Lecture 7

BST (Insertion & Deletion), Red-Black Trees

Modifying a BST: Insertion

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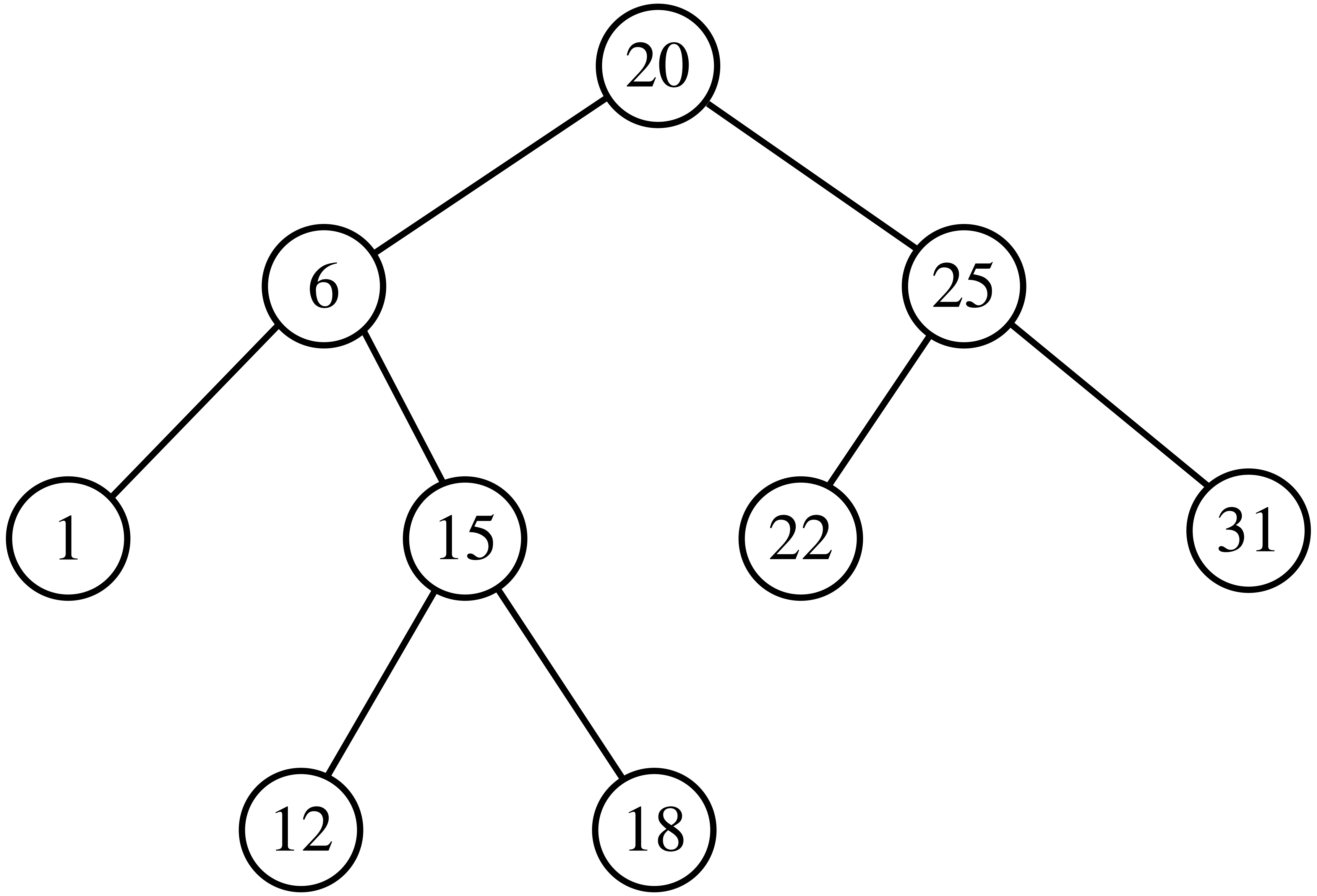
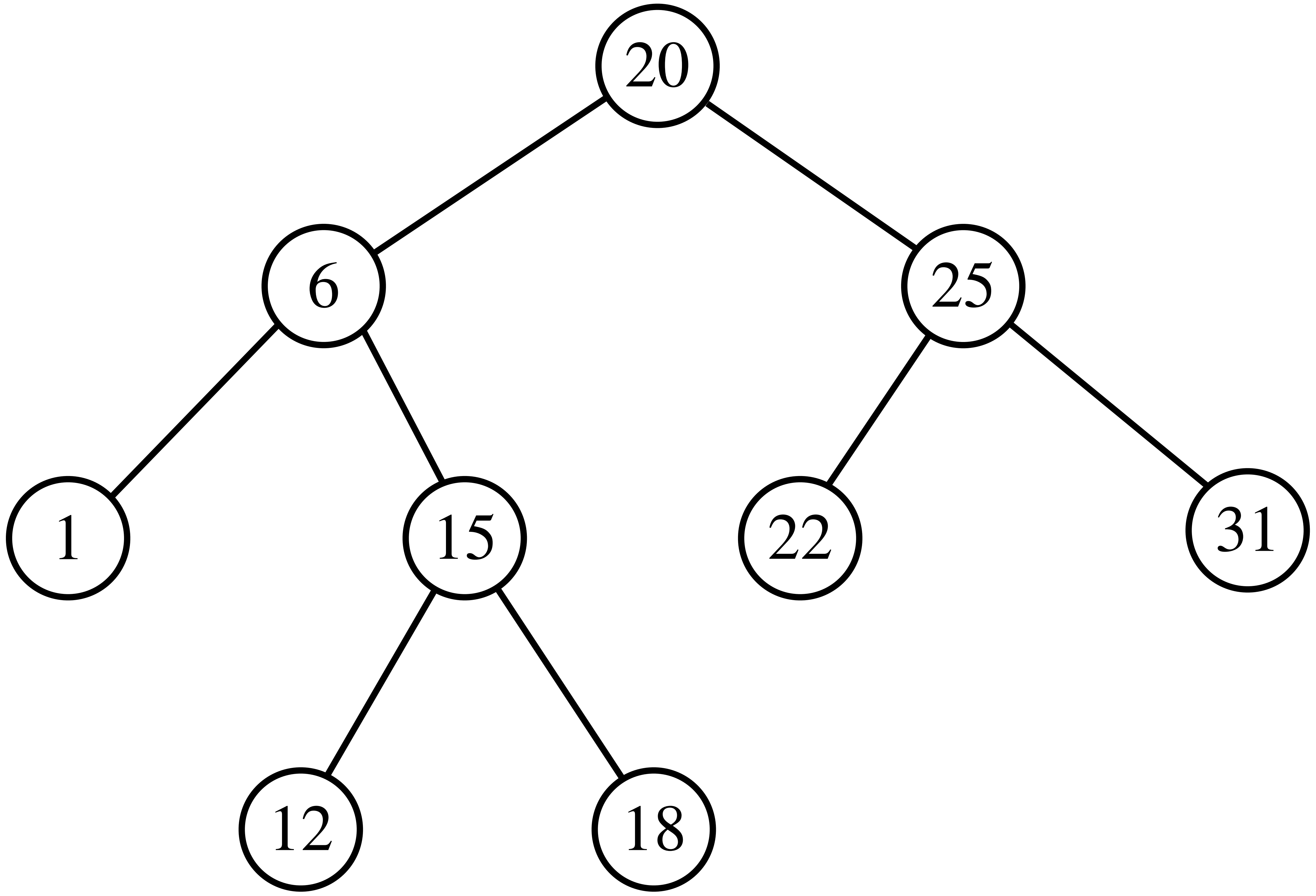


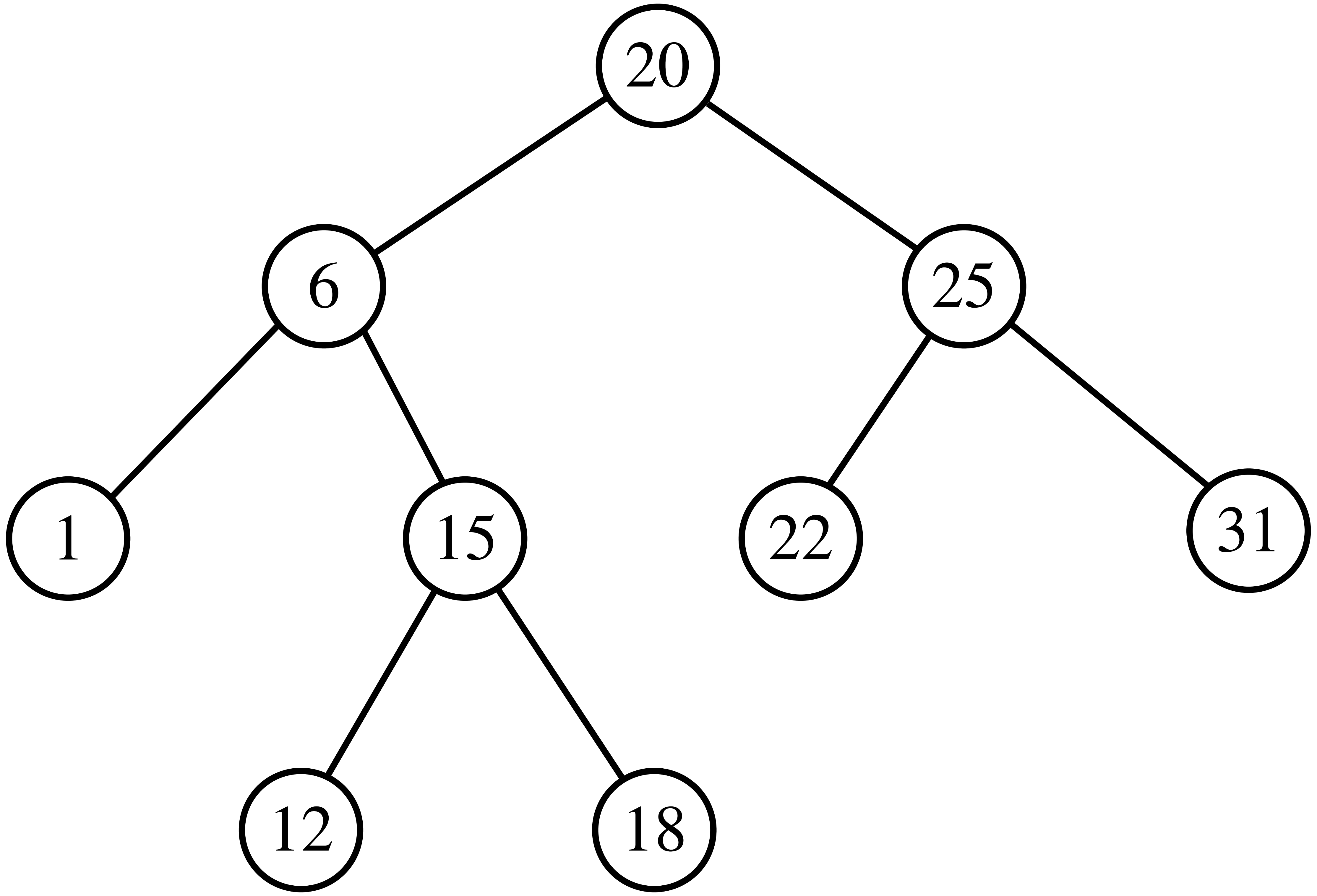
Modifying a BST: Insertion

Example:



Modifying a BST: Insertion

Example: Insert a node with 24 as key in the following BST.



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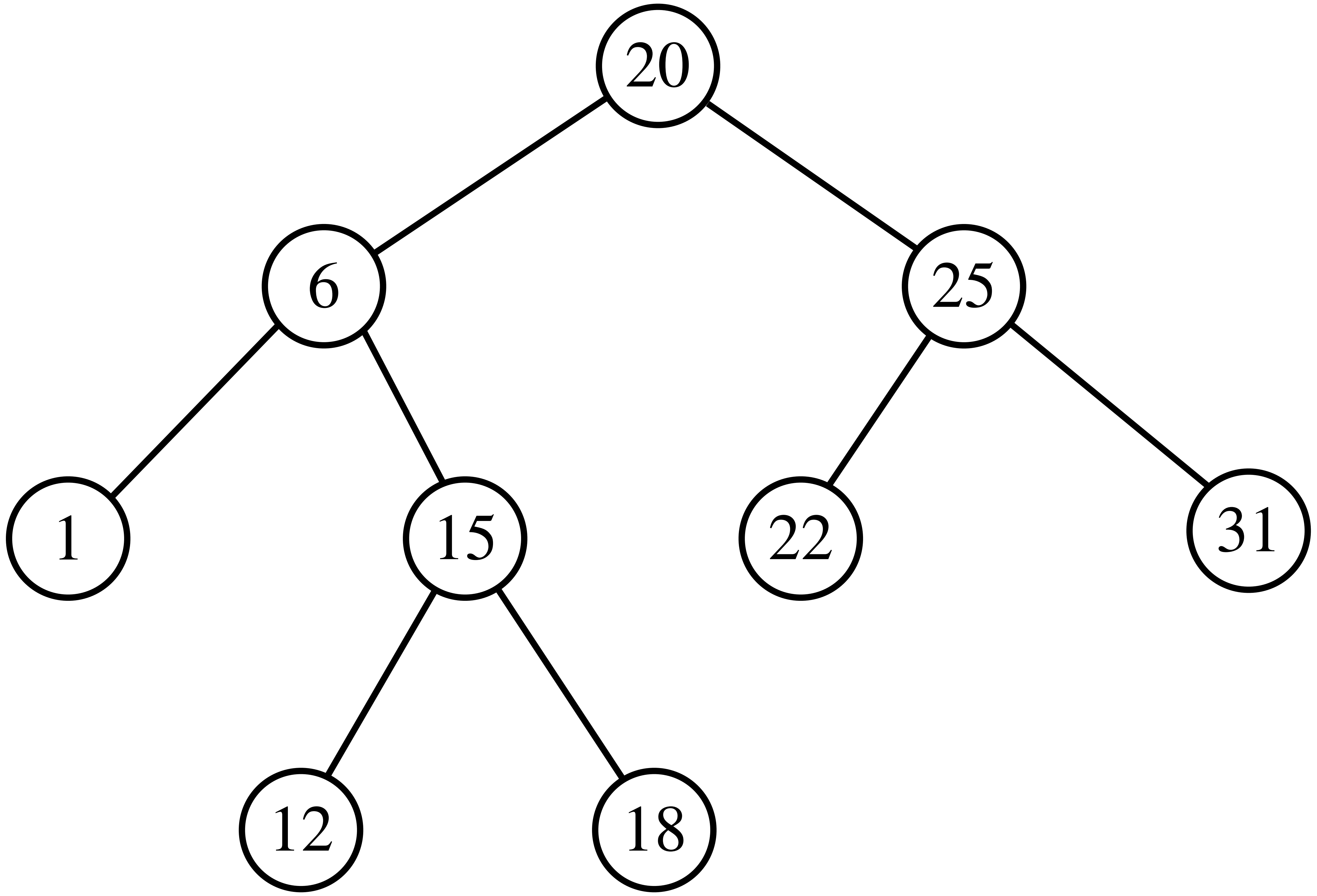

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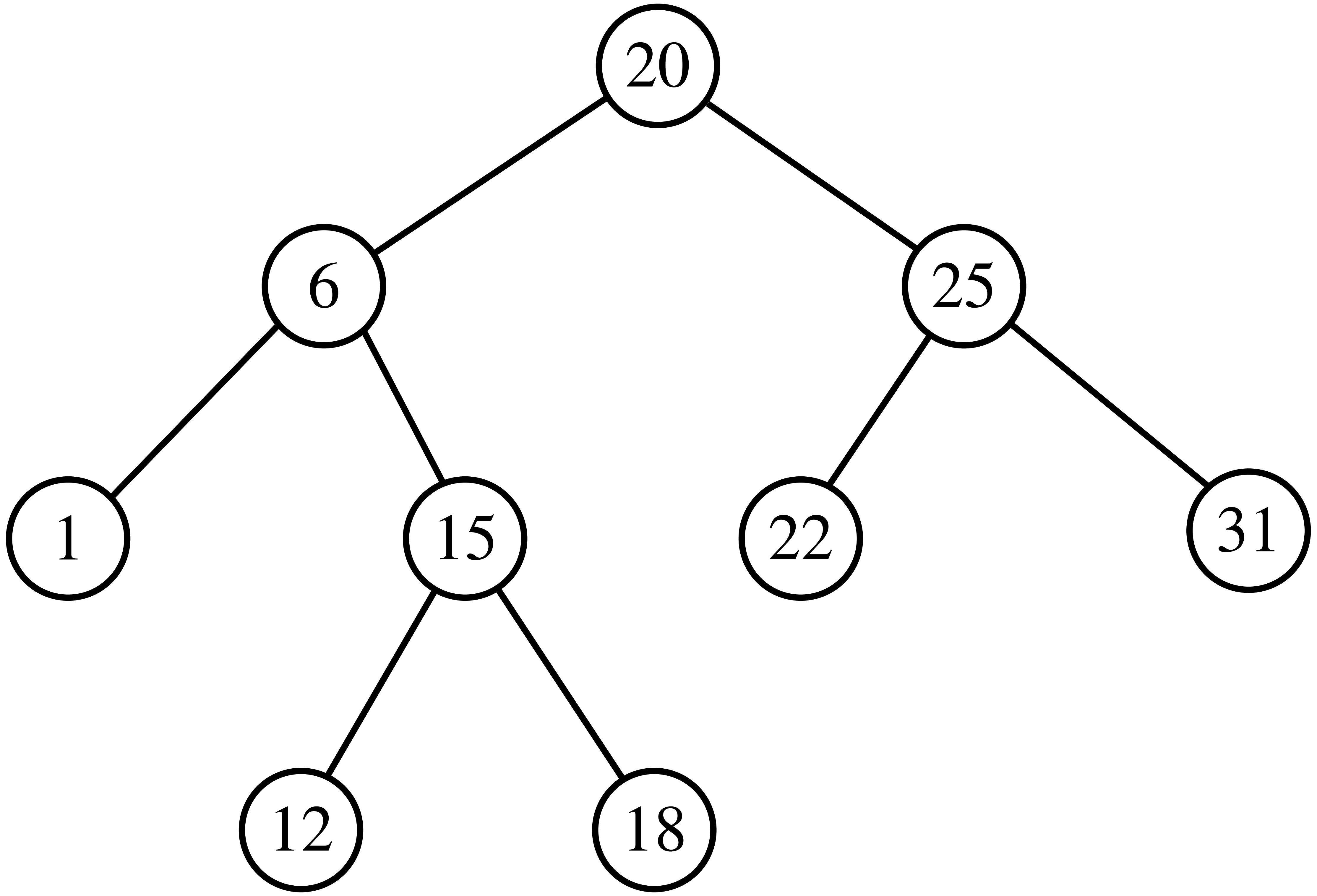
Example: Insert a node with 24 as key in the following BST.

Idea: Find the correct leaf where it can be inserted.



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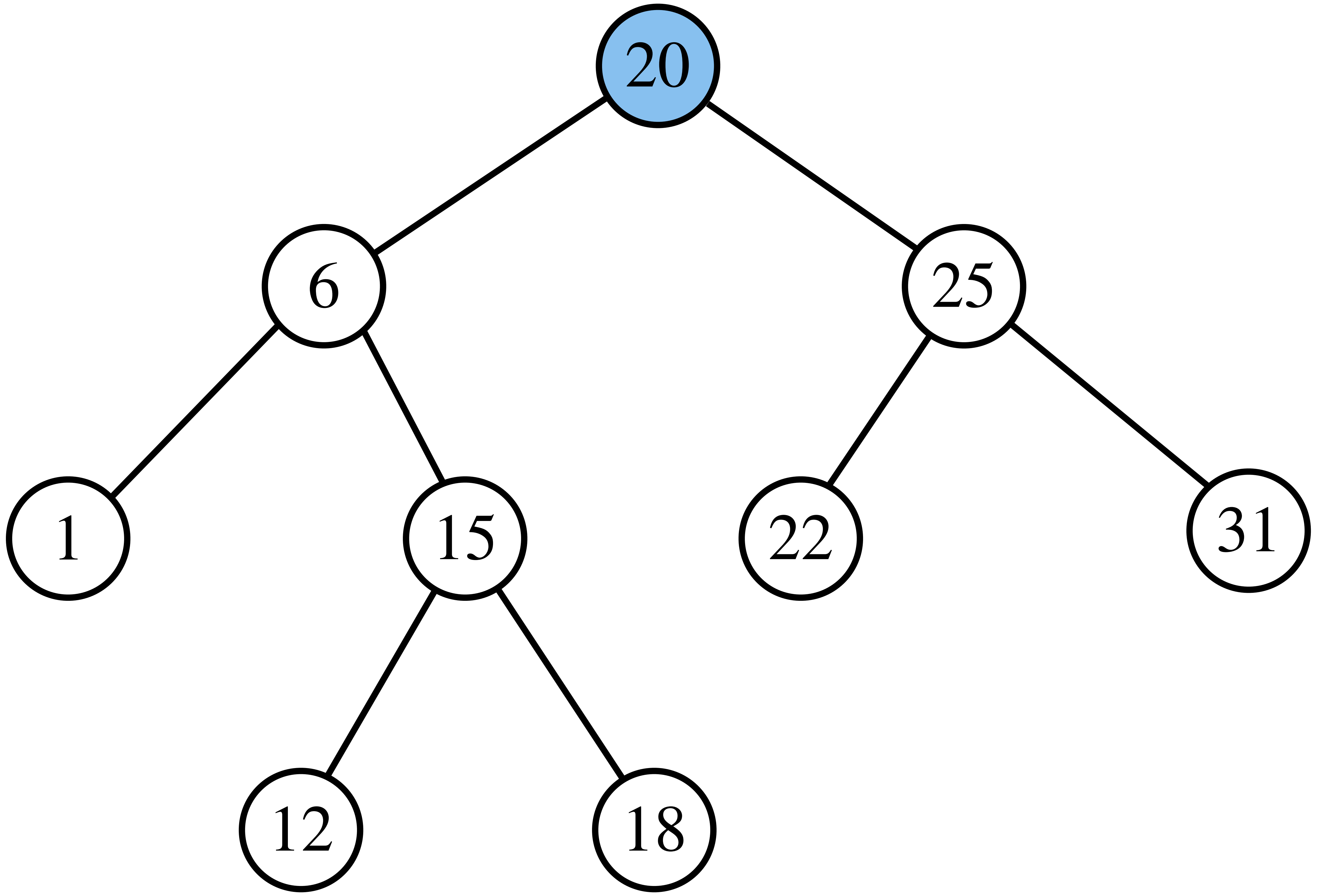

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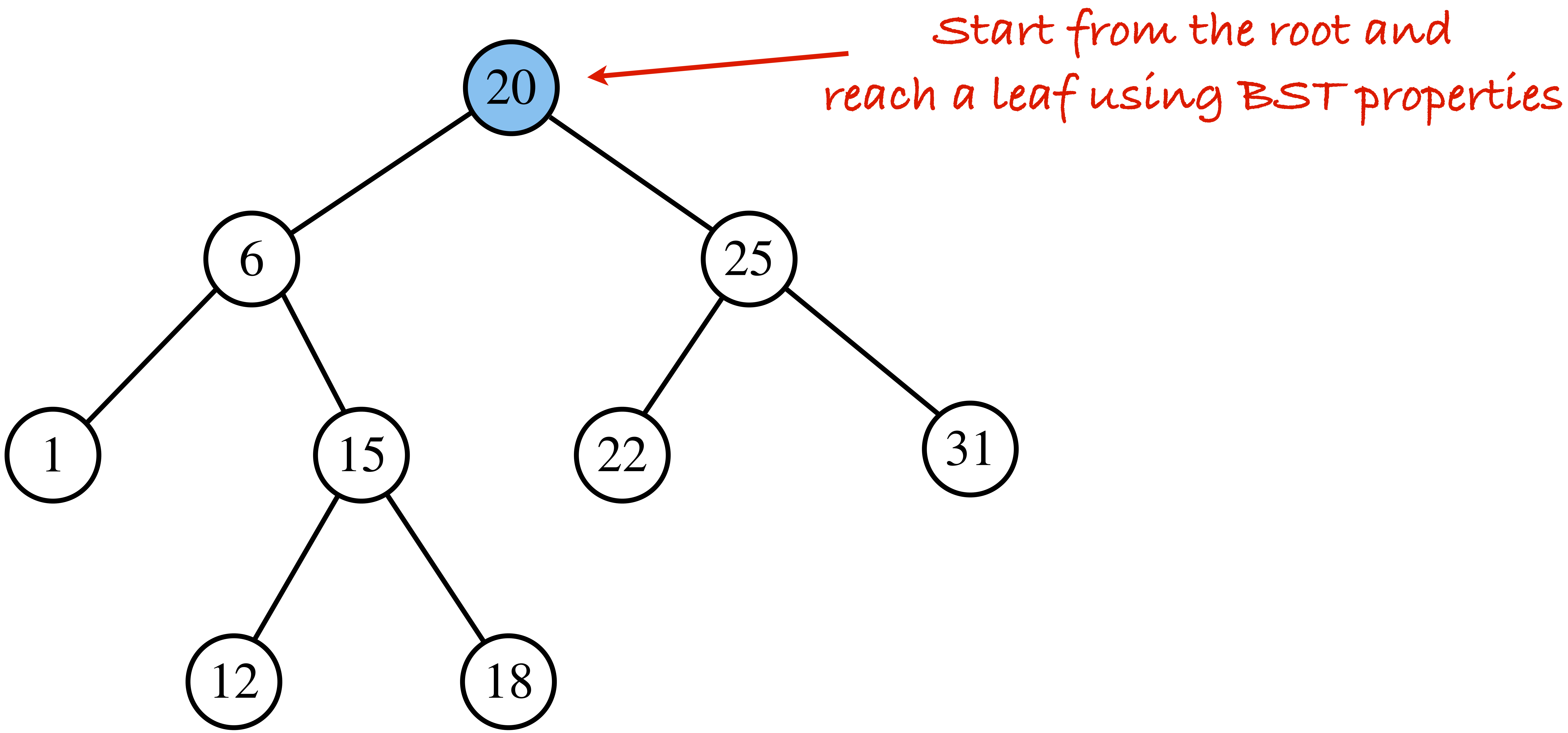

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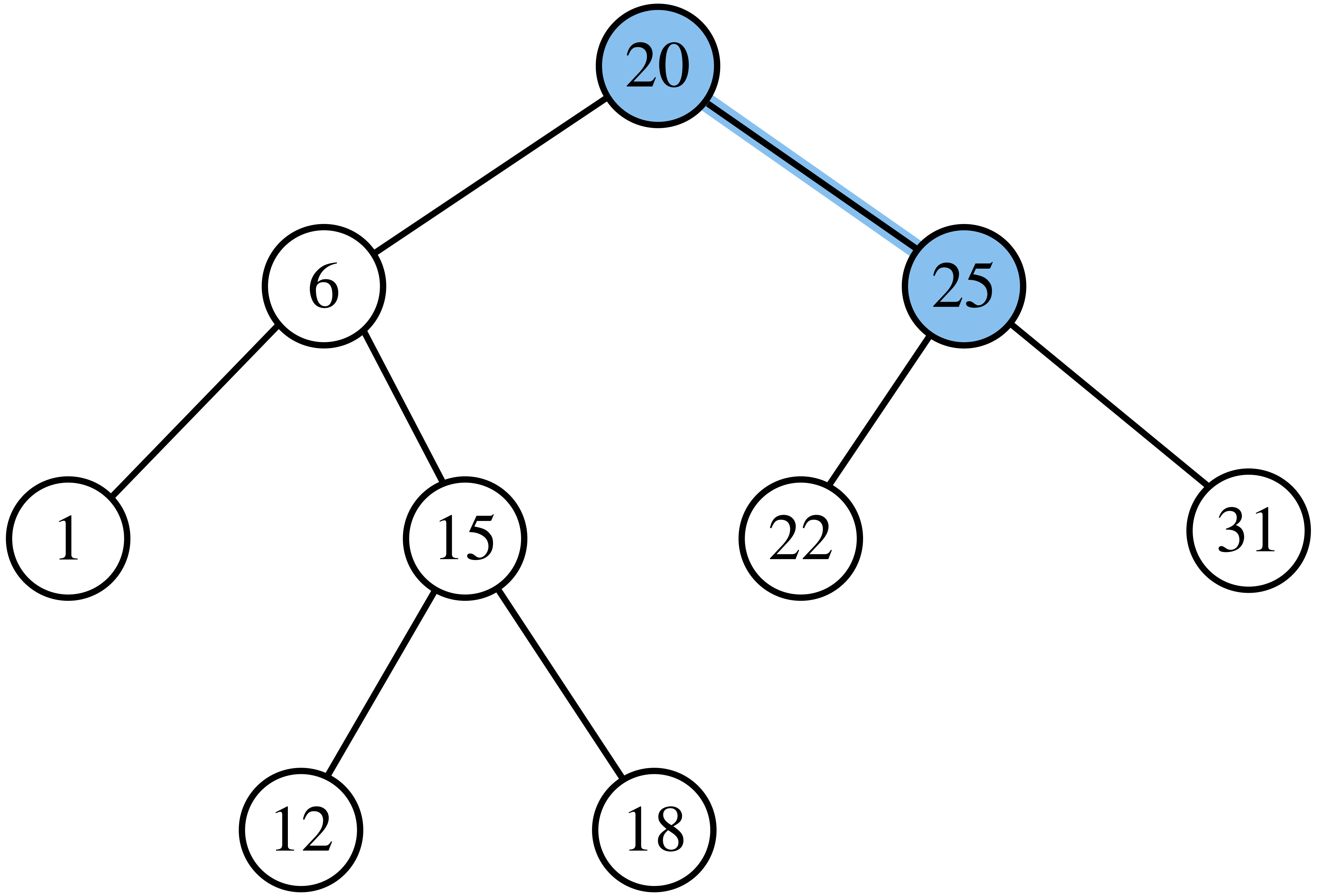

