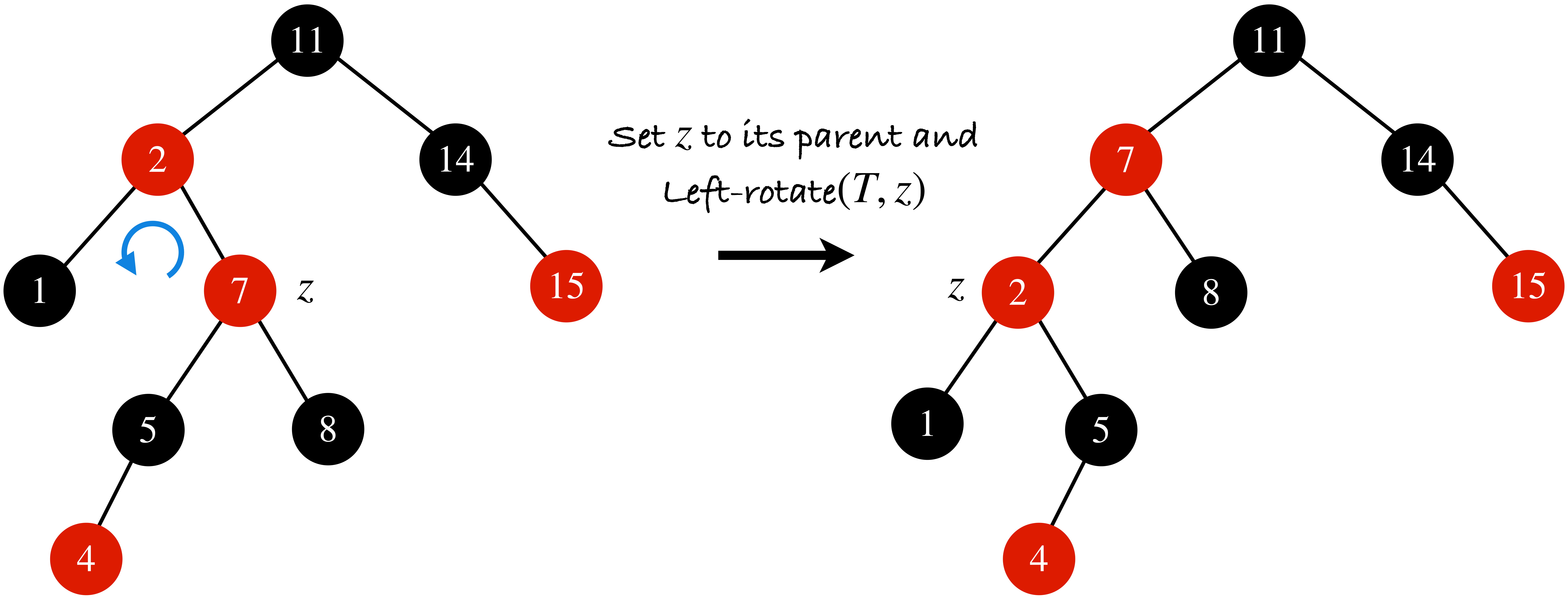
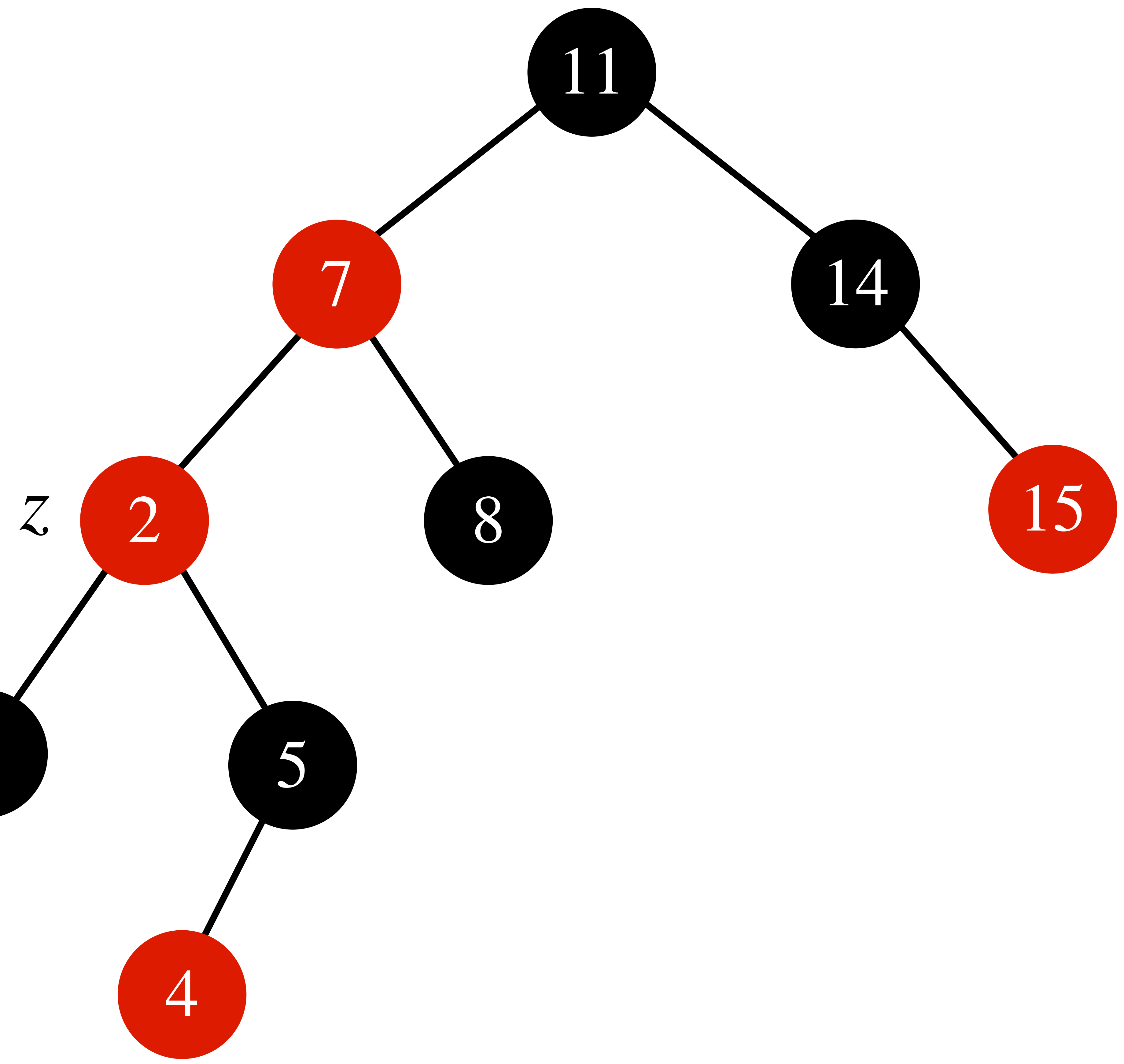


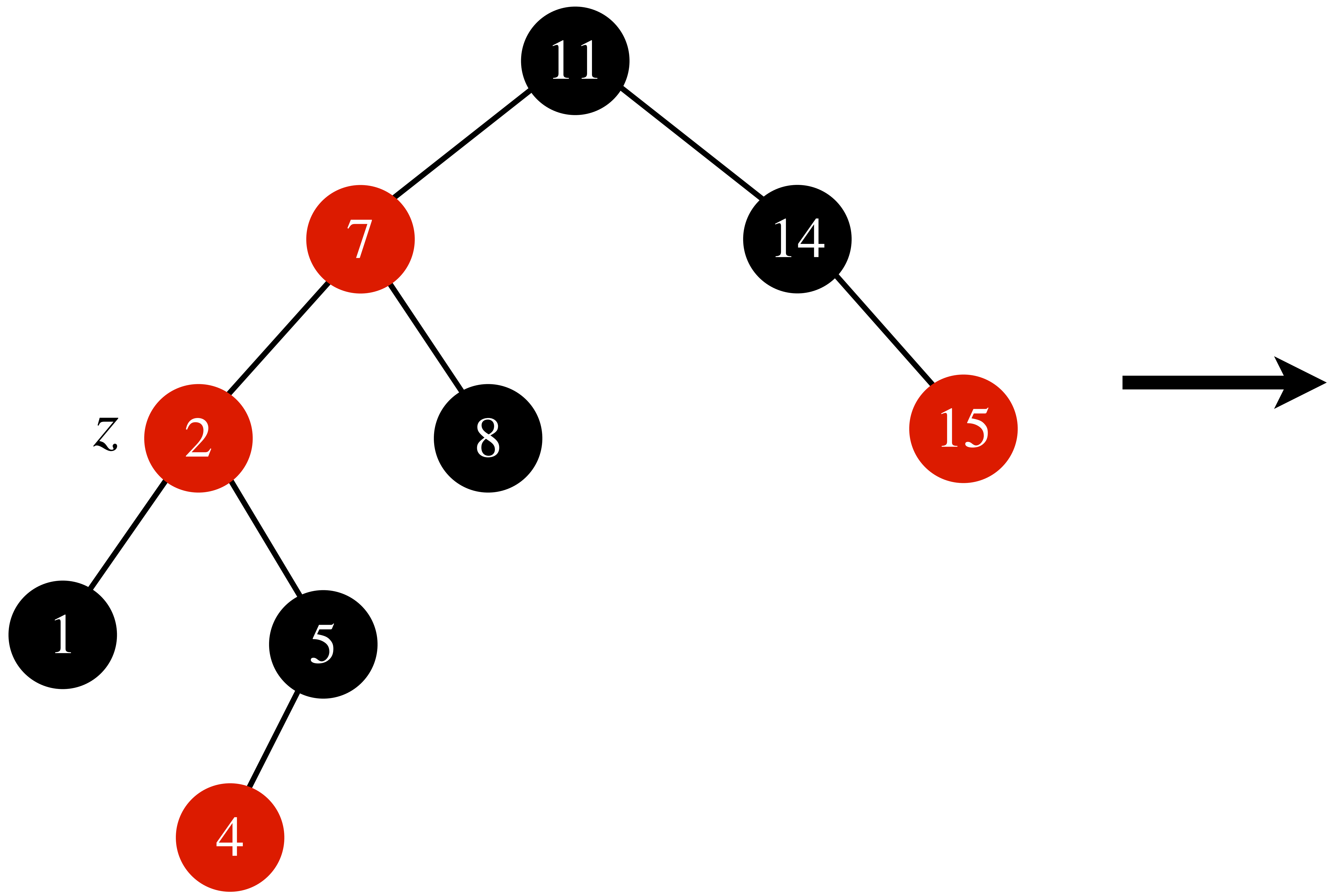
