Lecture 6

BST Terminology and Operations

BST: Basic Terminology

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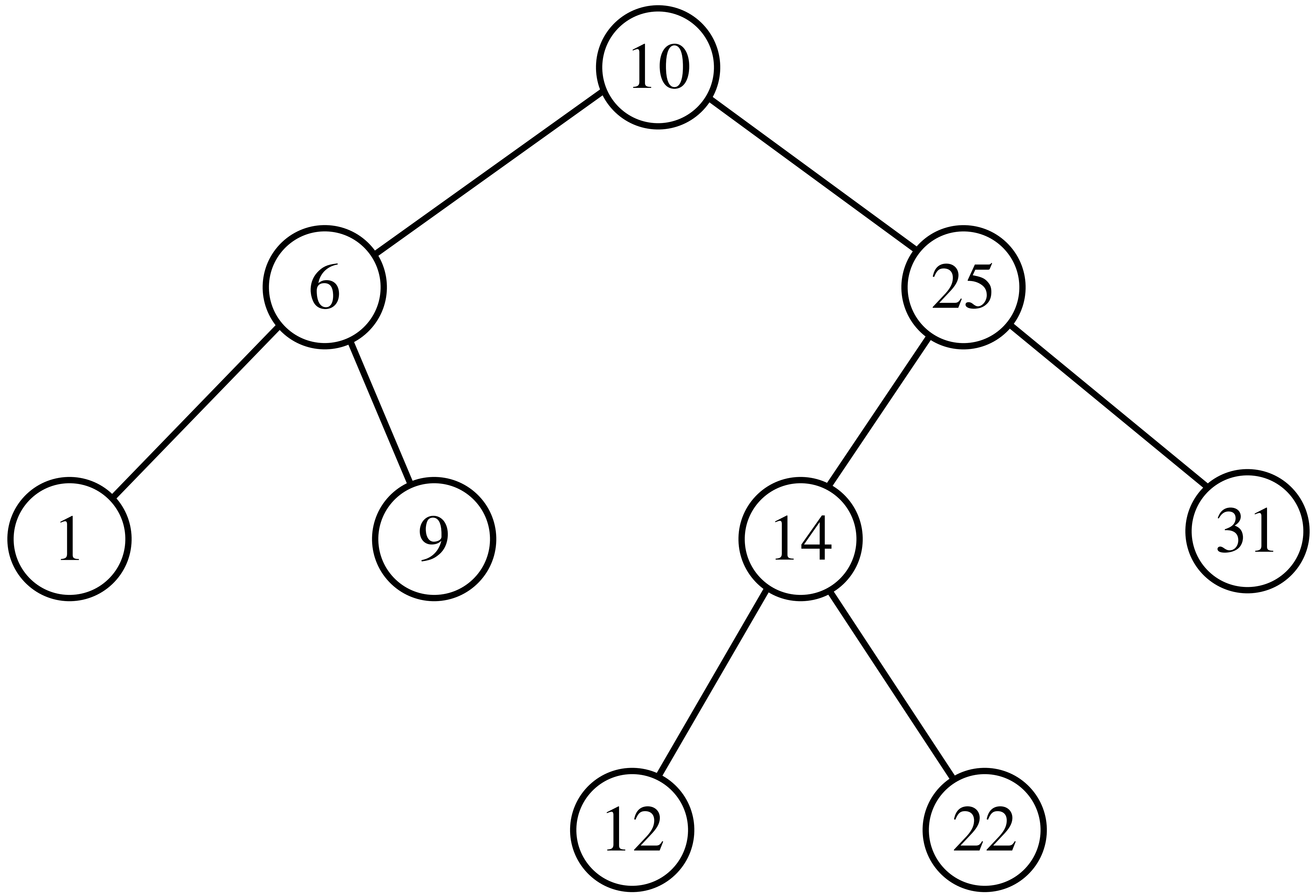

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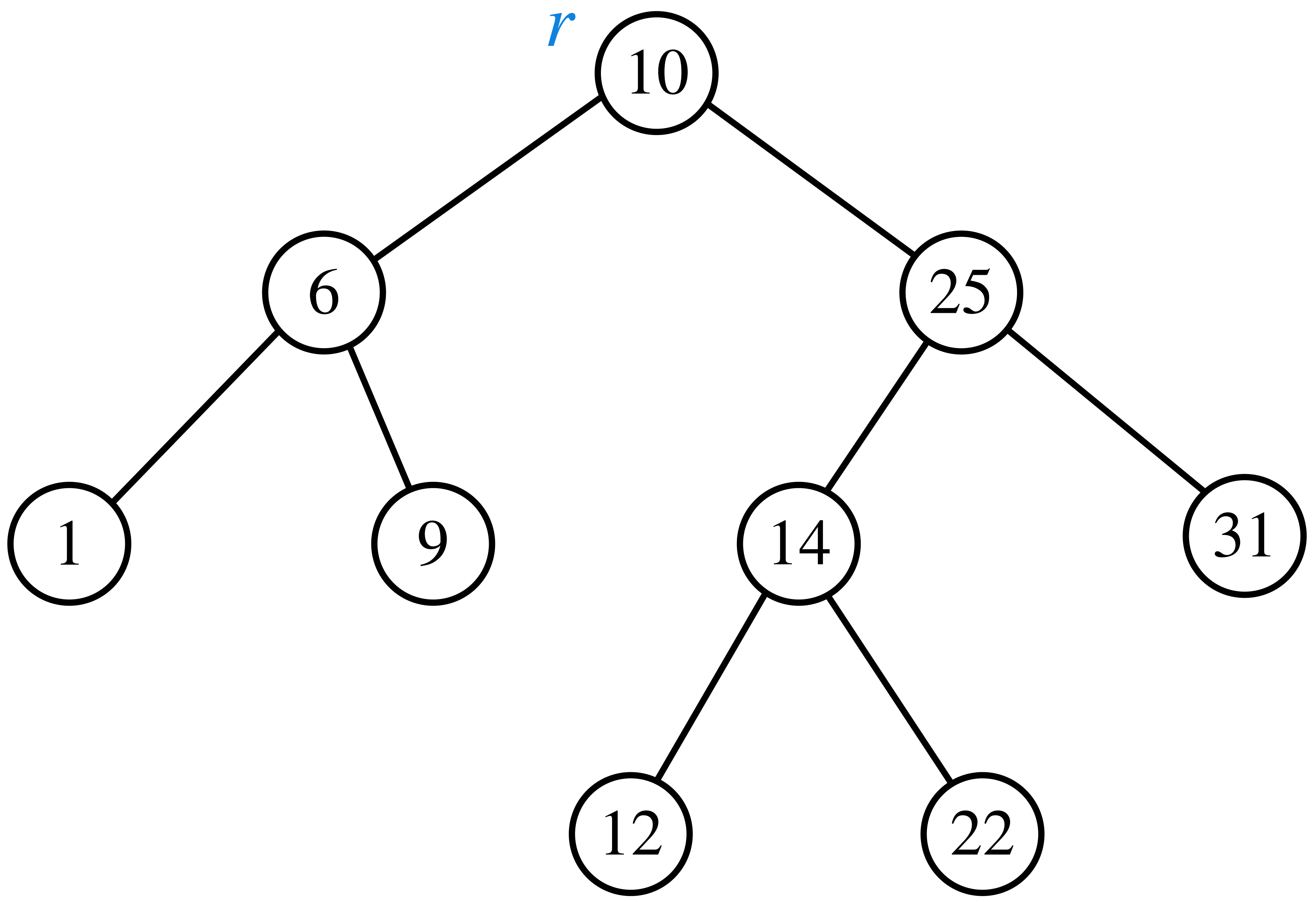

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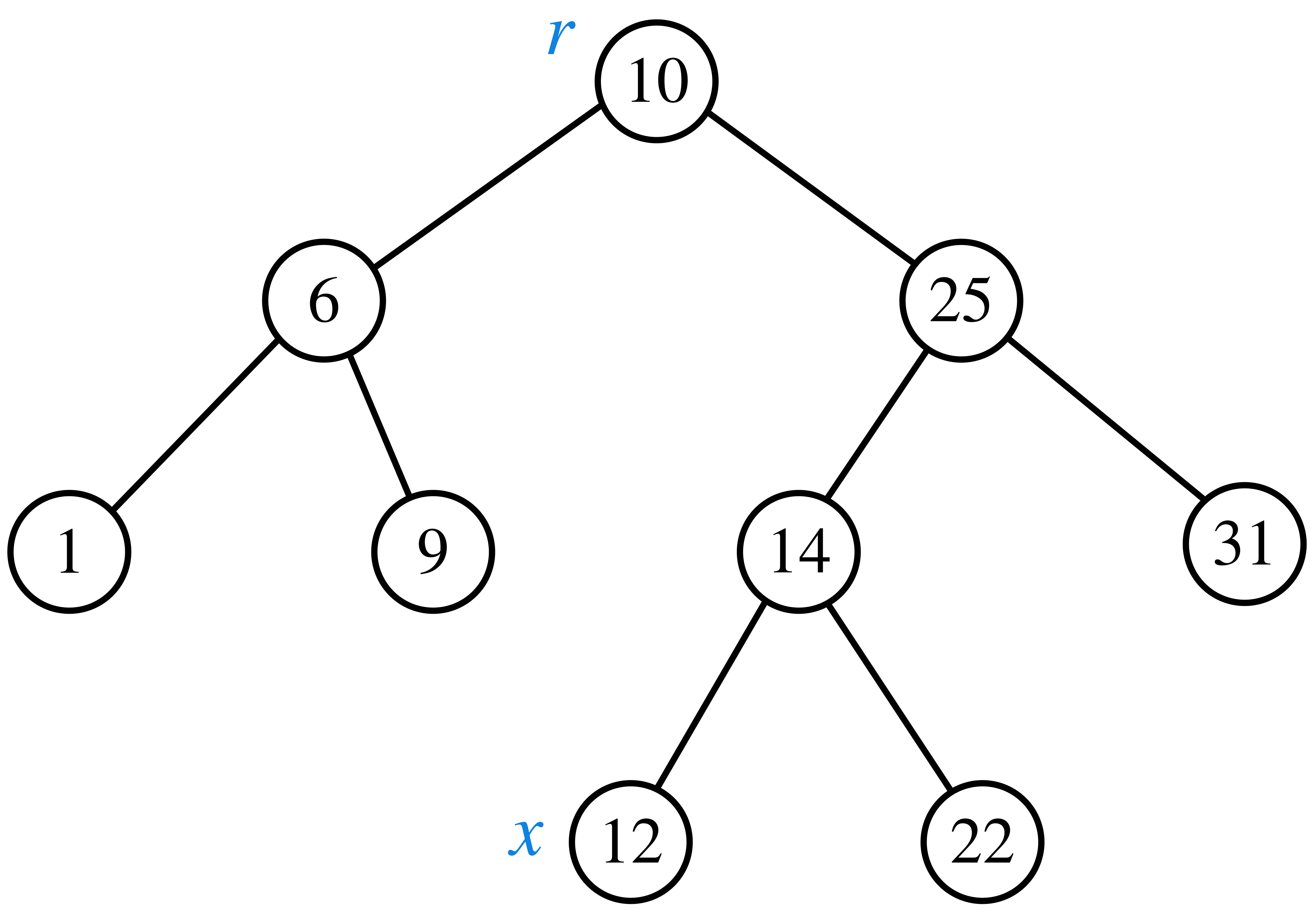

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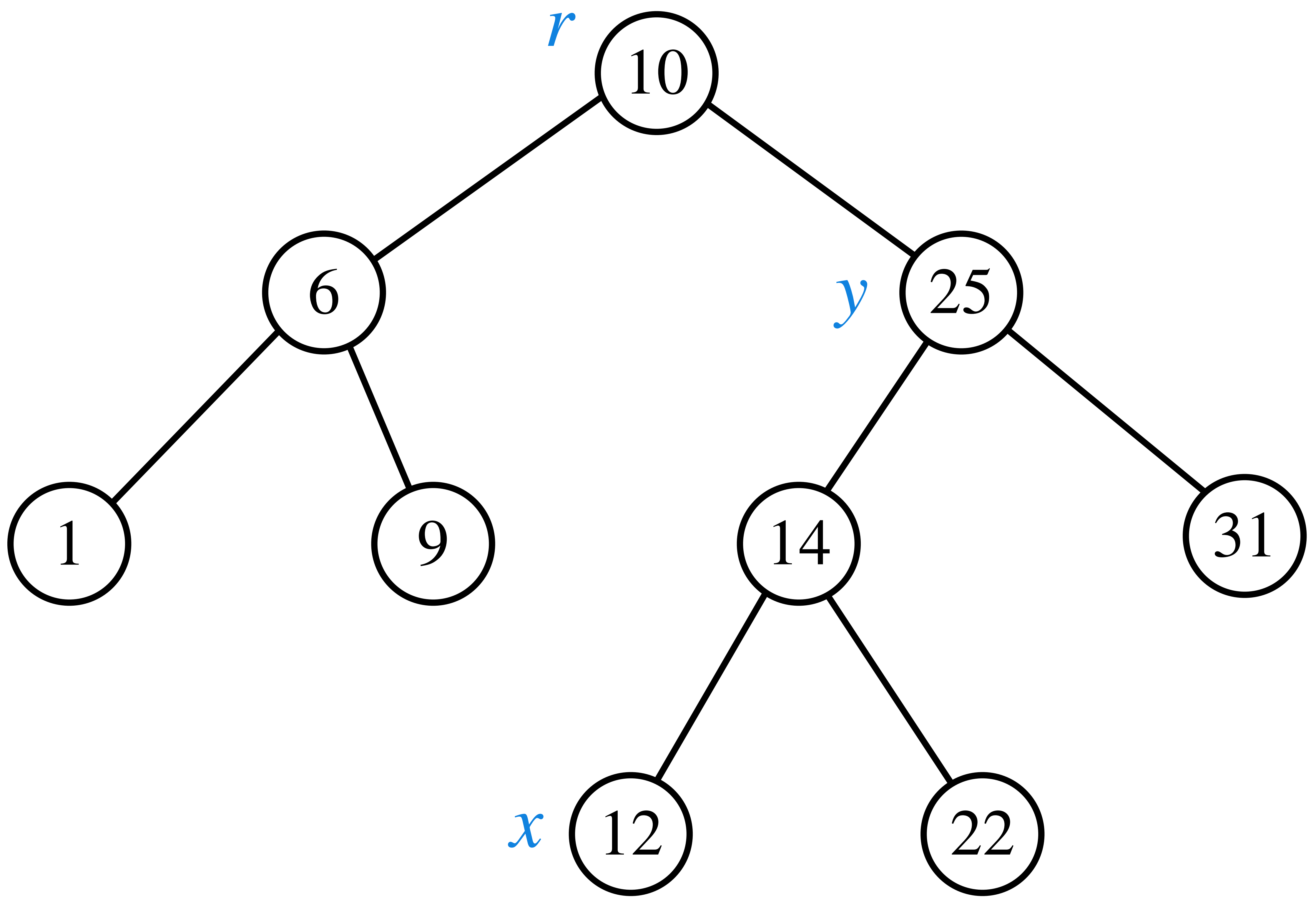

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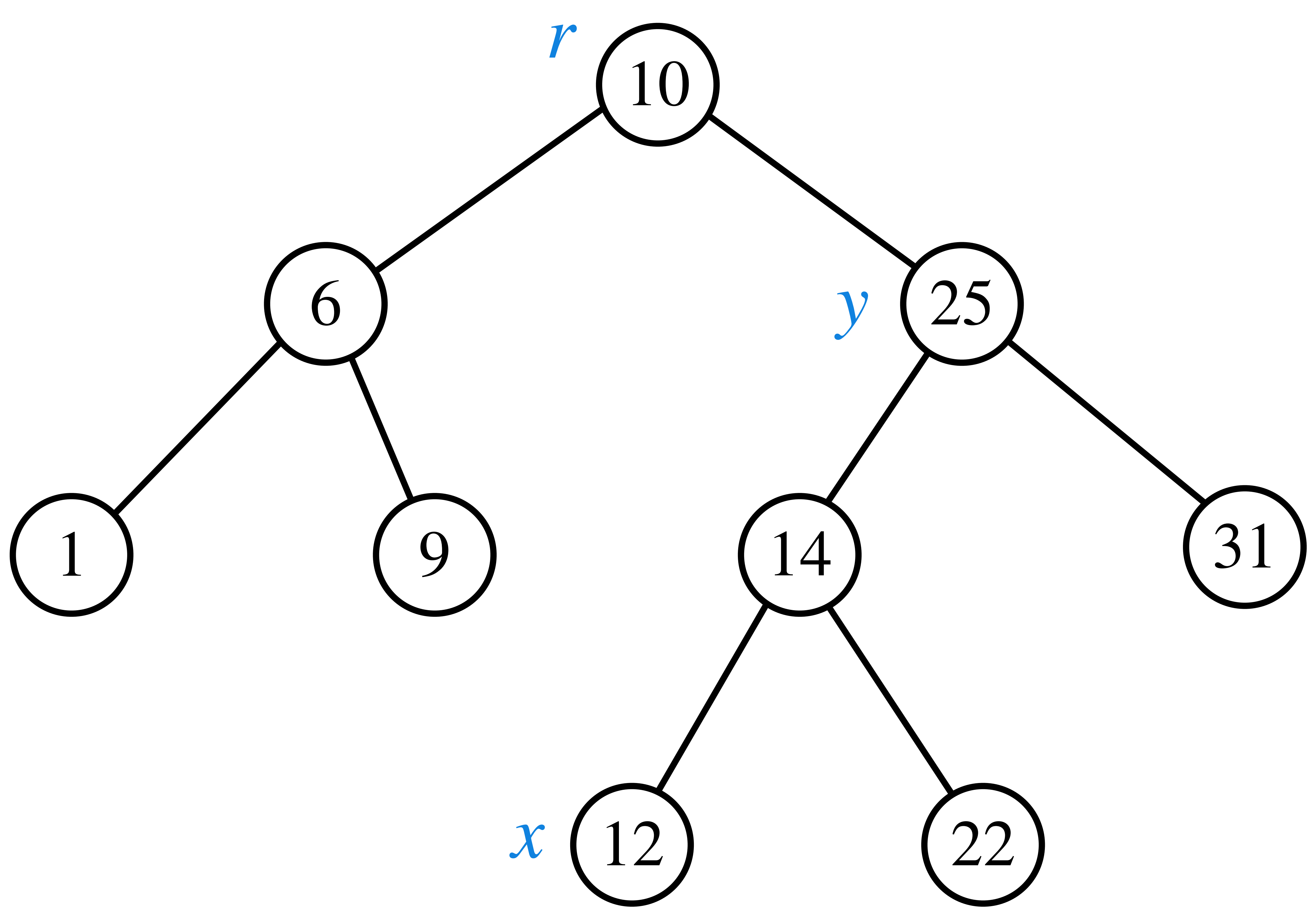