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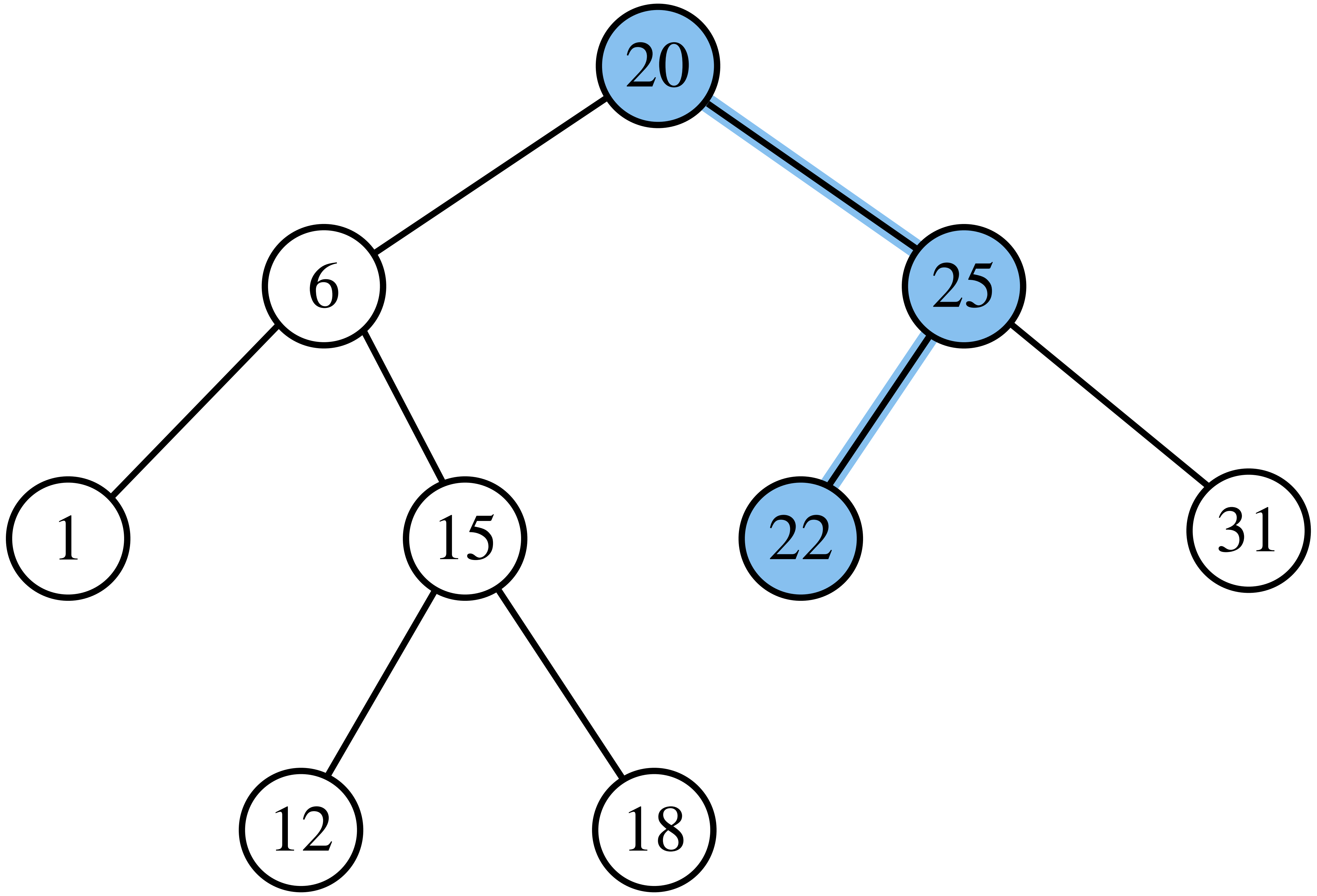

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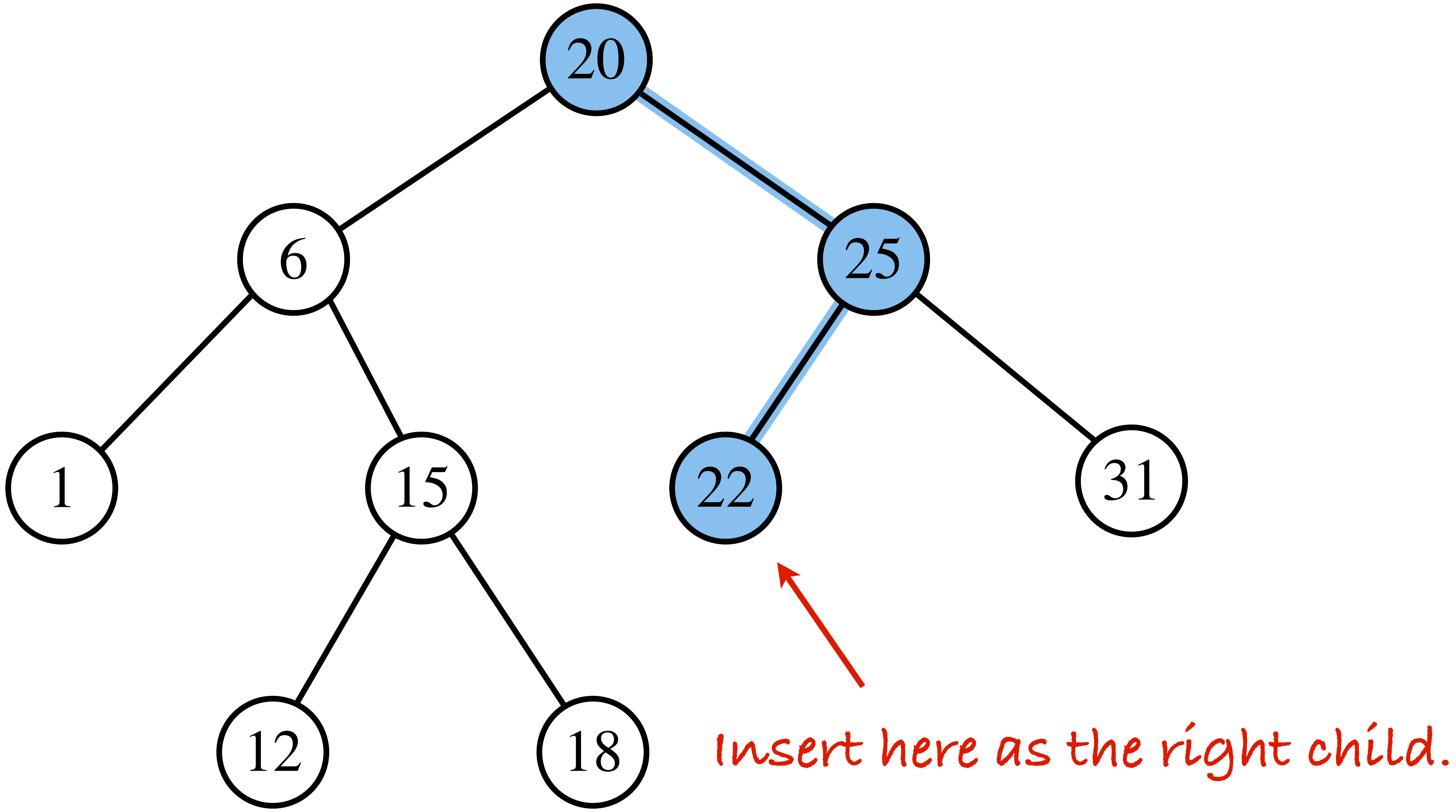

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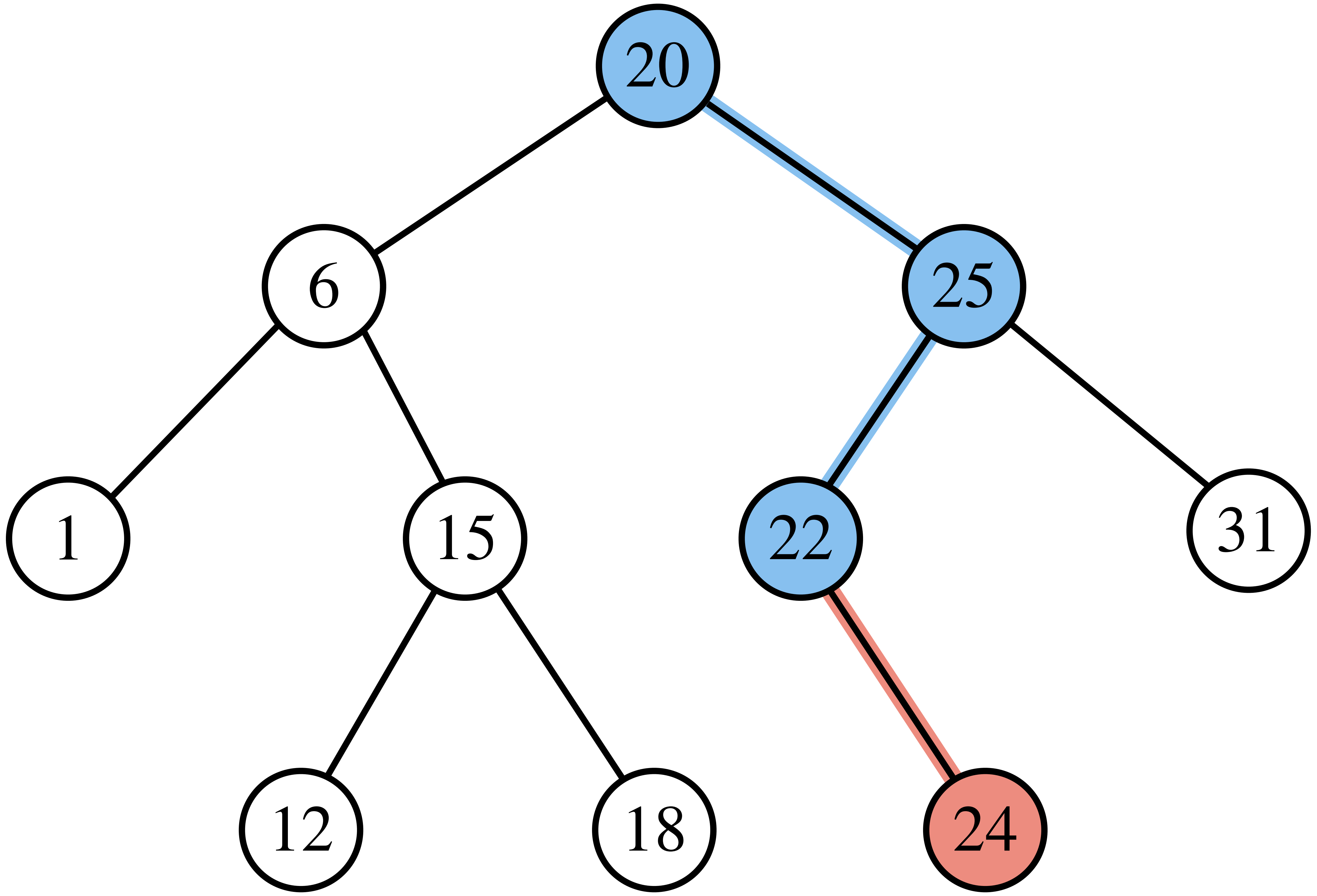

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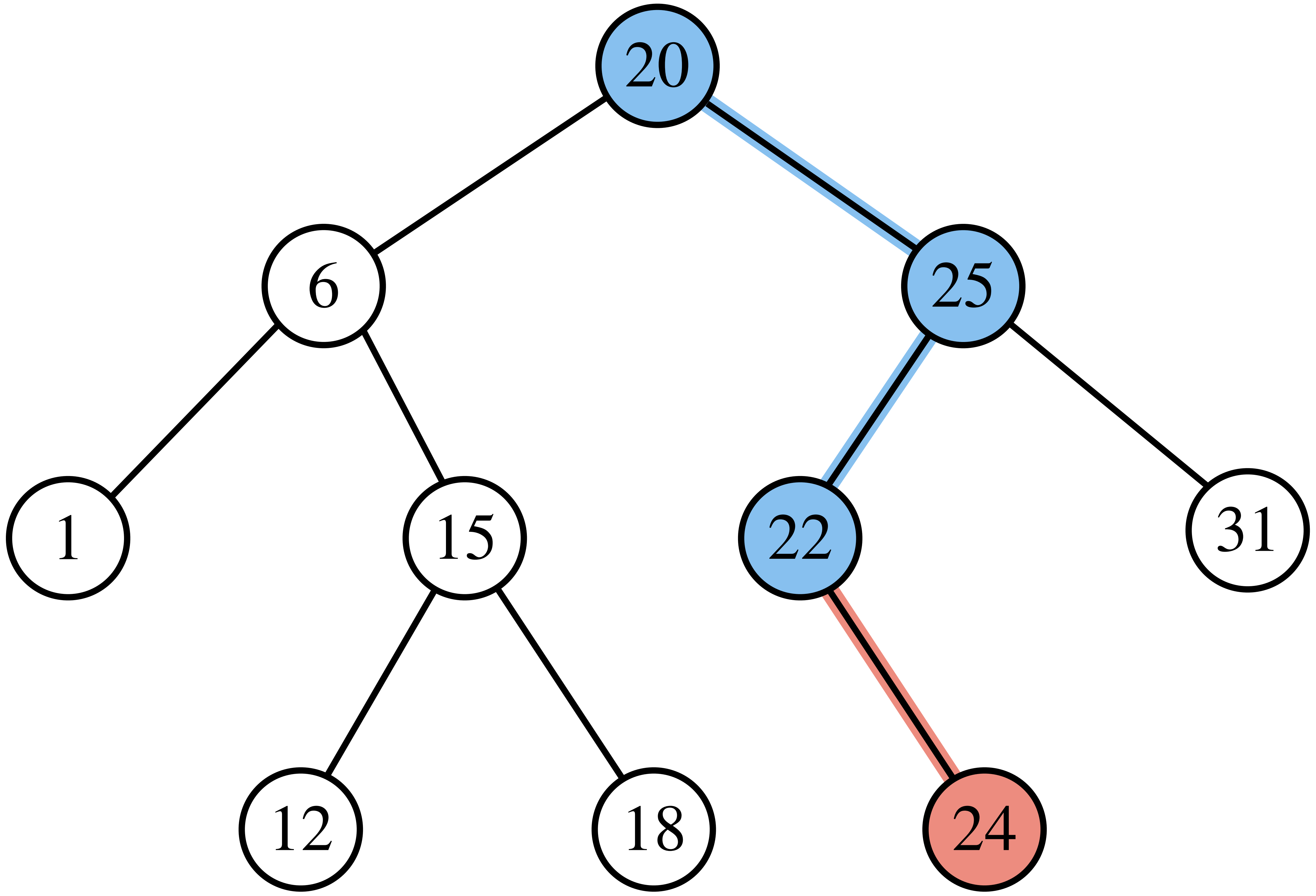

Searching for the correct leaf and insertion takes $O(h)$ time.

Modifying a BST: Deletion

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- **Case** 2: z has only single child.

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- **Case** 3: z has two children.

Tricky

Note: Node z is provided as input.

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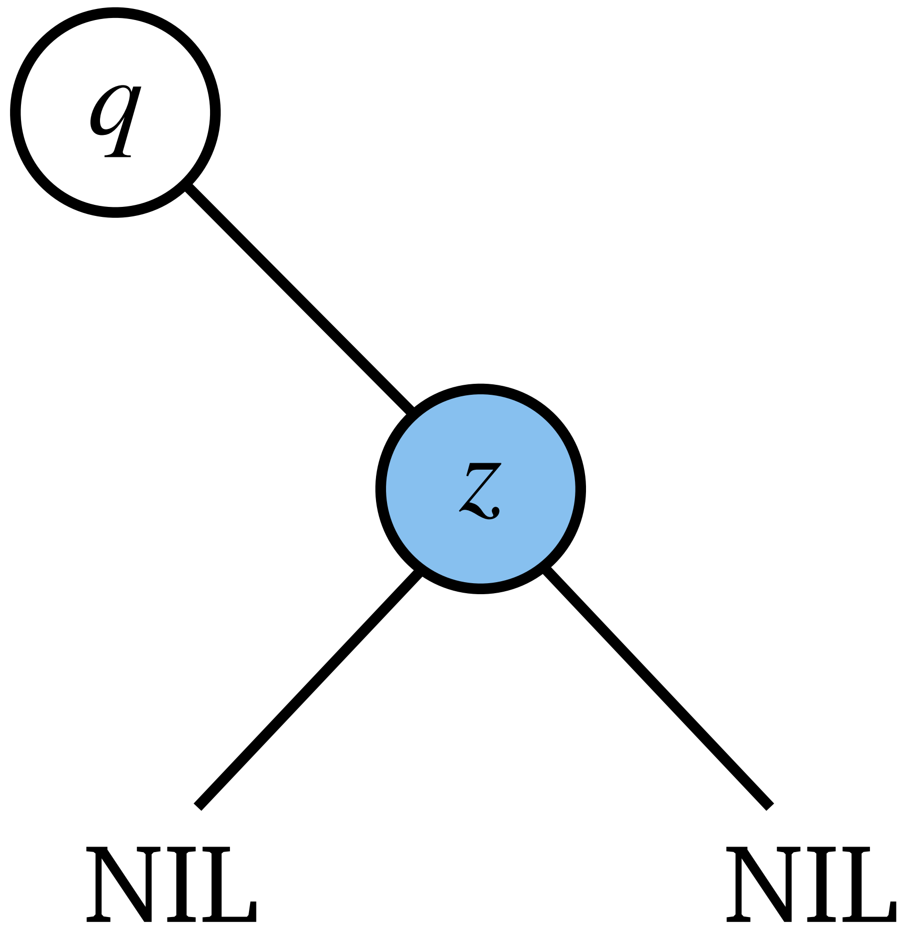

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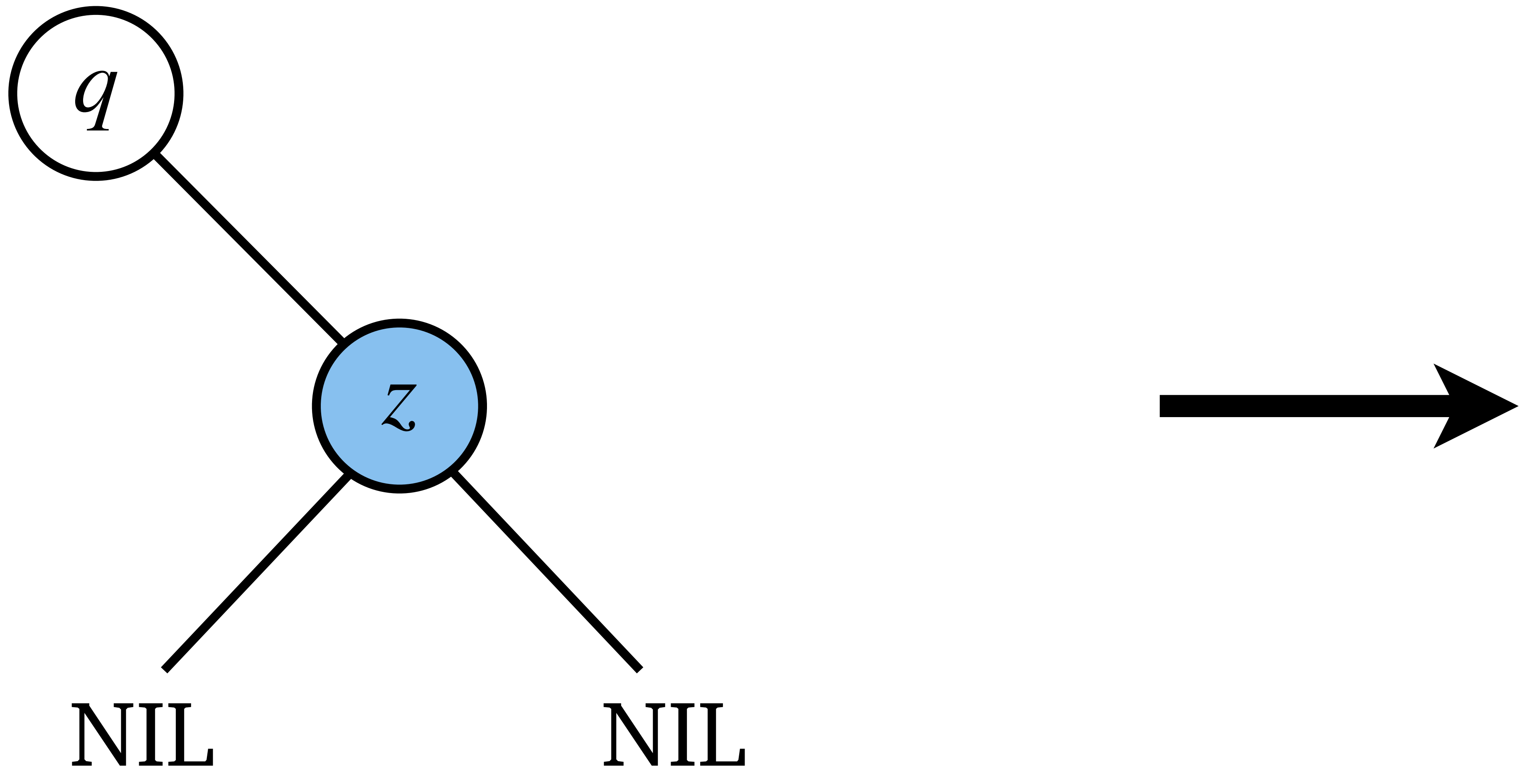

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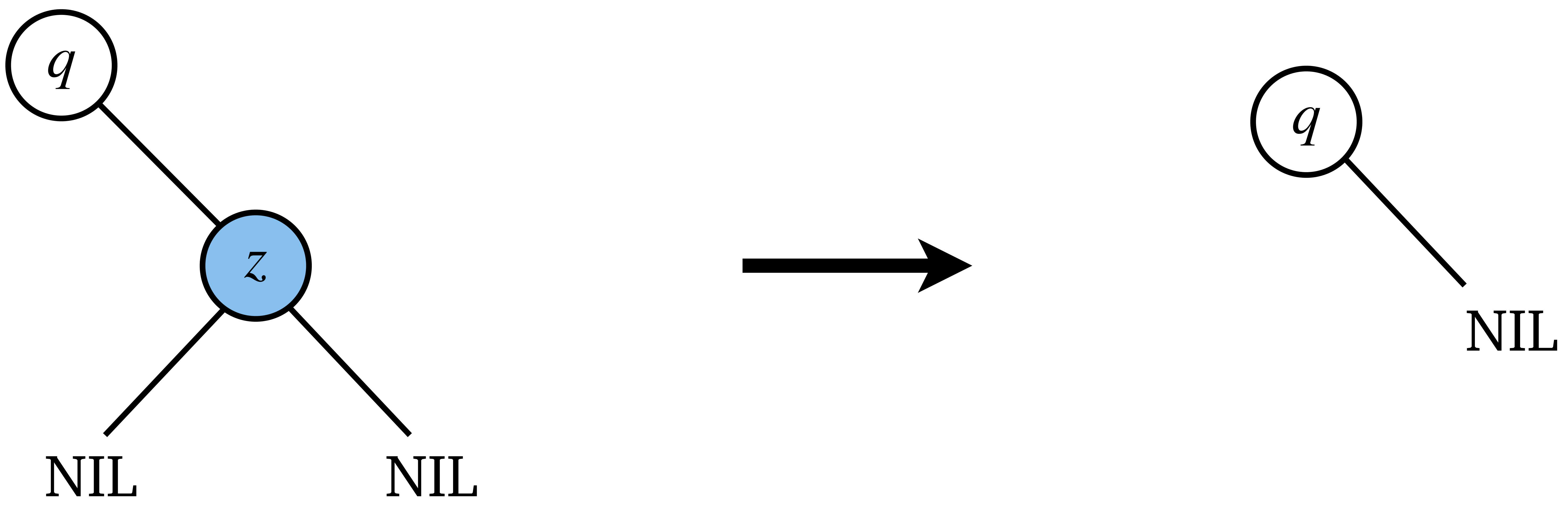

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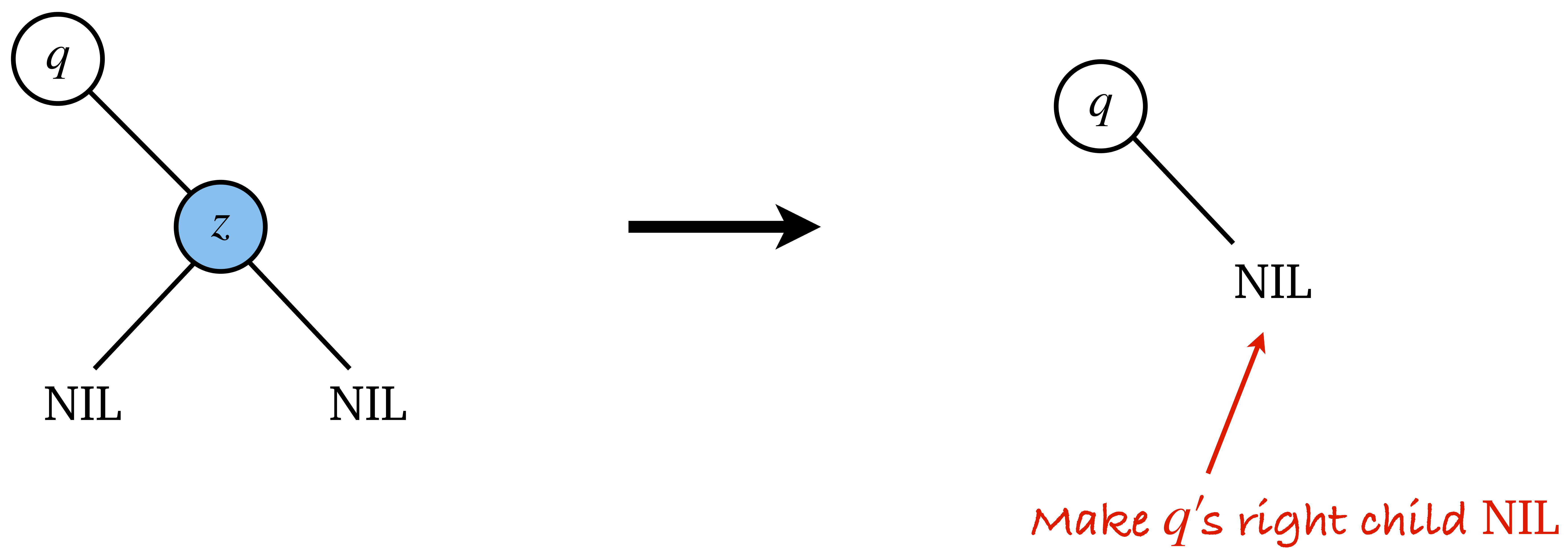

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Case 2: z has one child. (WLOG assume z is a right child.)

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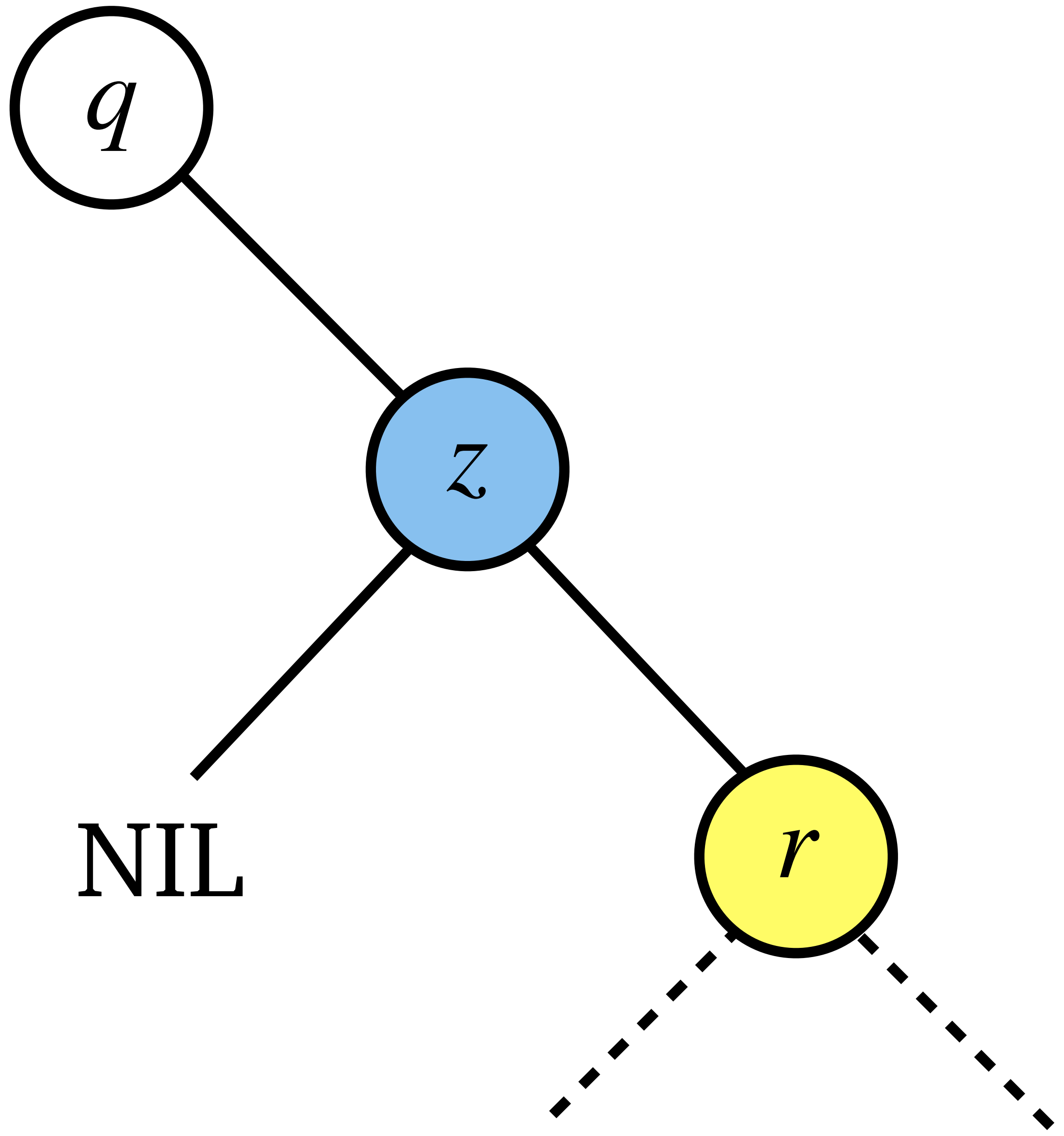