RB-Trees: Insertion Case 3

Case 3**:** z's uncle is black and z is a left child.



RB-Trees: Insertion Case 3

Case 3**:** z's uncle is black and z is a **left child**.



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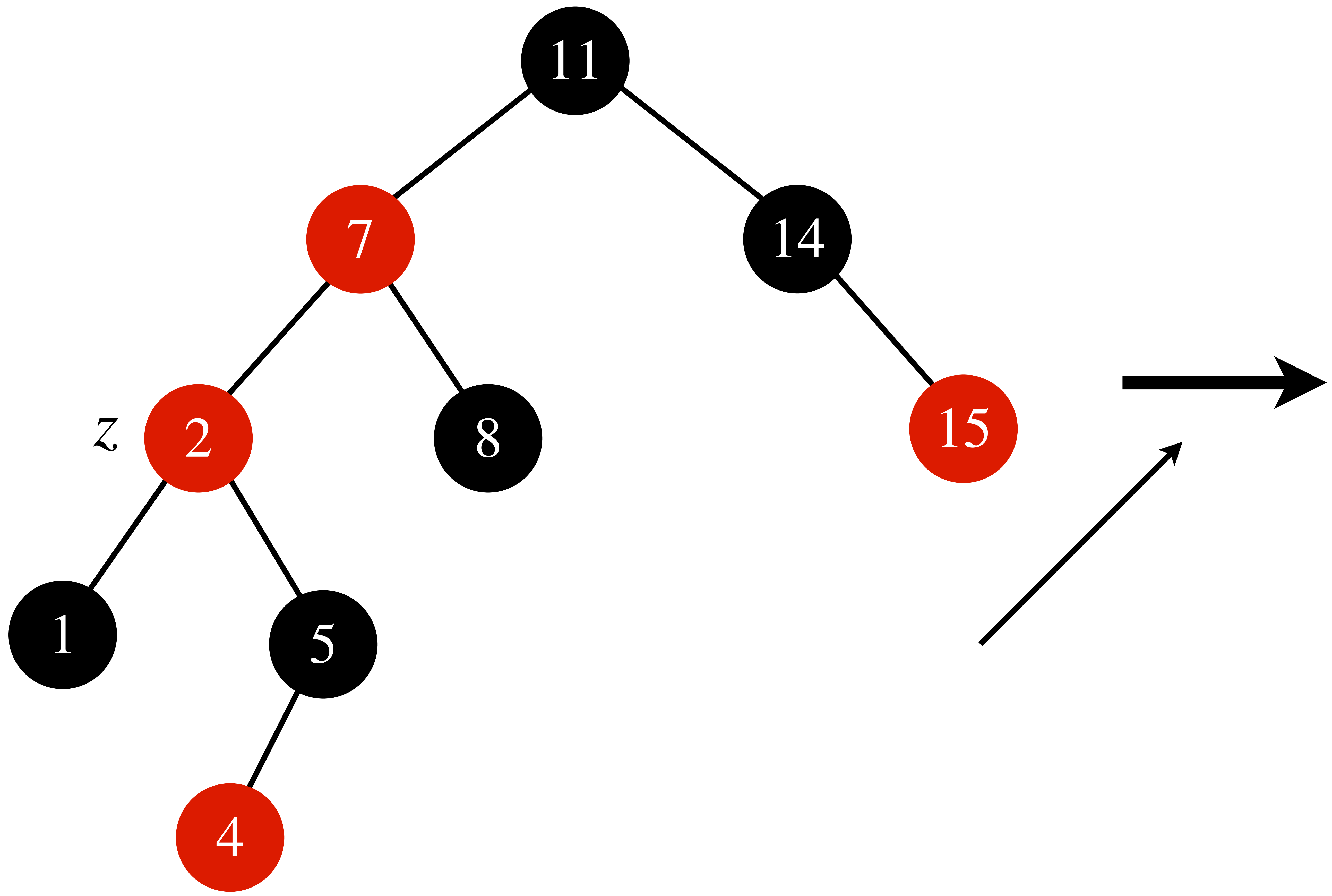

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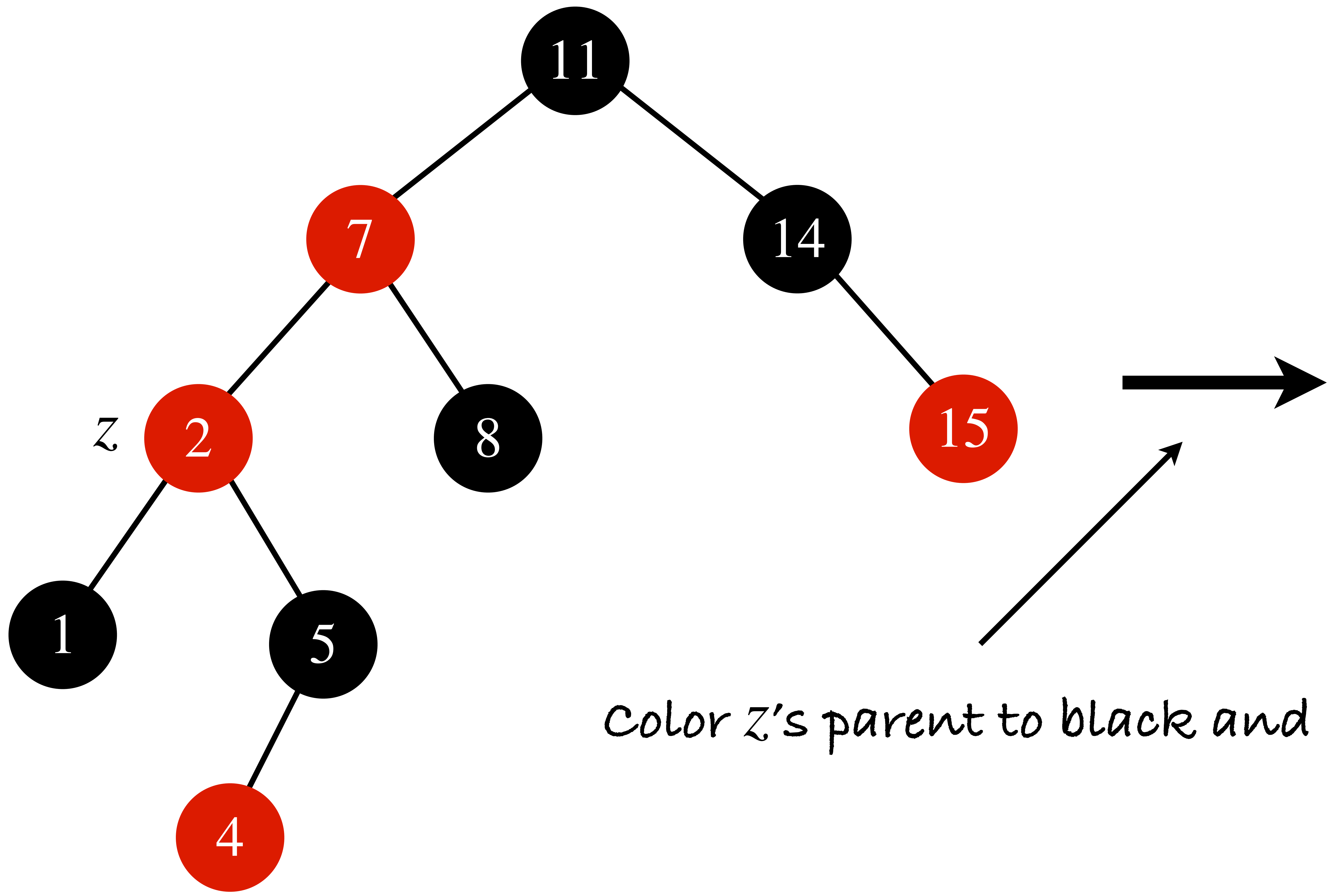