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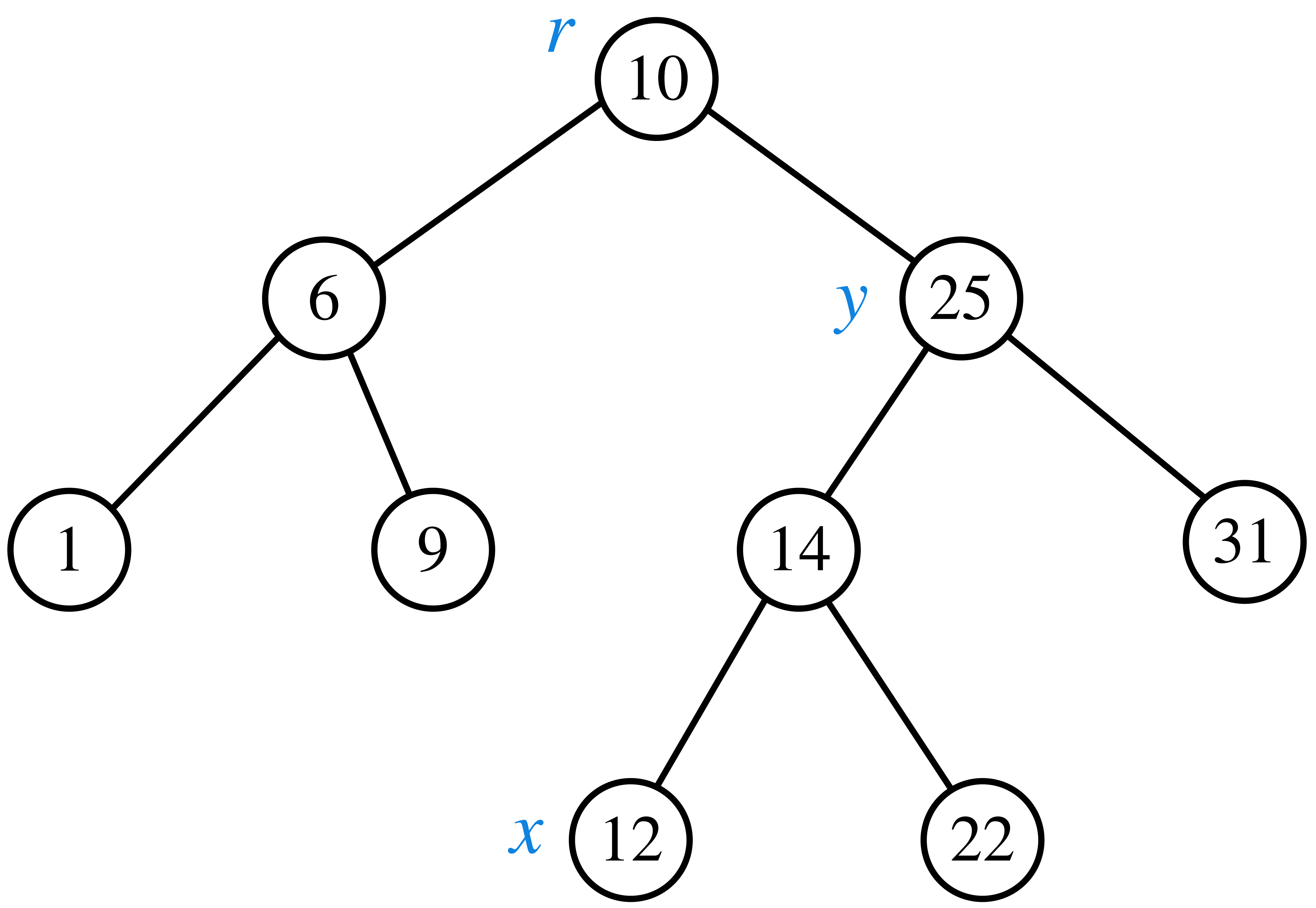

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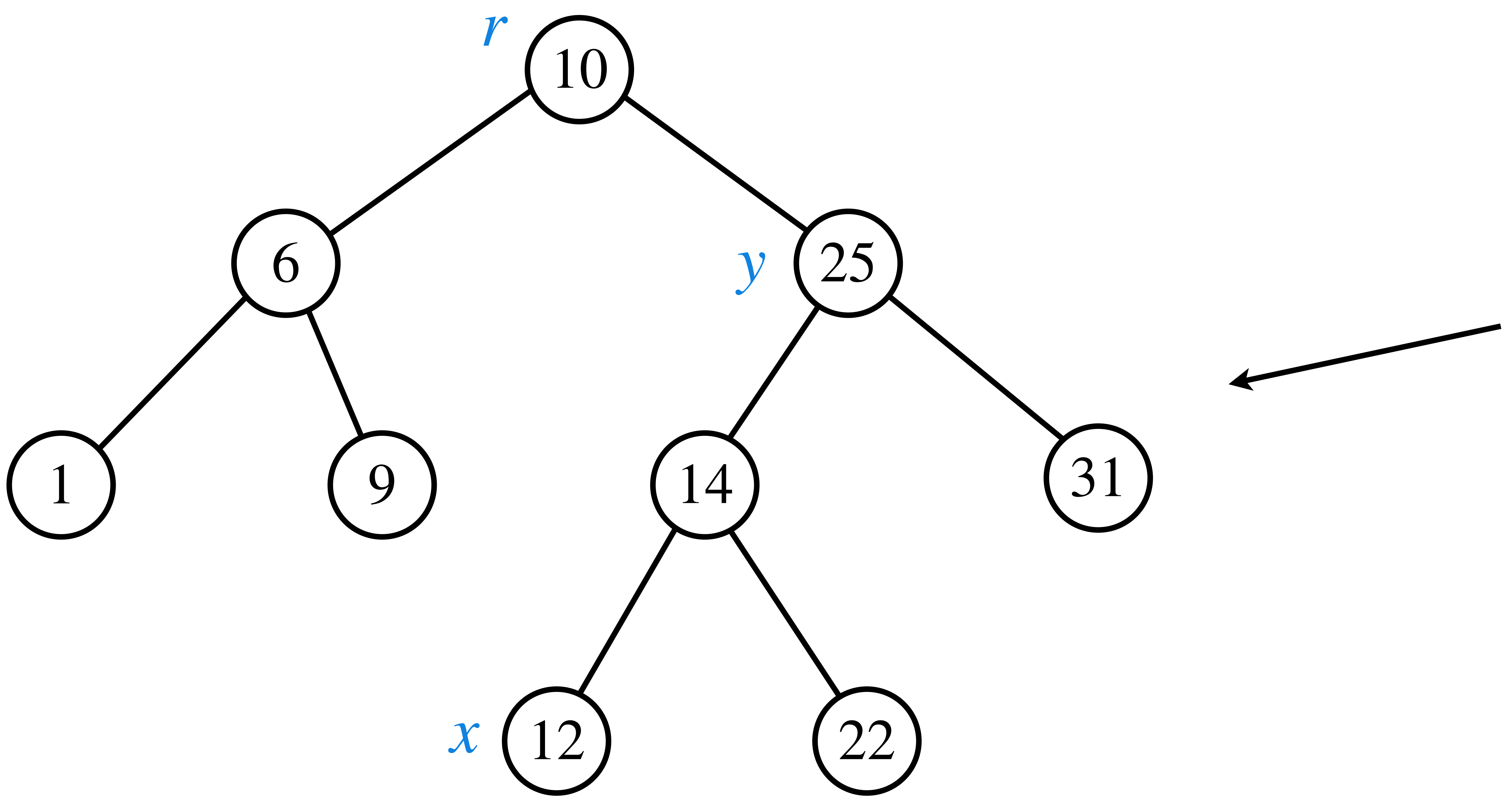

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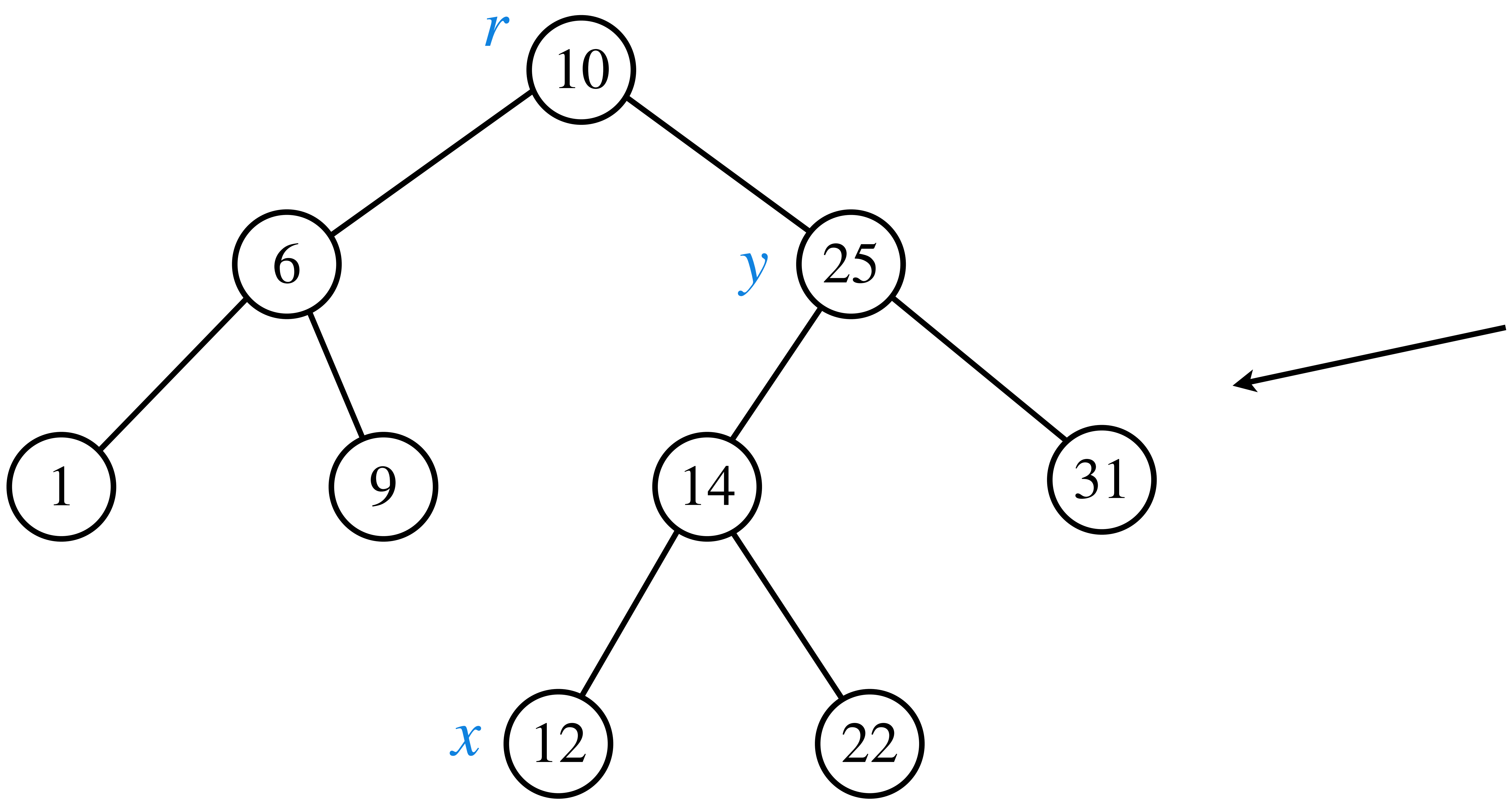

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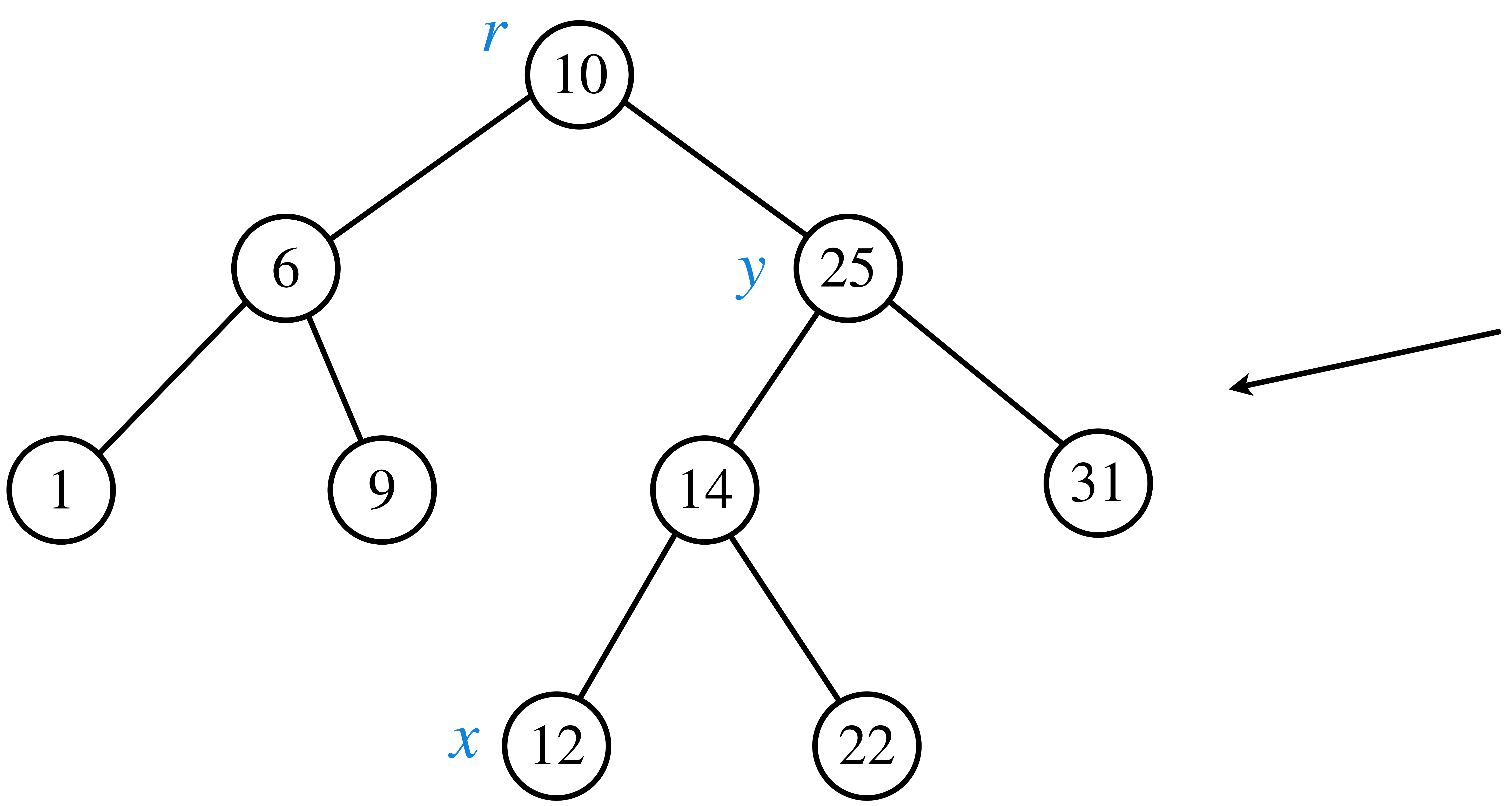

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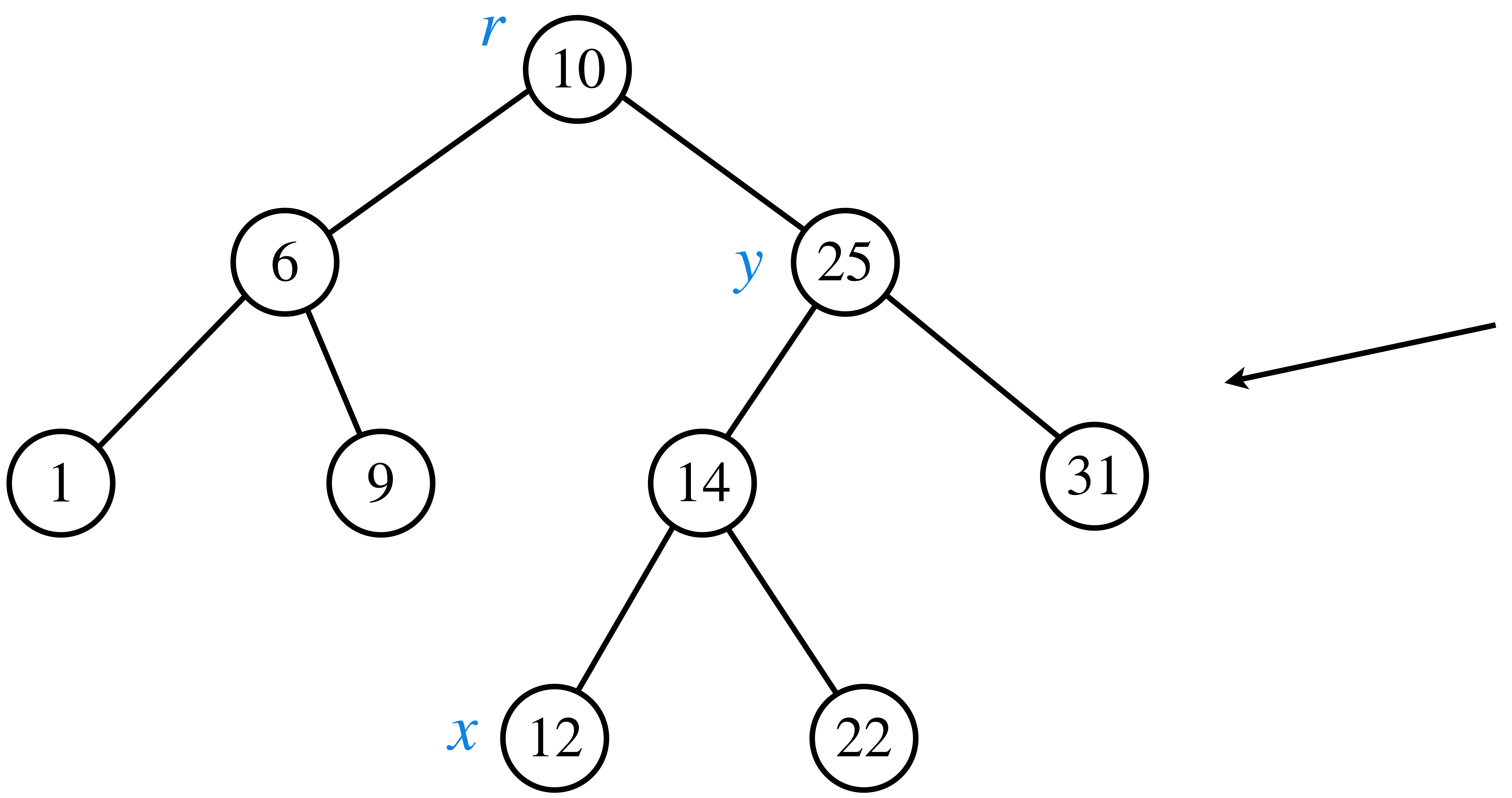

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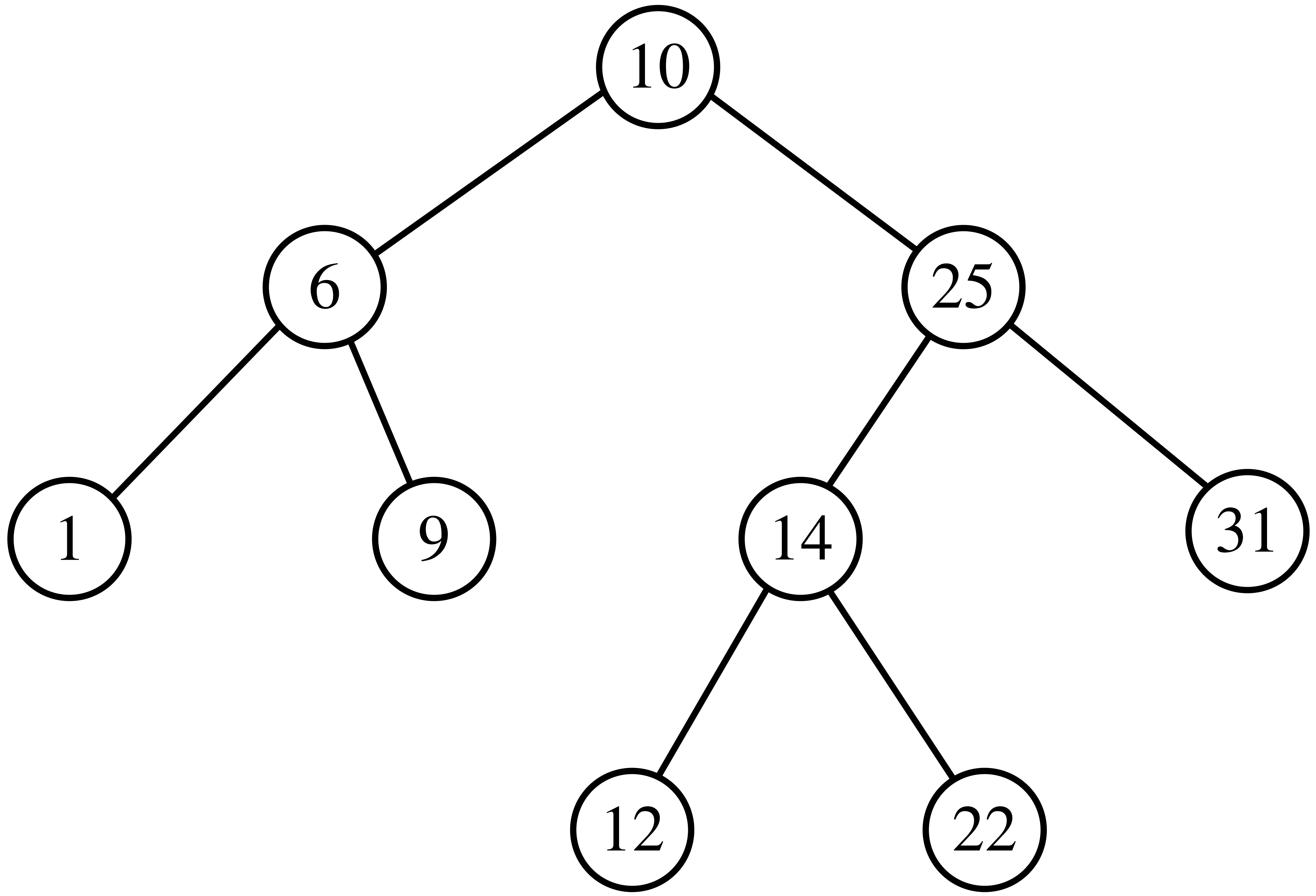