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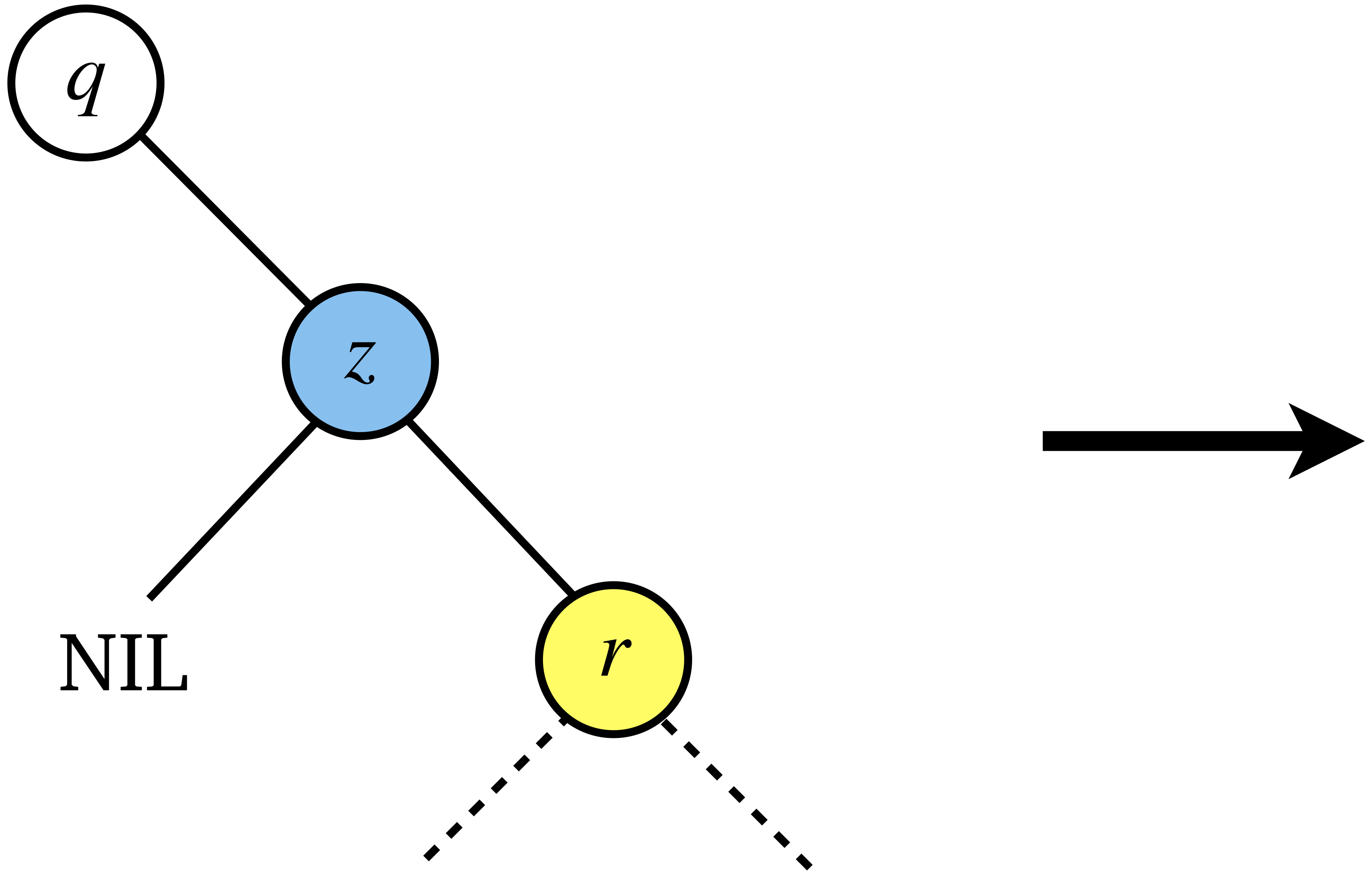

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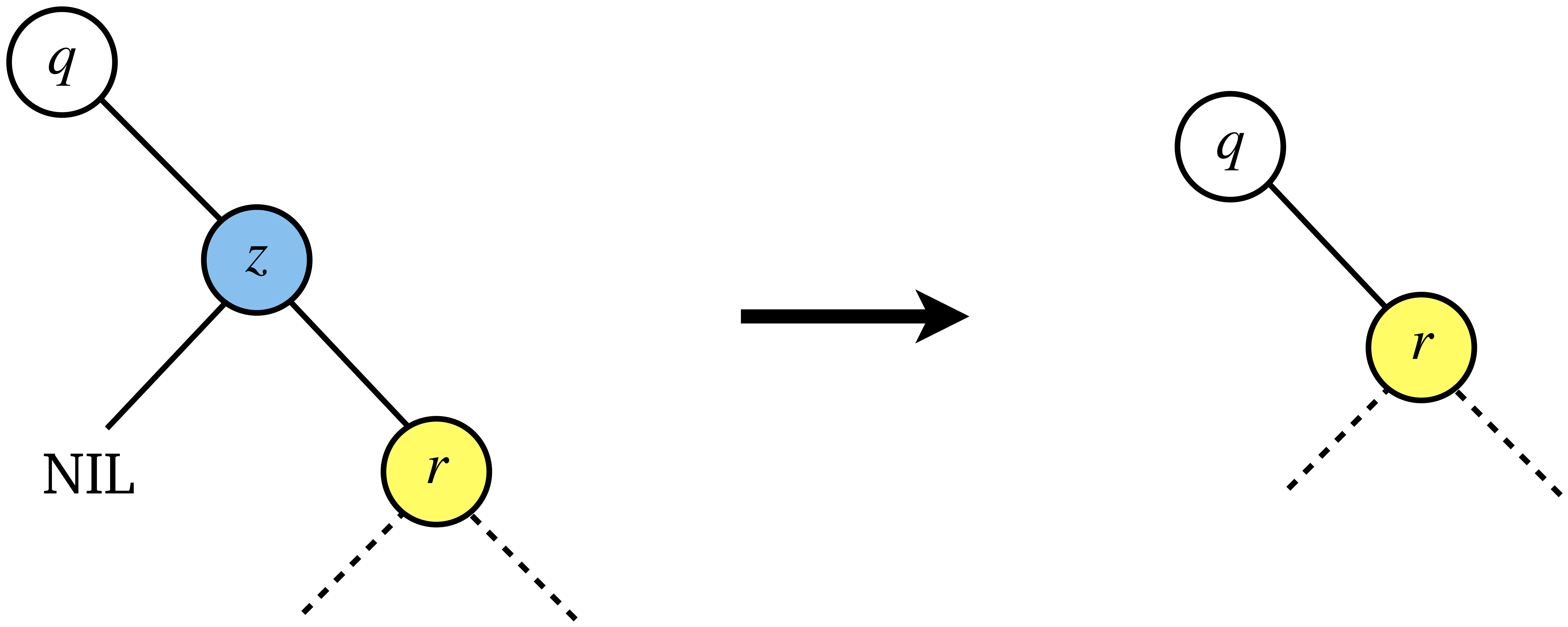

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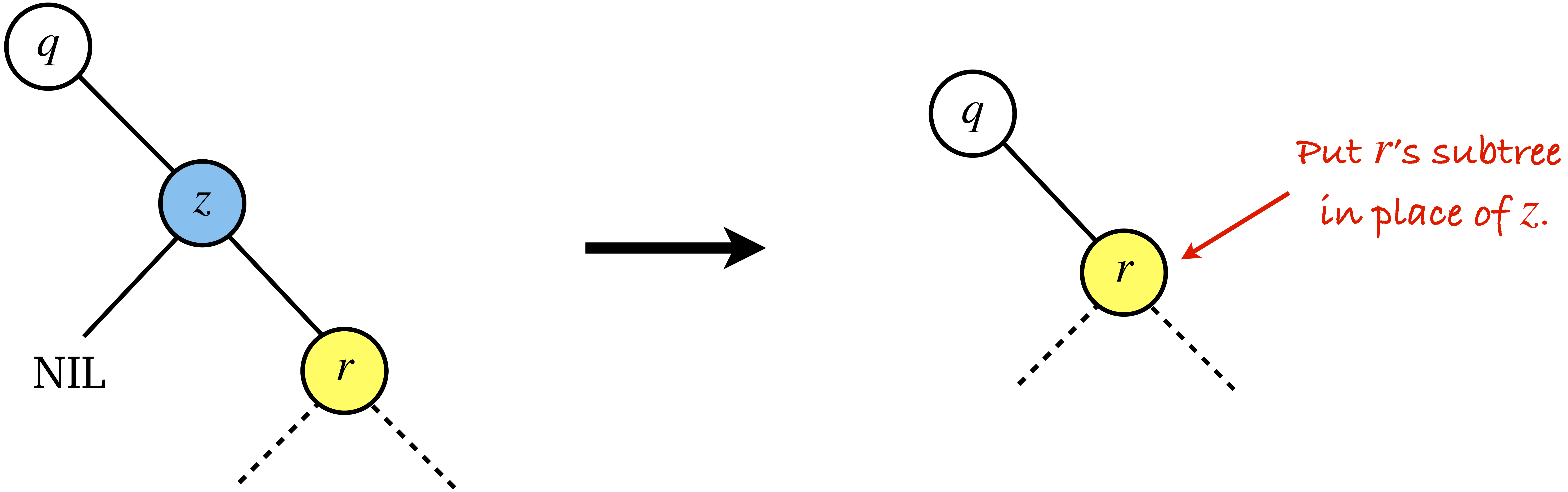

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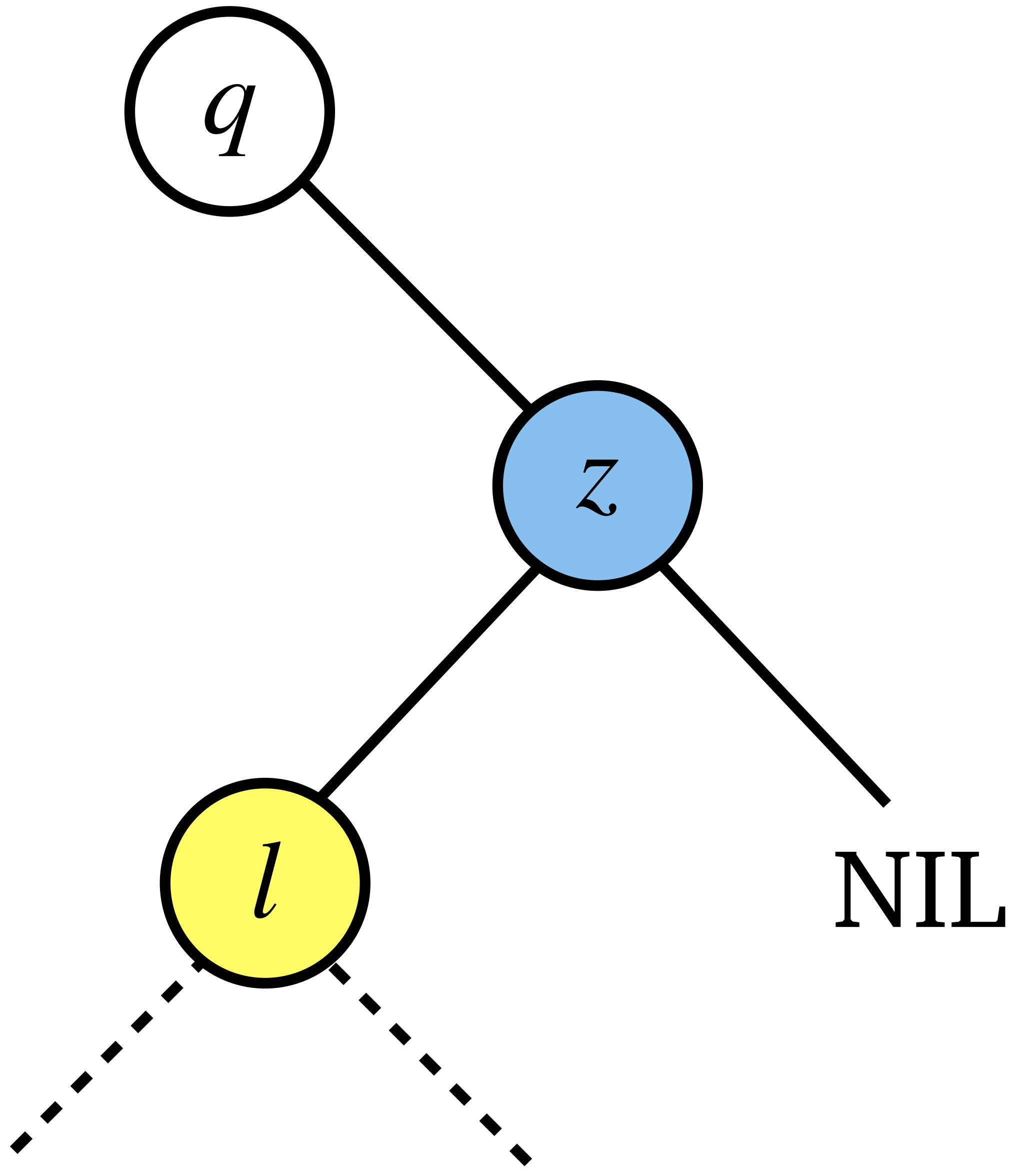

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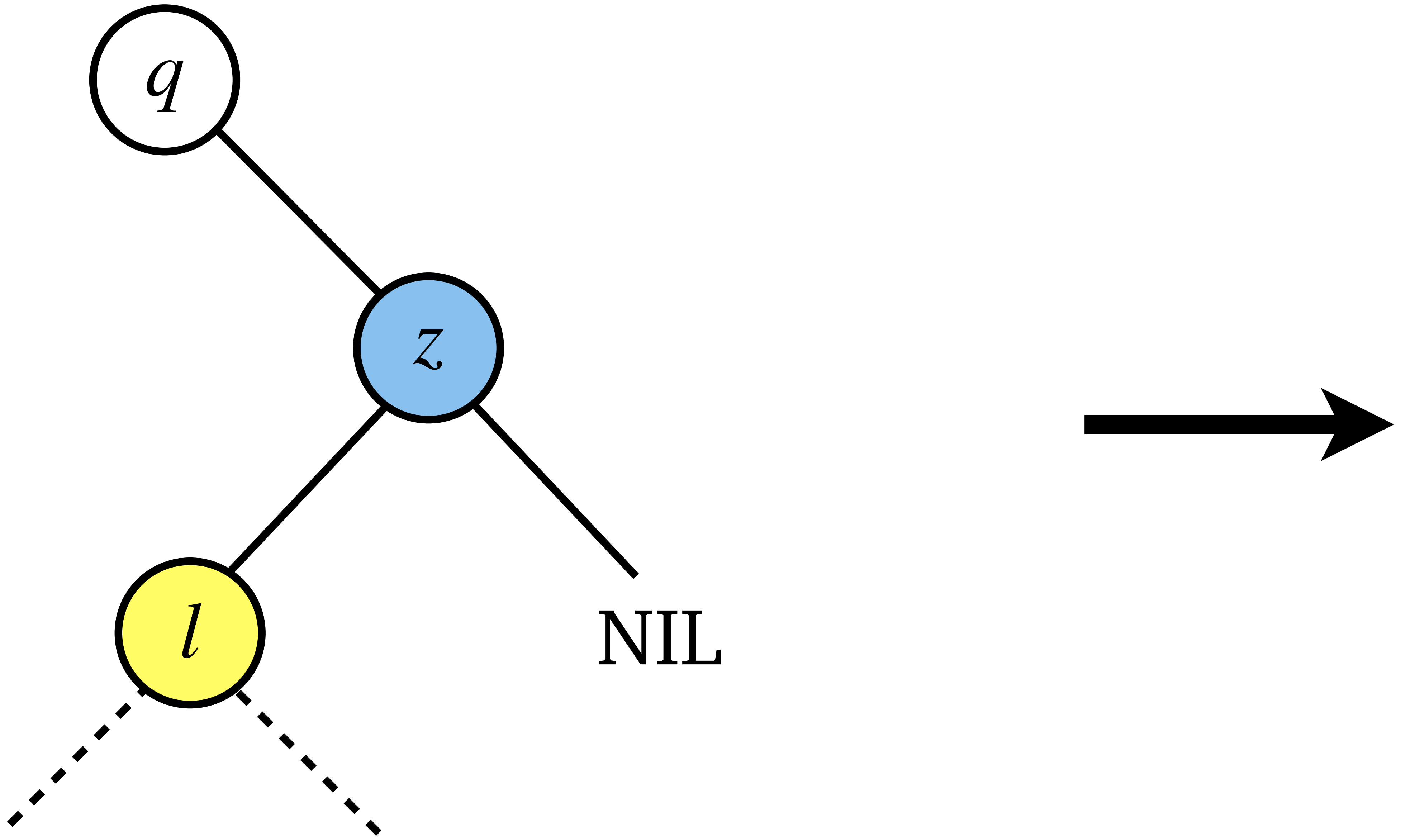

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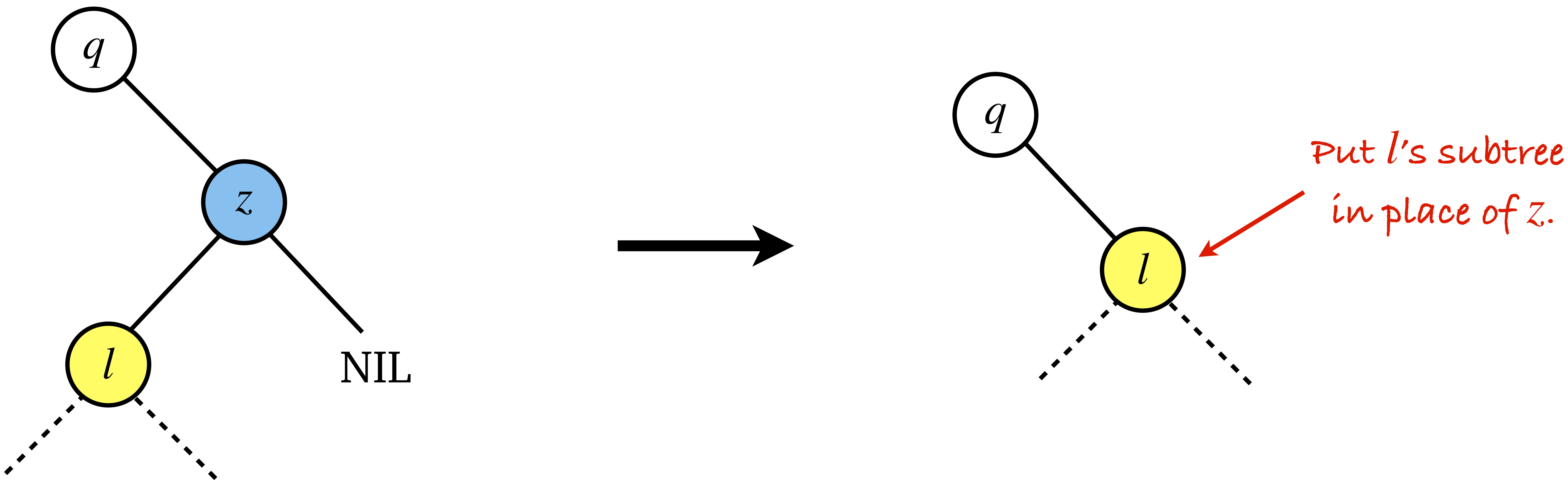

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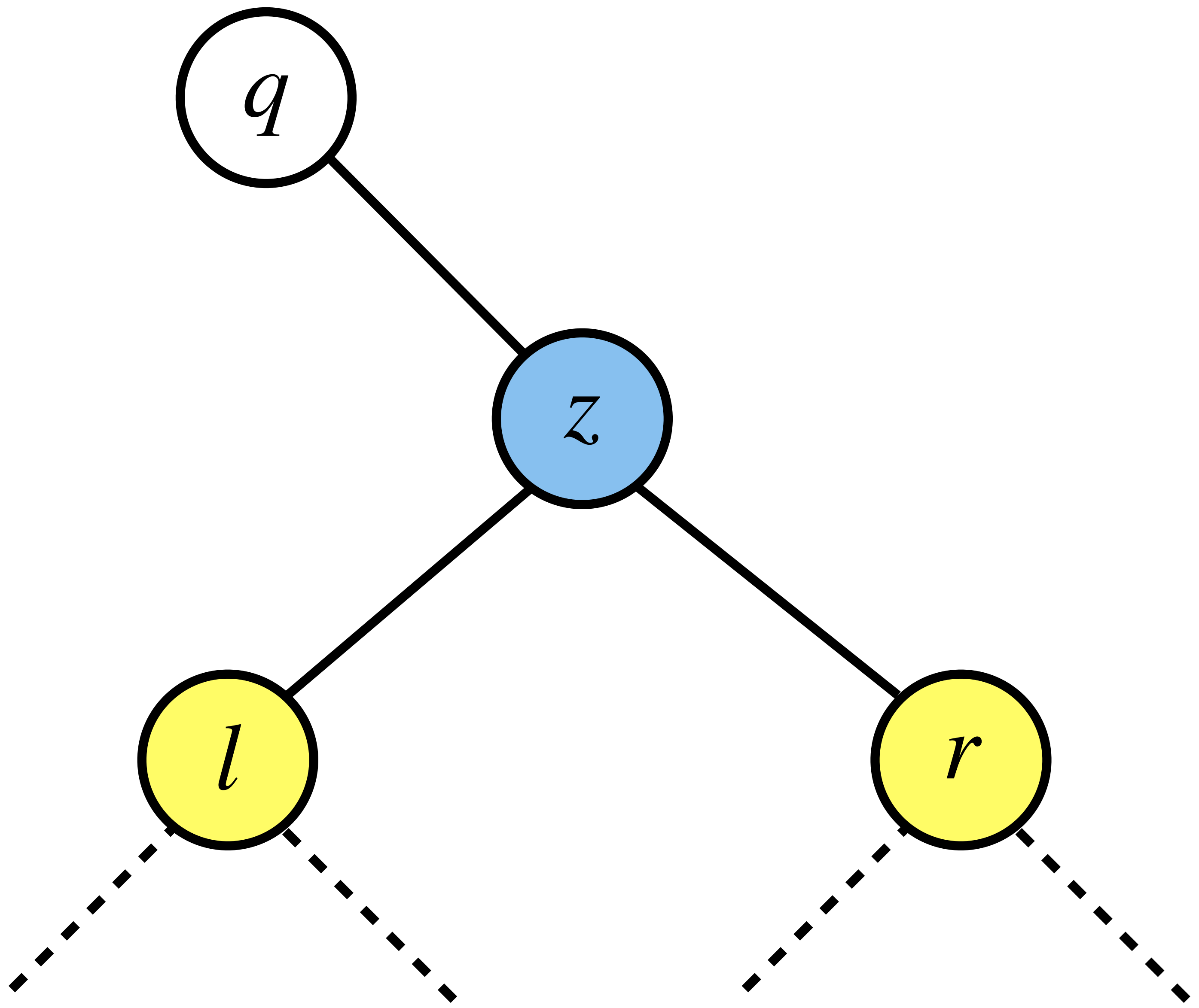

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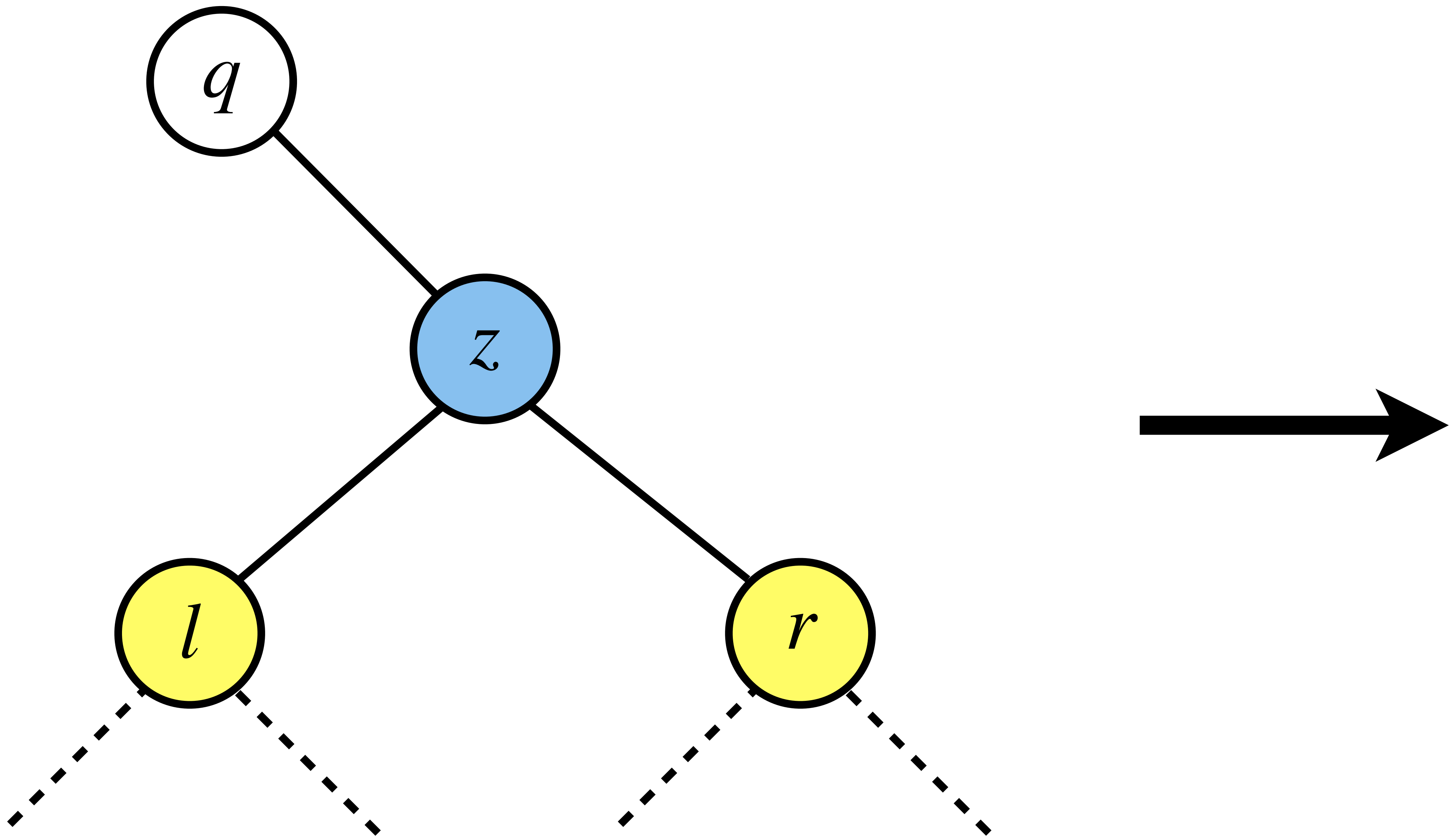

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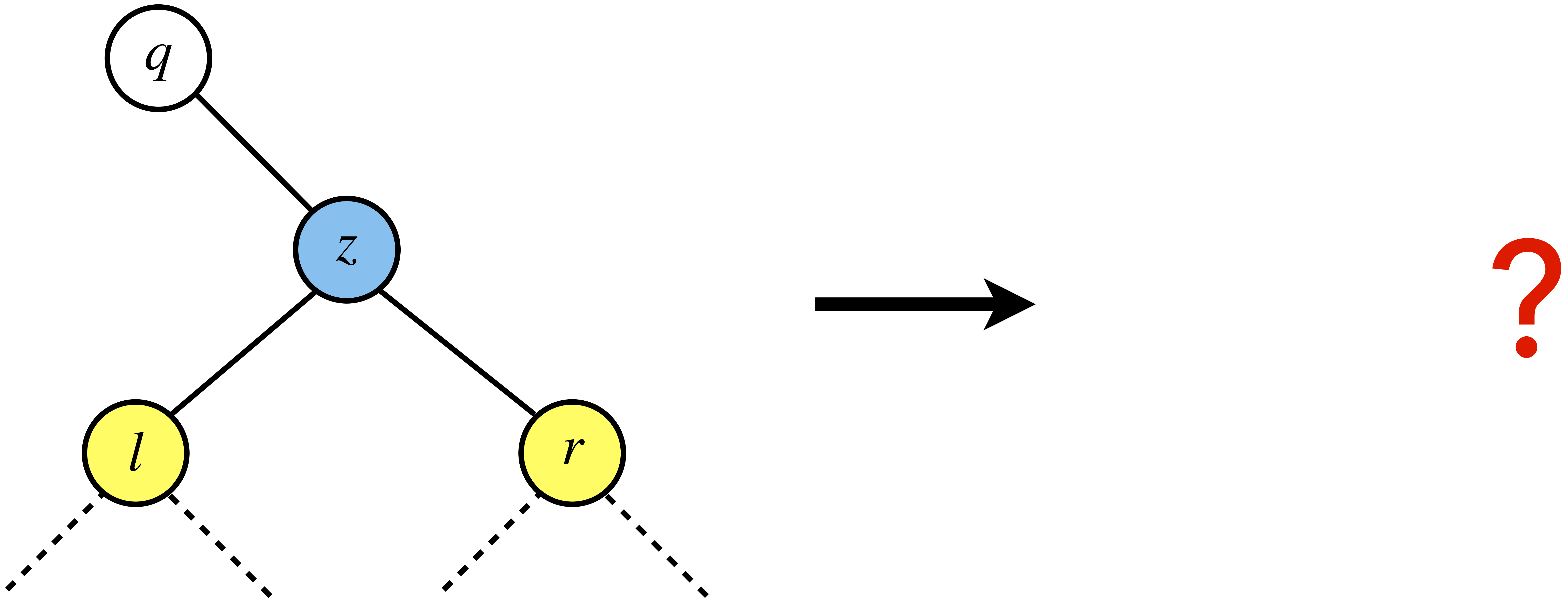

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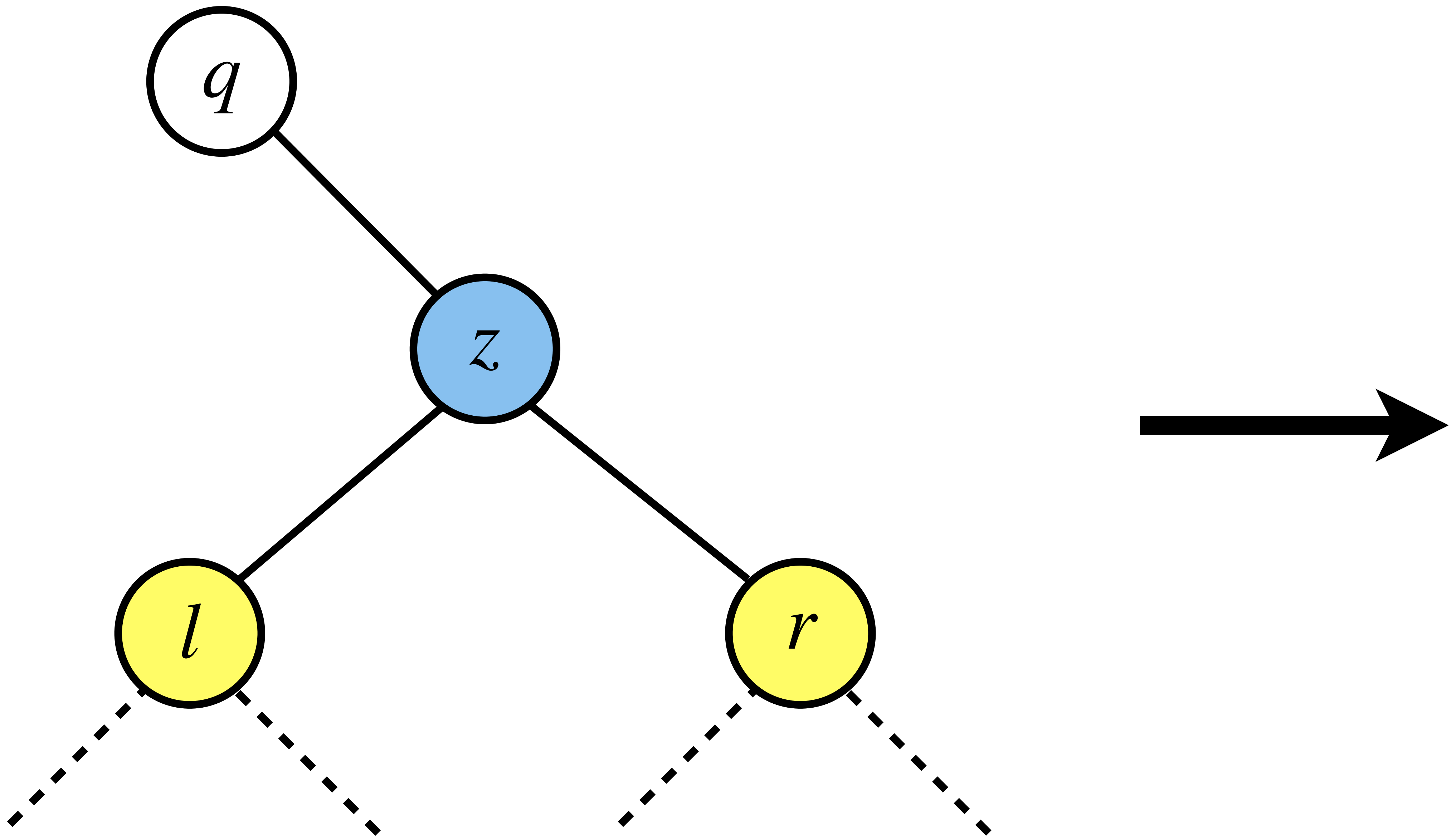

Two sub-cases:

- r has no left child.
- r has a left child.