Color z's parent to black and

RB-Trees: Insertion Case 3

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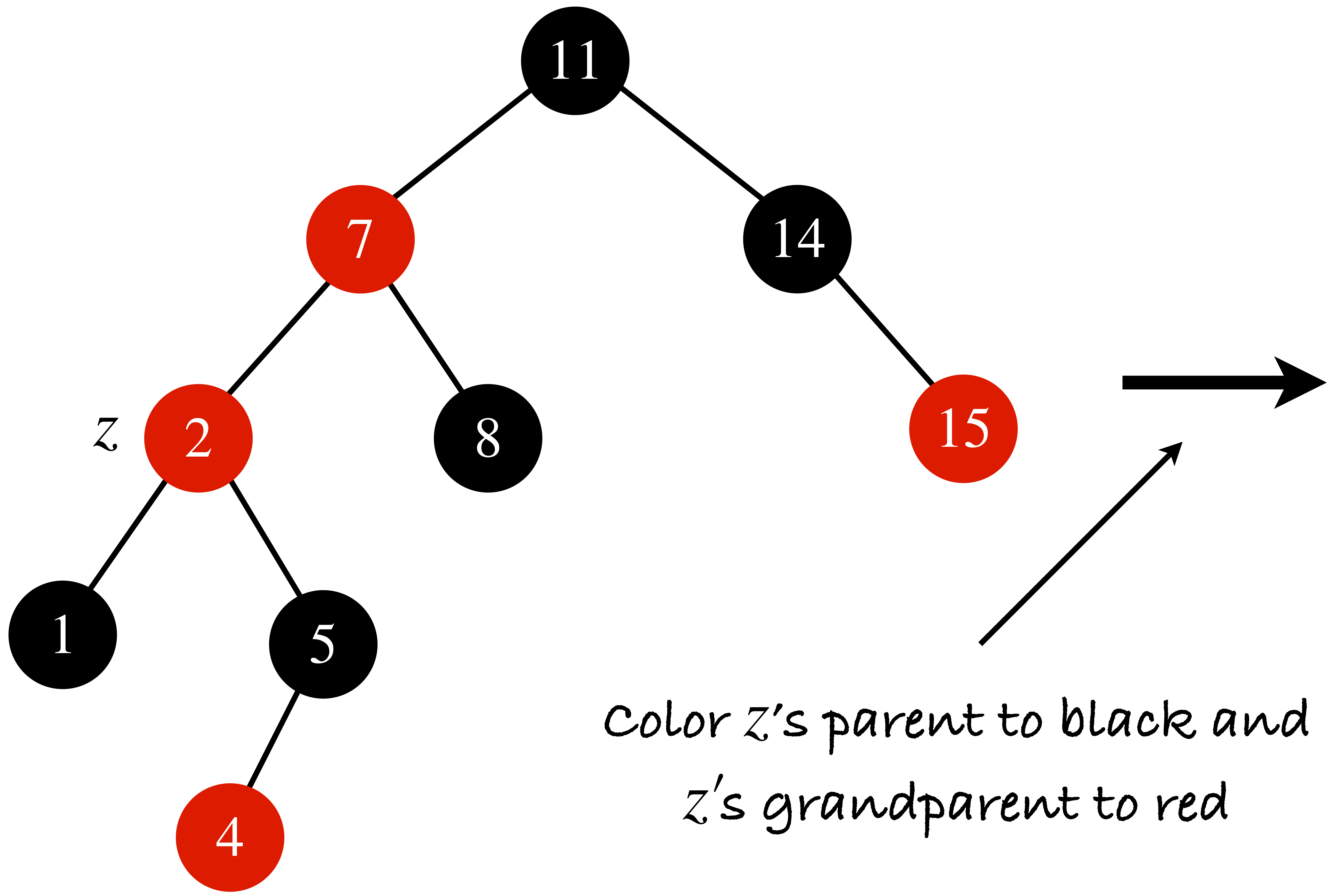

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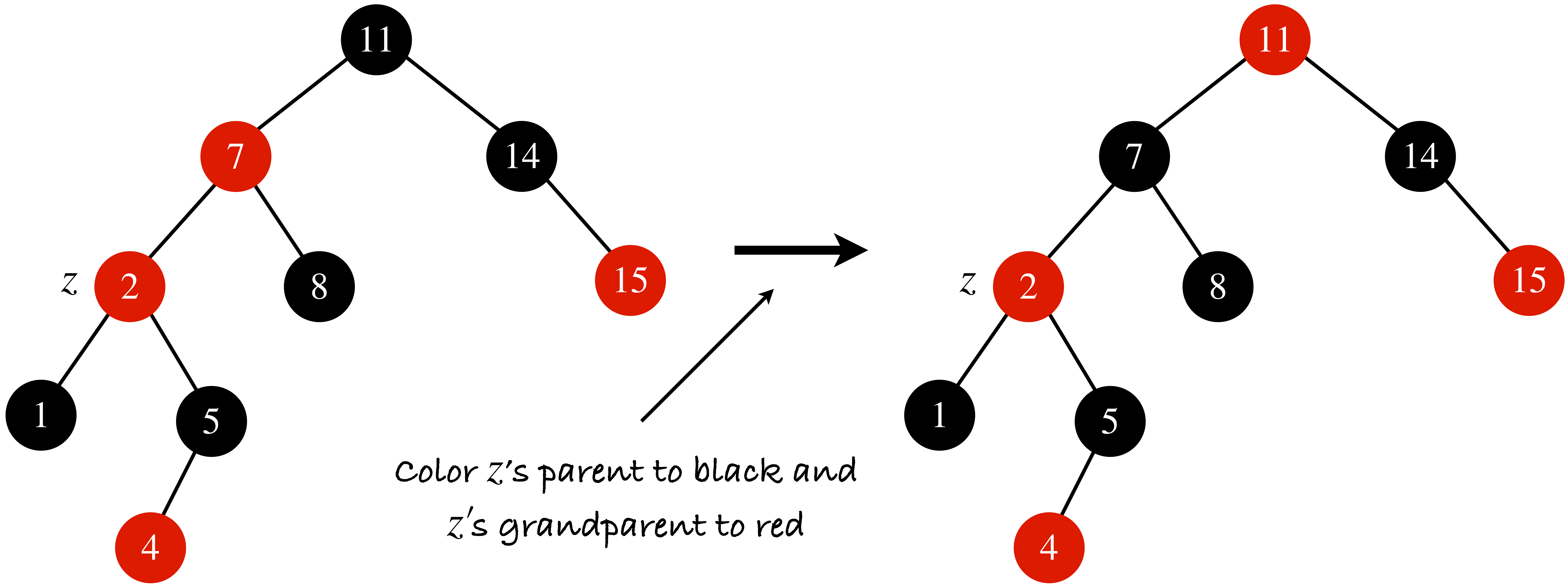