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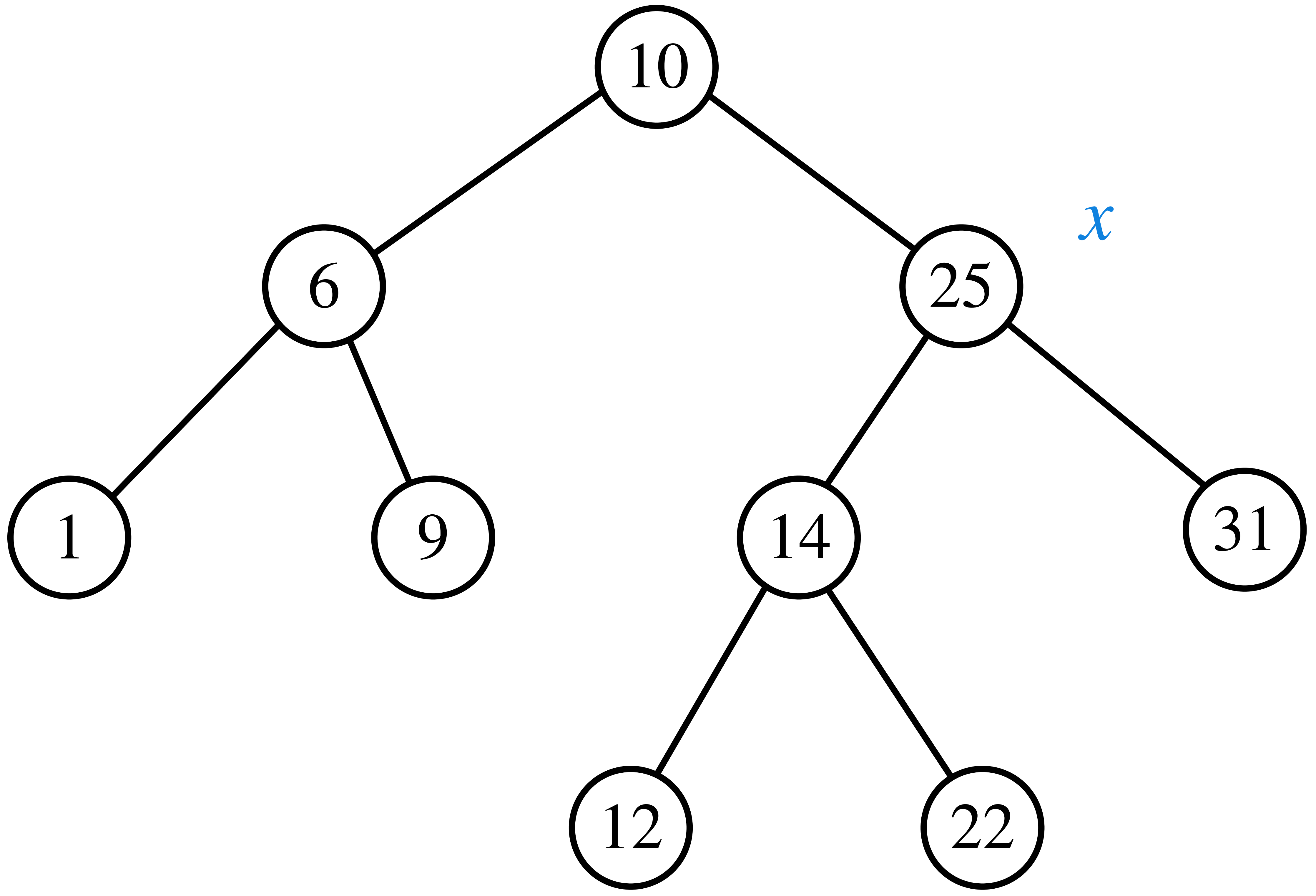

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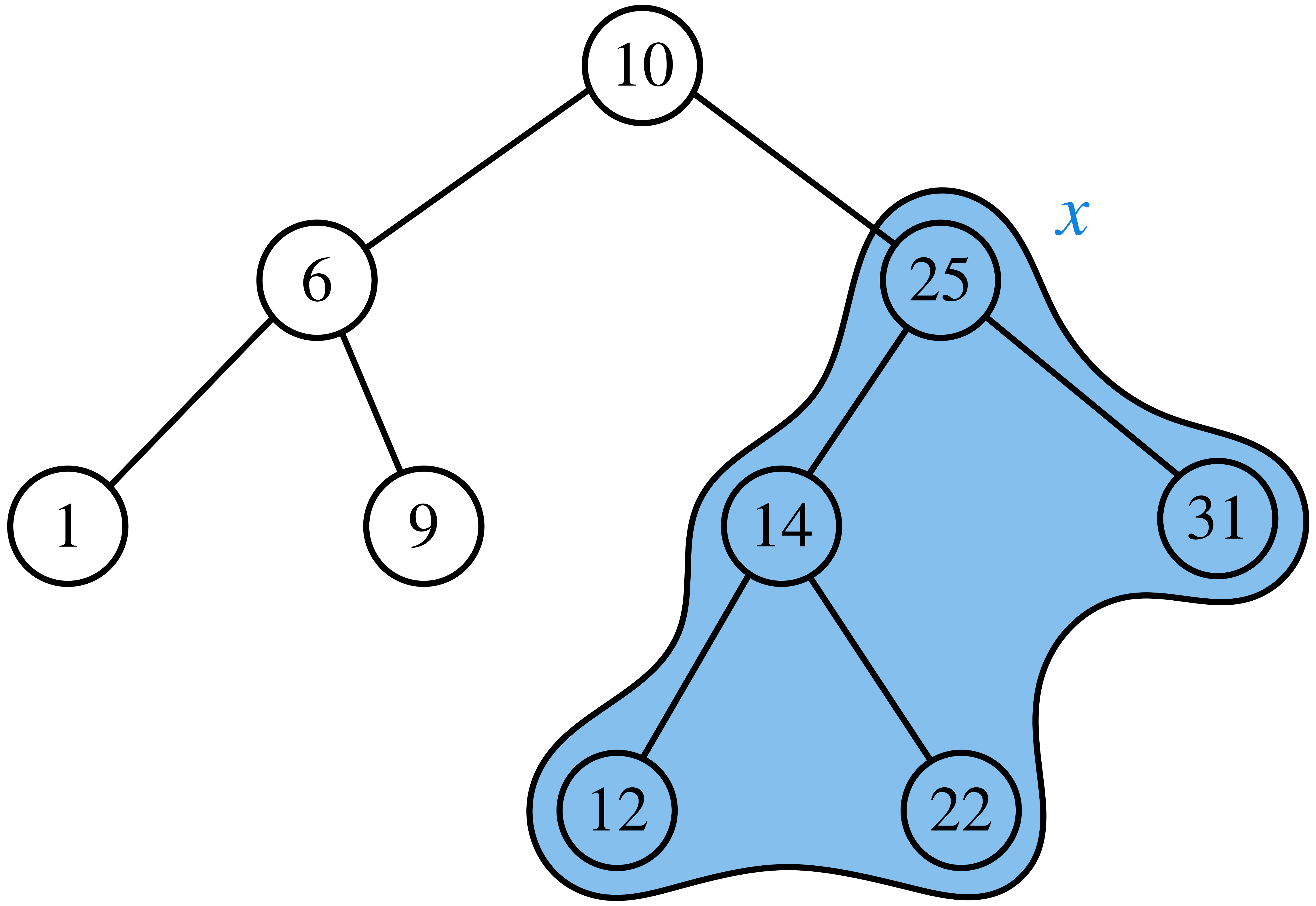

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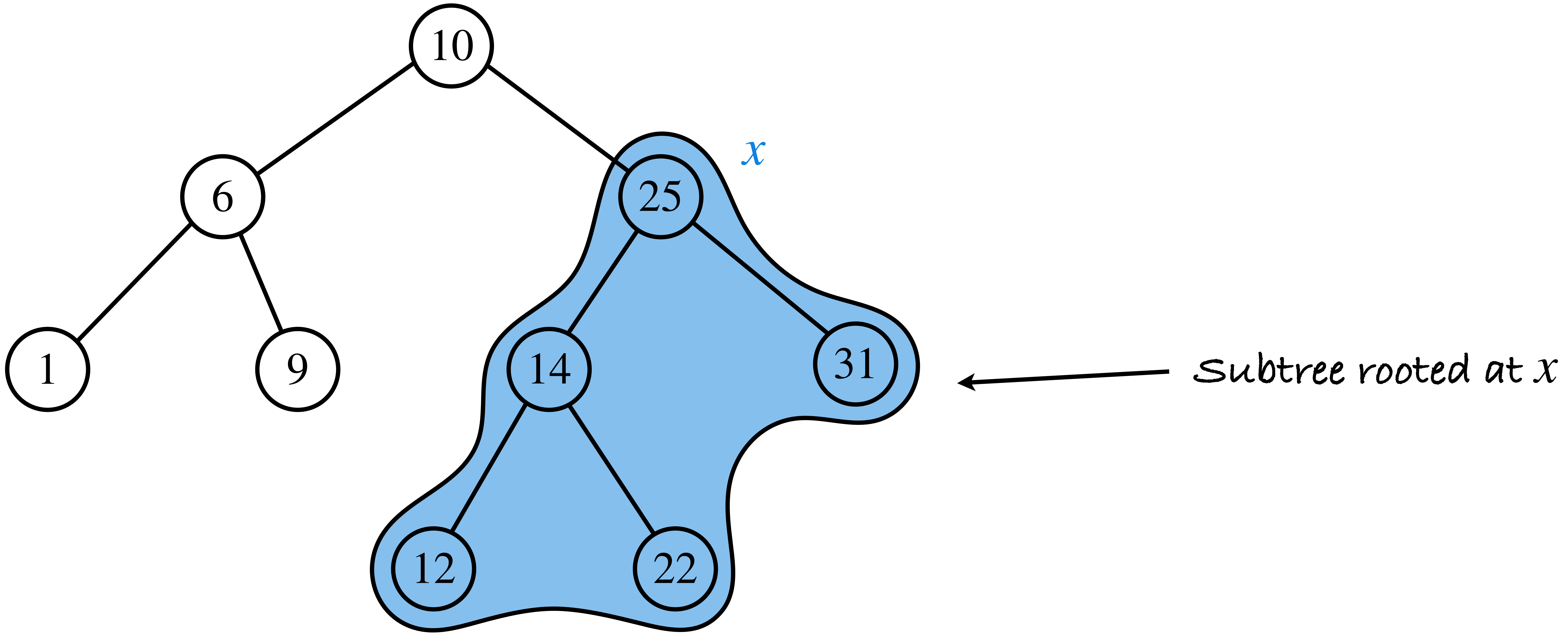

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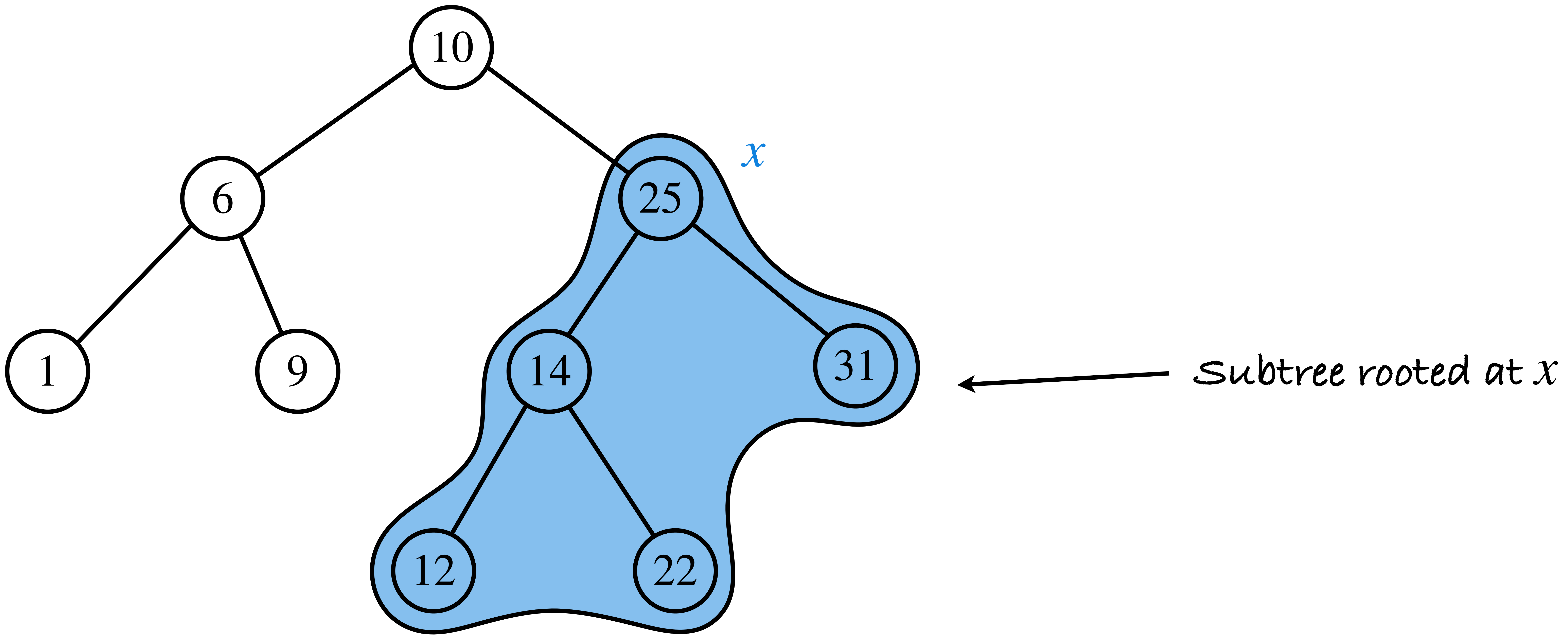
Defn: Subtree rooted at x is the tree containing only descendants of x.



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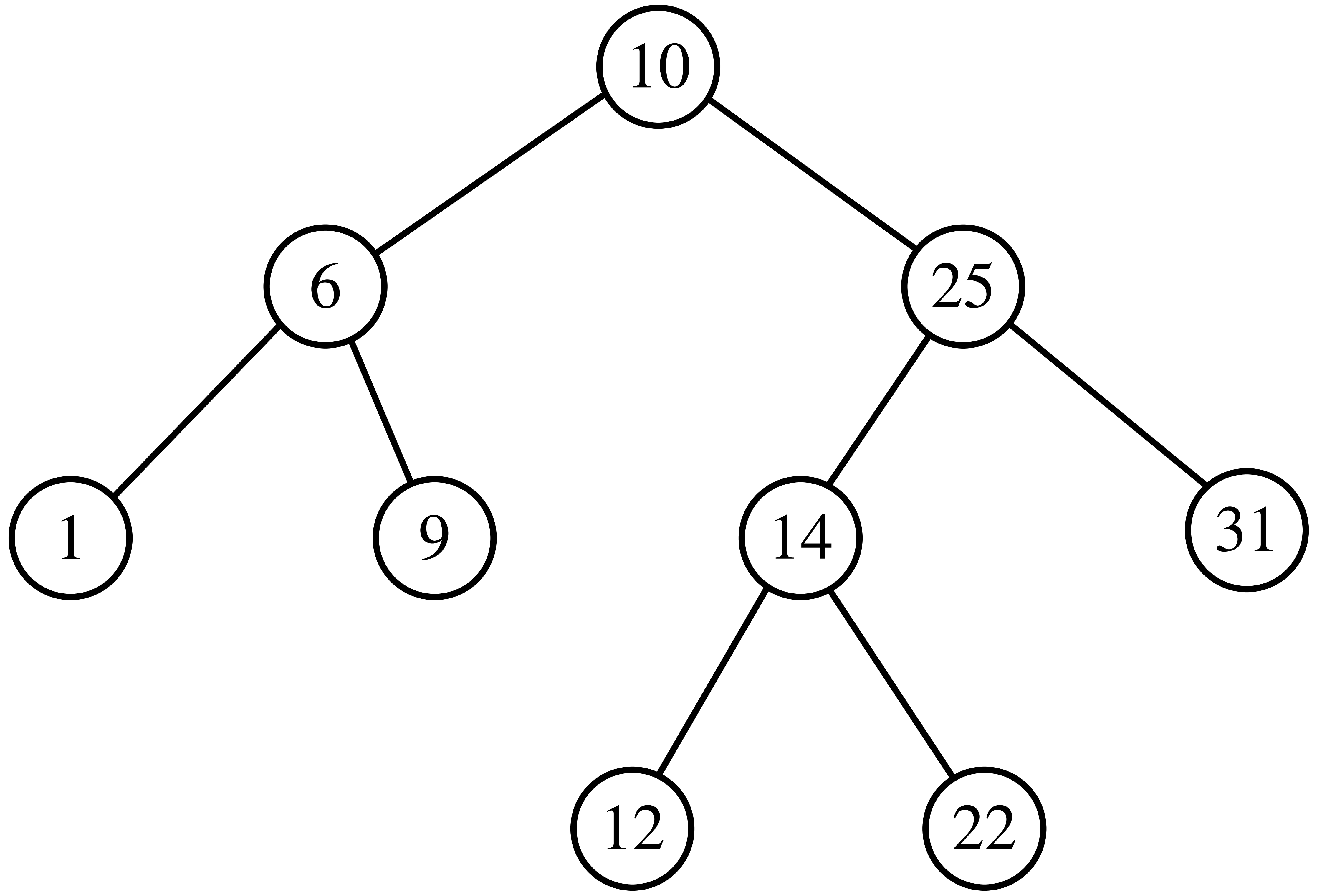

