DSAT (Jan - April 2025)

Assignment 1

Instructions:

1. *In questions asking for algorithm you should write pseudocode.*
2. *You are allowed to take help but do not copy-paste the answers.*
3. *Doing the problems on your own will help you in exams as extensions of some of these problems may come in exams.*
4. *All questions are of* 5 *marks, but they are not of equal difficulty level.*

1. Let $A[1:n]$ be an array of n distinct numbers. If $i < j$ and $A[i] > A[j]$, then the pair (i, j) is called an inversion of A. Give an $O(n \log n)$-time algorithm that determines the number of inversions in A.

2. Consider inserting the keys 10,22,31,4,15,28,17,88,59 into a hash table of length $m = 11$ using open addressing. Illustrate the result of inserting these keys using linear probing with $h(k, i) = (k + i) \% m$ and using double hashing with $h_1(k) = k$ and $h_2(k) = 1 + (k \% (m - 1))$.

3. Suppose you are given a permutation p of the integers 1 to n, and seek to sort them to be in increasing order [1,…, n]. The only operation at your disposal is $reverse(p, i, j)$, which reverses the elements of a subsequence $p_i, \ldots, p_j$ in the permutation. For the permutation [1,4,3,2,5] one reversal (of the second through fourth elements) suffices to sort. Suppose that the cost of $reverse(p, i, j)$ is equal to its length, the number of elements in the range, $|j - i| + 1$. Design an algorithm that sorts in $O(n \log^2 n)$ cost.

4. Describe an $O(n \log n)$ time and $O(n)$ space algorithm that, given a set of n integers and another integer x, determines whether S contains two elements that sum exactly x. *(Hint: Read about binary search)*

5. Let T be a BST. Describe an $O(n)$ time algorithm that on input $T.root$ can find the minimum absolute difference of any two keys of T. For instance, if keys of T are 3,8,1,12,7,15, then answer will be $8 - 7 = 1$.

6. Consider a BST T. Let x and y be two of the keys of T. Is it the case that BST we get after deleting x and then y is the same as the BST we get after deleting y and then x? Either argue for it or give a counterexample.