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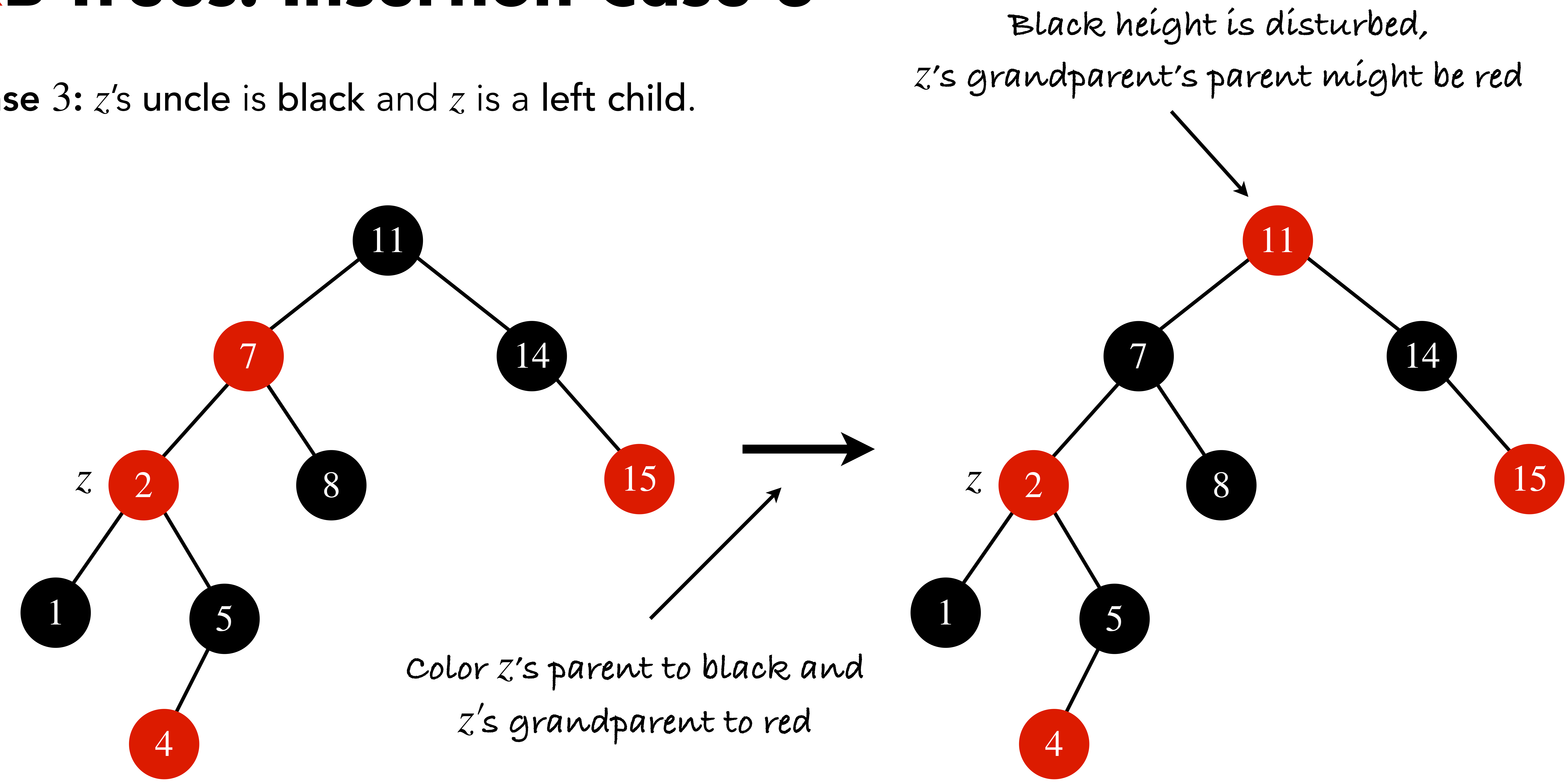