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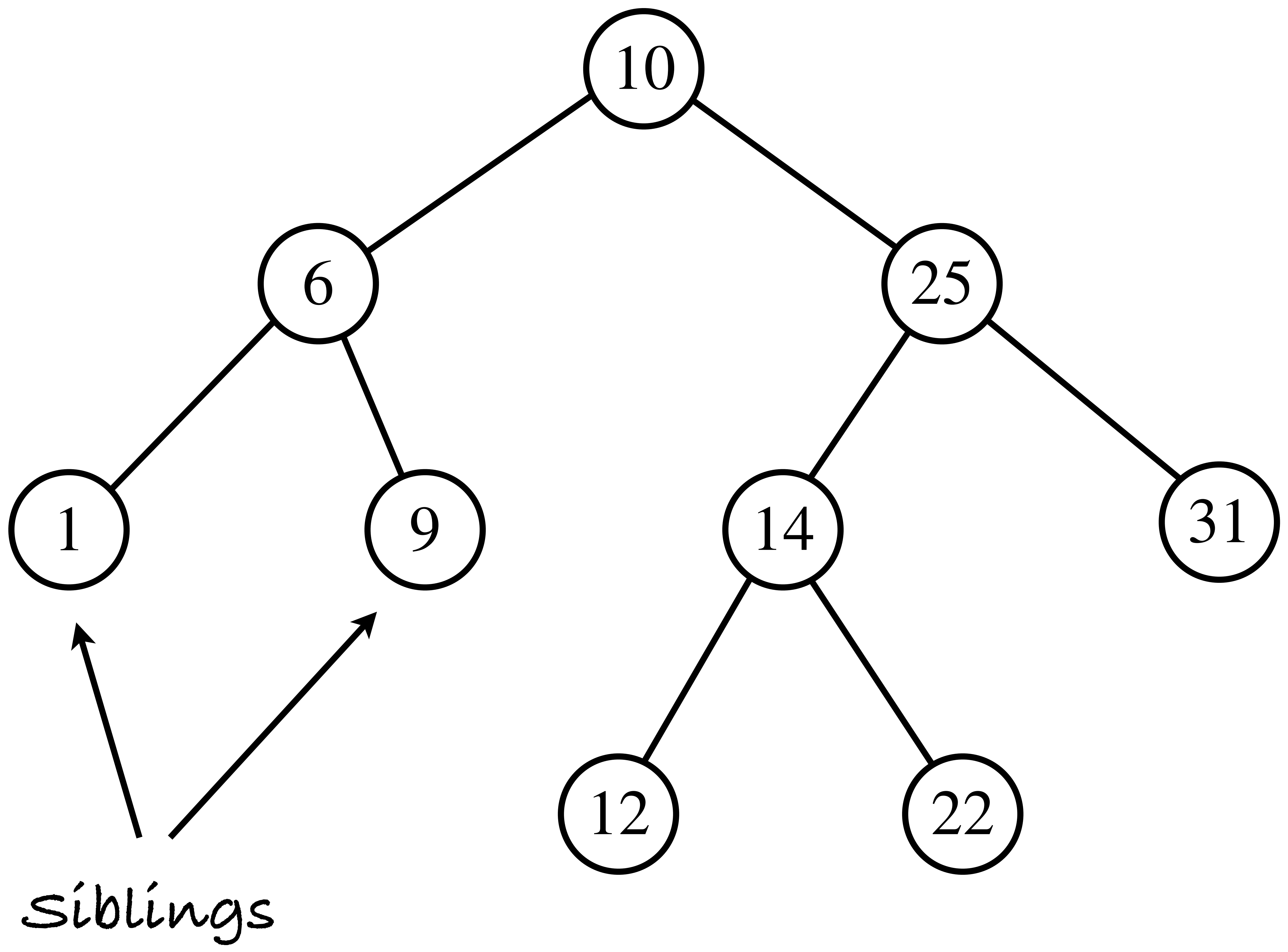

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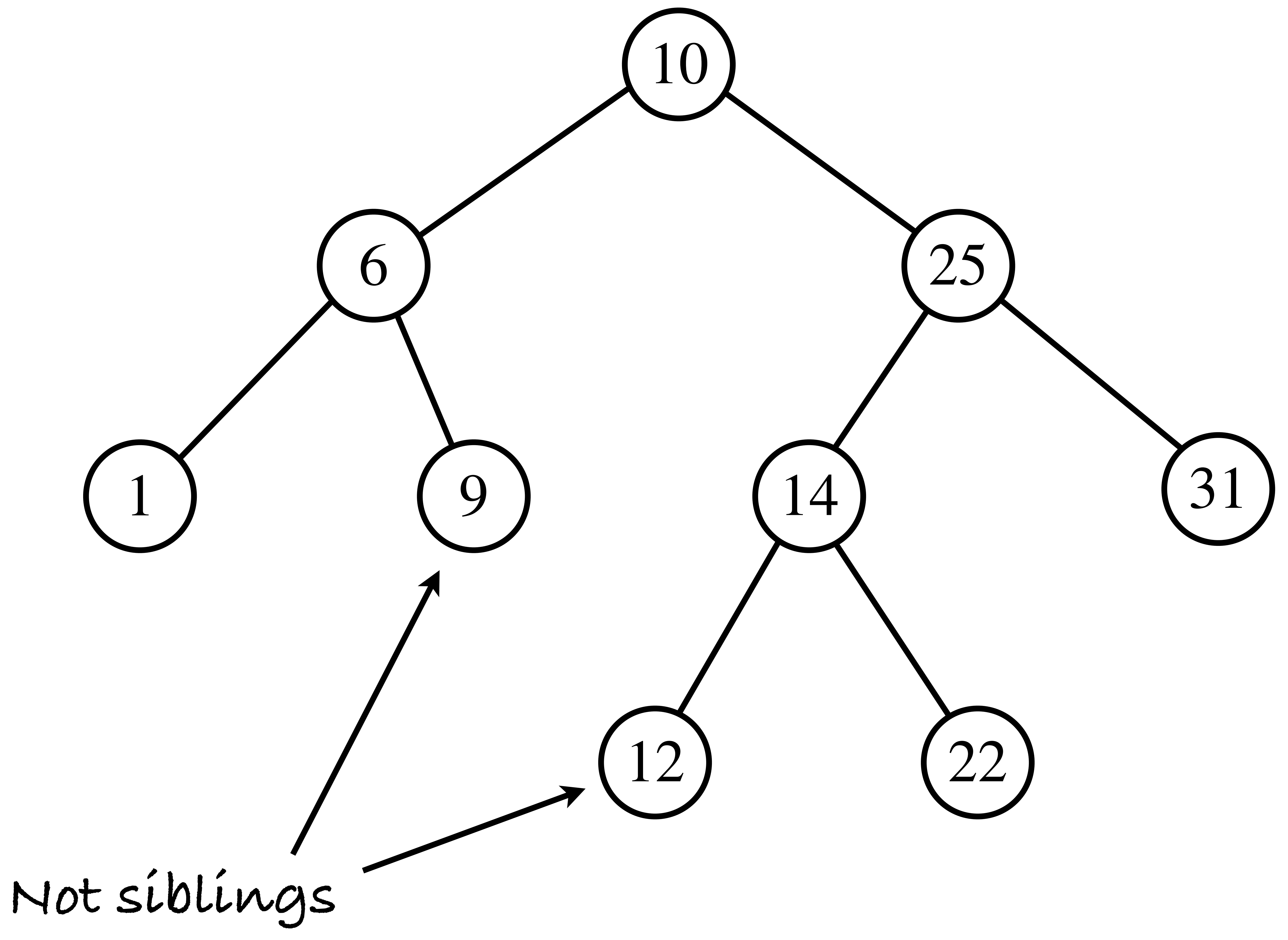

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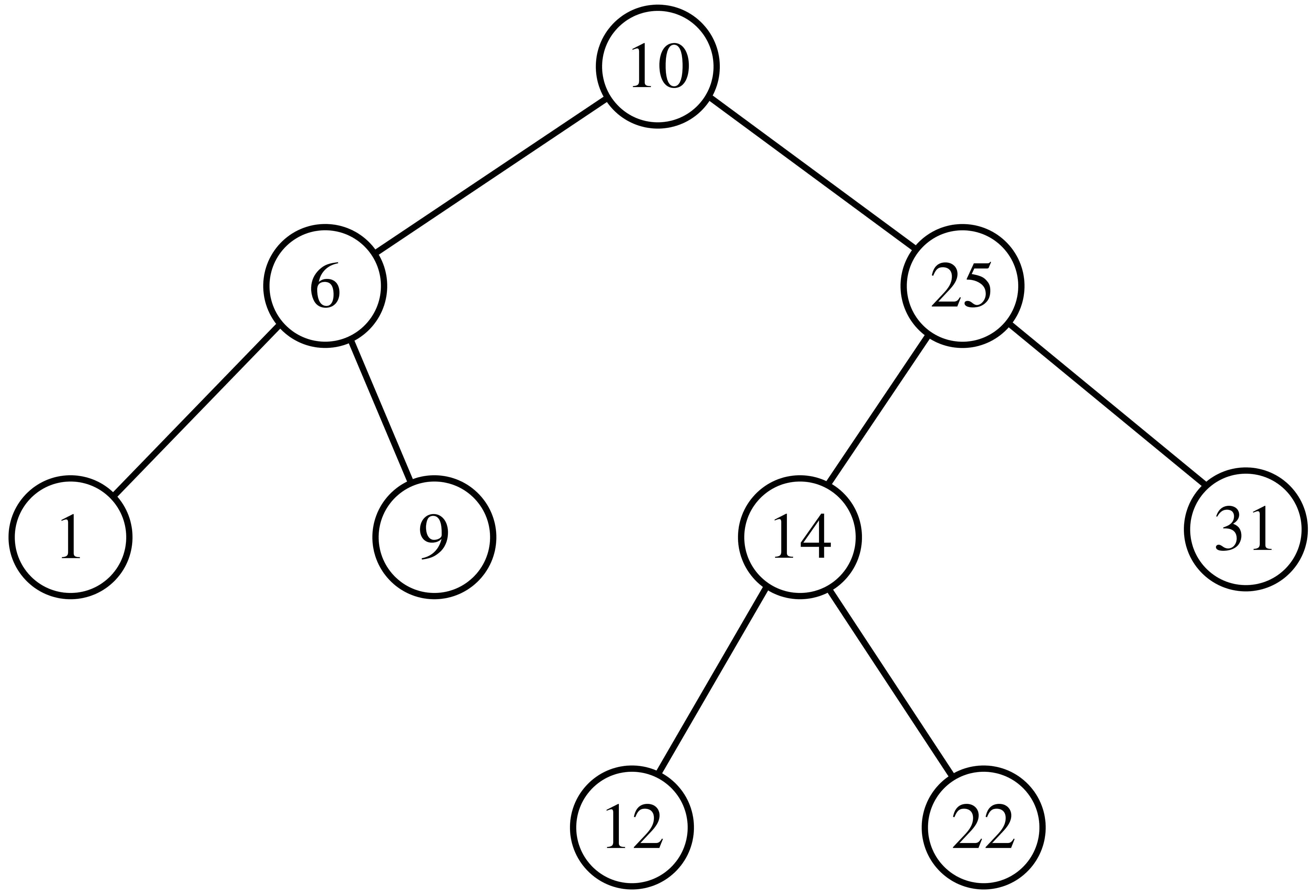

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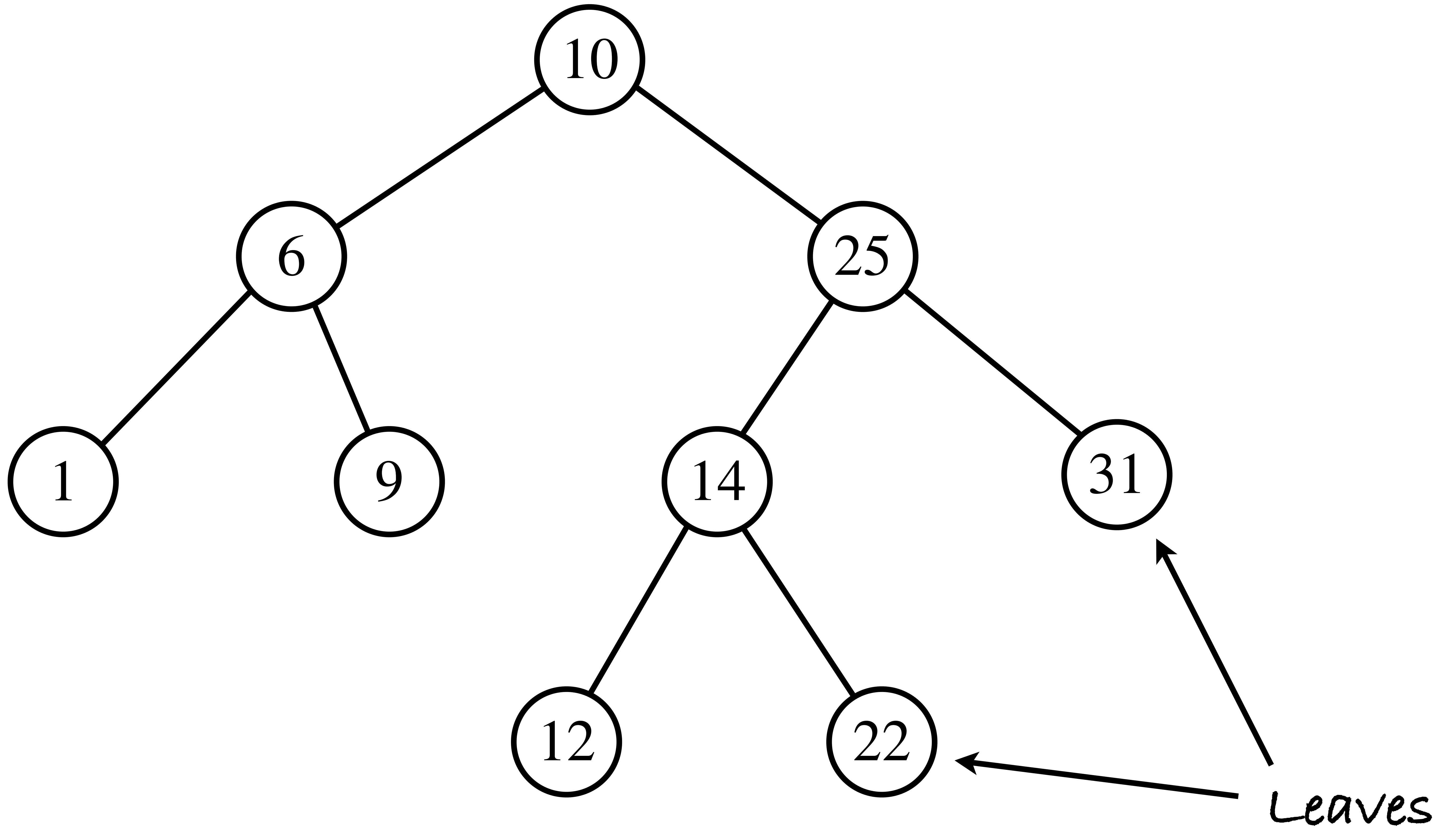

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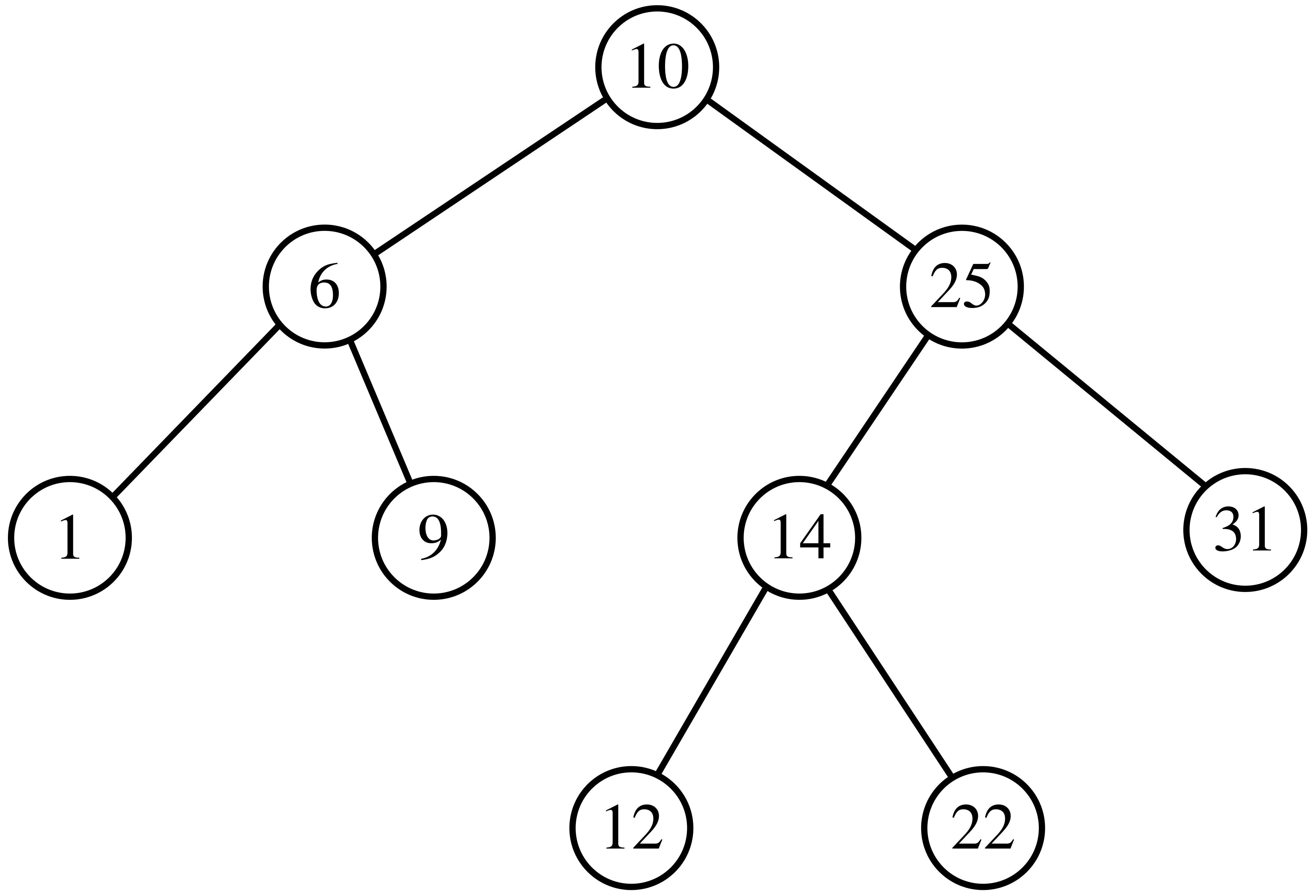

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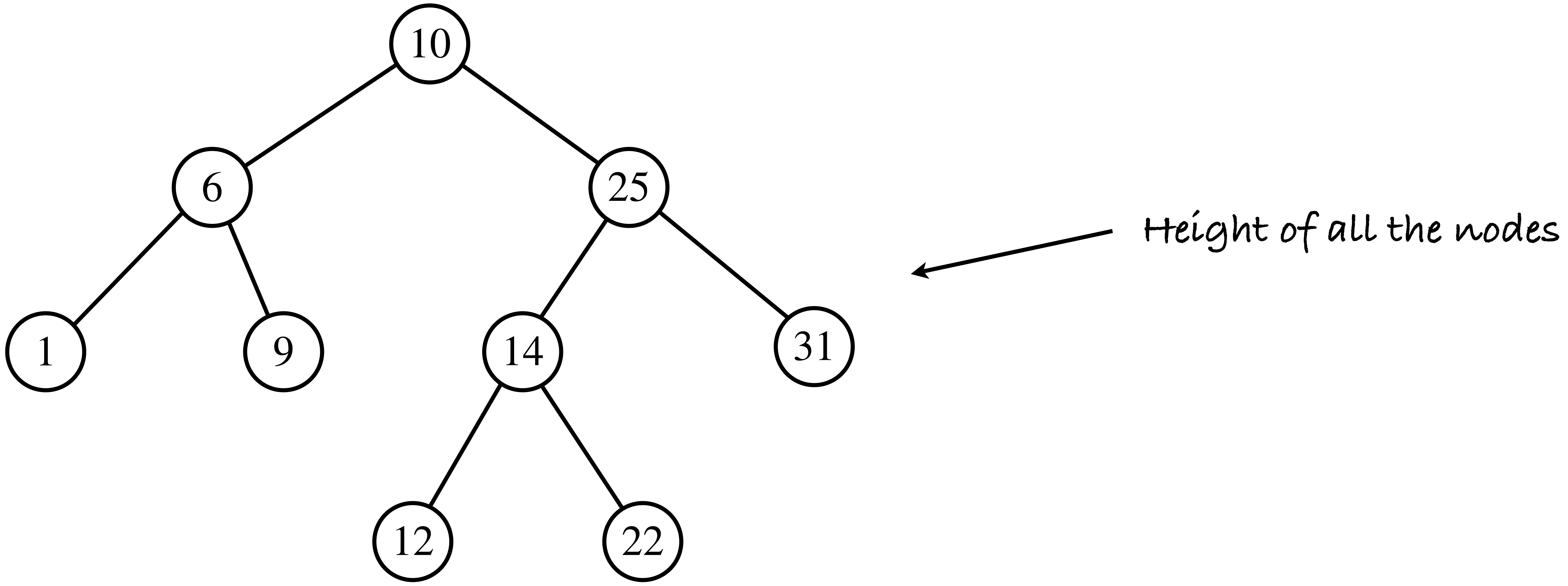