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Case 3a: z has two children where its right child has no left child.

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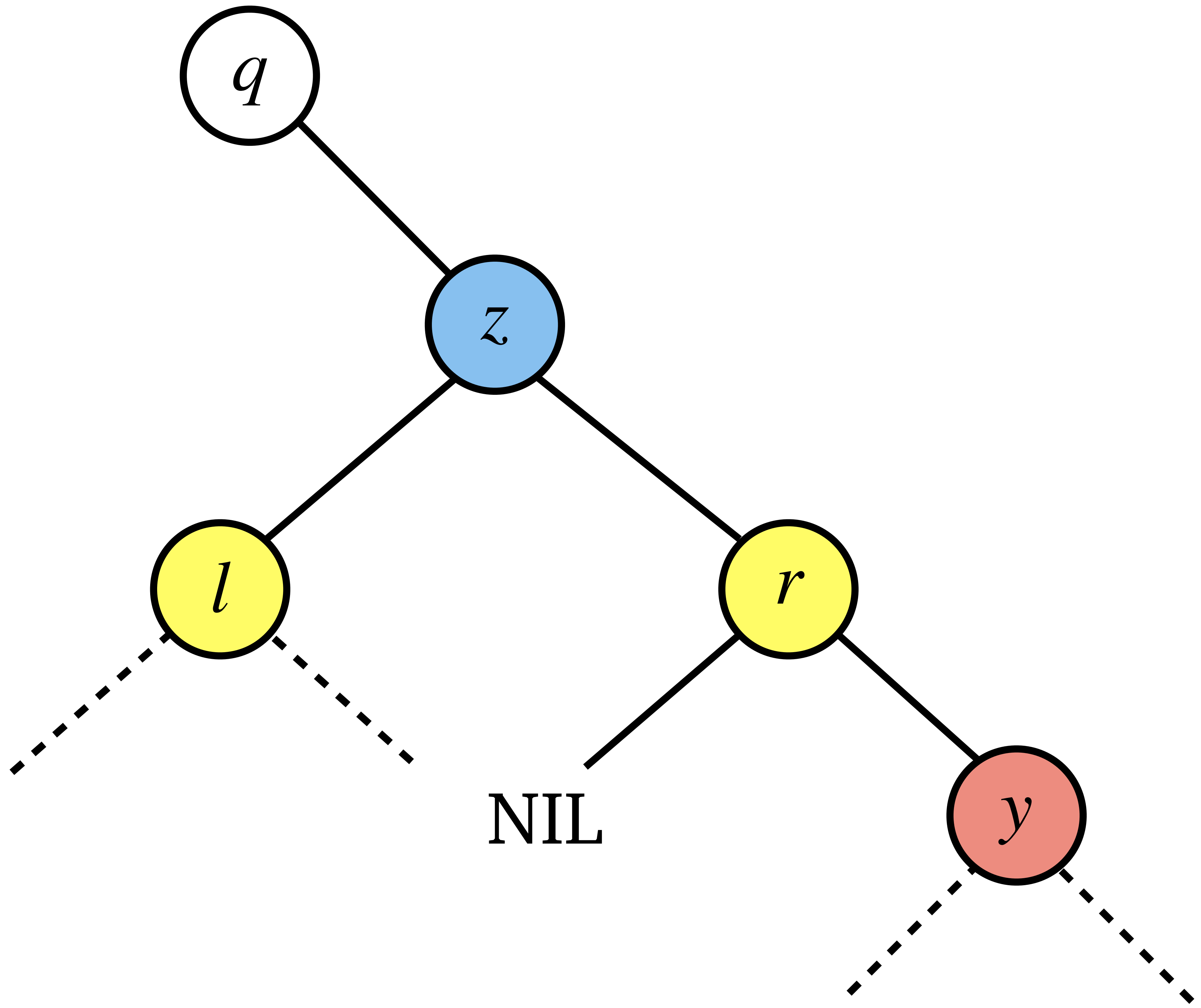
Case 3**a**: z has two children where its right child has no left child.



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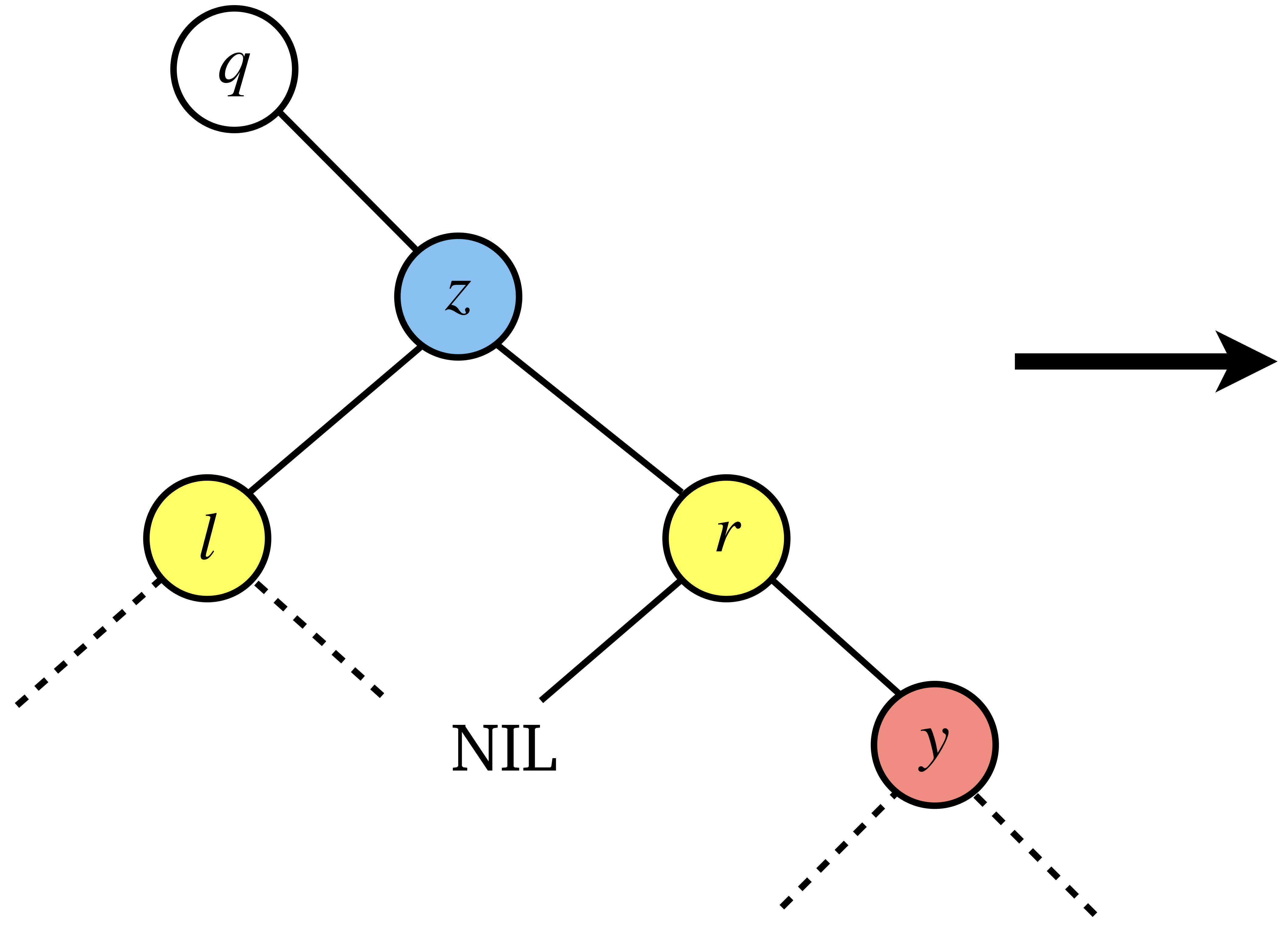

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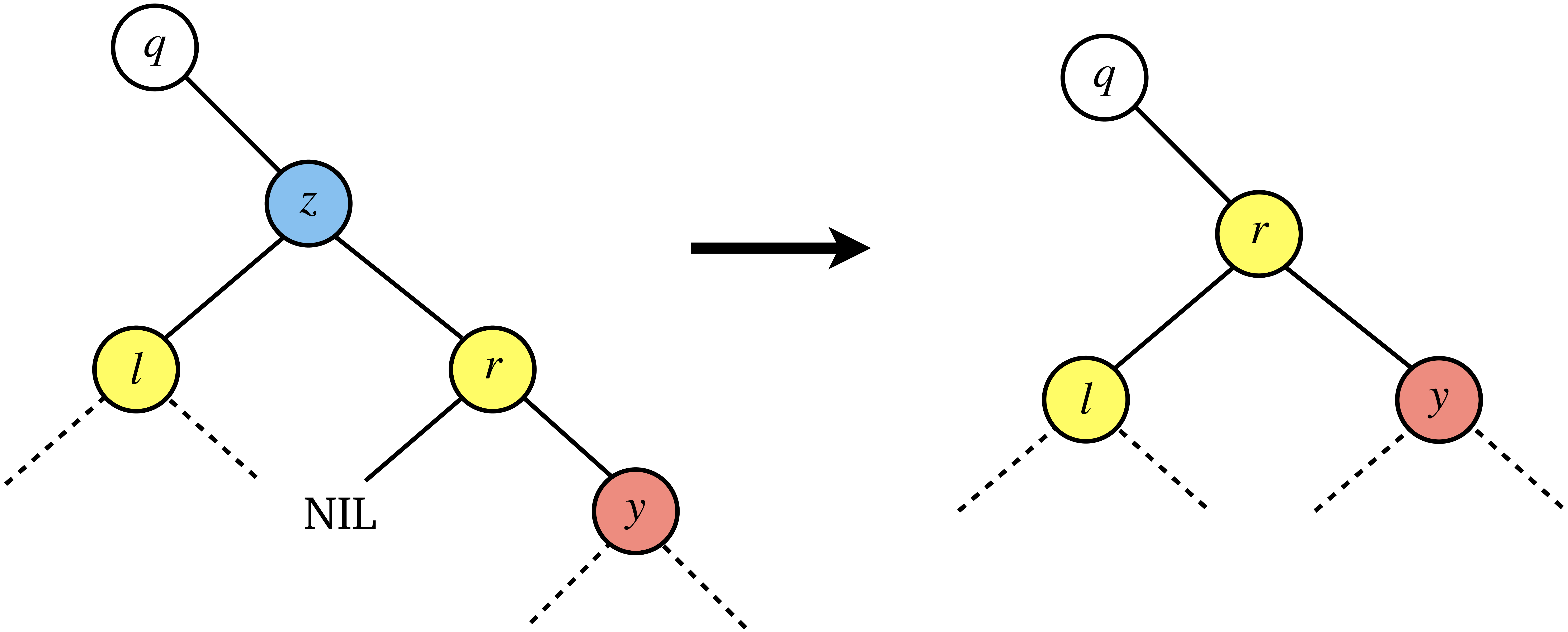

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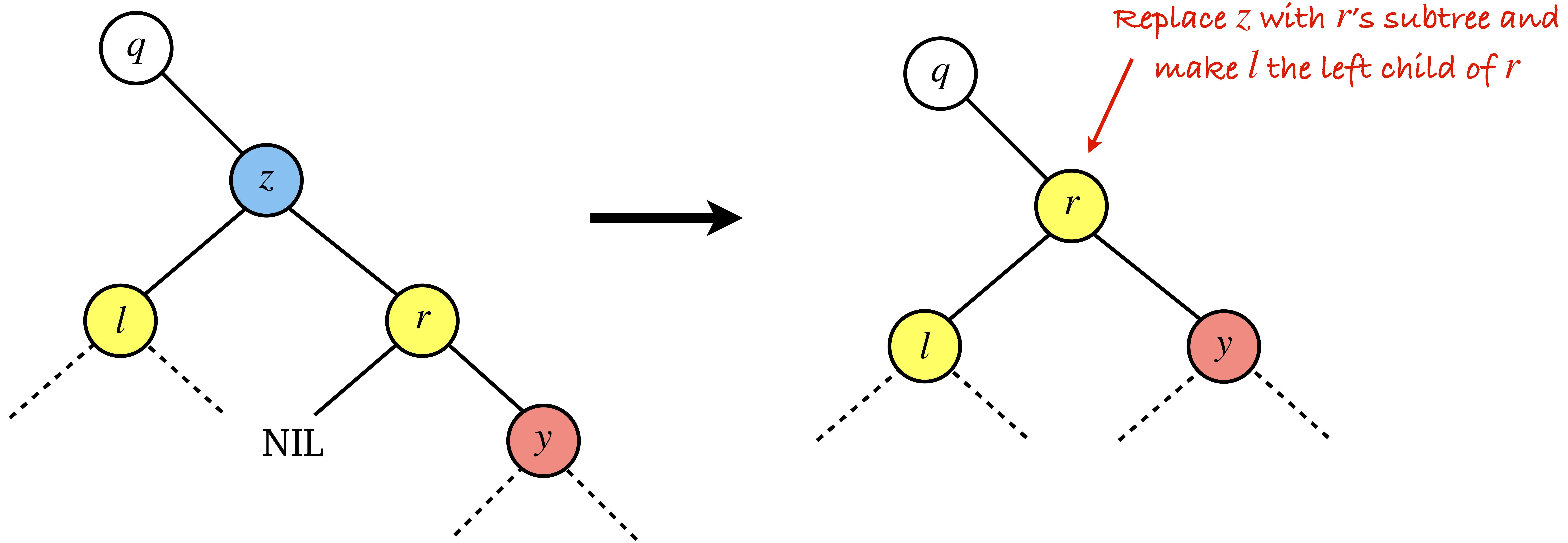

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Case 3b: z has two children where its right child has a left child.

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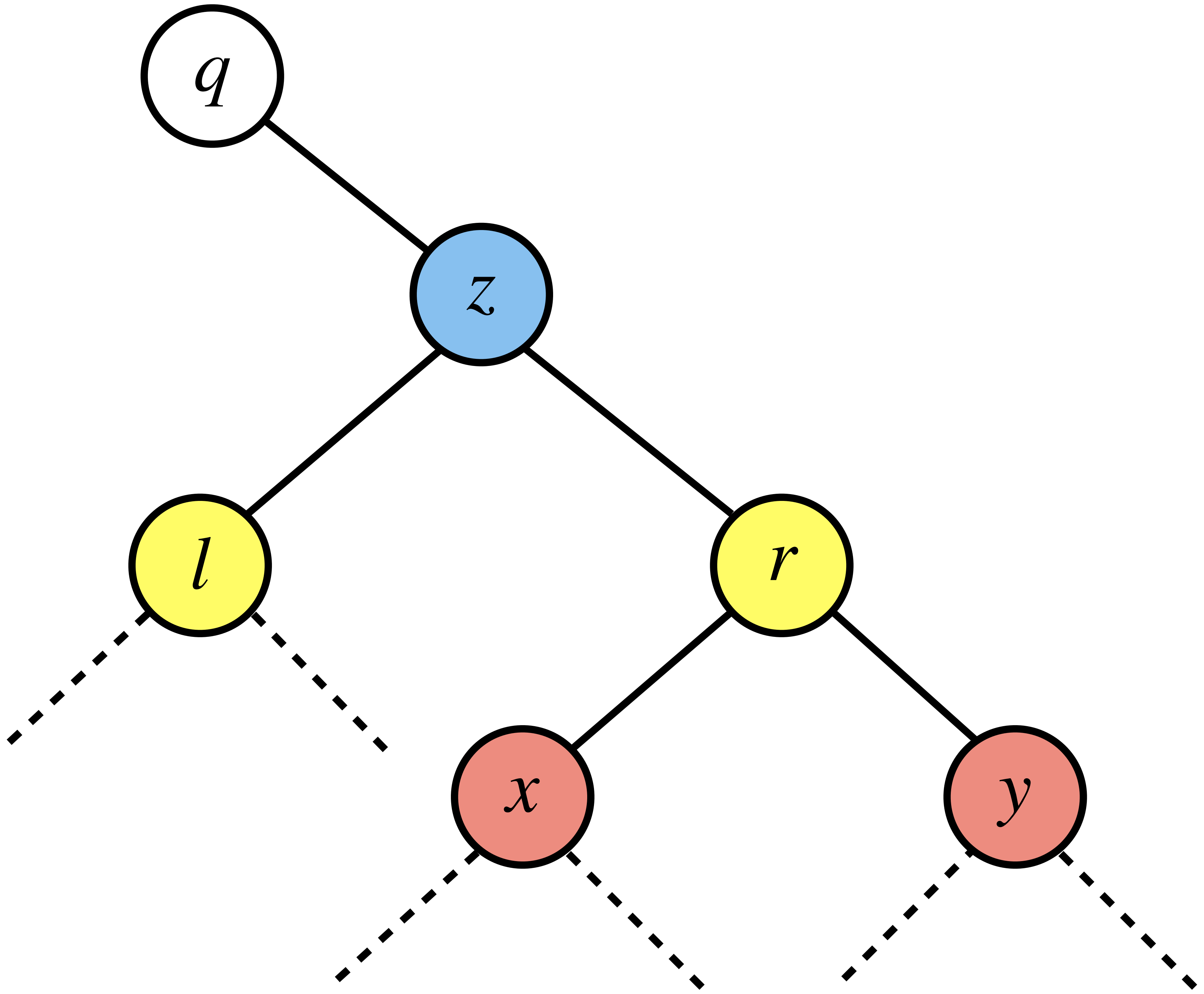

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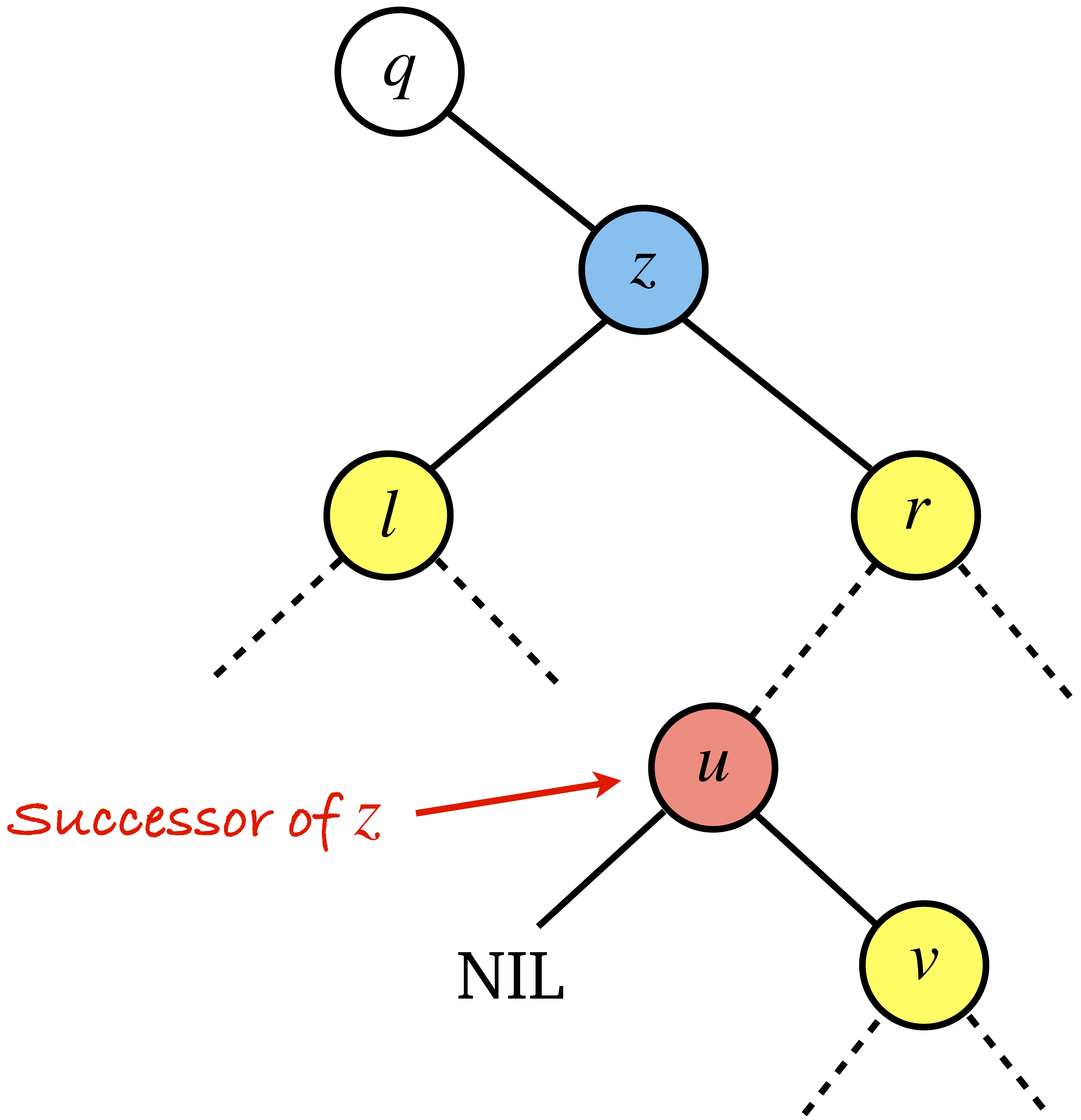
Case 3**b**: z has two children where its right child has a left child.



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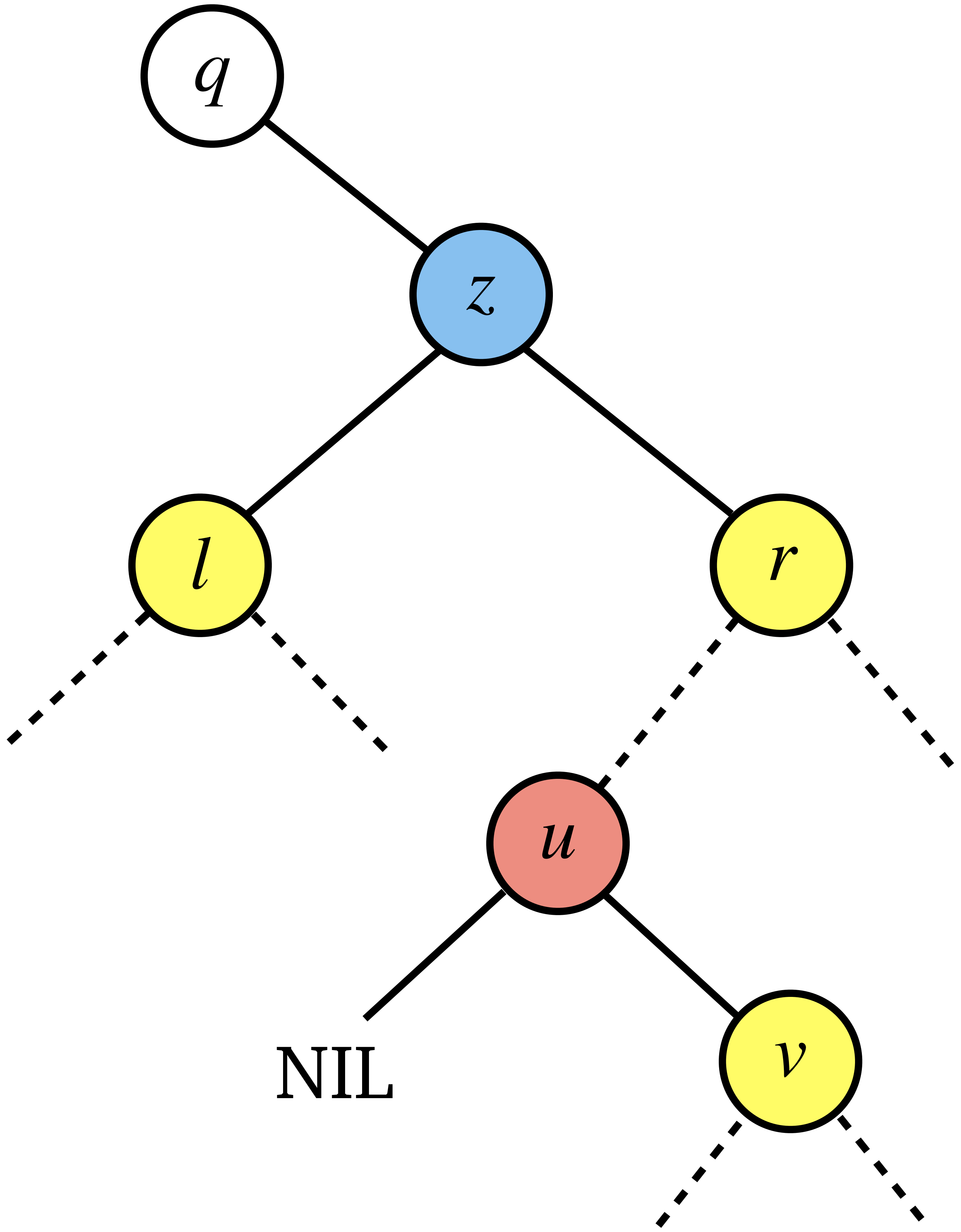

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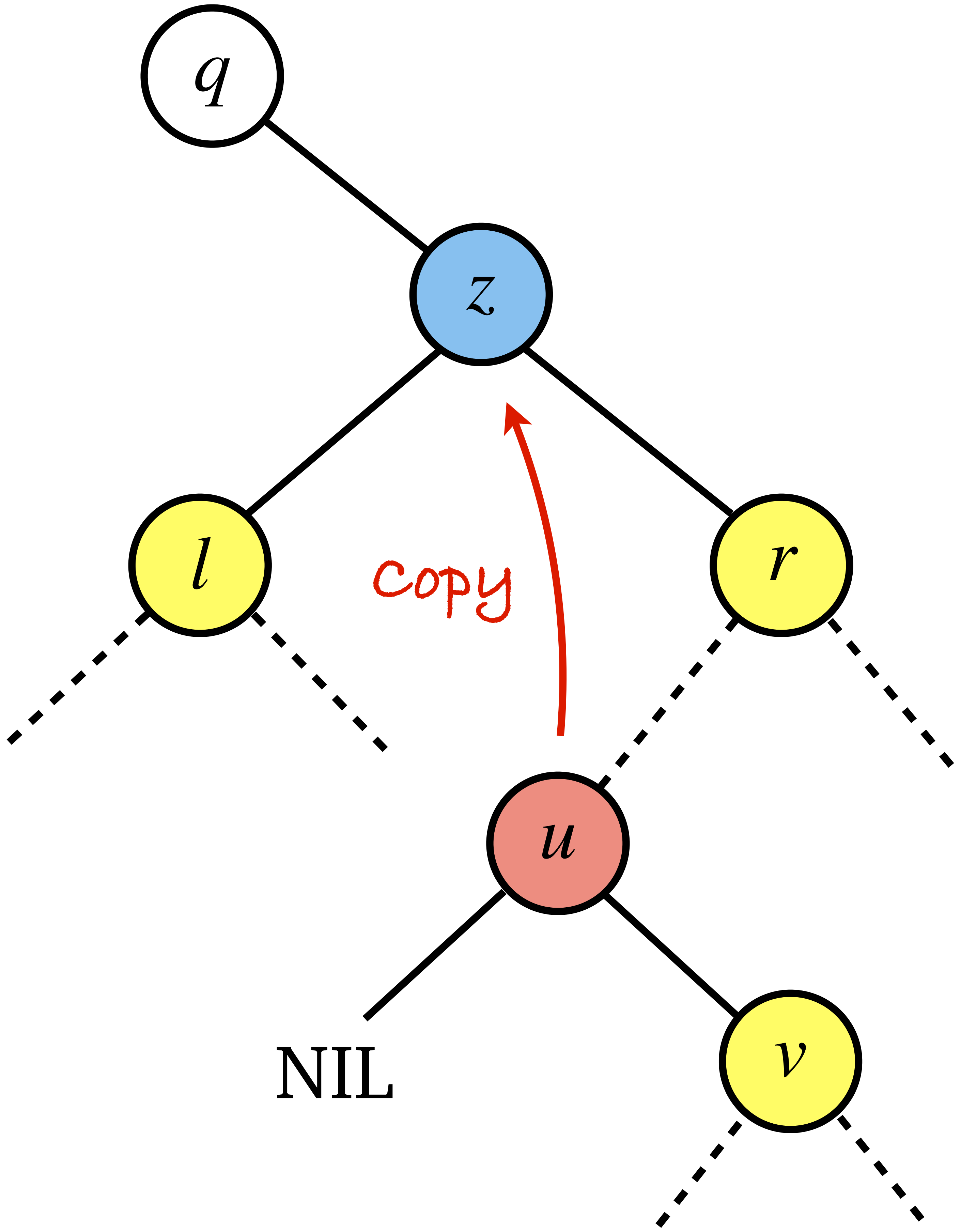