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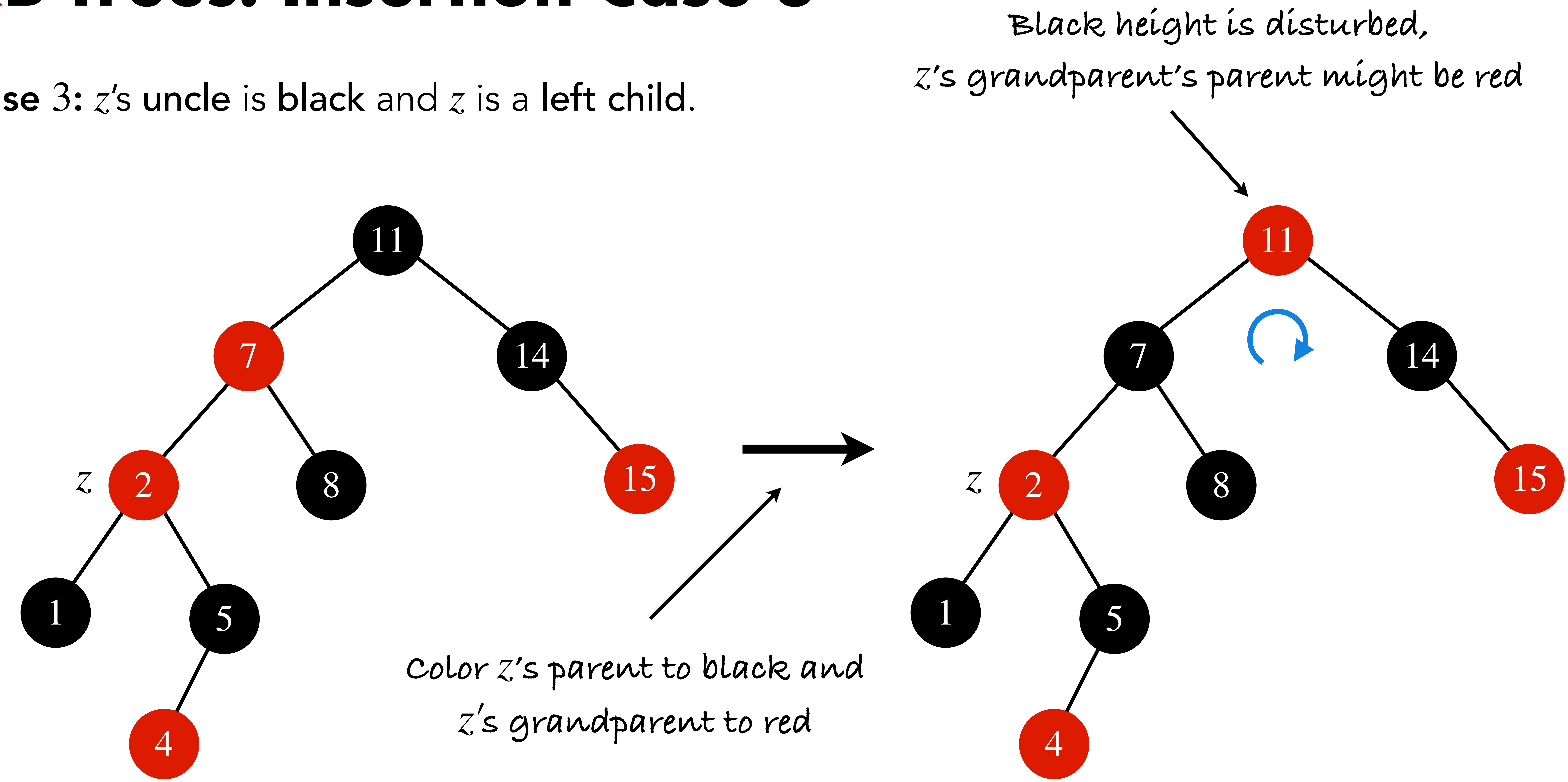

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