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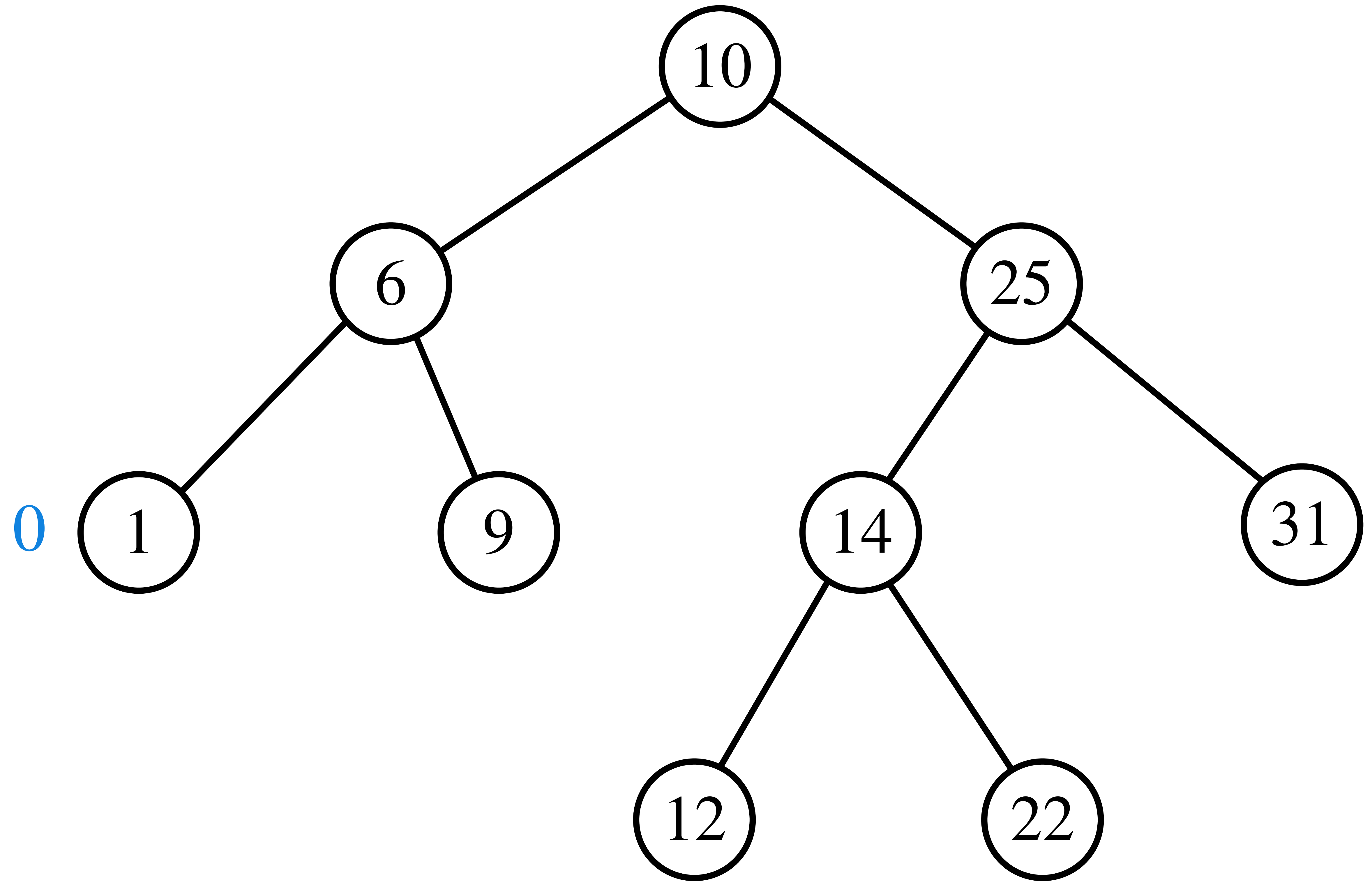

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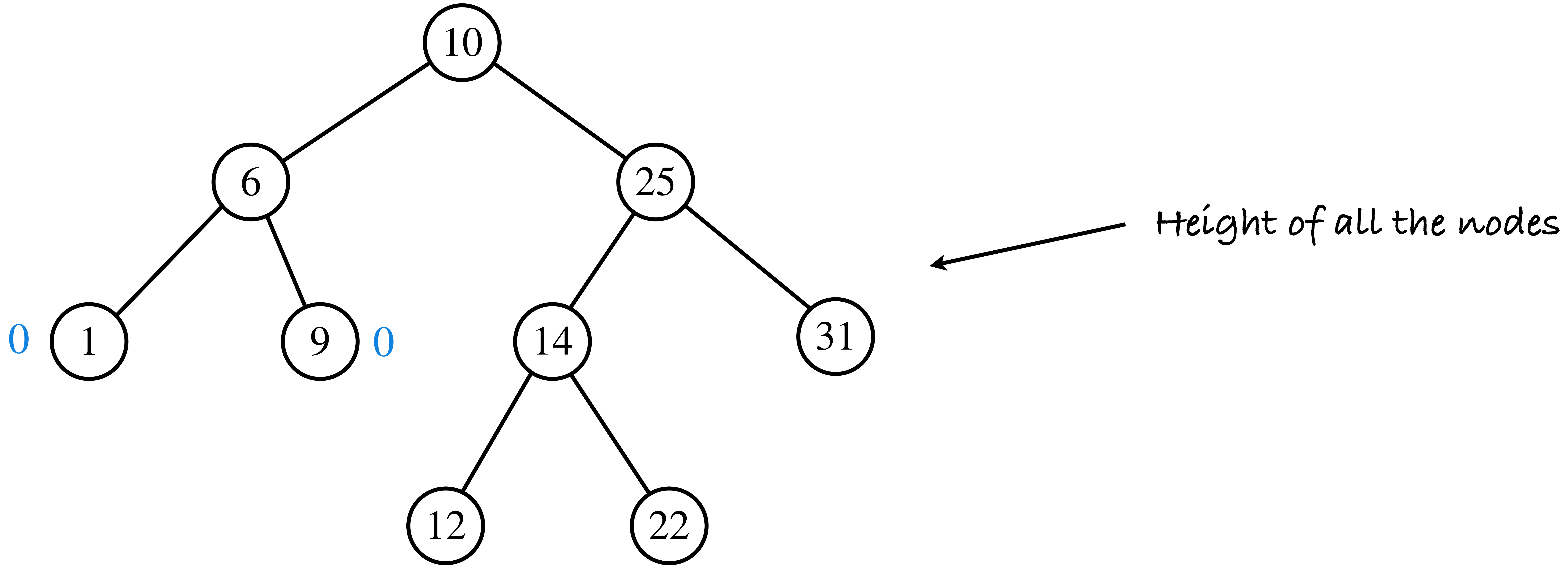

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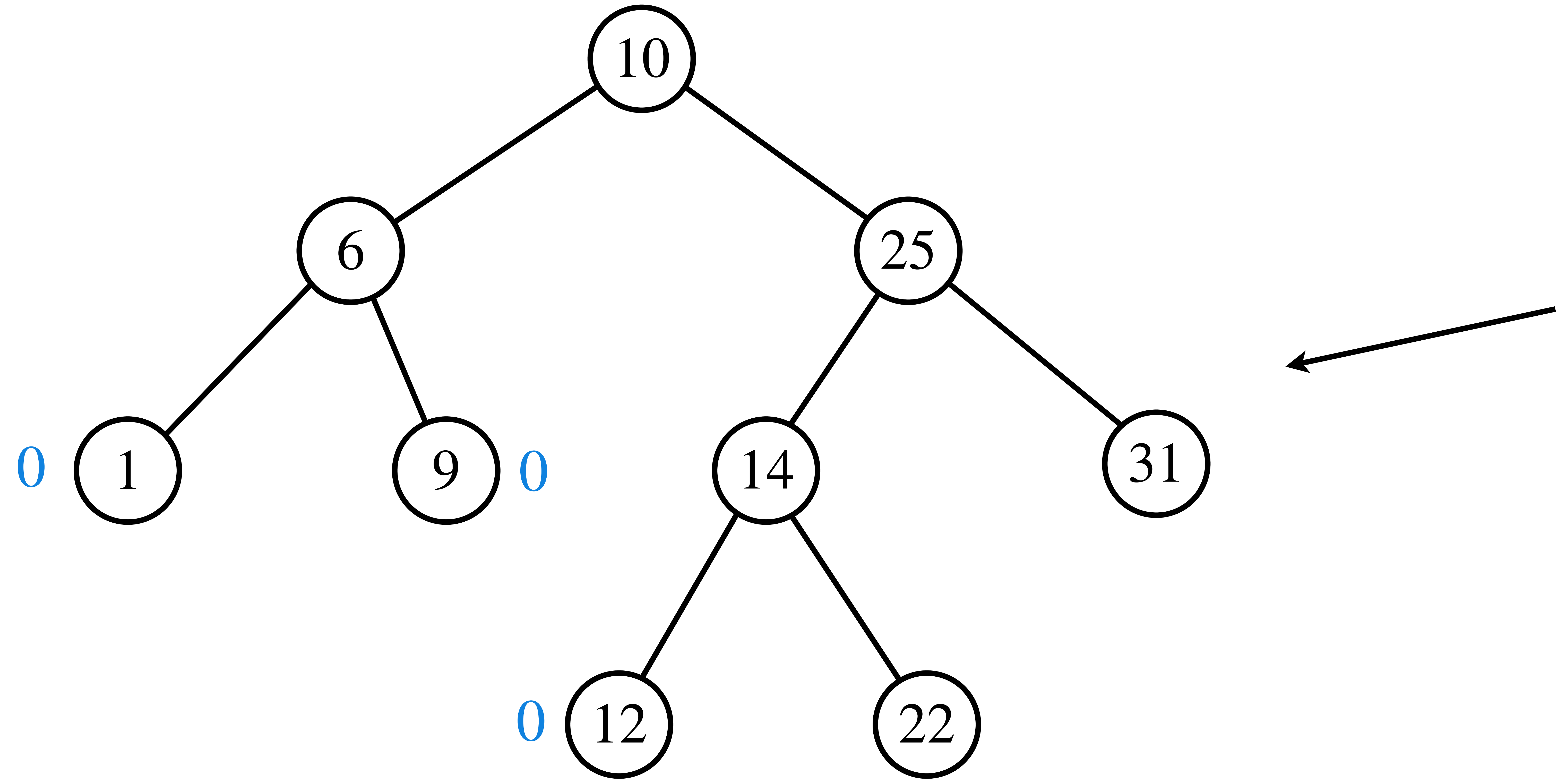

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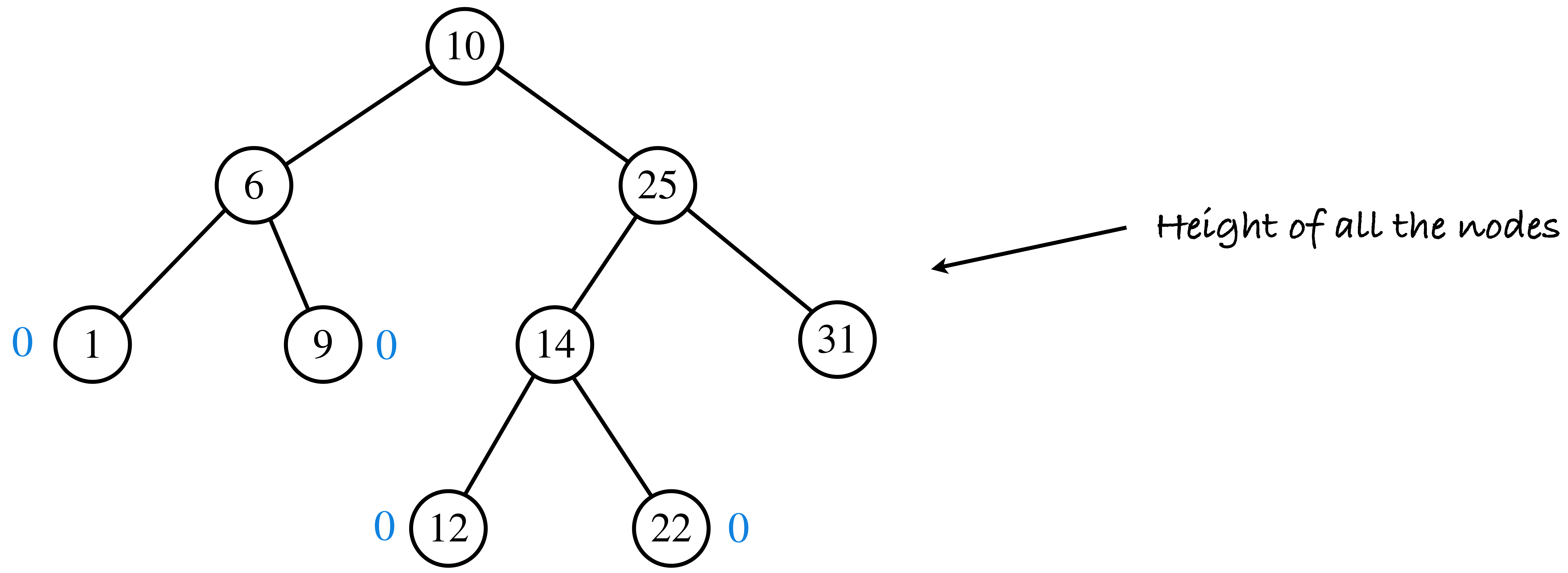

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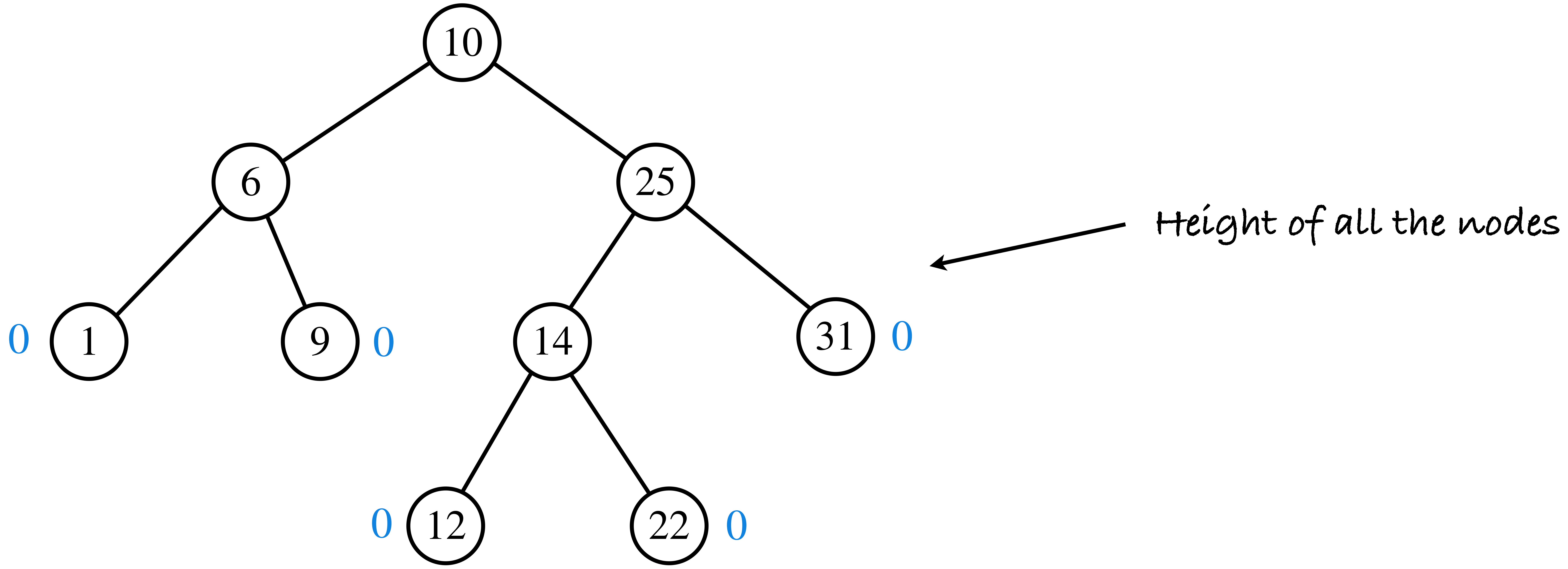

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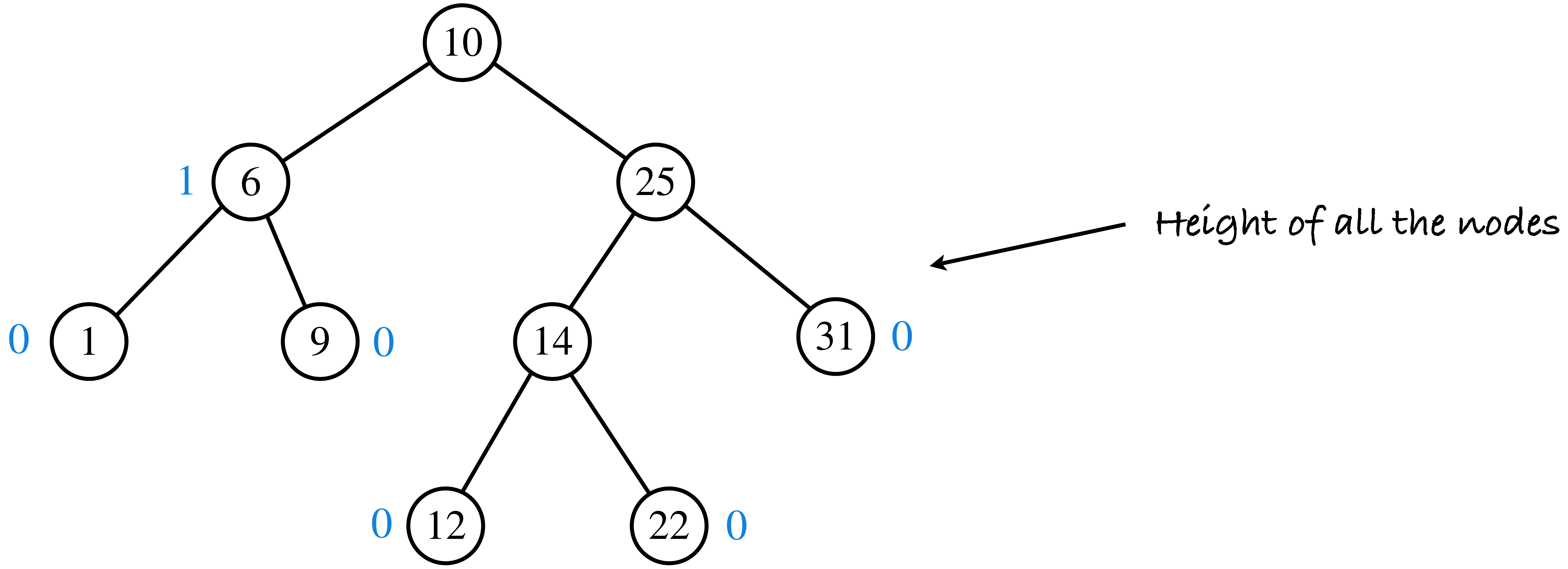