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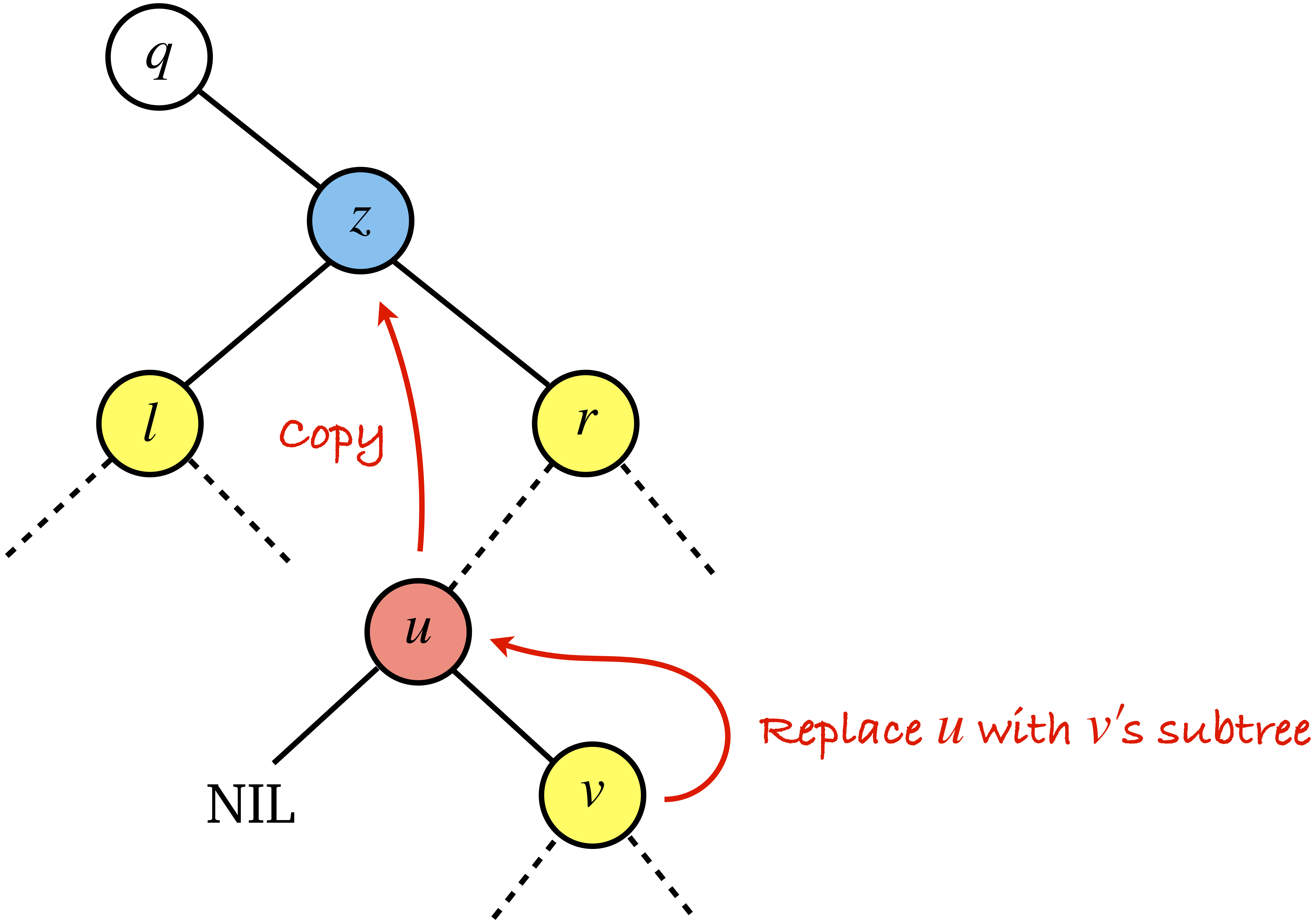

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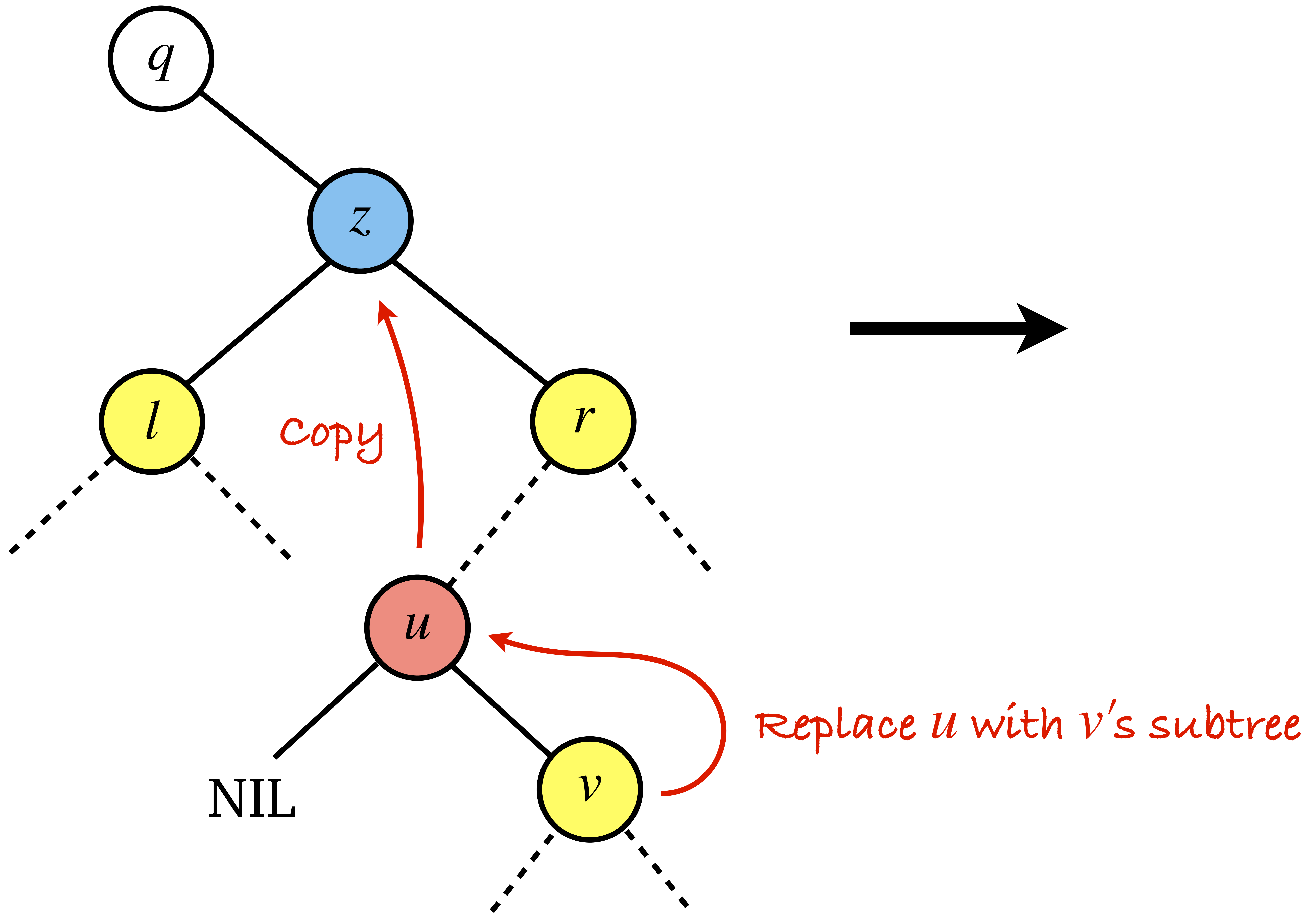

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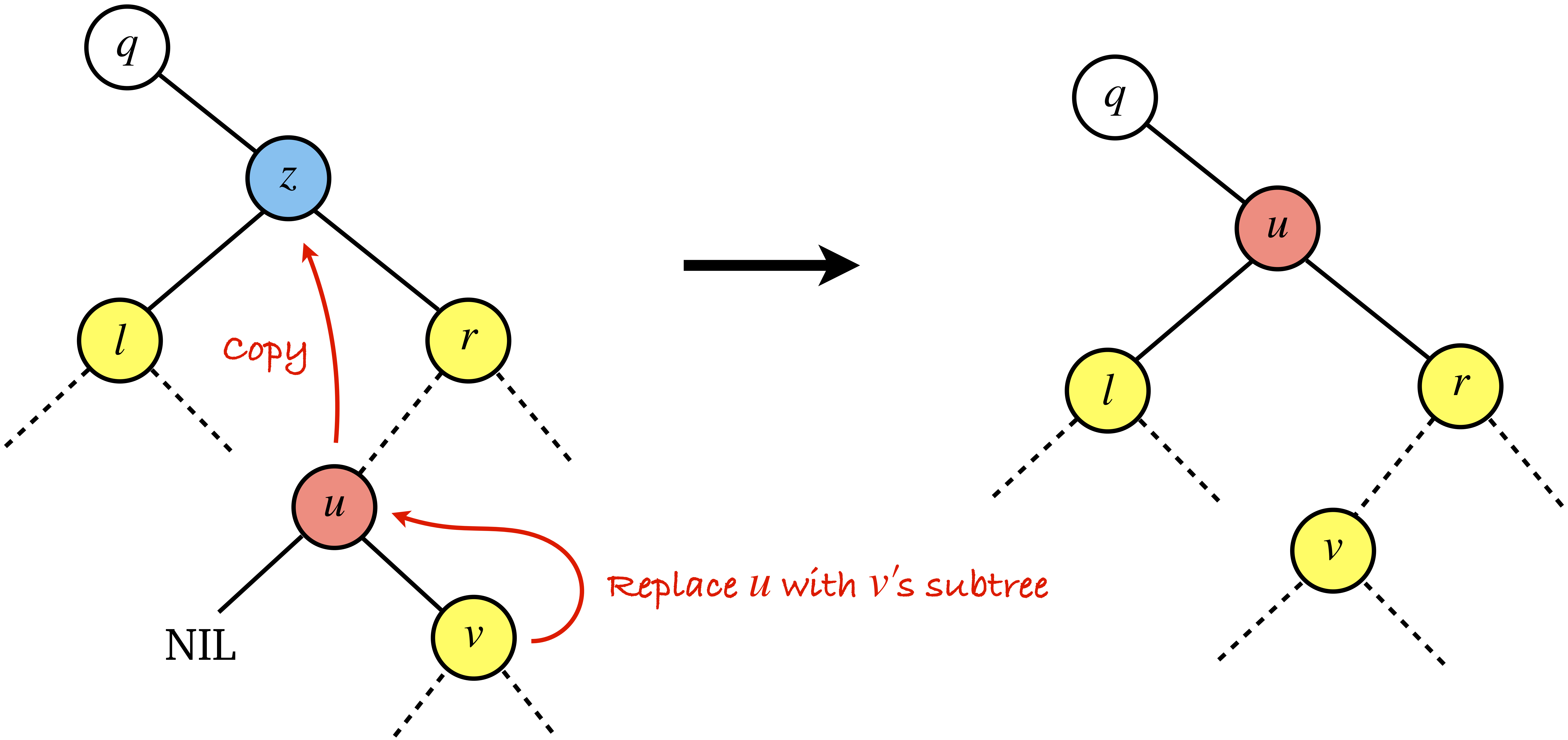

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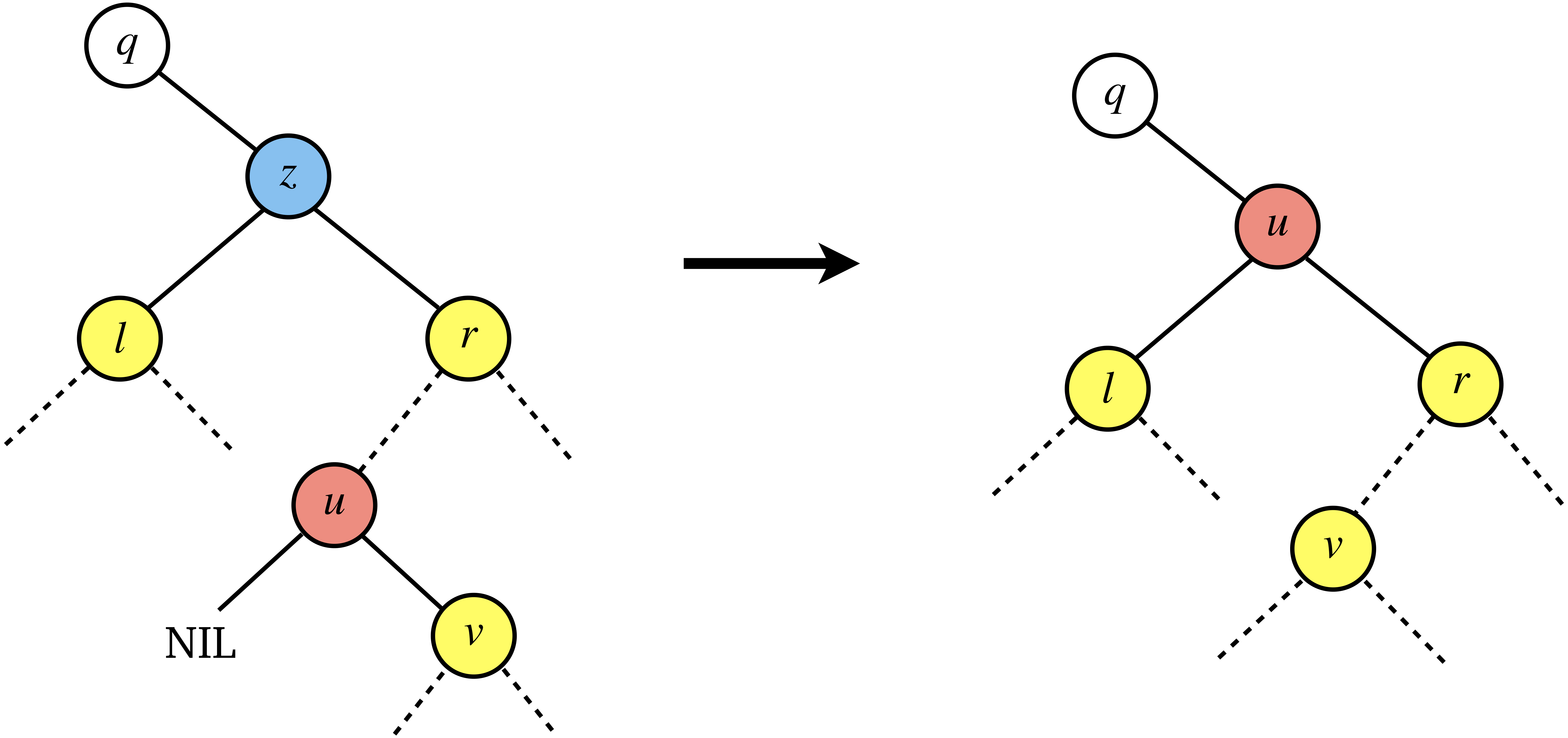

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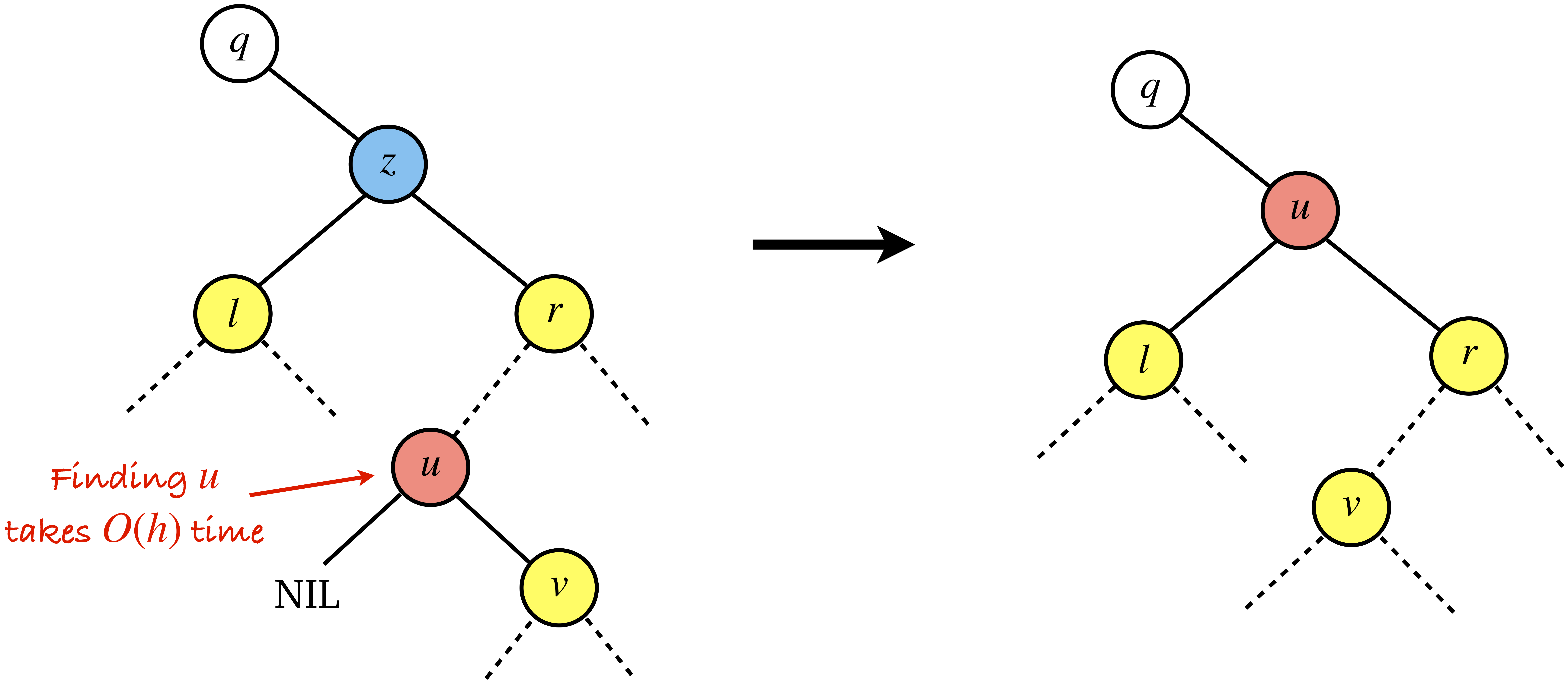

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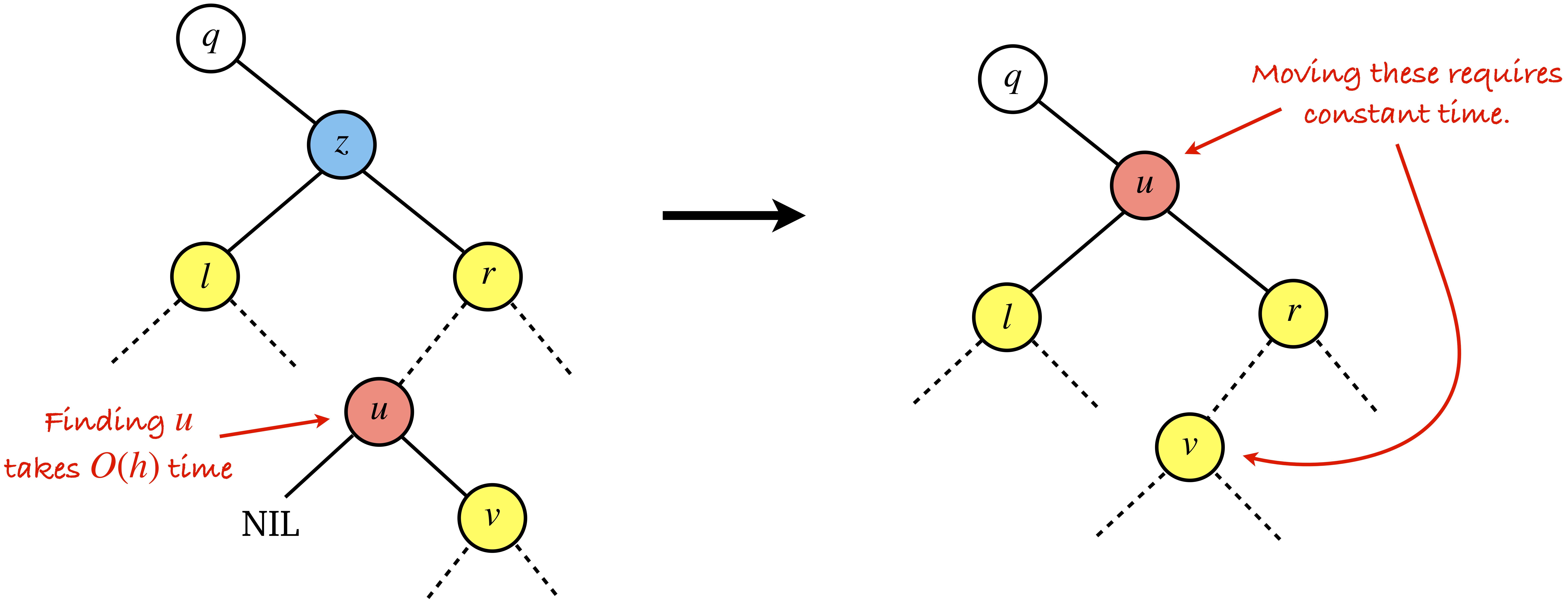

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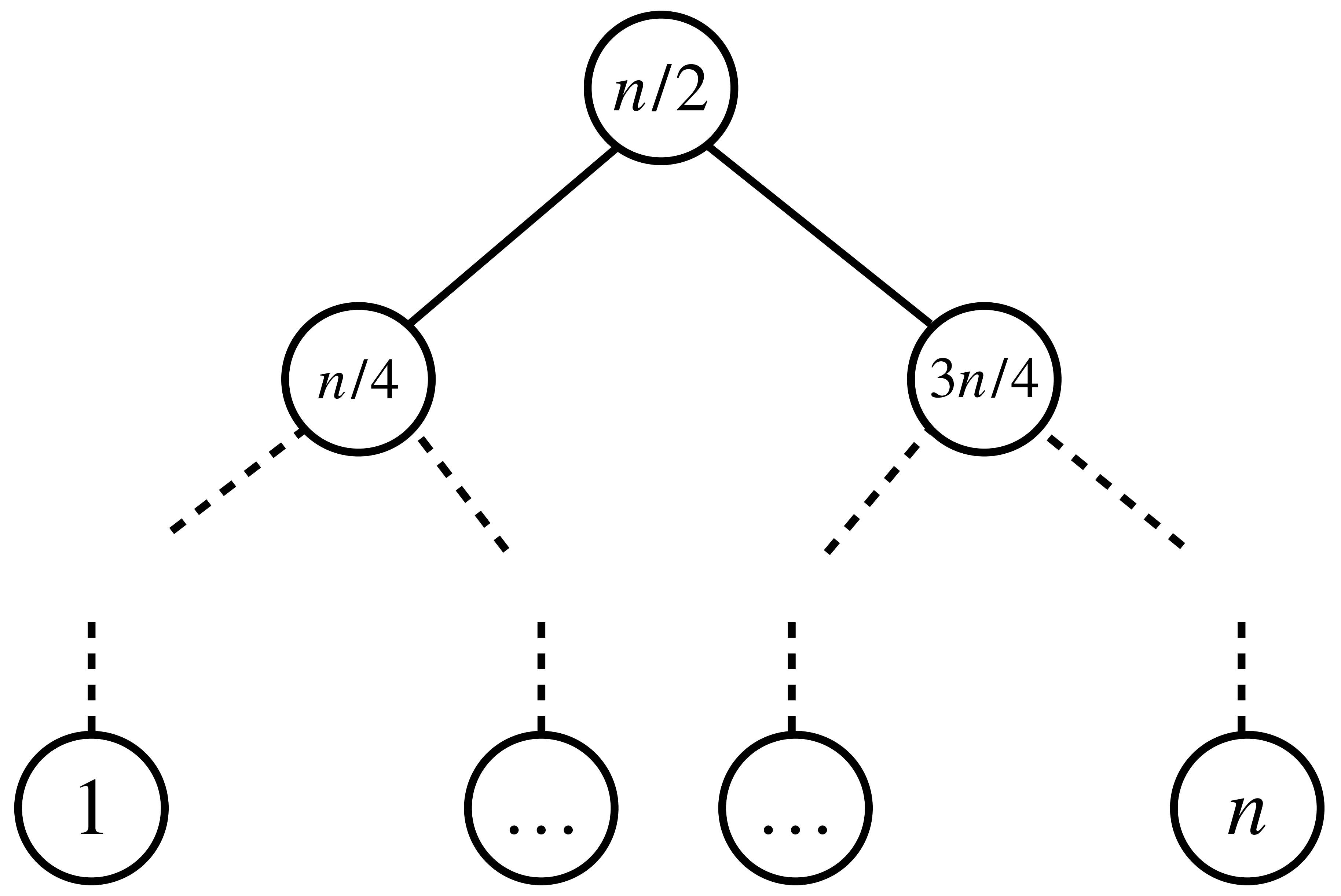

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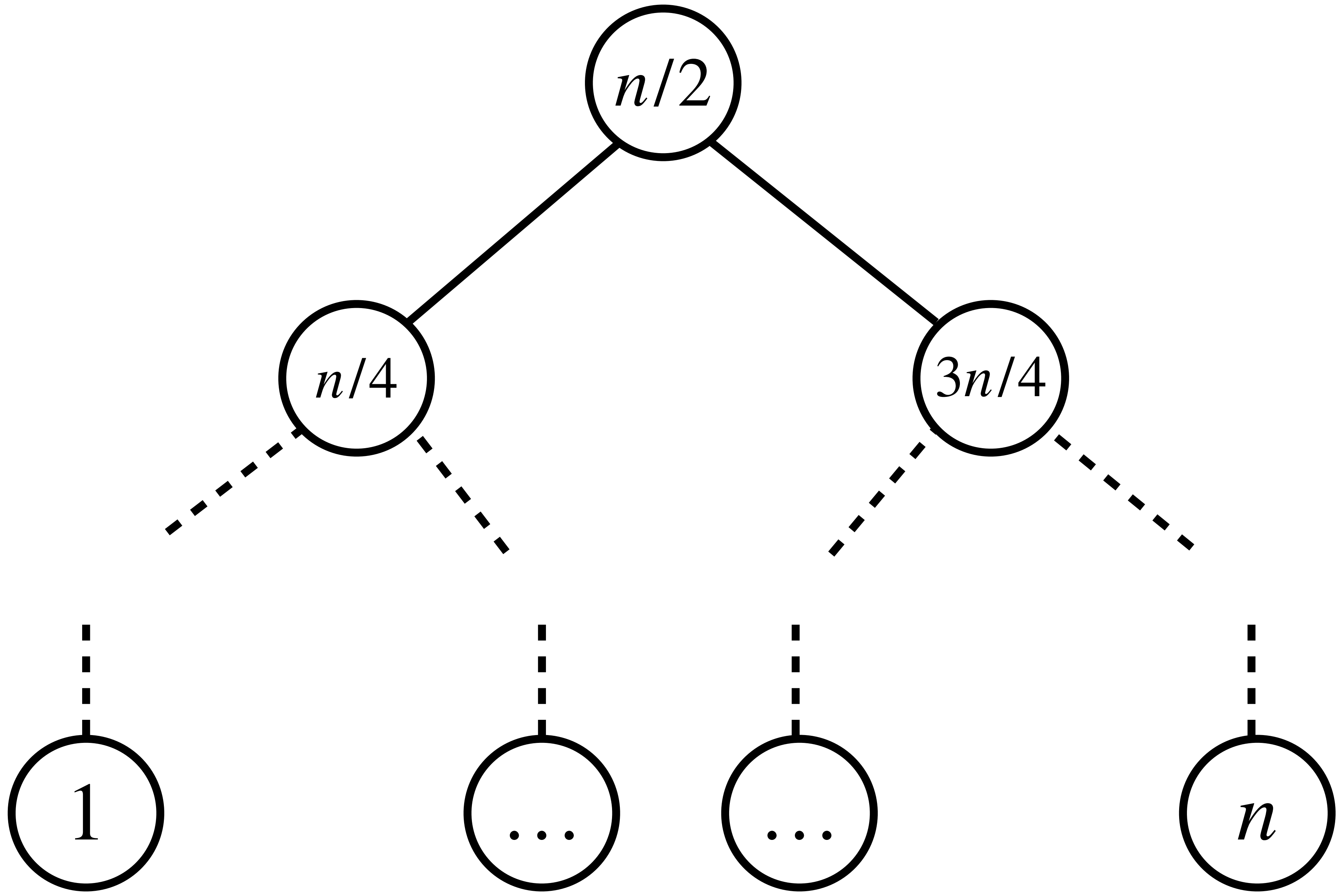

Best case: $h = O(\log n)$

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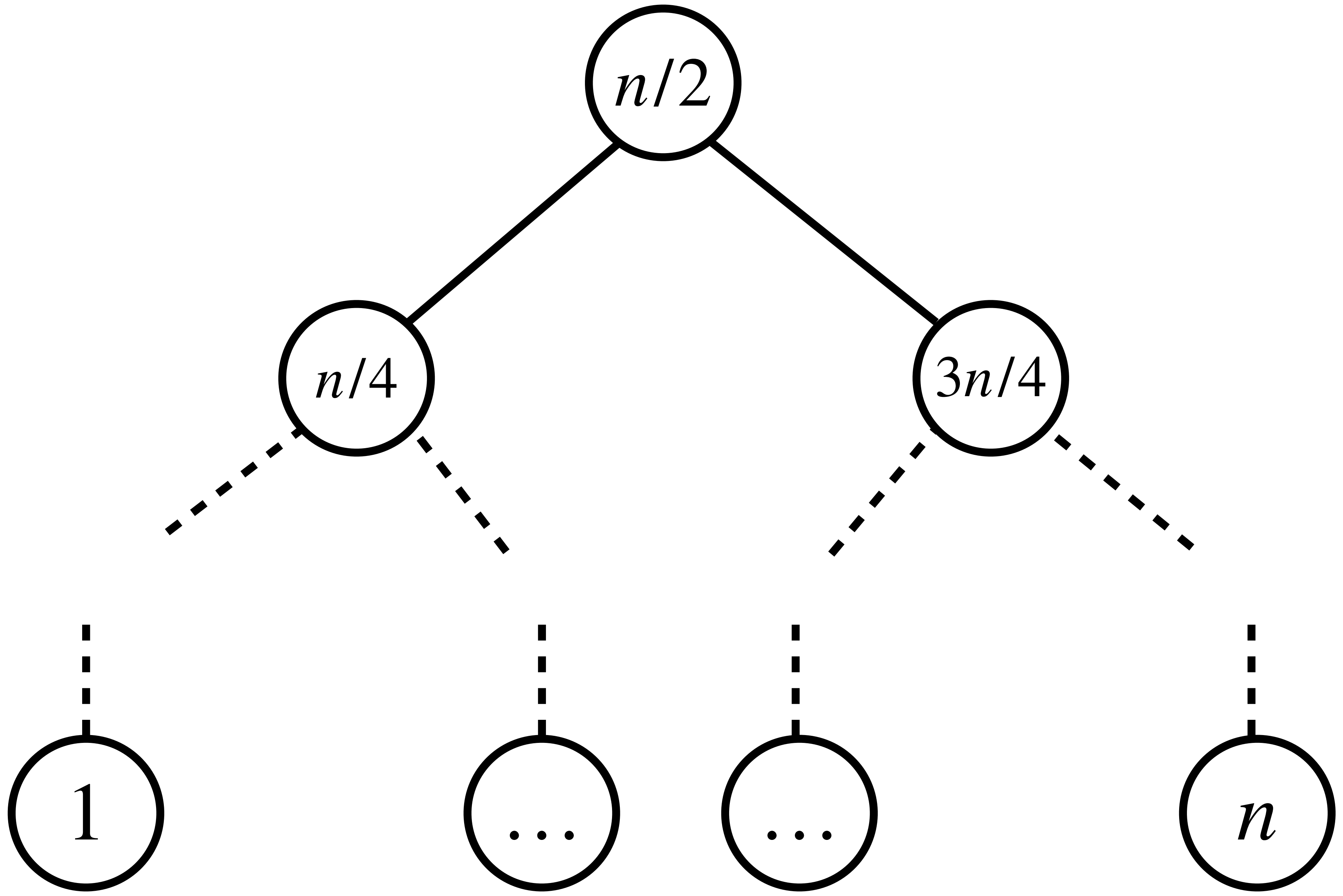

