Optimization Methods Quiz

Time: 30 minutes

Total Marks: 20

Section A: Newton's Method

1. What is the primary advantage of Newton's method over gradient descent?

   a. Lower computational cost per iteration
   b. Quadratic convergence rate
   c. No requirement for second derivatives
   d. Guaranteed global convergence

2. True/False: Newton's method requires the Hessian matrix to be positive definite for minimization problems.

○ True ○ False

3. Short Answer: Why might Newton's method fail to converge for non-convex functions?

(2 marks)

Section B: Steepest Descent

4. The steepest descent direction for minimizing $f(x)$ is:

   a. $\nabla f(x)$
   b. $-\nabla f(x)$
   c. $\nabla^2 f(x)^{-1} \nabla f(x)$
   d. $\nabla f(x) \times \nabla^2 f(x)$

5. True/False: The steepest descent method converges linearly for quadratic functions.

○ True ○ False

6. Calculate the steepest descent direction for $f(x,y) = x^2 + 3y^2$ at $(1, -1)$.

(2 marks)

Section C: Directional Derivatives

7. The directional derivative of $f(x,y) = xy$ at $(2,3)$ in the direction $u = (1/\sqrt{2}, 1/\sqrt{2})$ is:

   a. $5/\sqrt{2}$
   b. $3/\sqrt{2}$
   c. 2.5
   d. 6

8. True/False: The maximum directional derivative of a function equals the magnitude of its gradient.

○ True ○ False

Section D: Quasi-Newton Methods

9. BFGS and DFP are examples of:

   a. Line search methods
   b. Hessian approximation methods
   c. Stochastic optimization
   d. Penalty function methods

10. Why are quasi-Newton methods preferred over Newton's method for large-scale problems?

(2 marks)

Section E: Application

11. For $f(x) = x^4 - 3x^2 + 2$:

Perform **one iteration of Newton's method** starting at $x_0 = 1$.

(4 marks)