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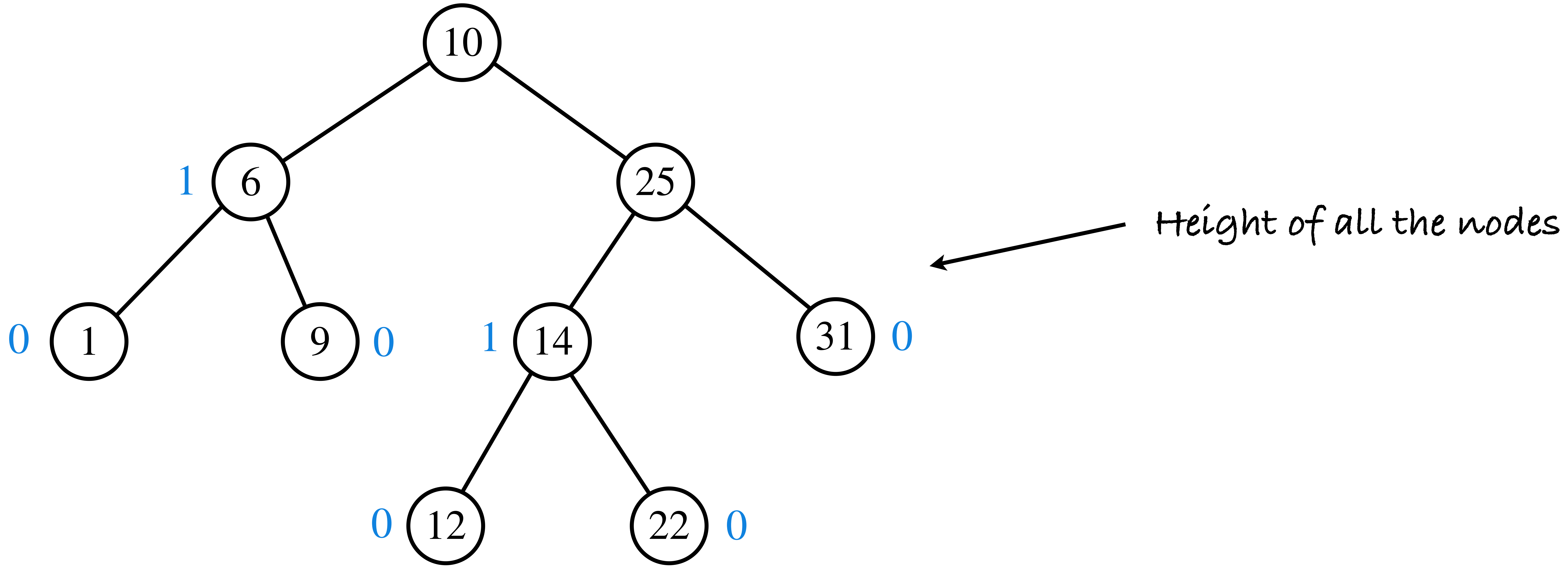

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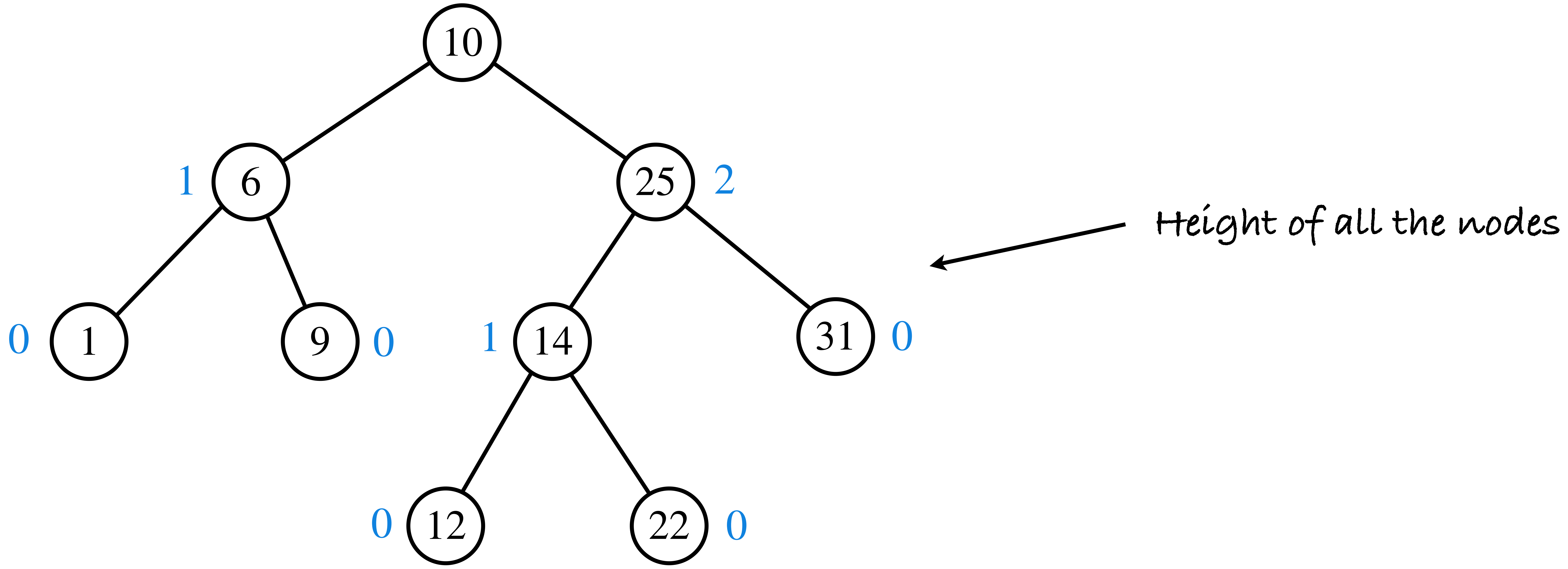

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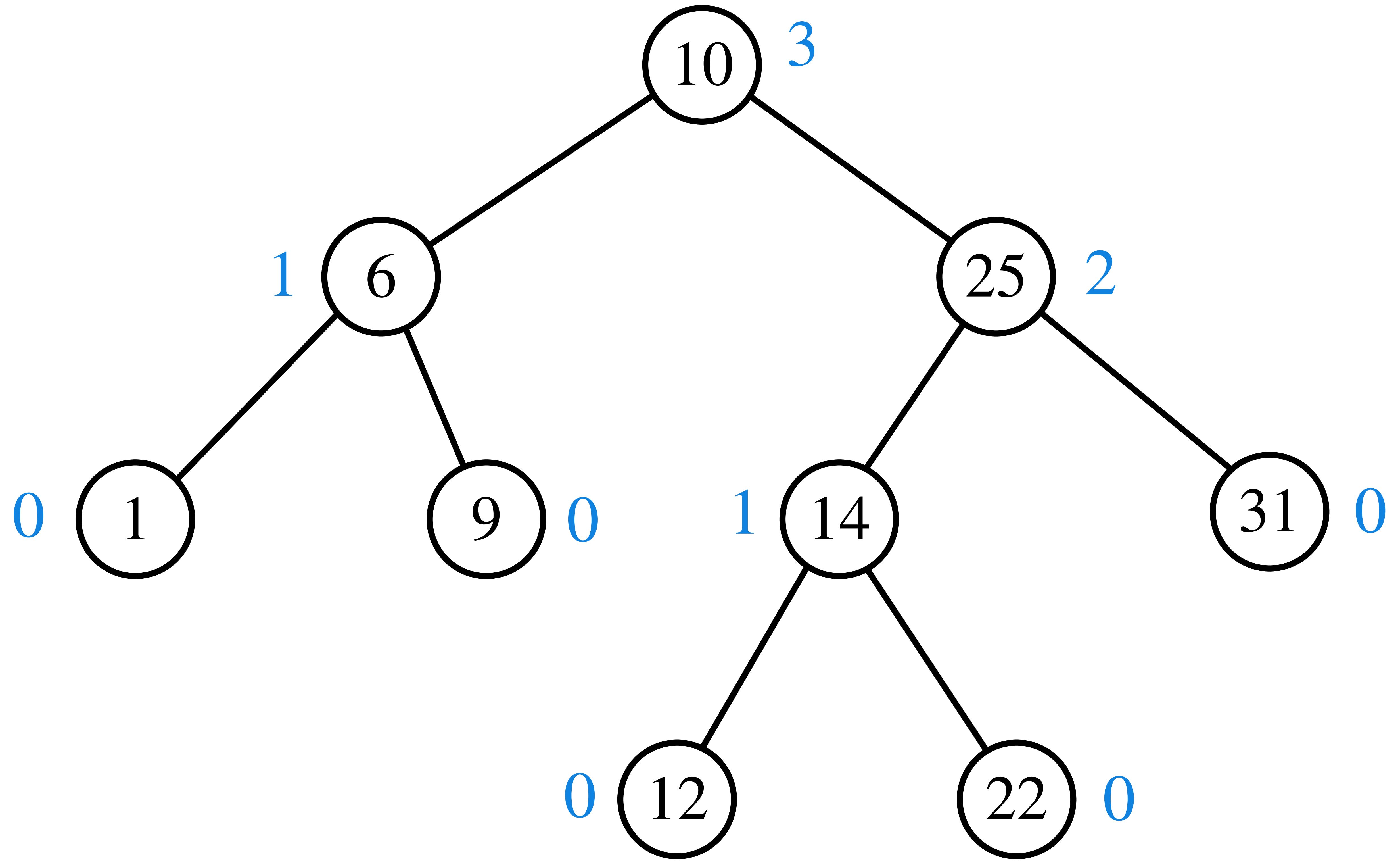

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Proof of Correctness: Can be proven using induction on the number of nodes in the tree.

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Runtime: $O(n)$, where n is the number of nodes in the tree. Each node gets printed once.

Can be formally proven by solving recurrence using substitution method.

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Goal: Given a pointer to the root of a BST, search for an element with the key k in it.

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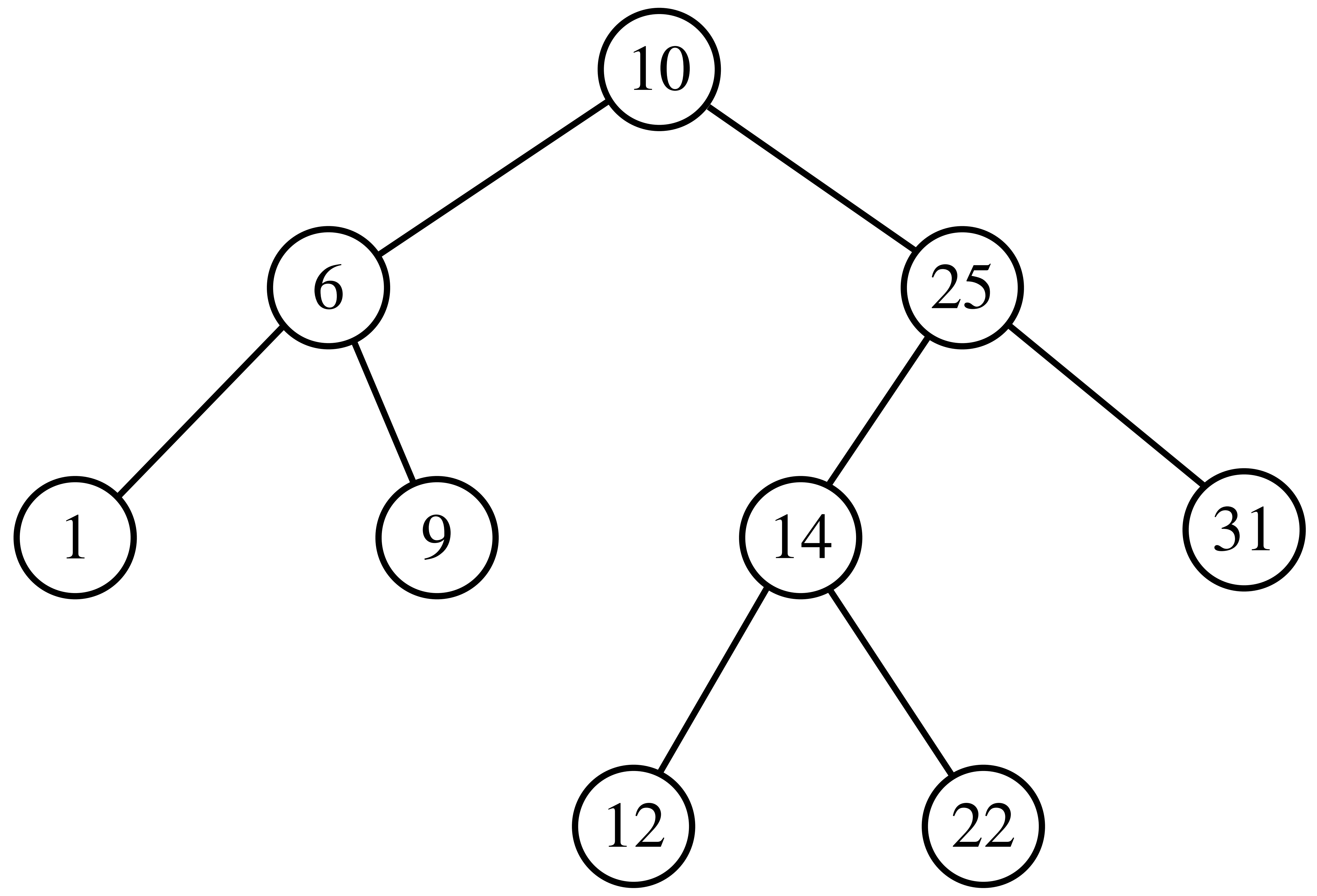

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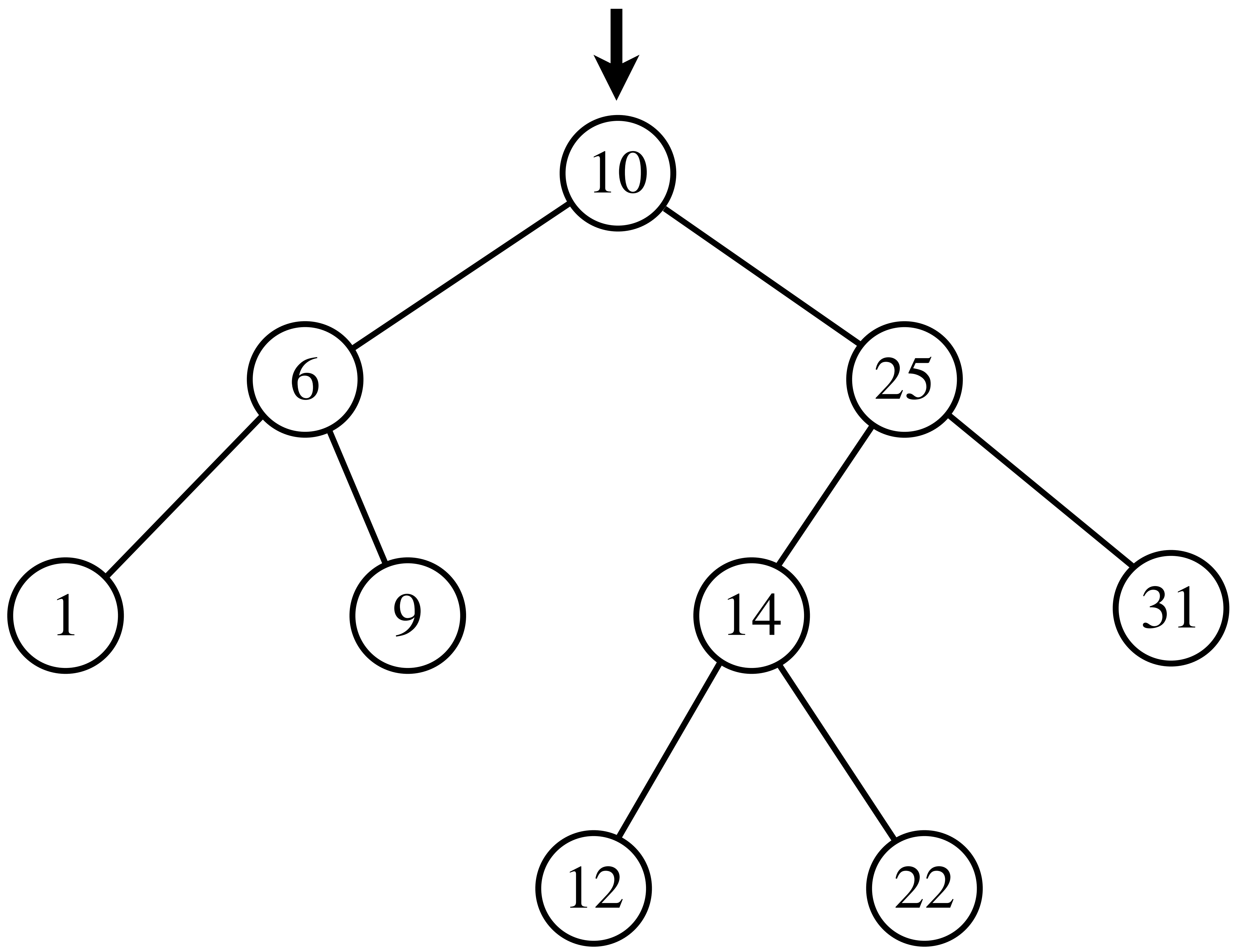

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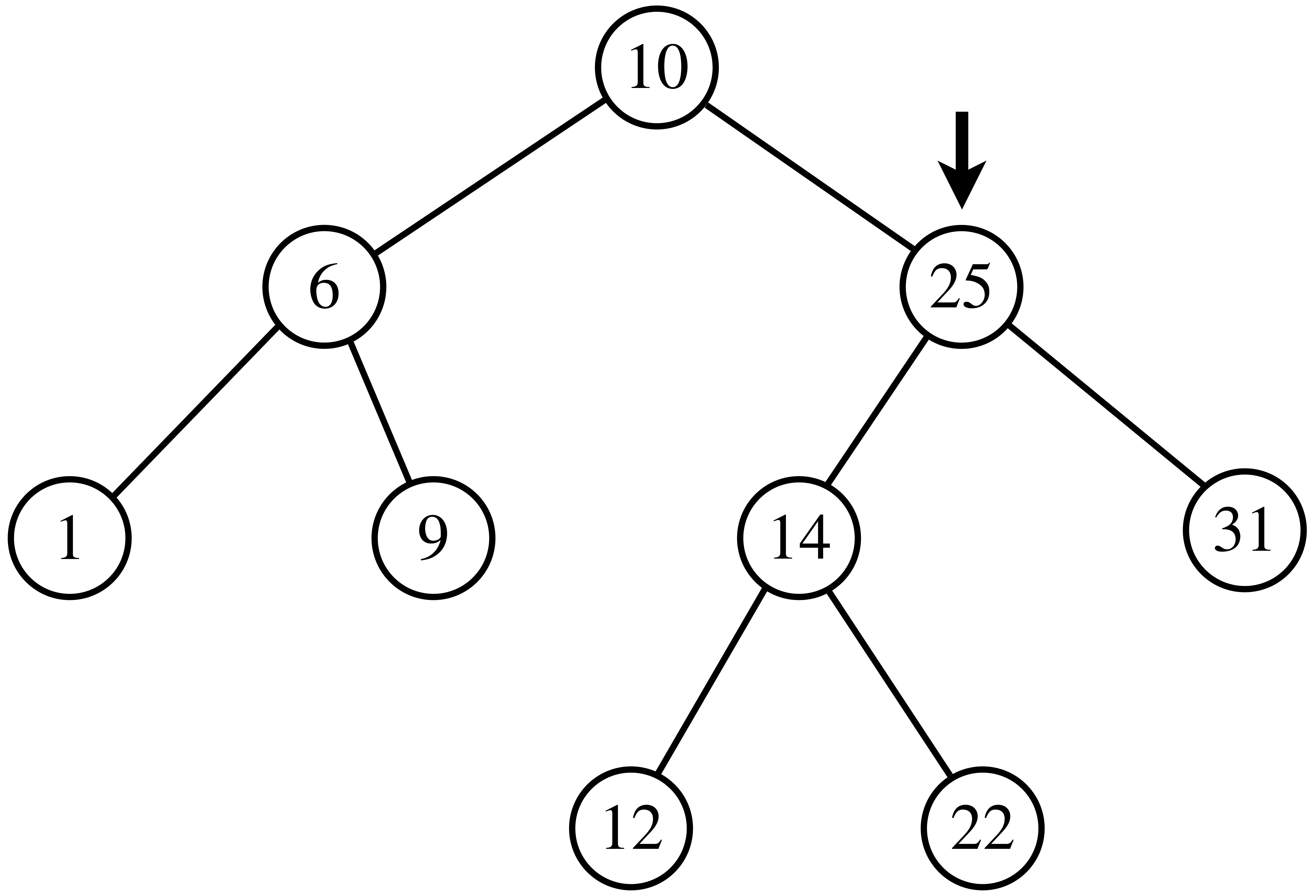