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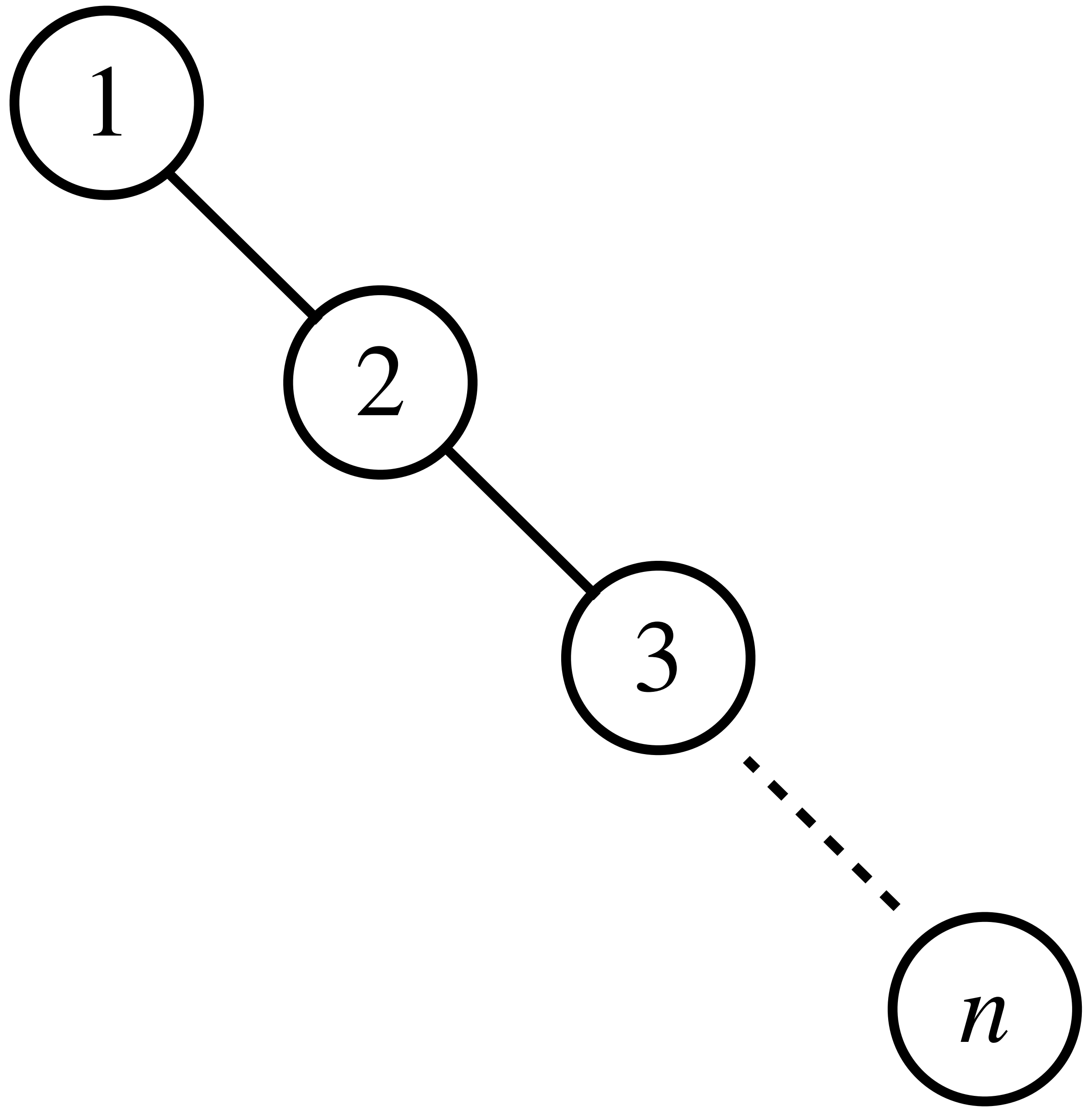

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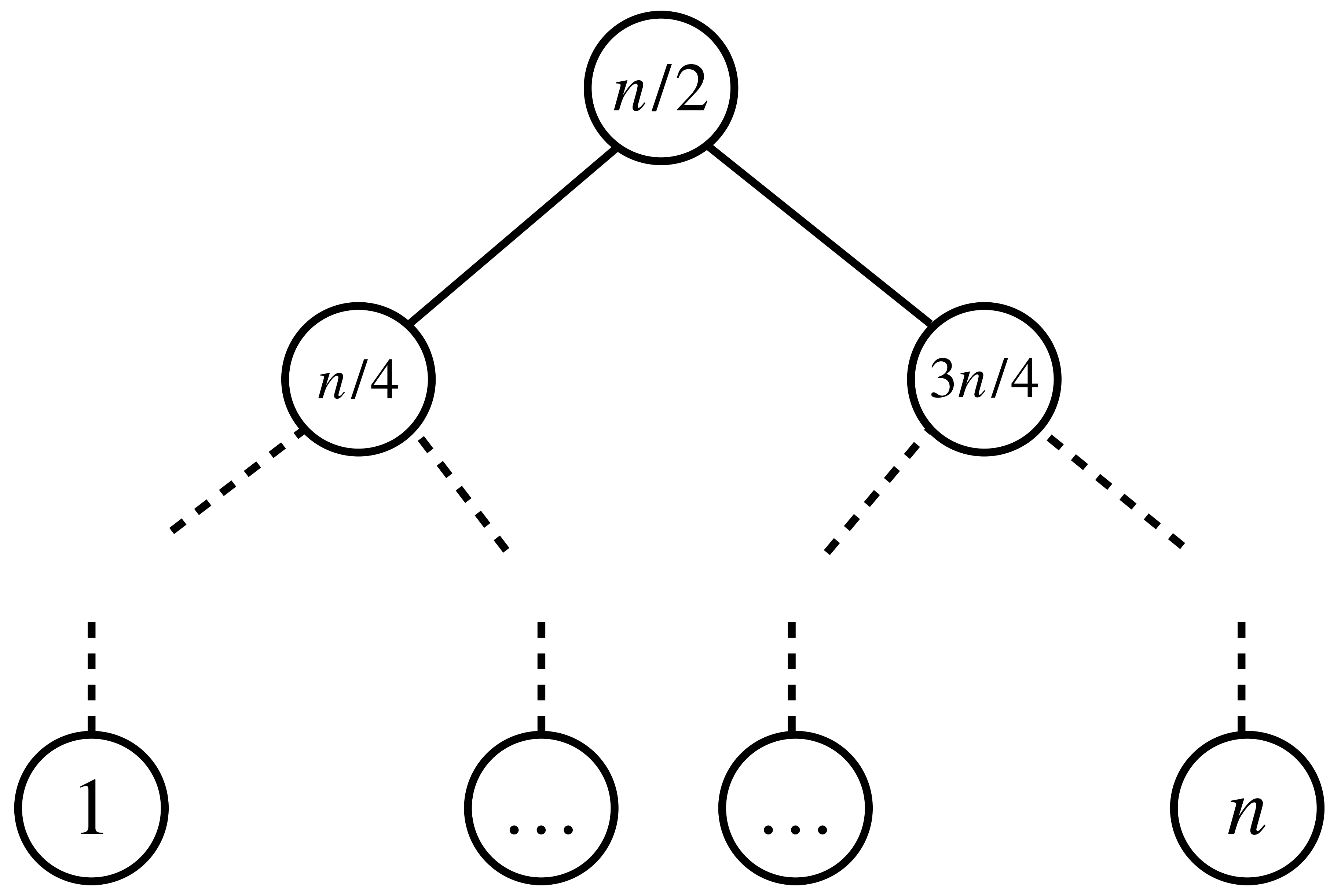

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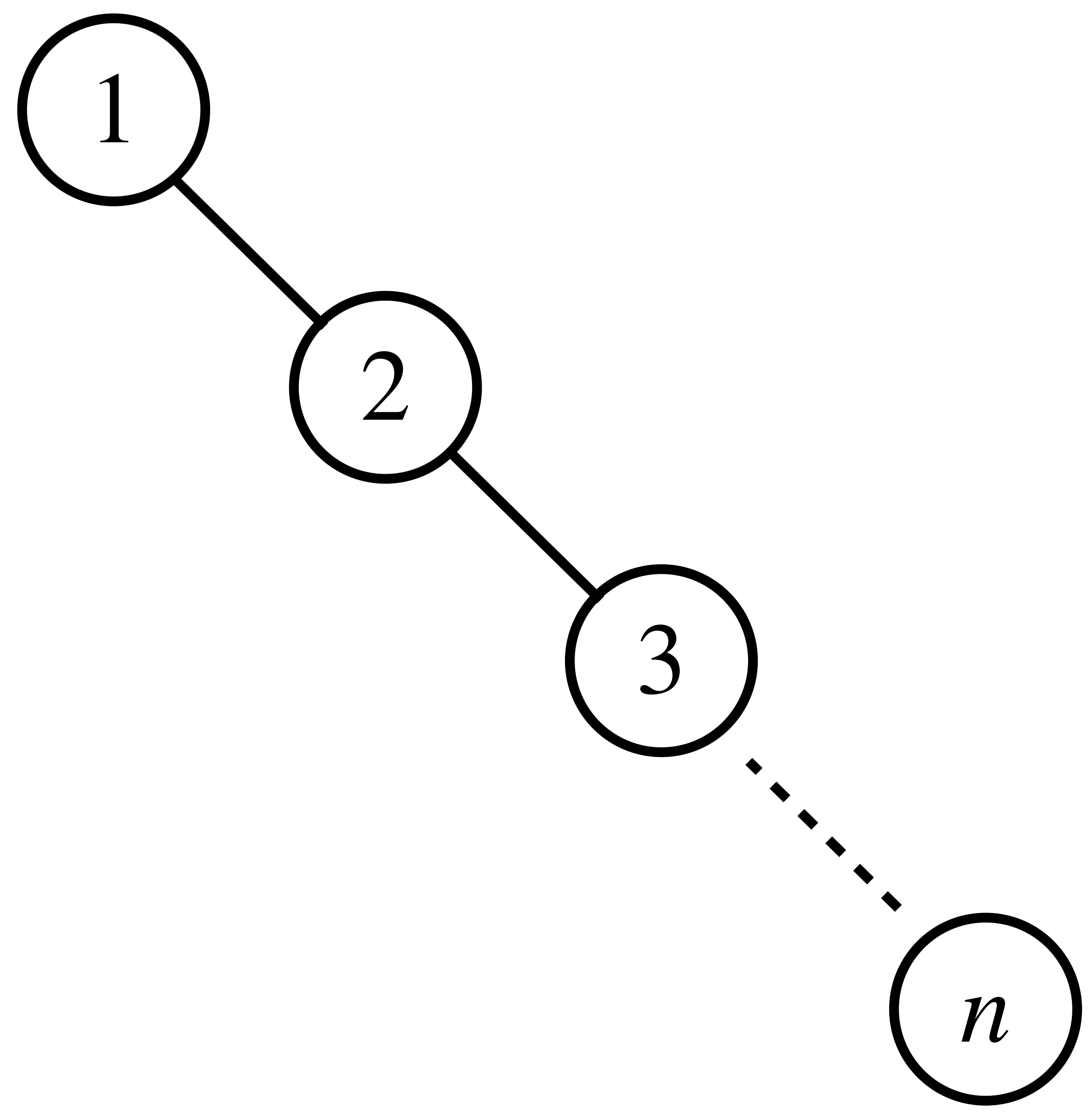

Worst case: $h = O(n)$

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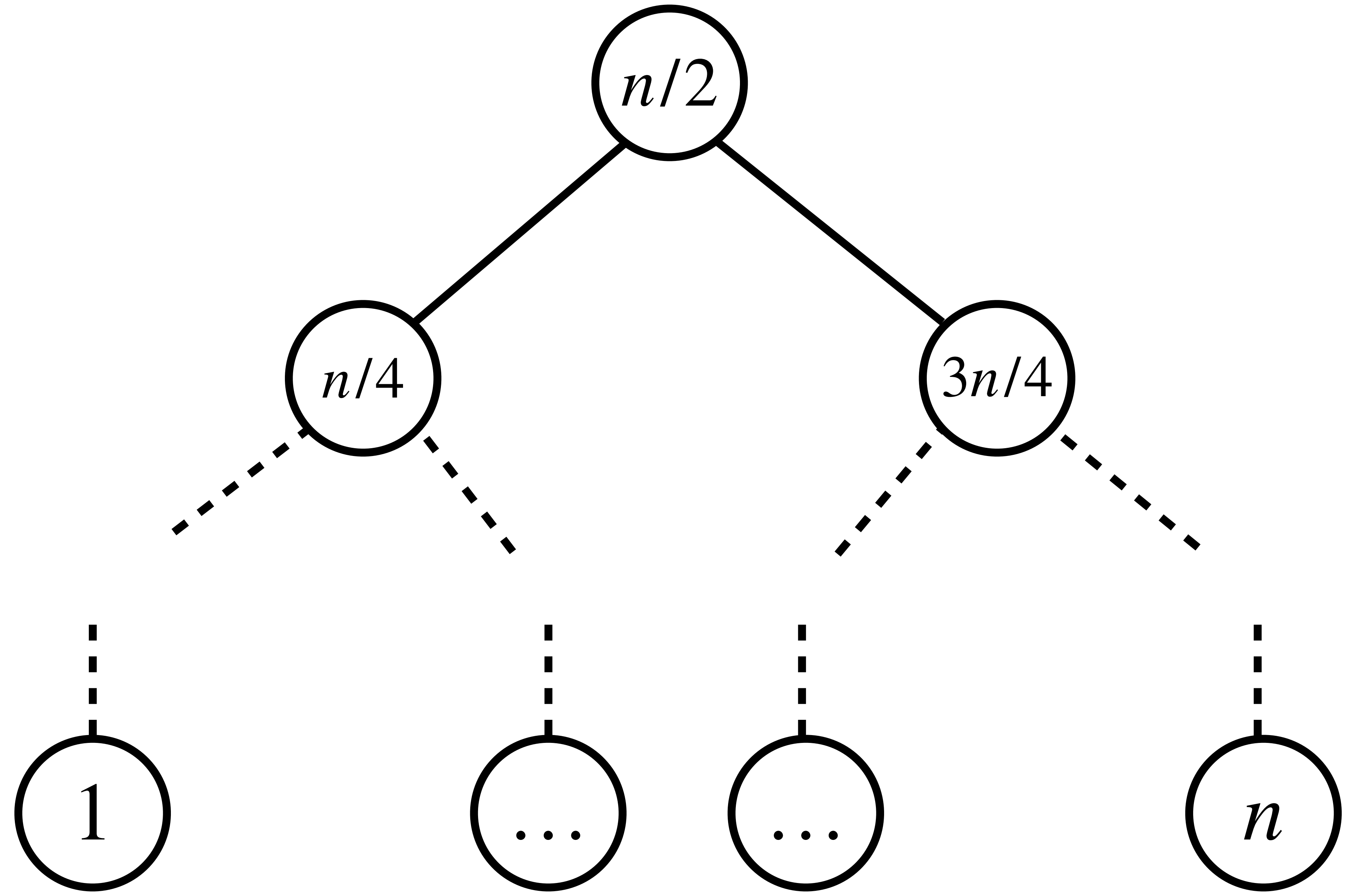

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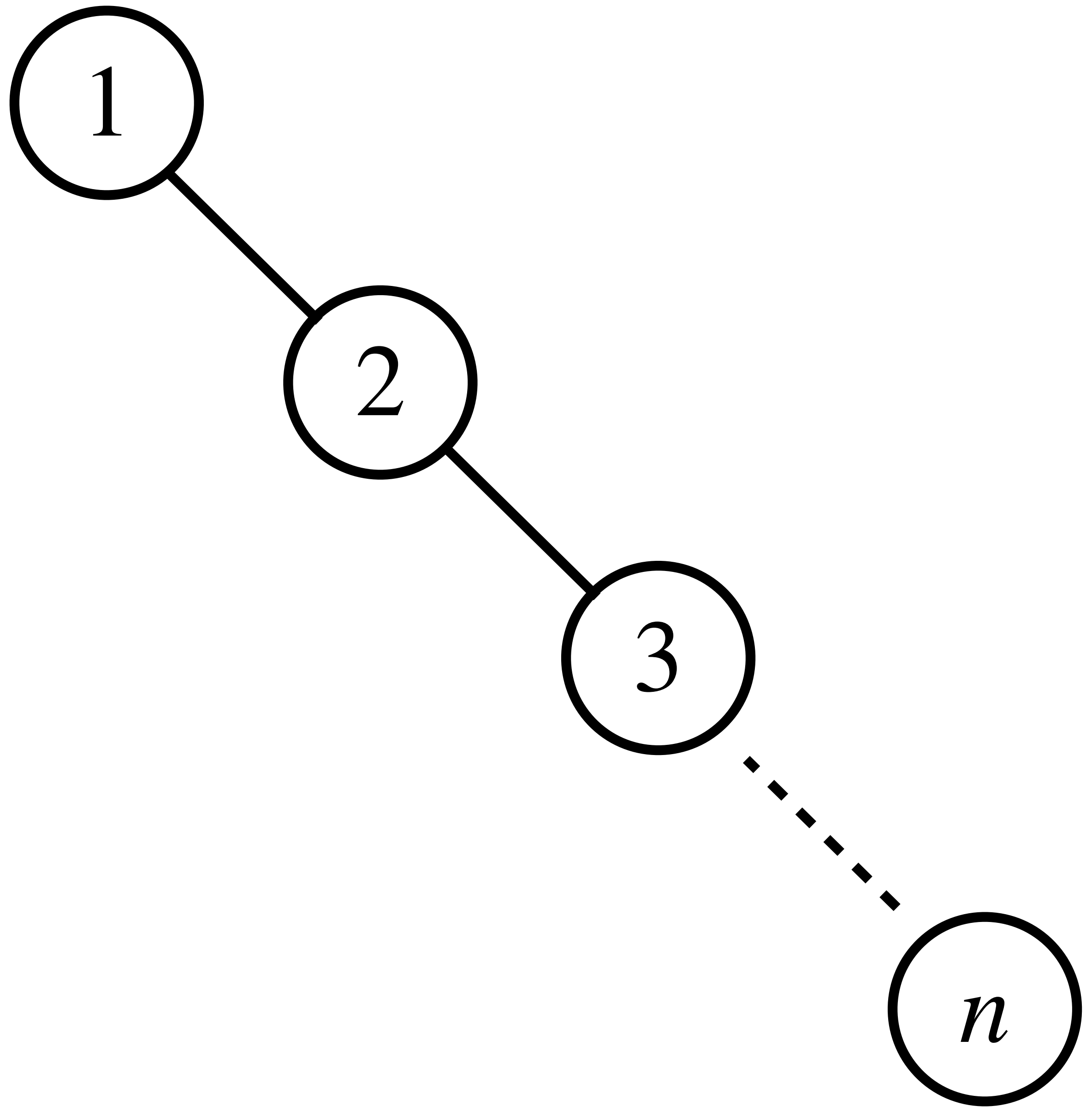

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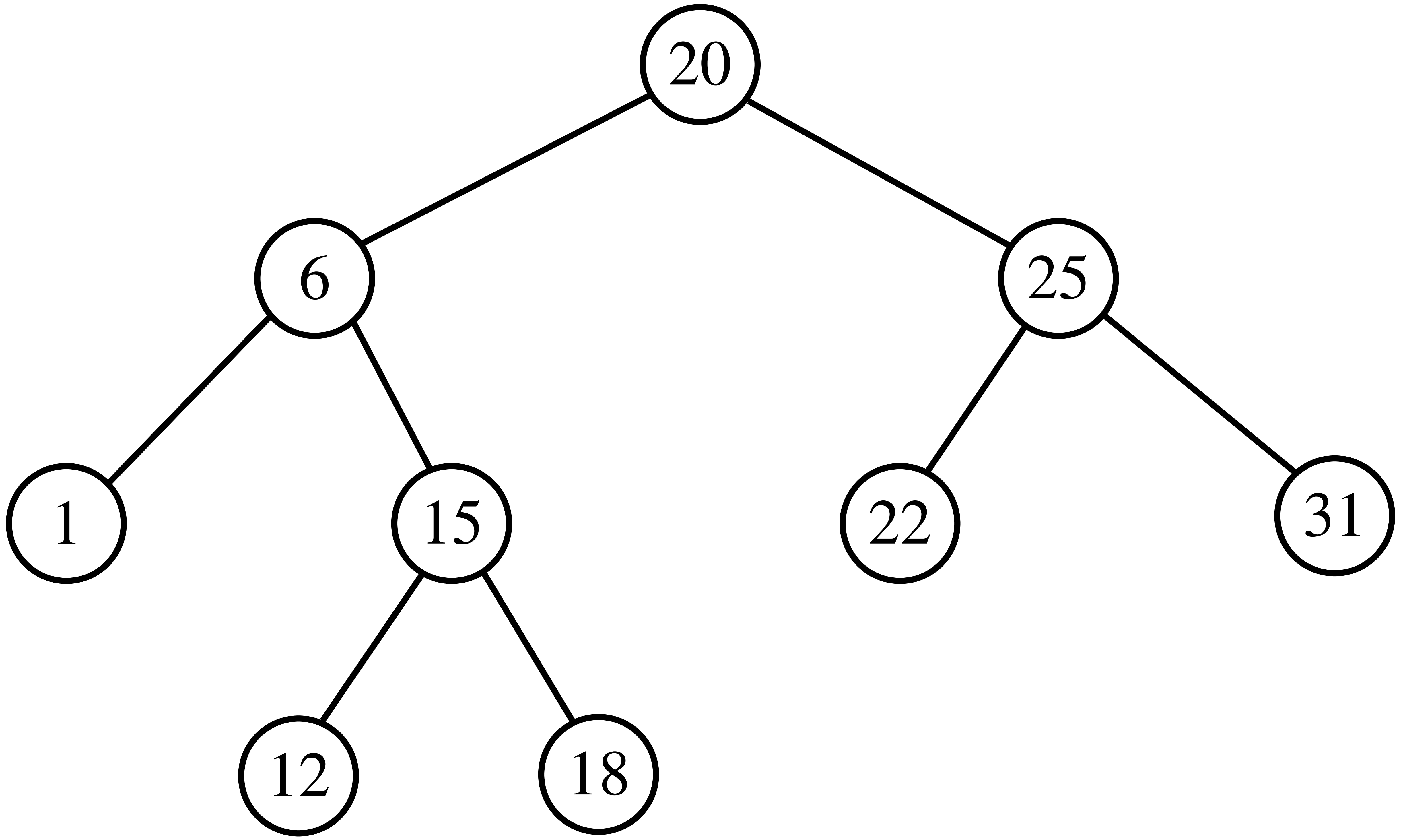
Red-black (RB) trees provide a way keep height $O(\log n)$.

RB-Trees: How do they look like?



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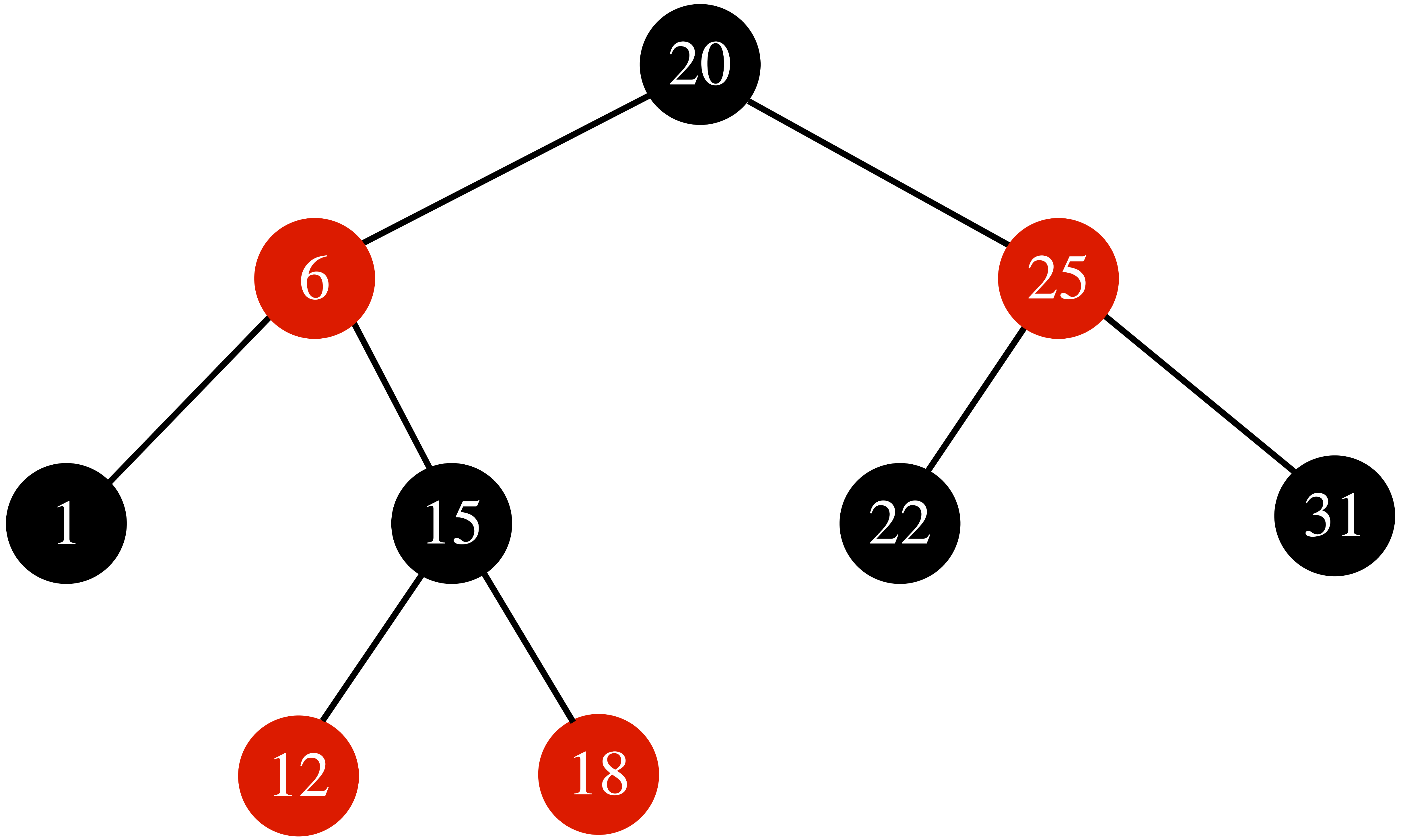

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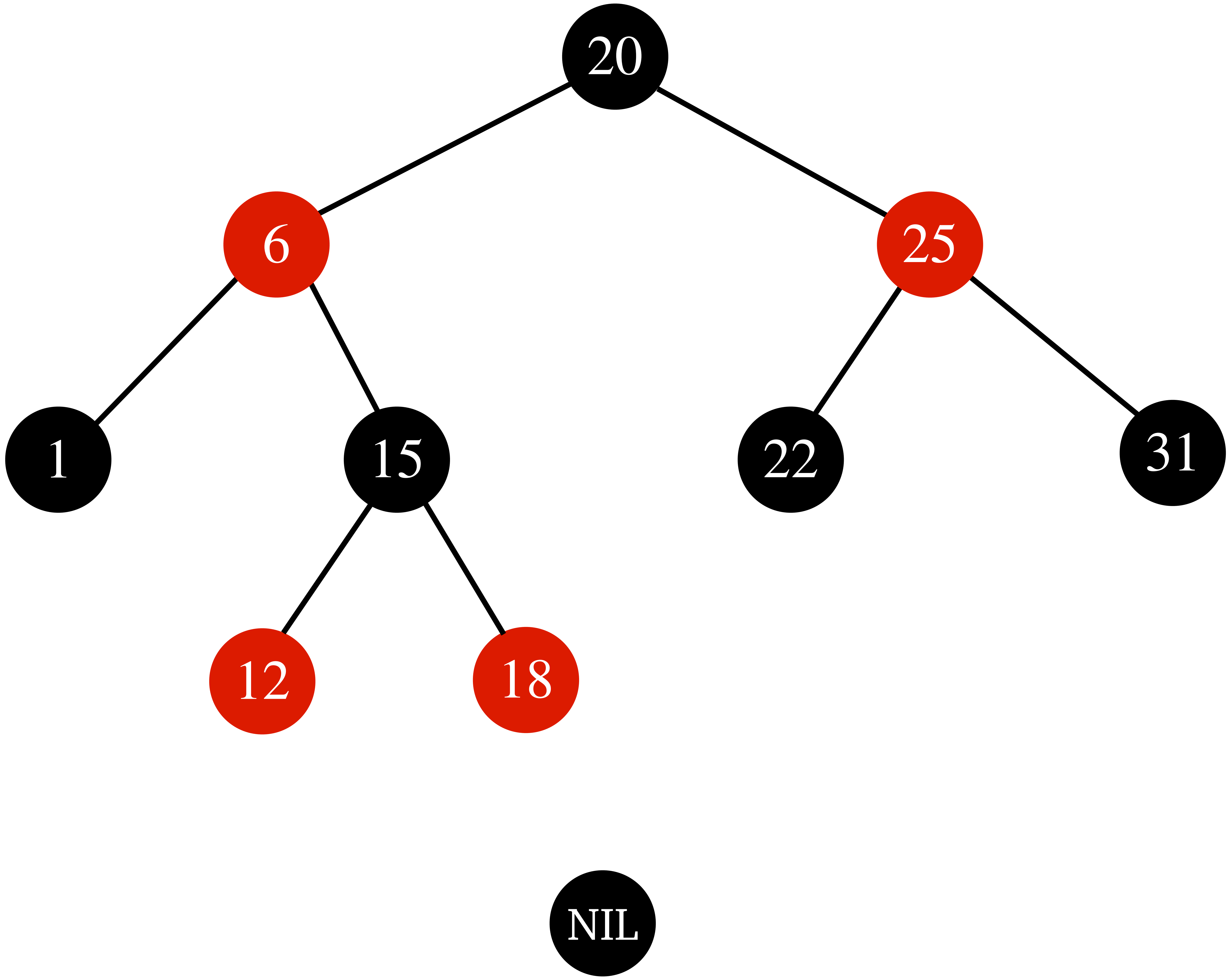