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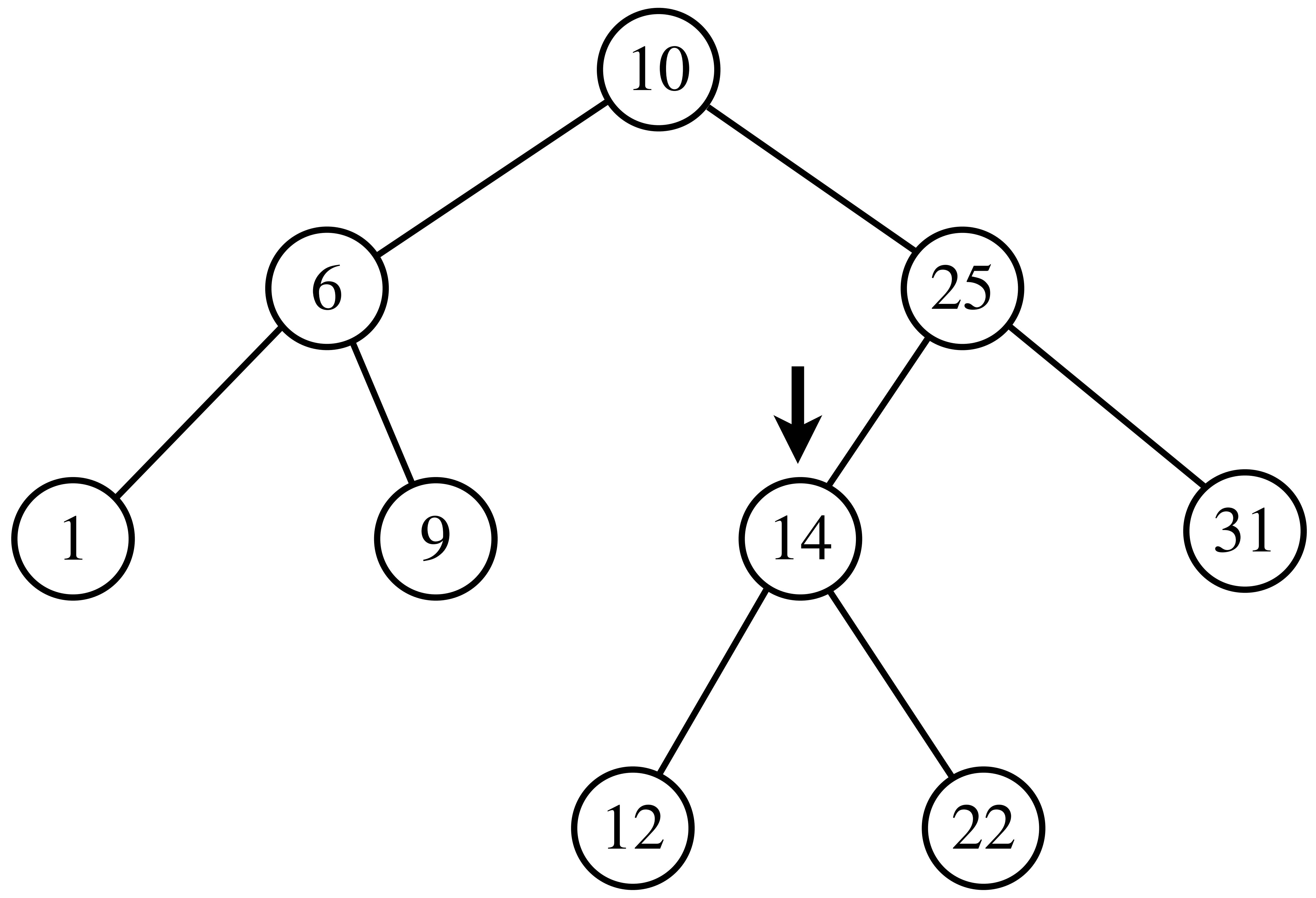

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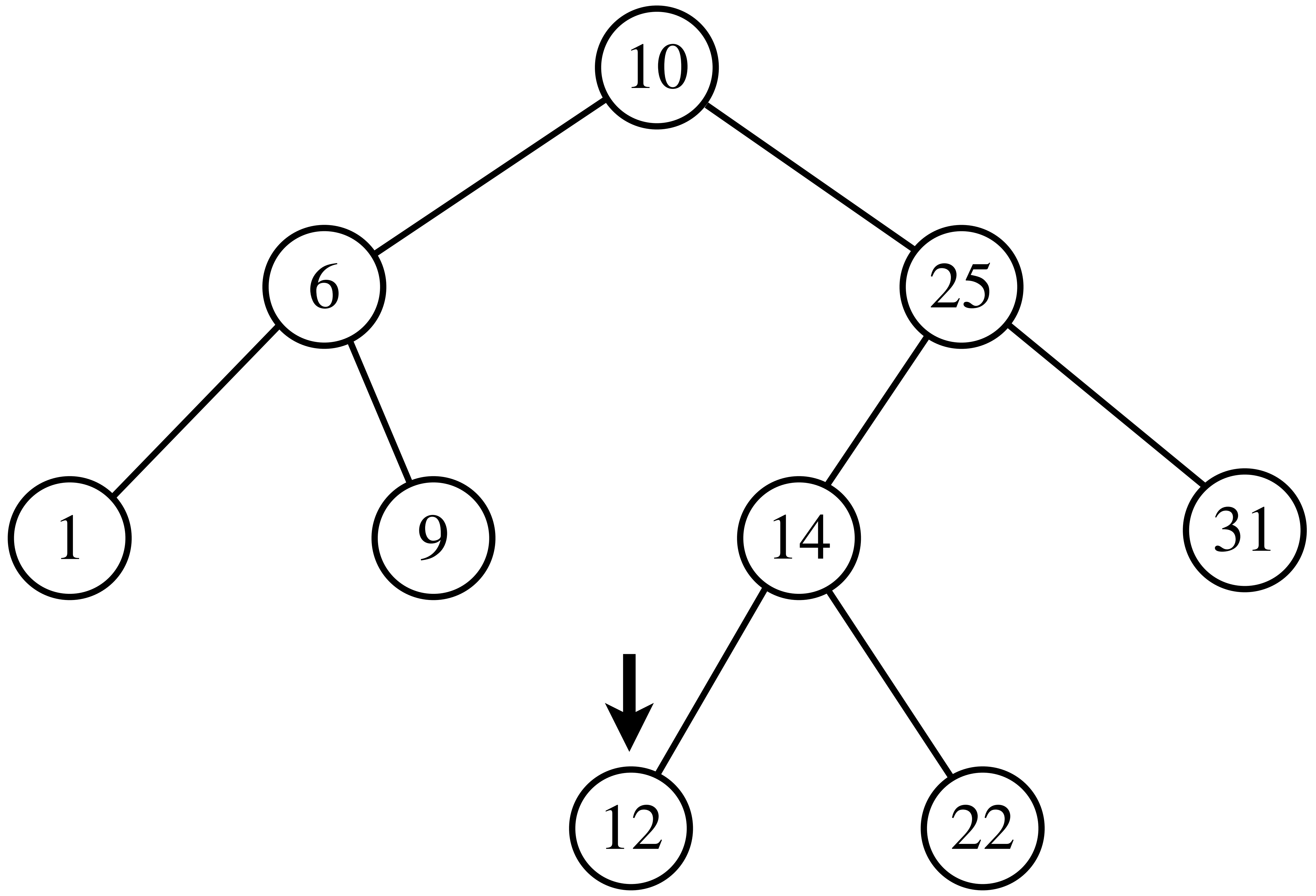

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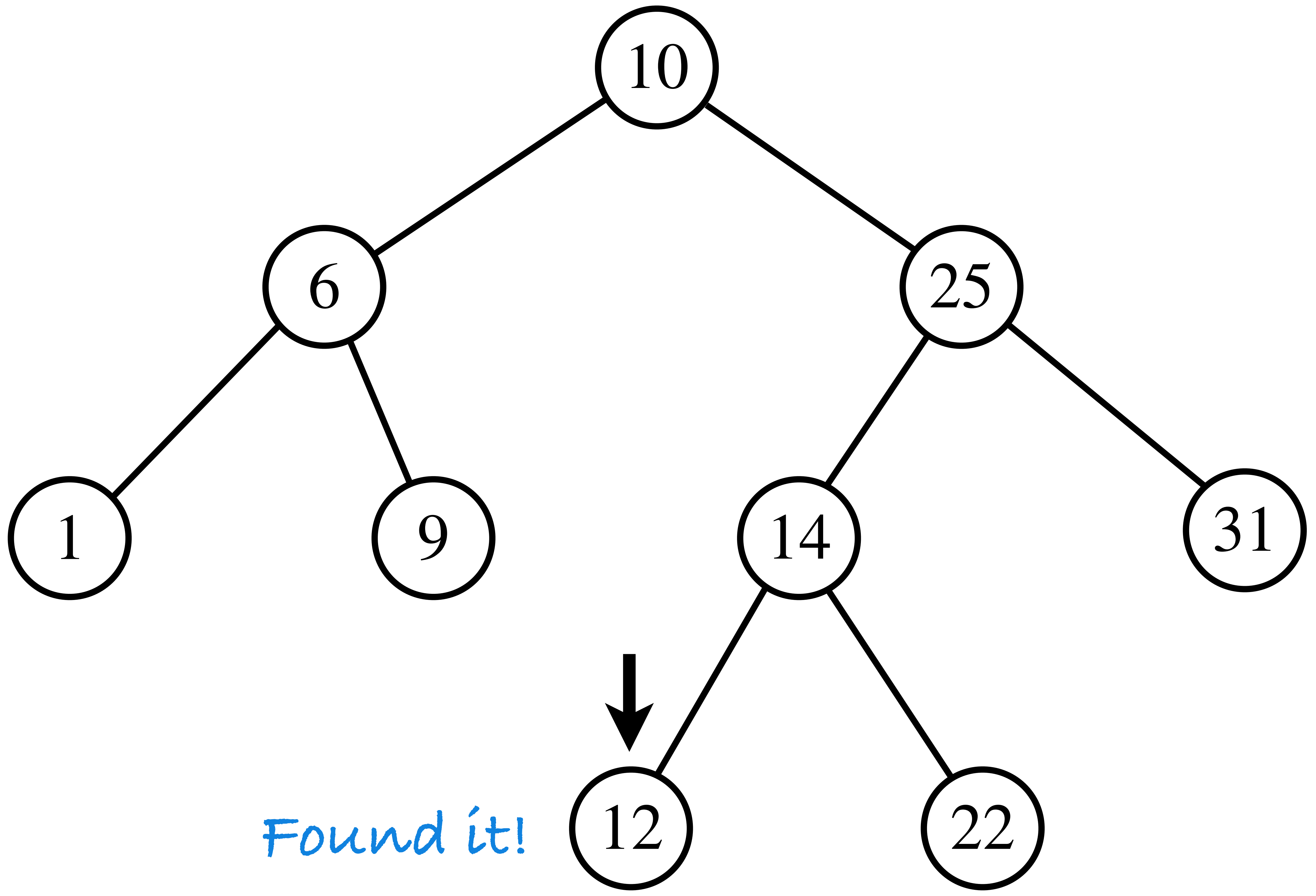

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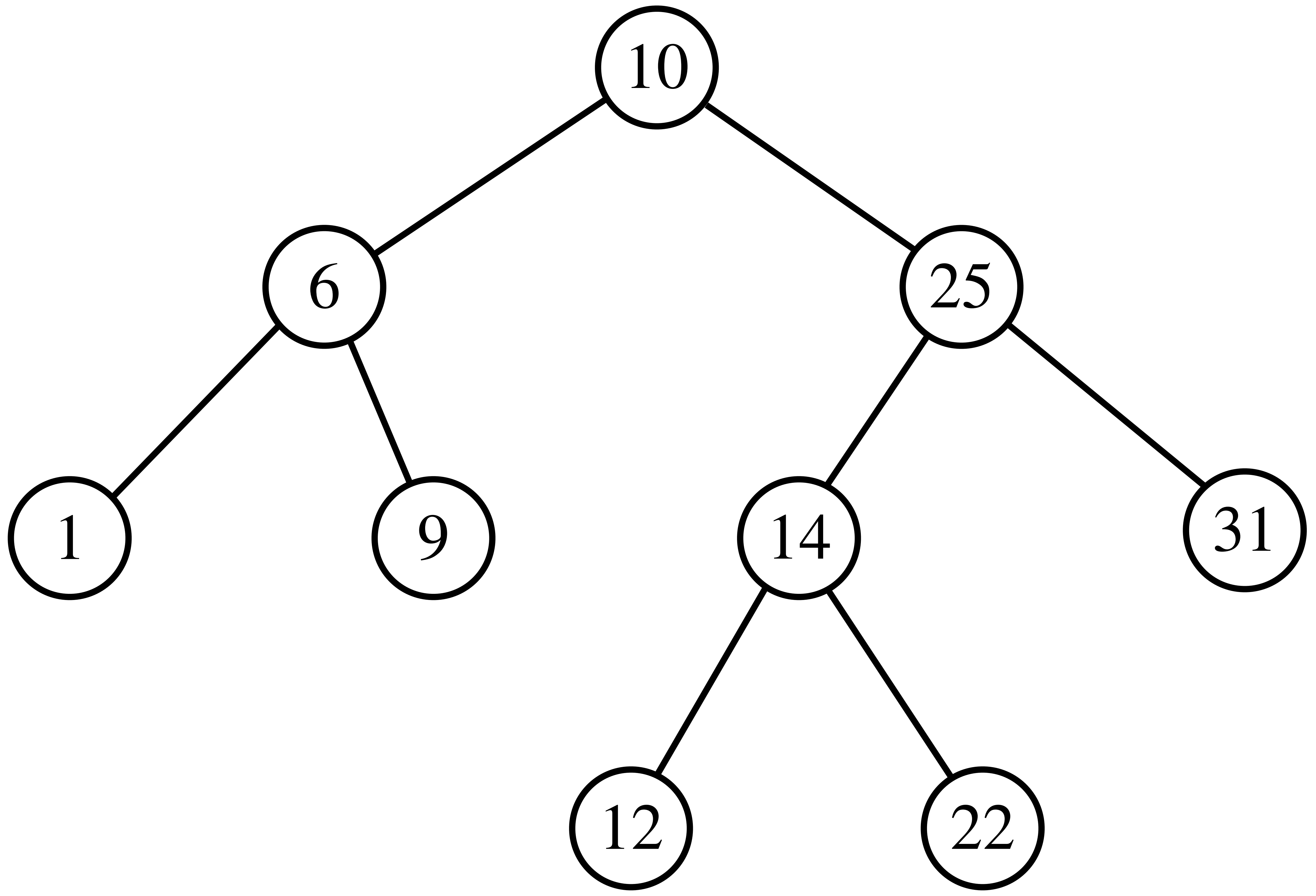
Illustration: Searching for 5 in below BST.



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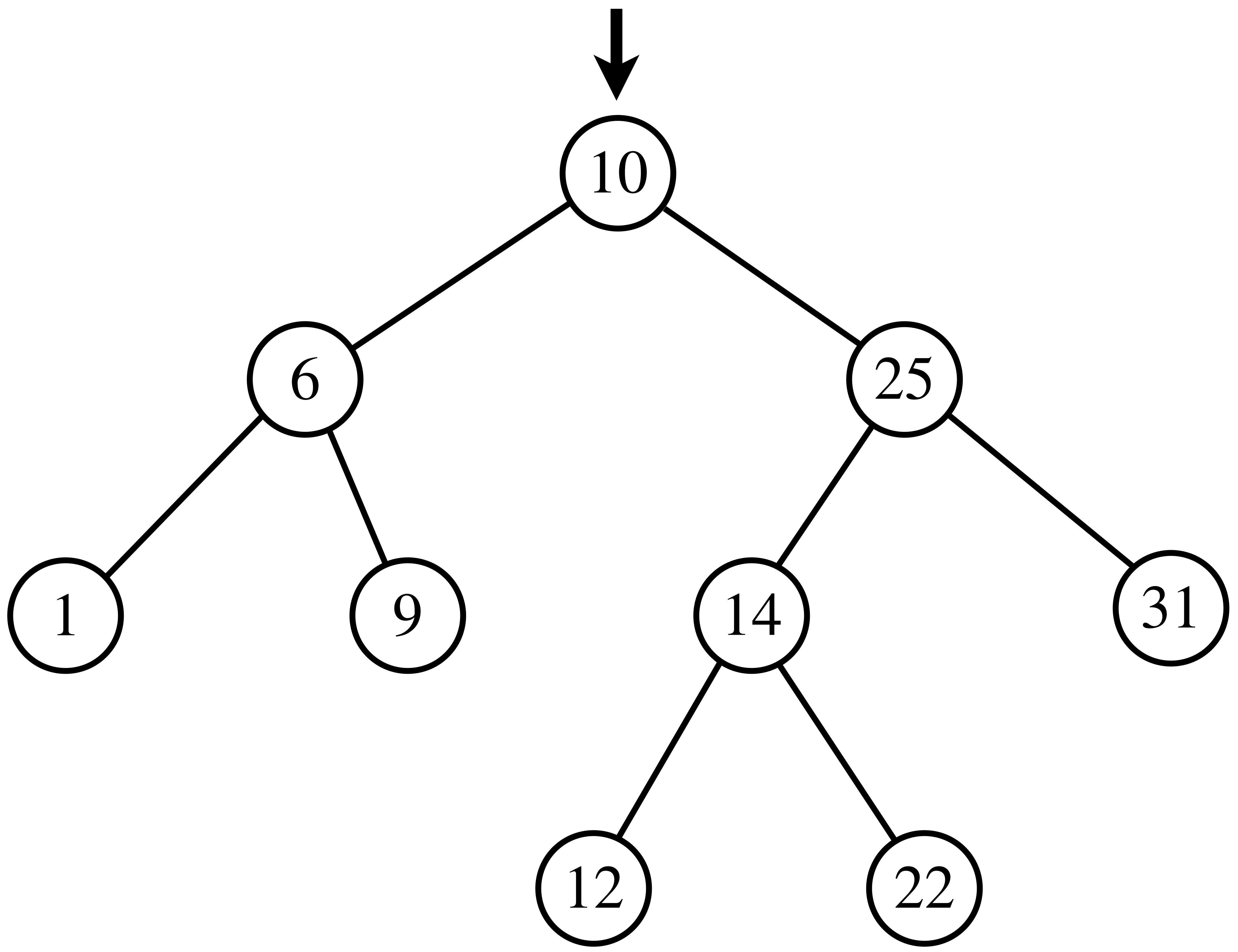

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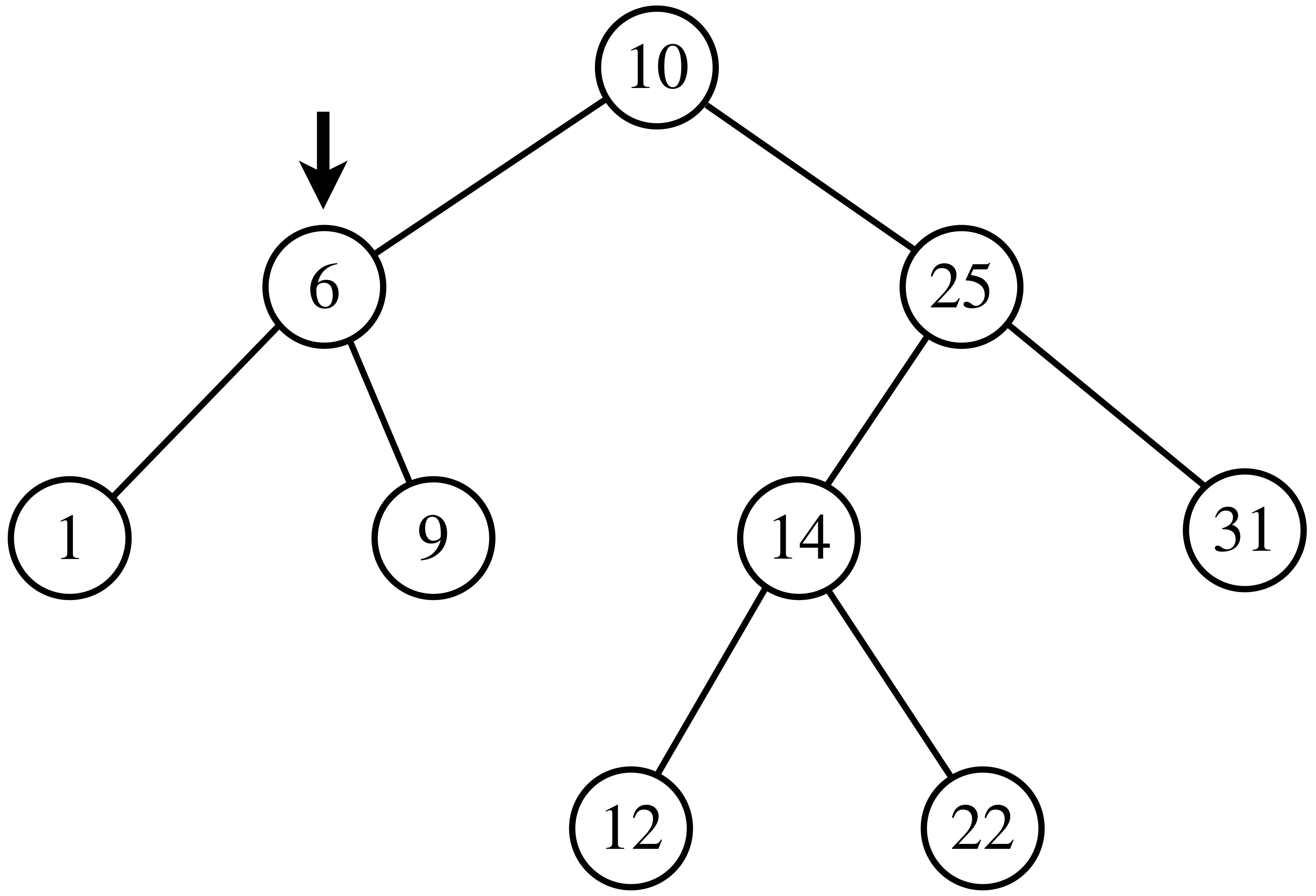