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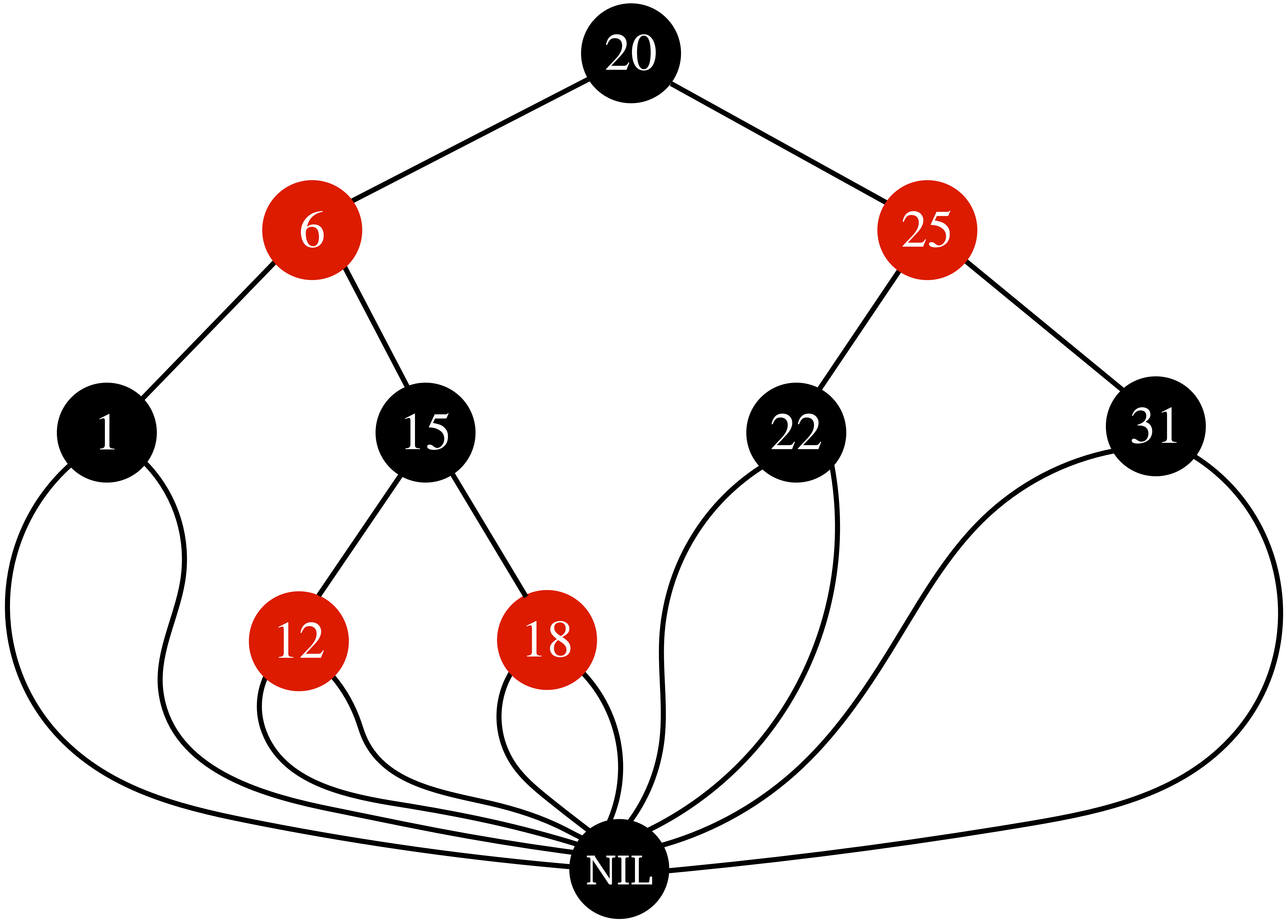

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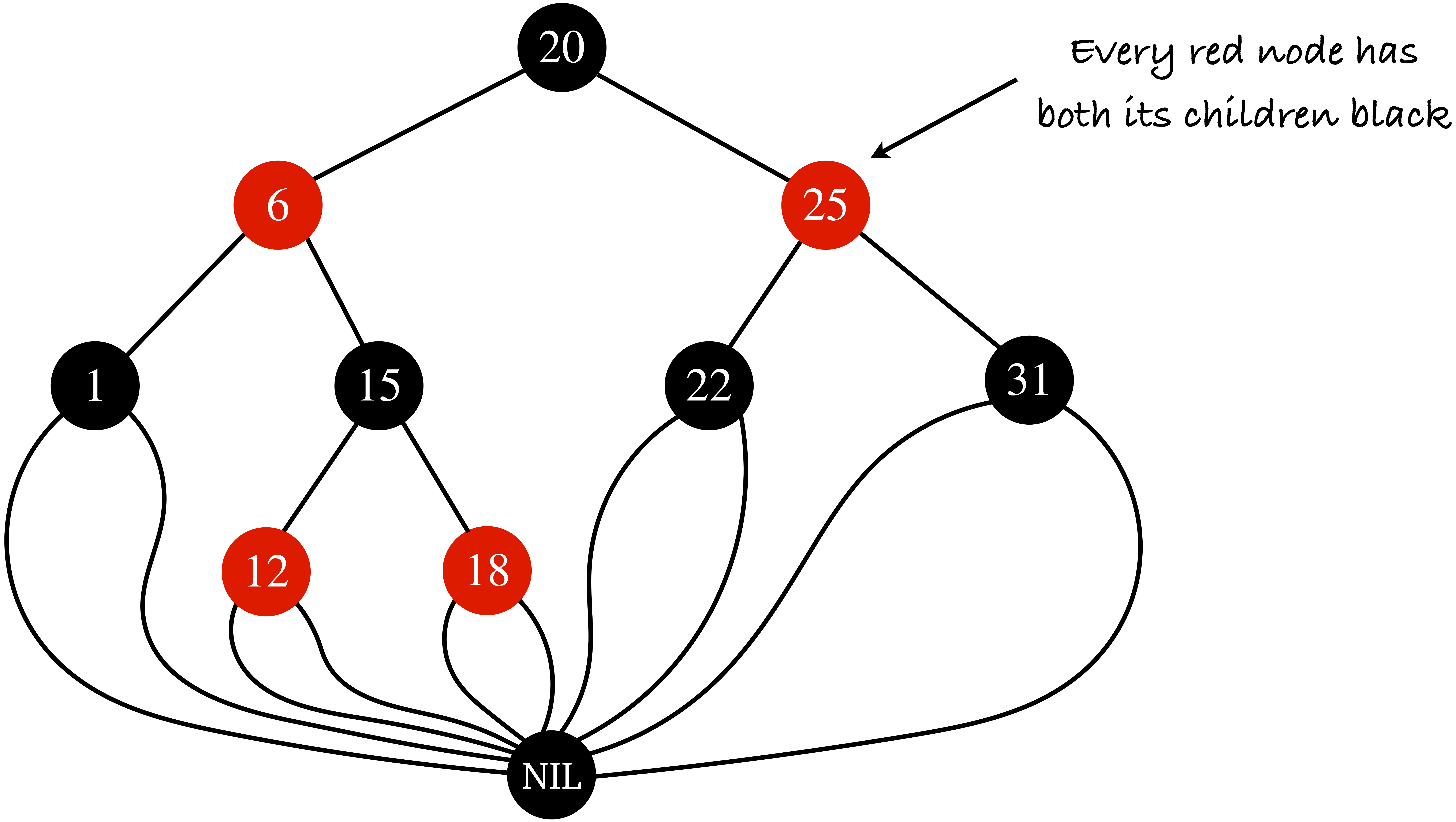

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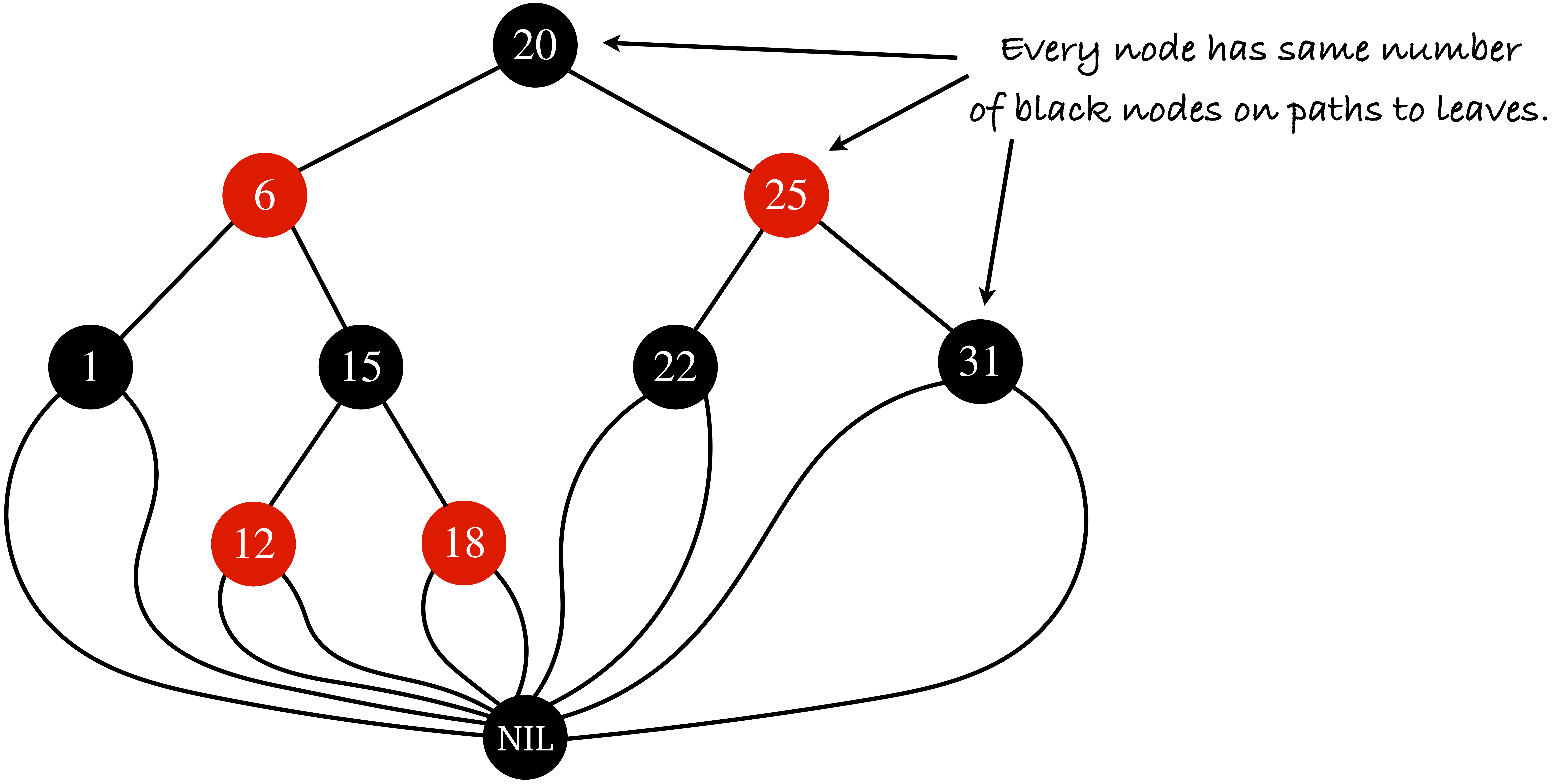

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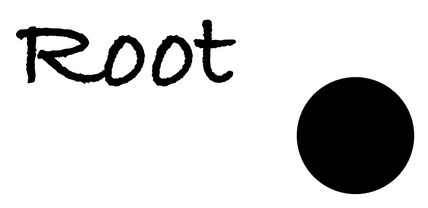

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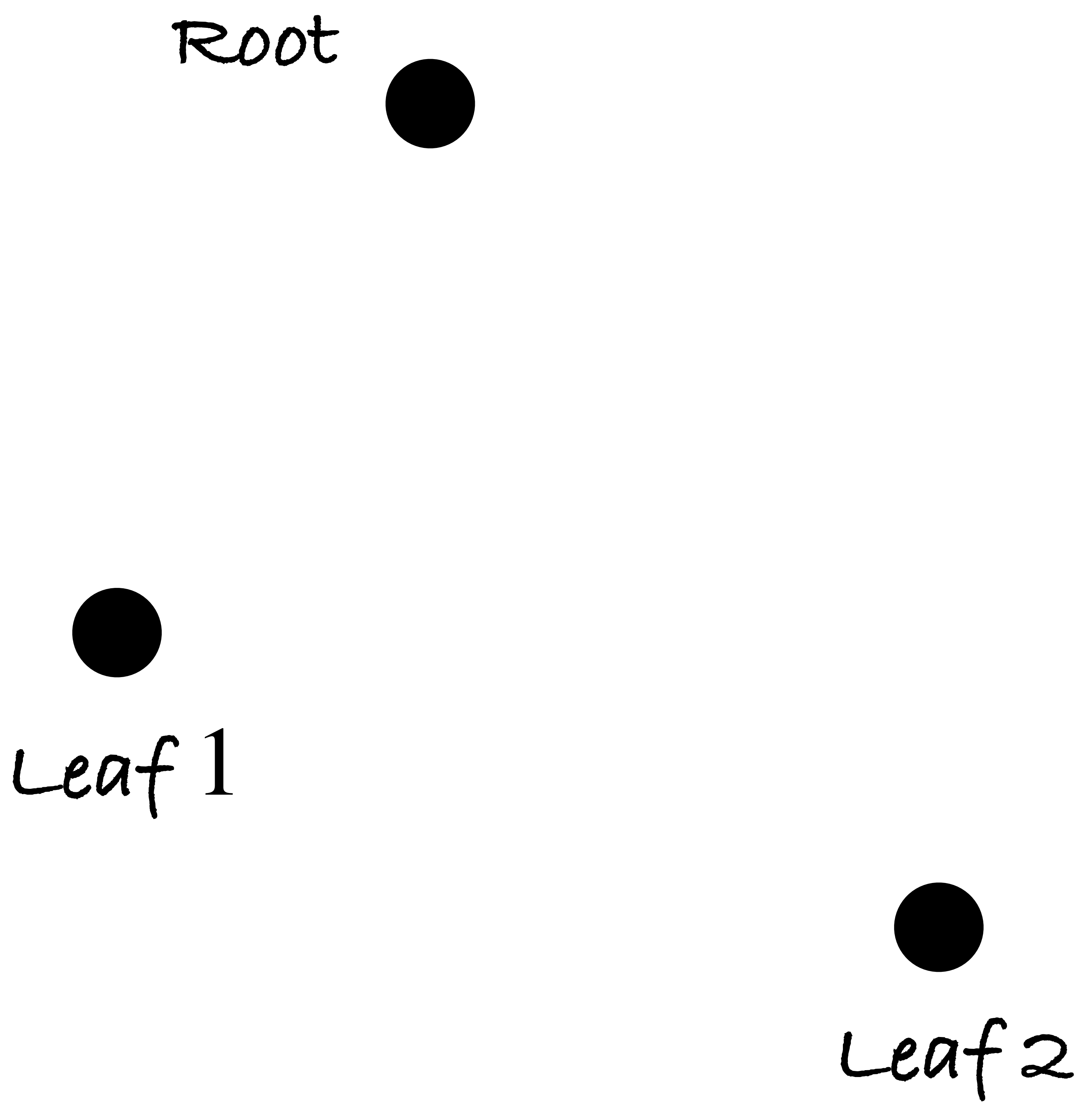

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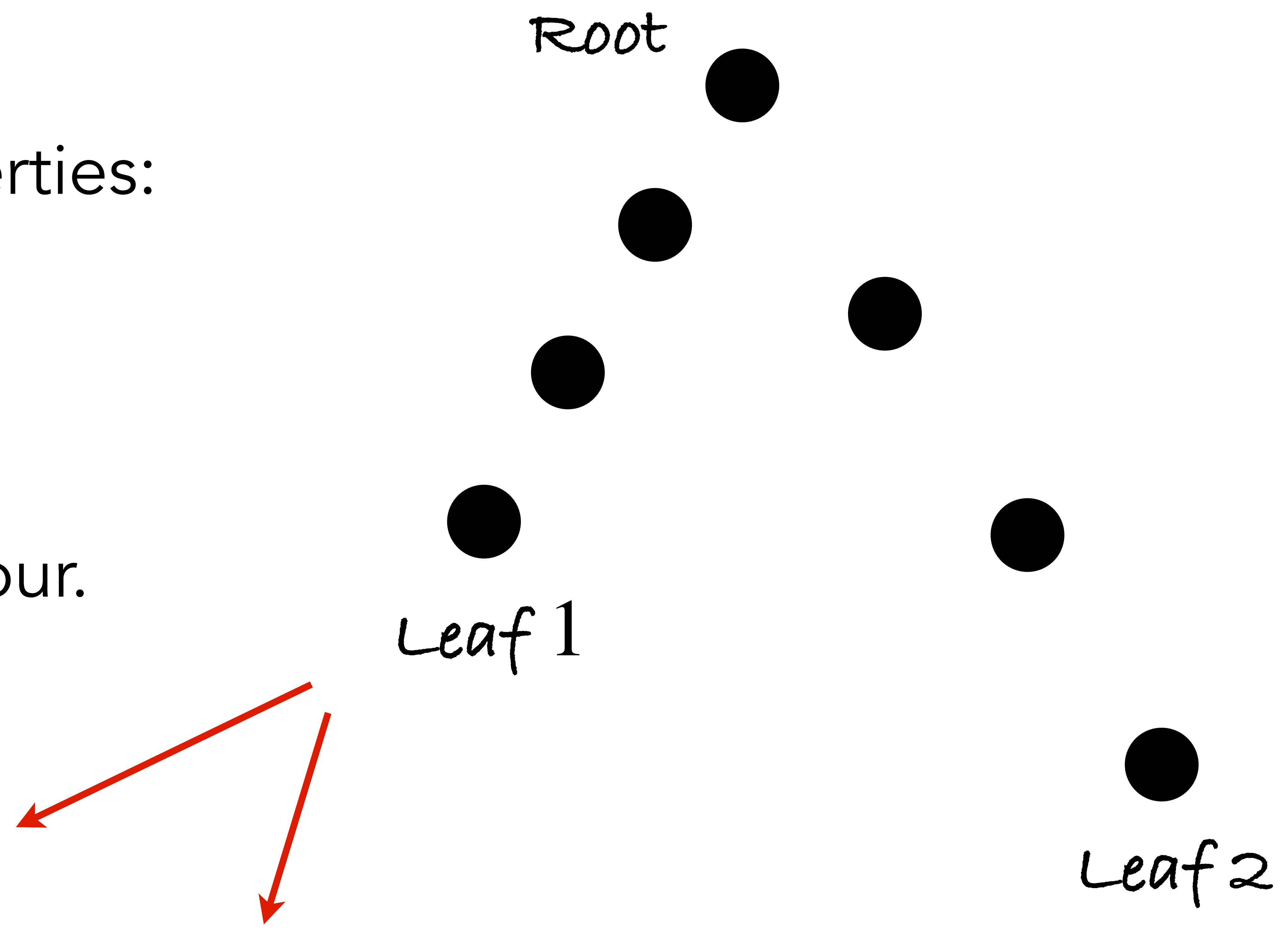

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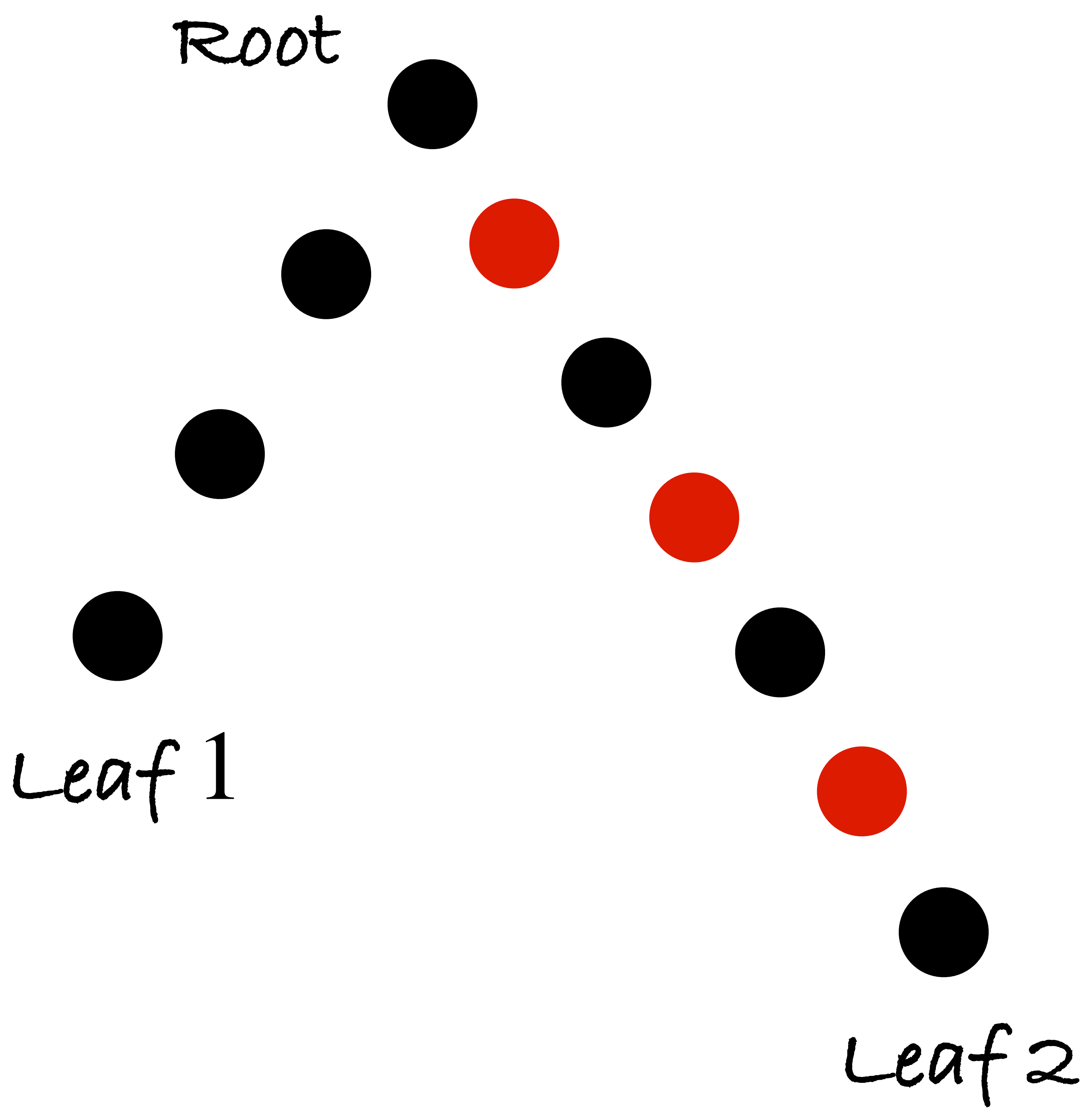

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Both these properties ensure that no path from root to a leaf is more than twice as long as any other.

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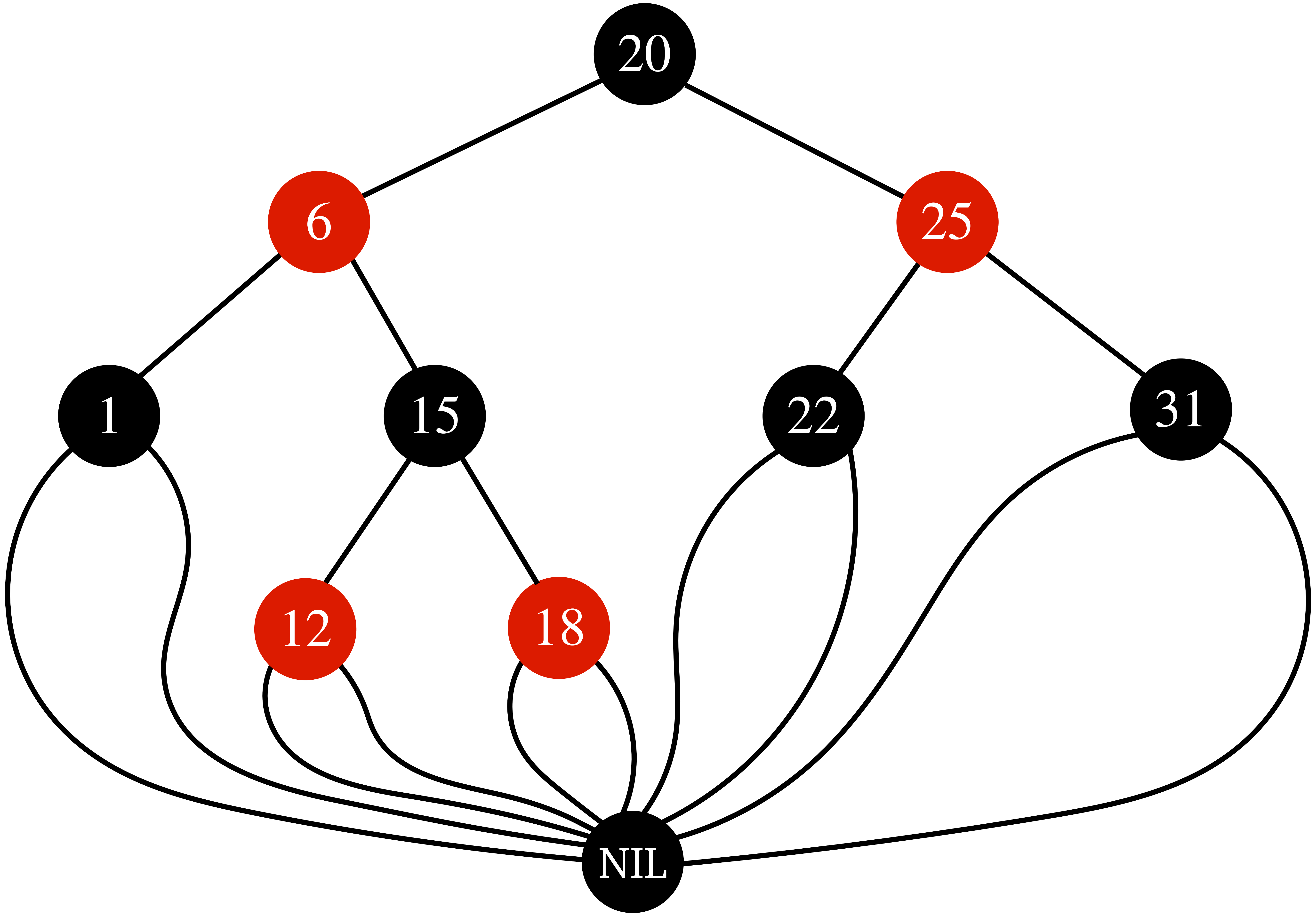

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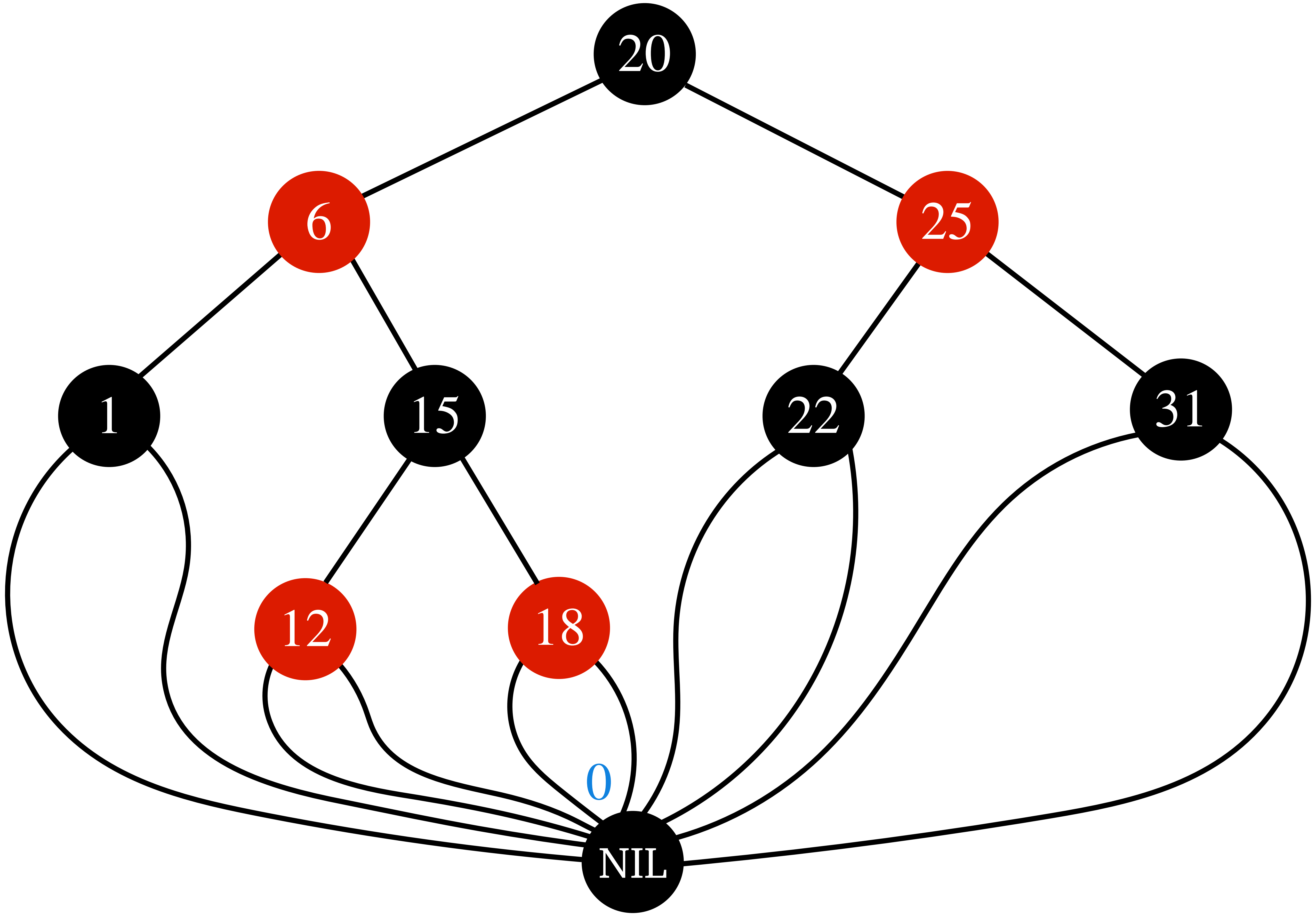

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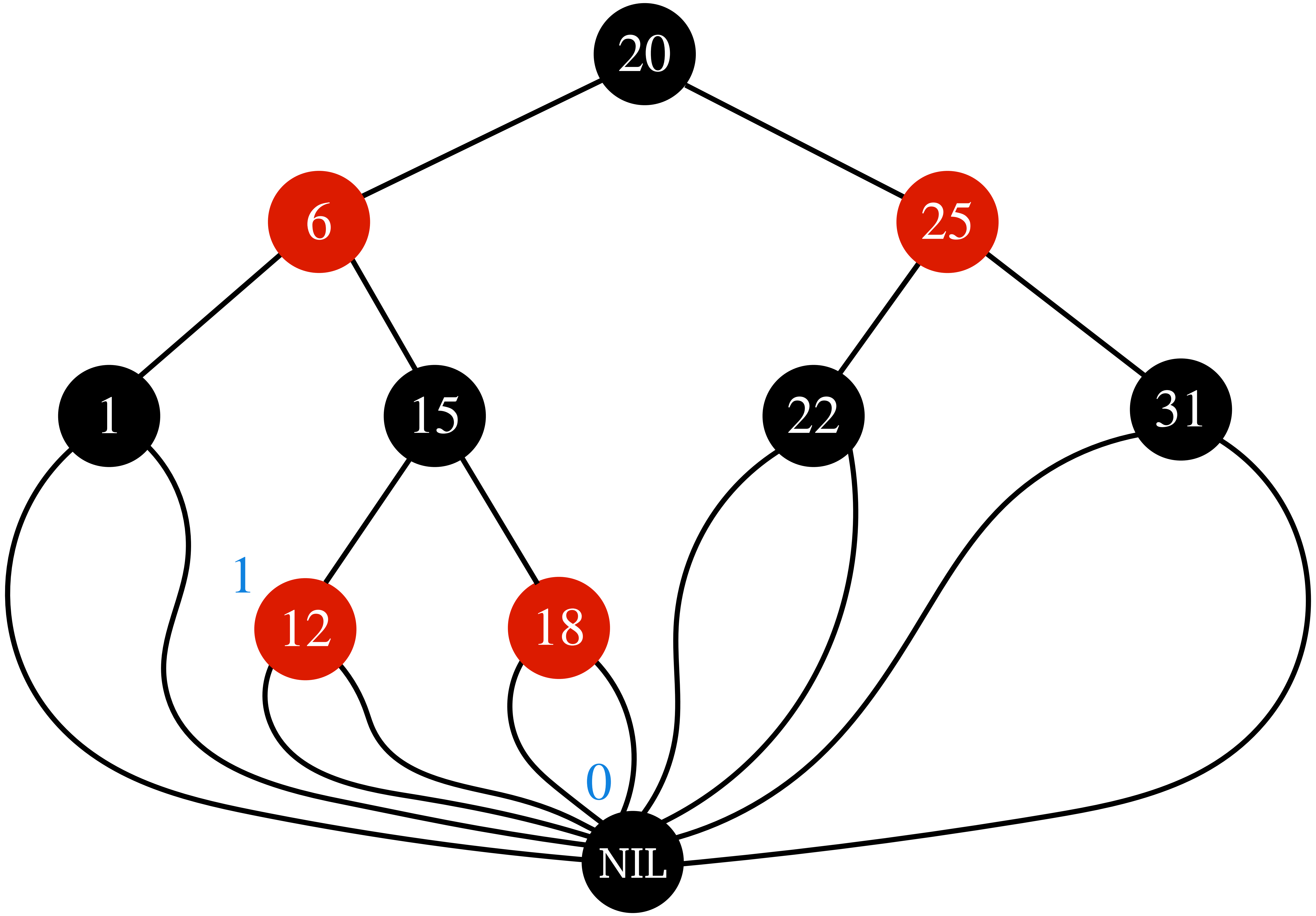

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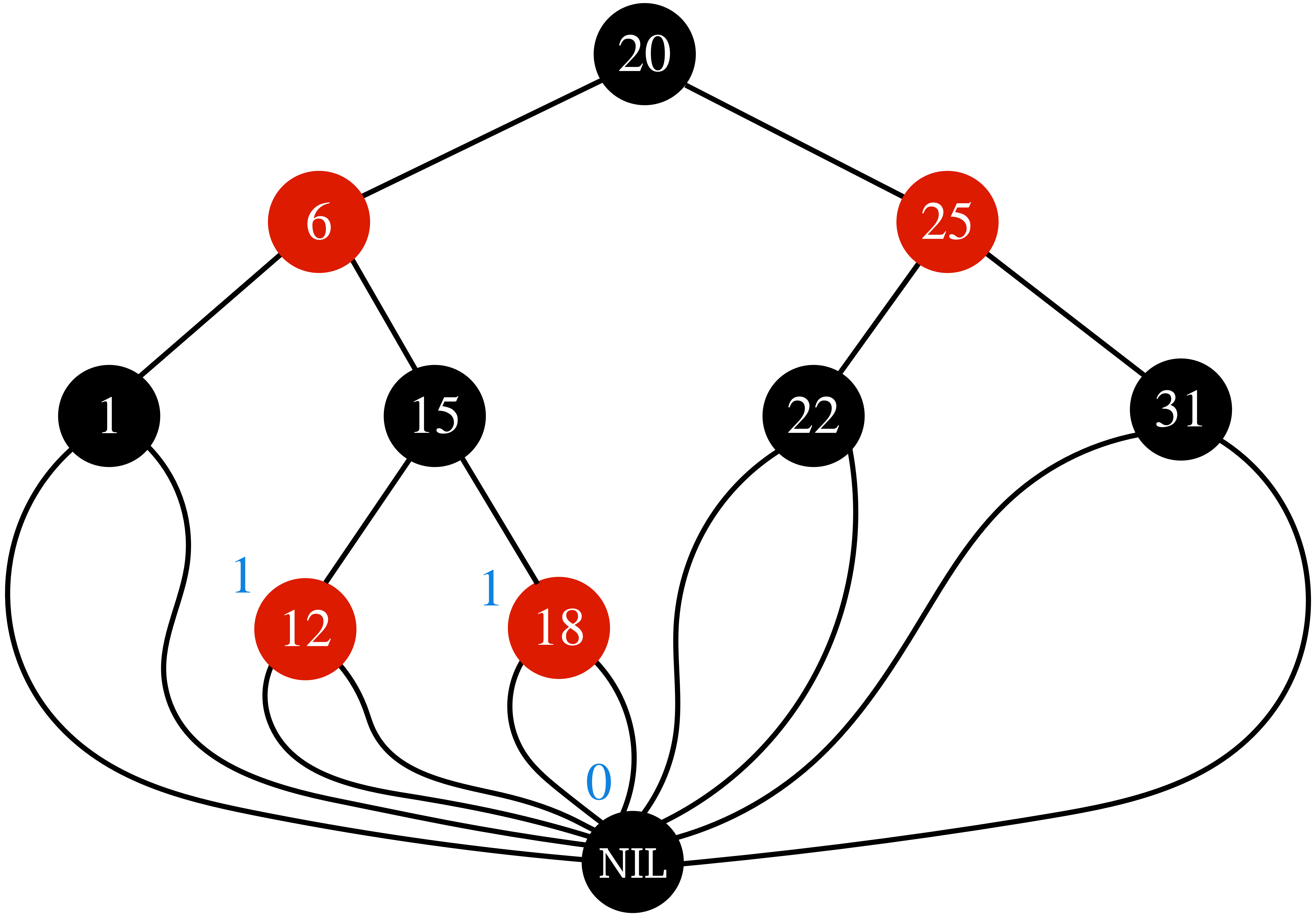

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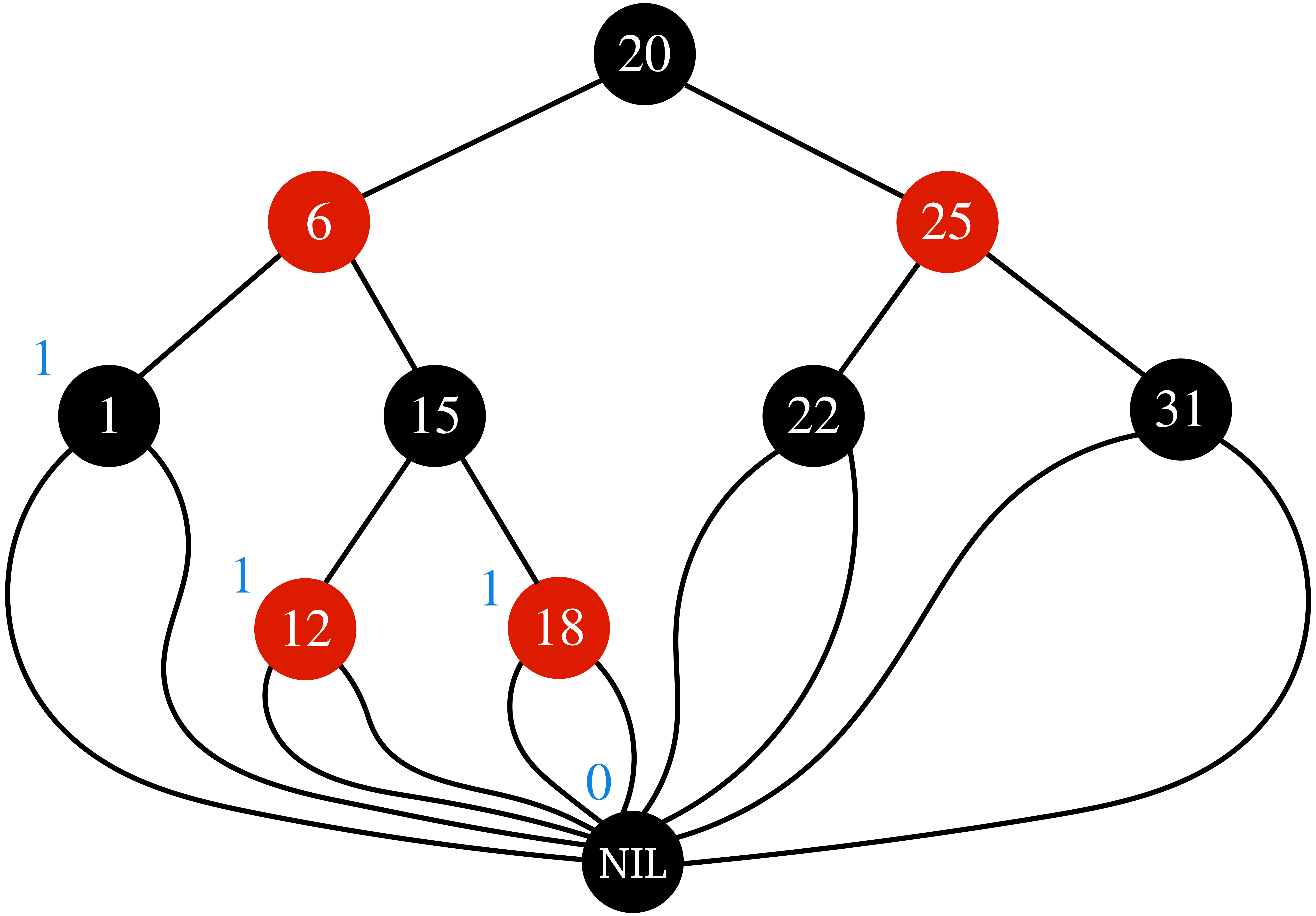

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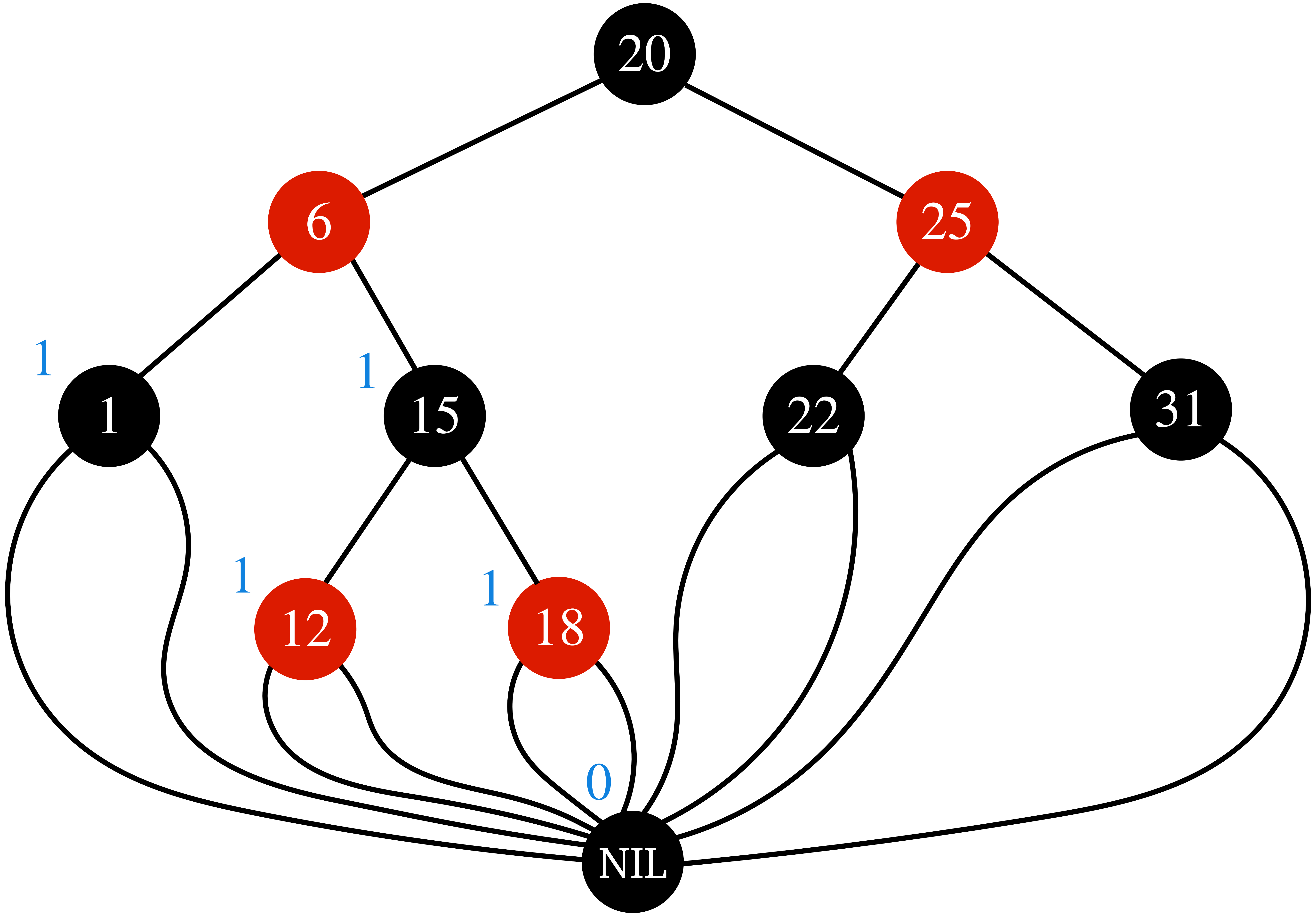

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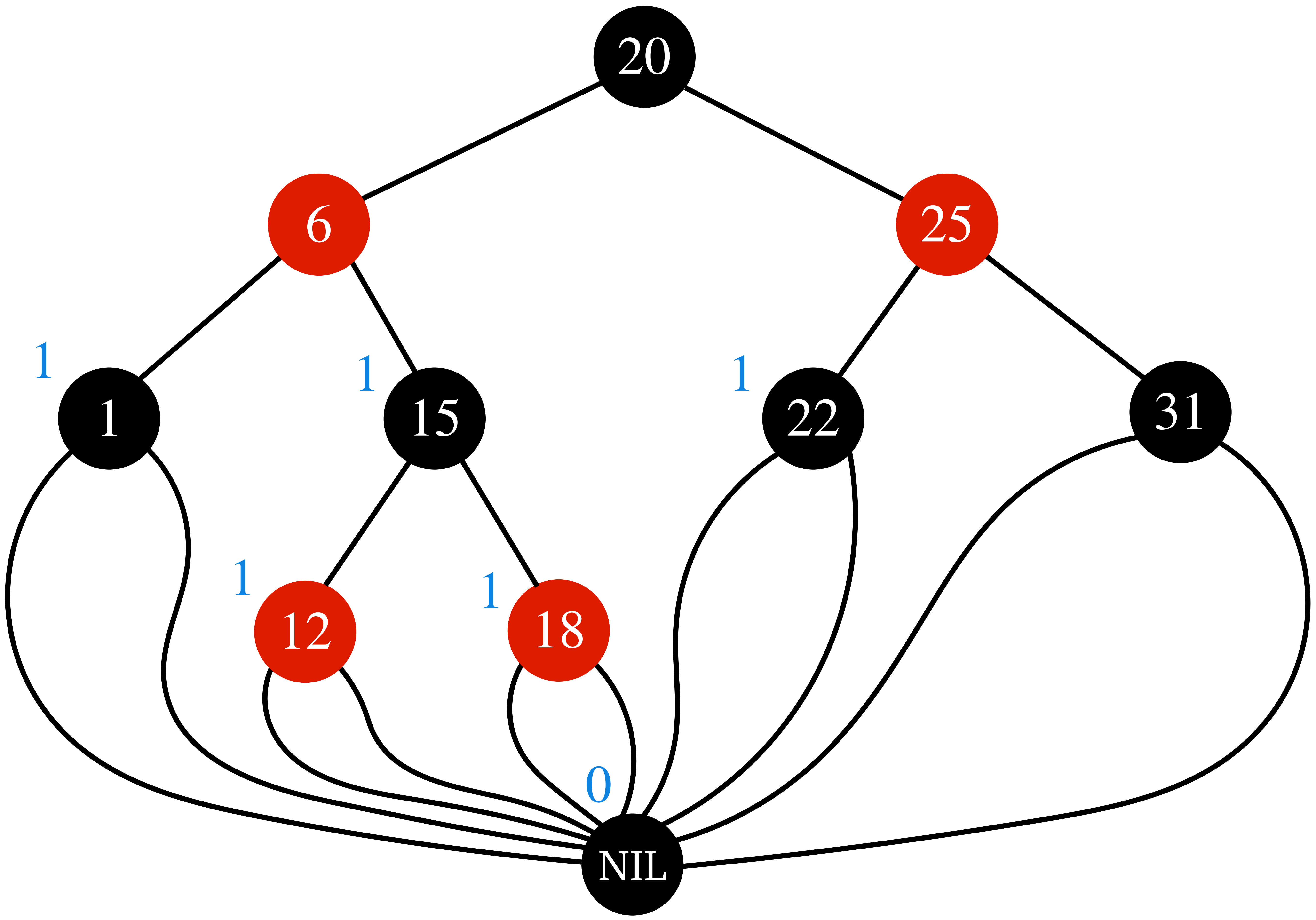

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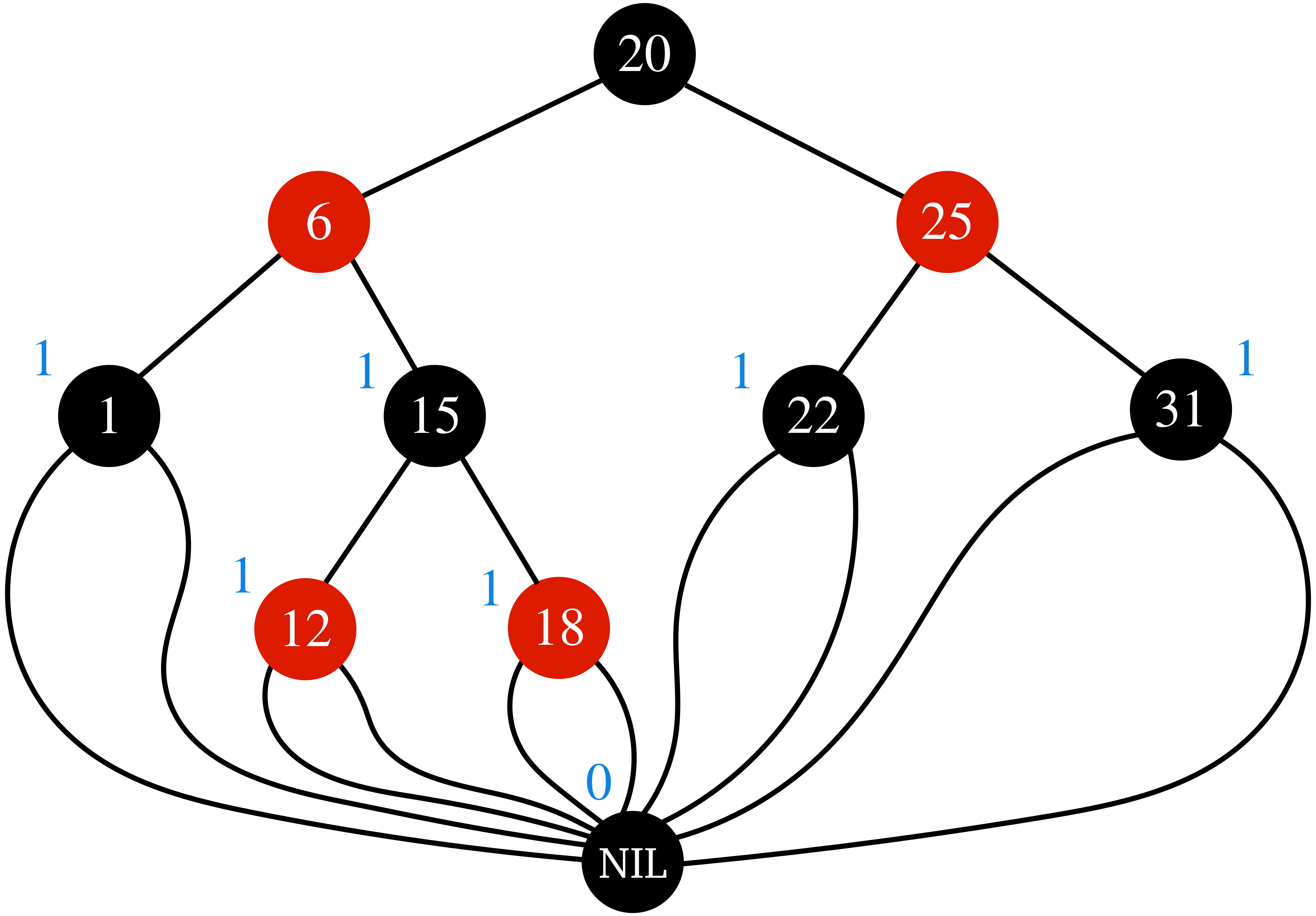

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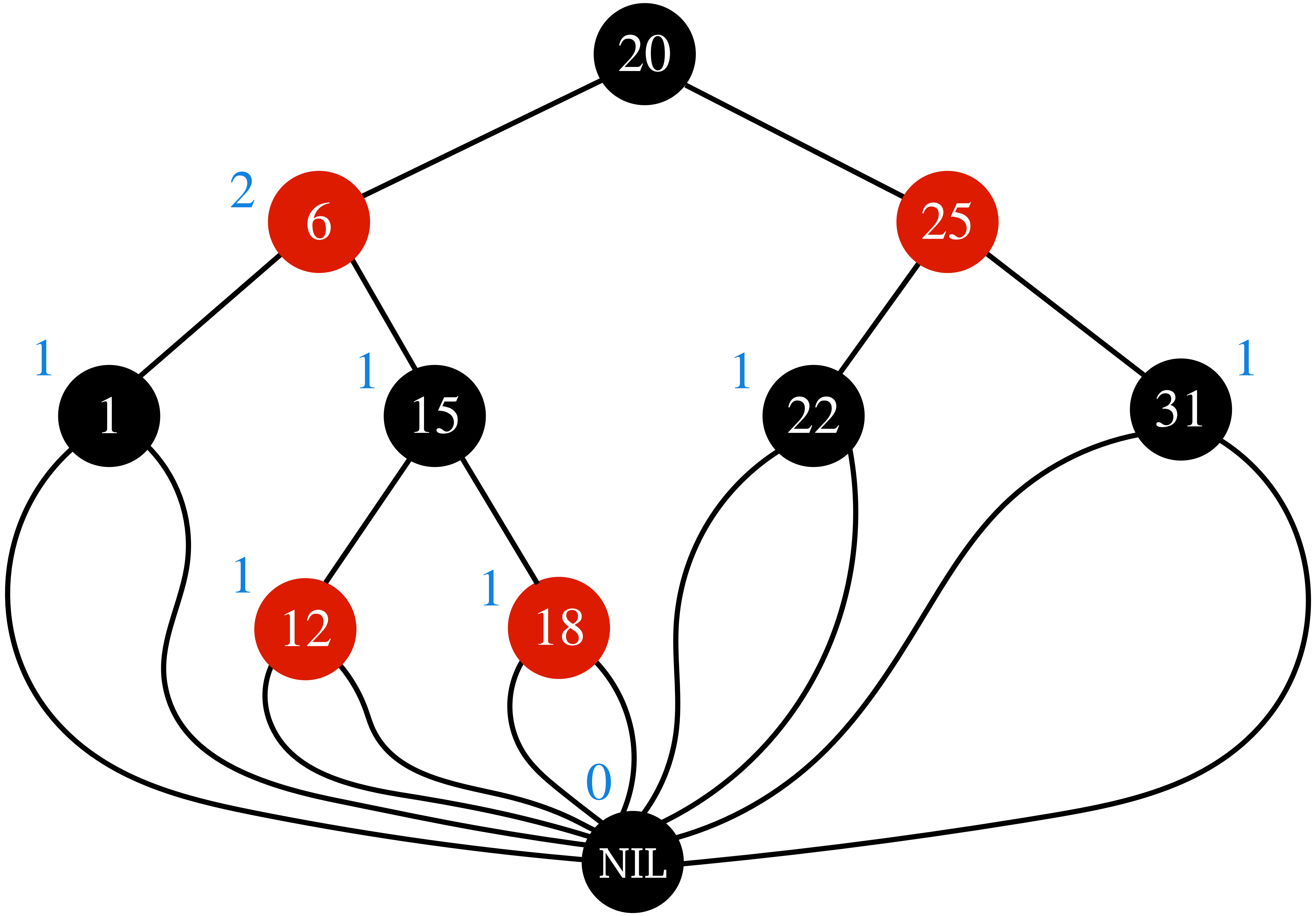

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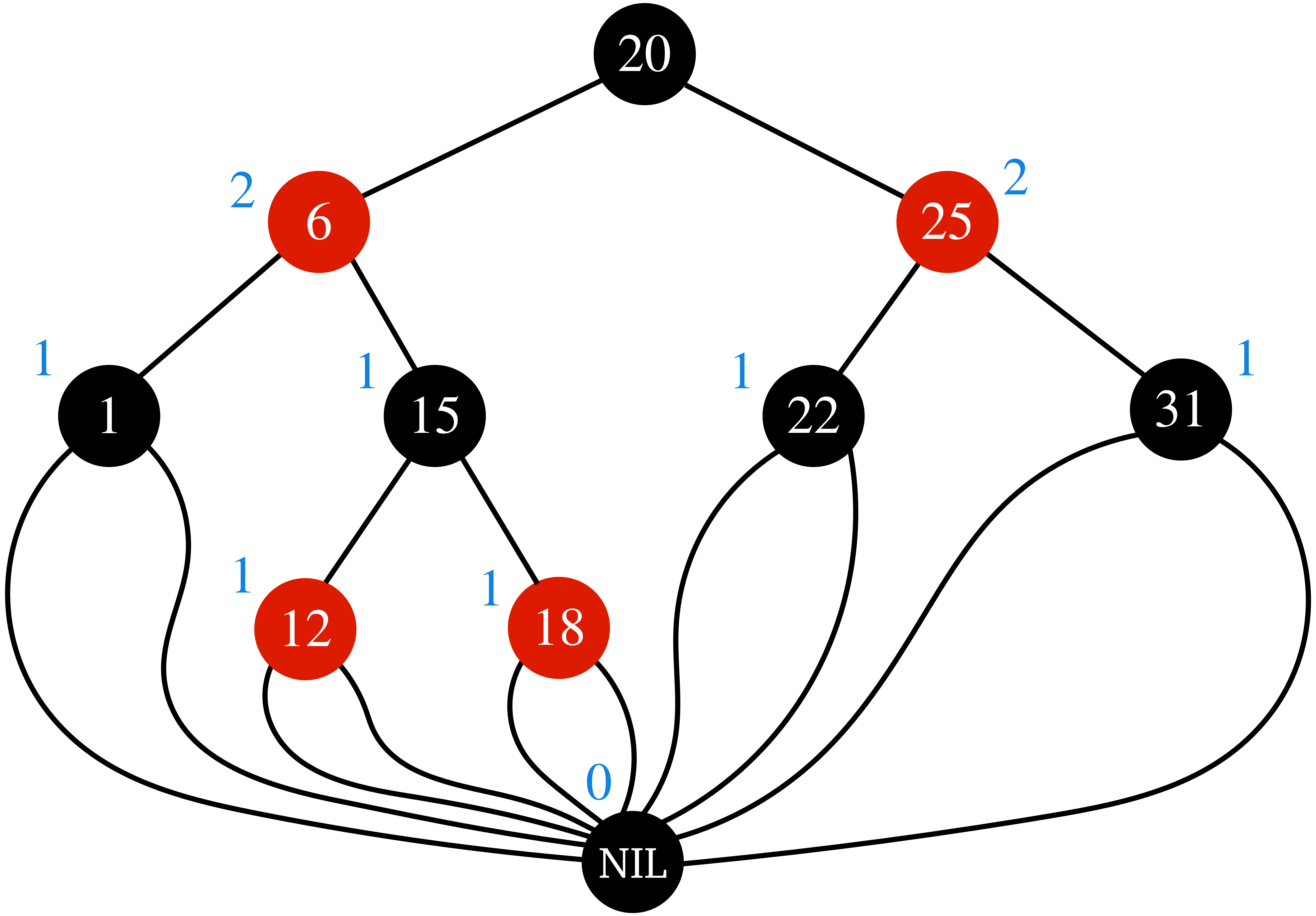

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