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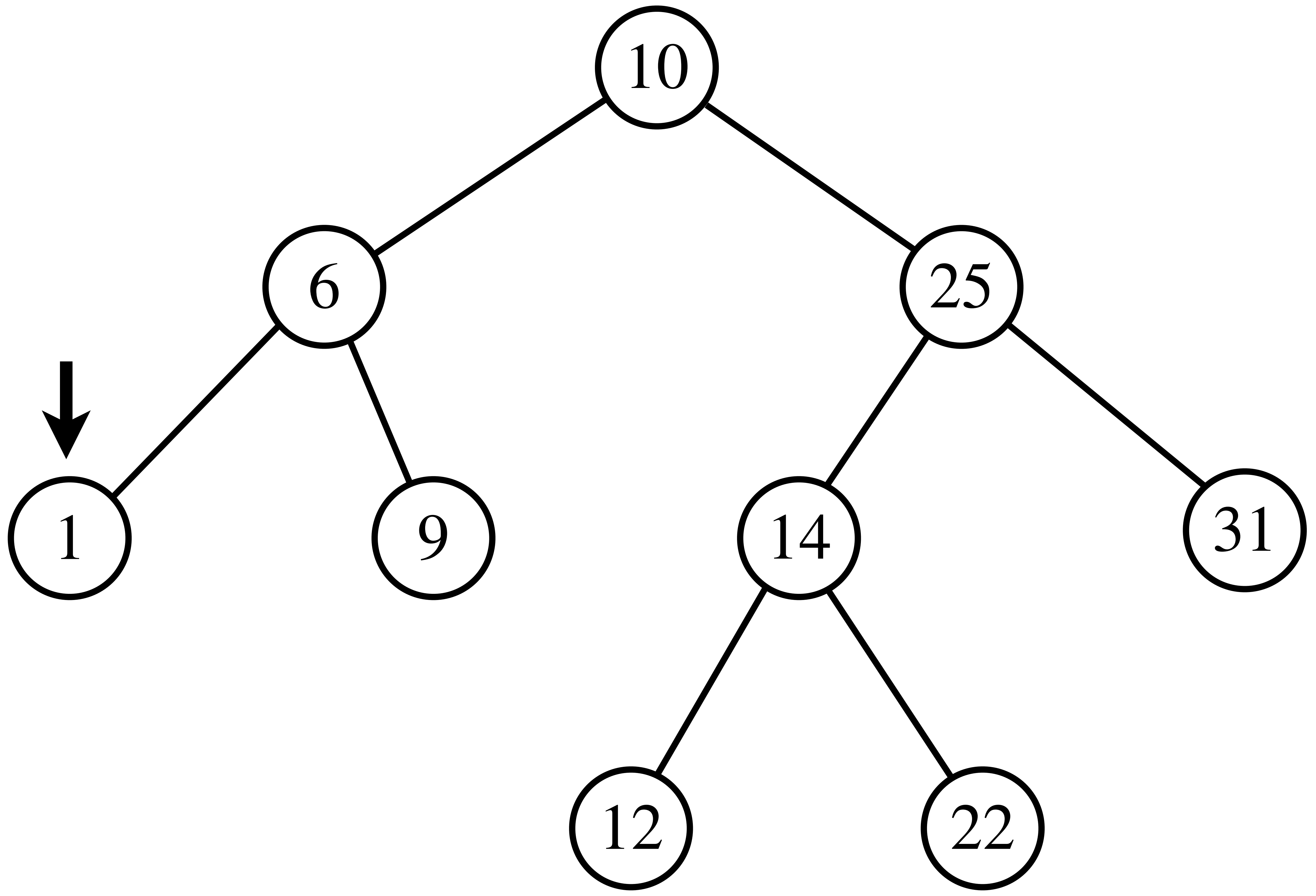

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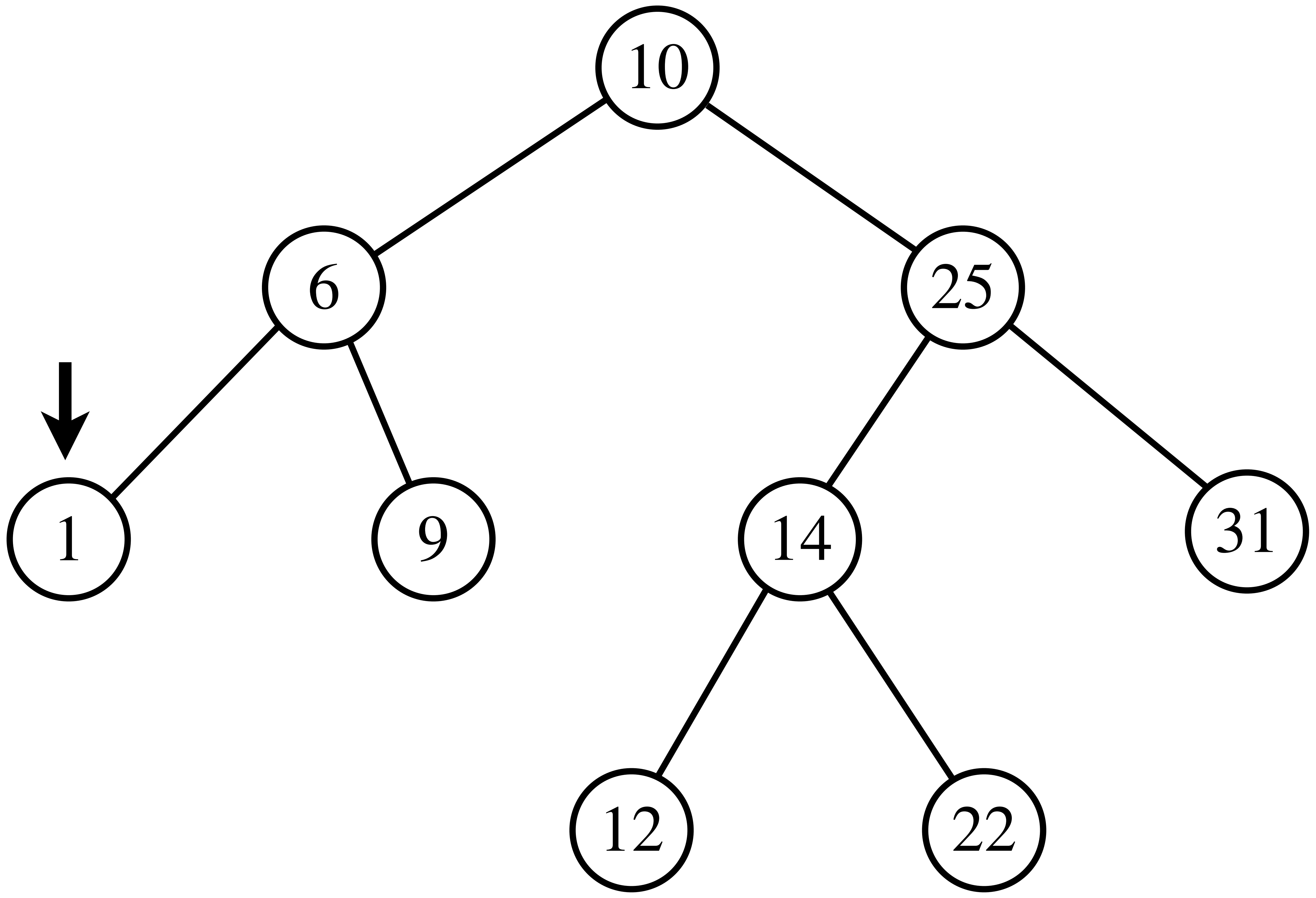

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Algorithm: Call **Tree-Search**$(T.root, k)$ to search for element with key k in tree T.

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Tree-Search $(x, k)$:
1.    while $x \neq$ NIL and $k \neq x\,.\,key$
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4.     else
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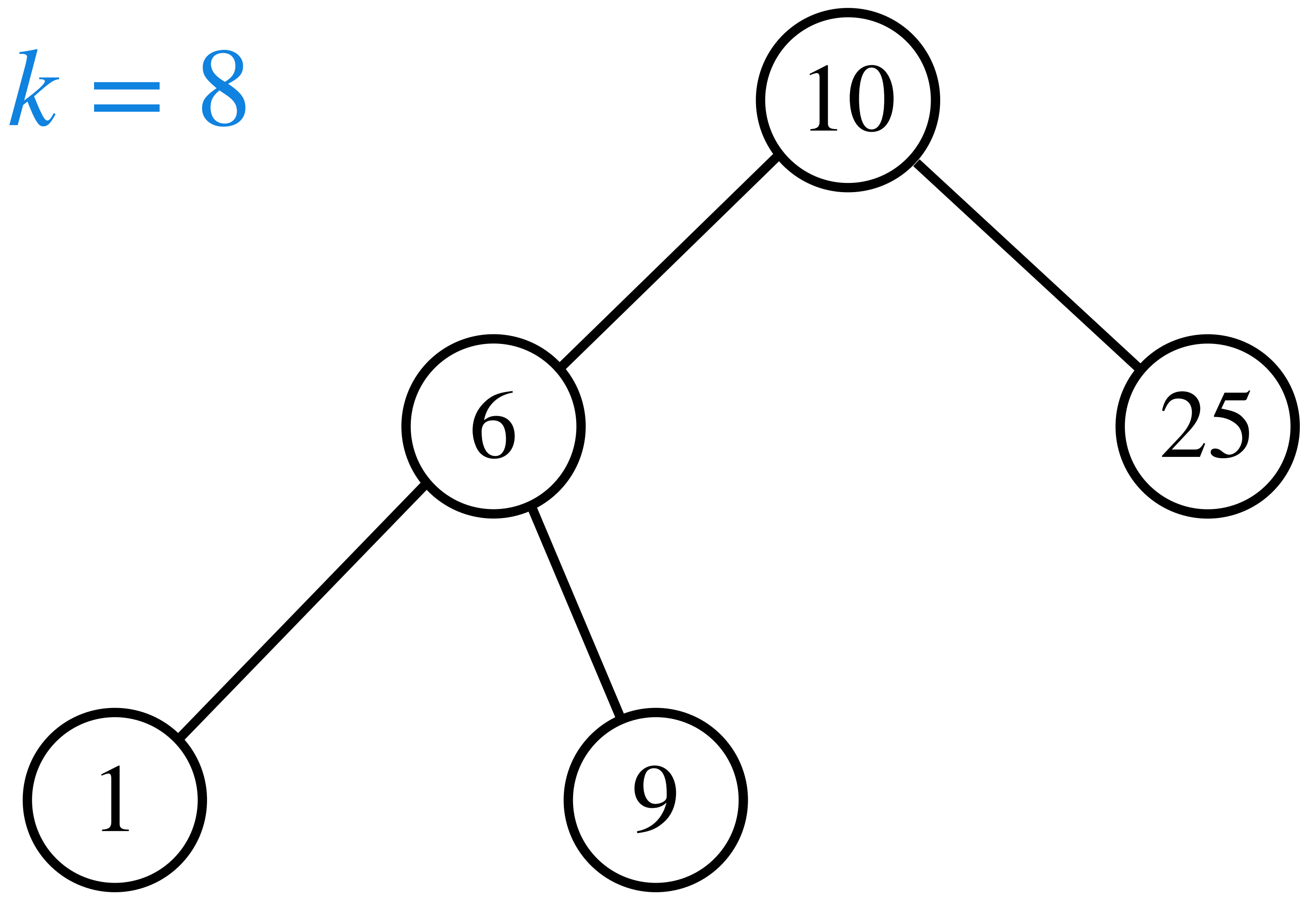

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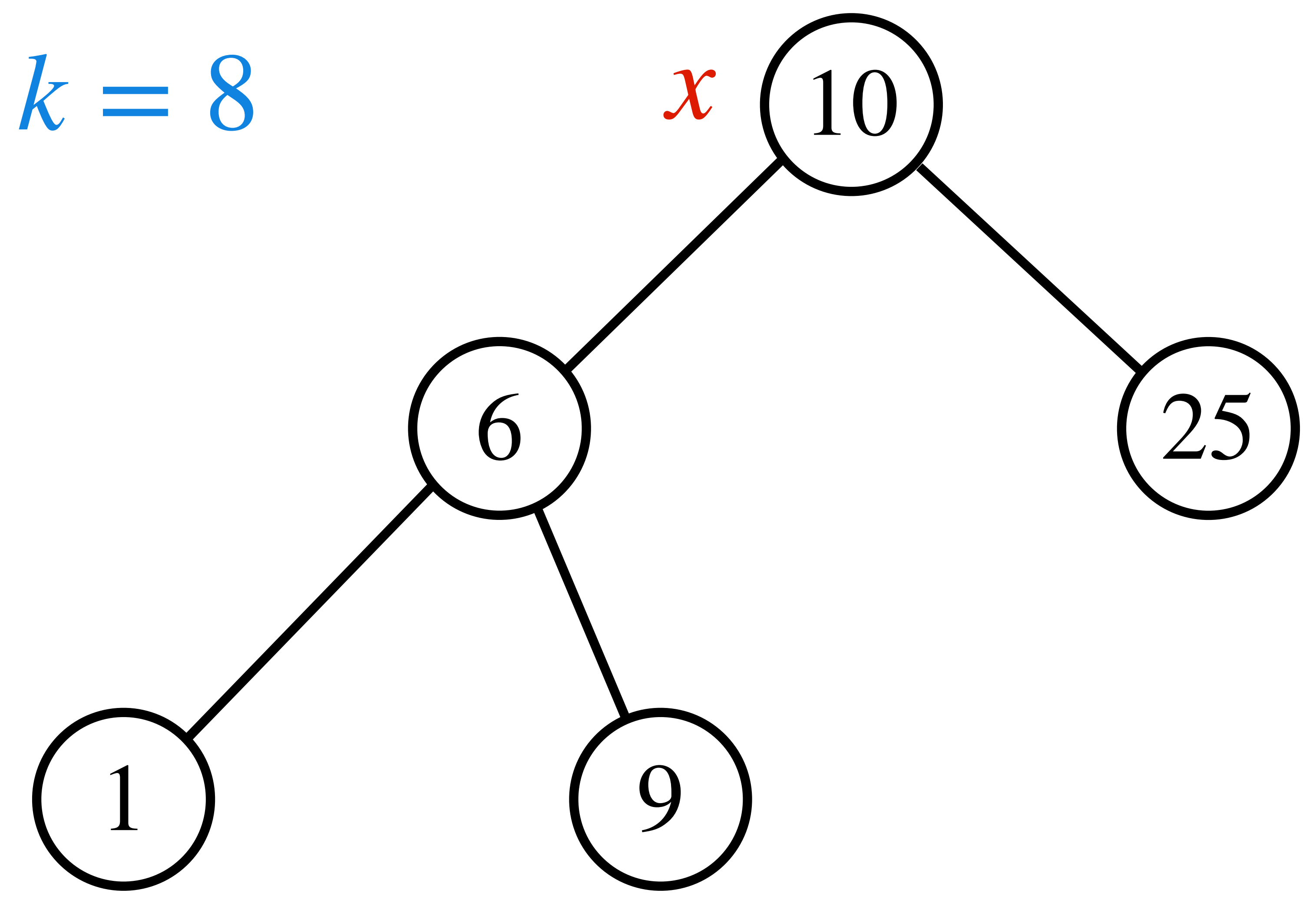

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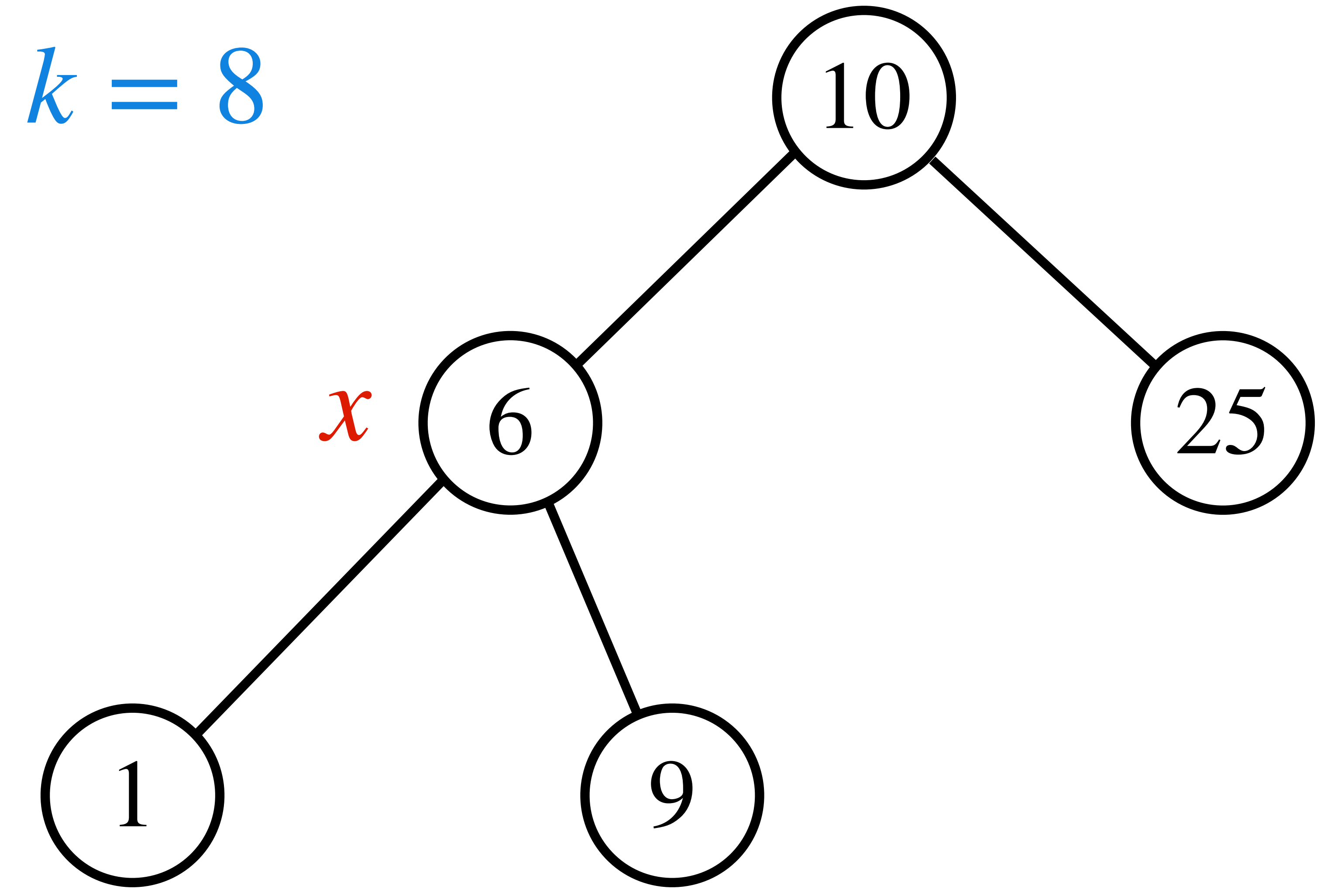

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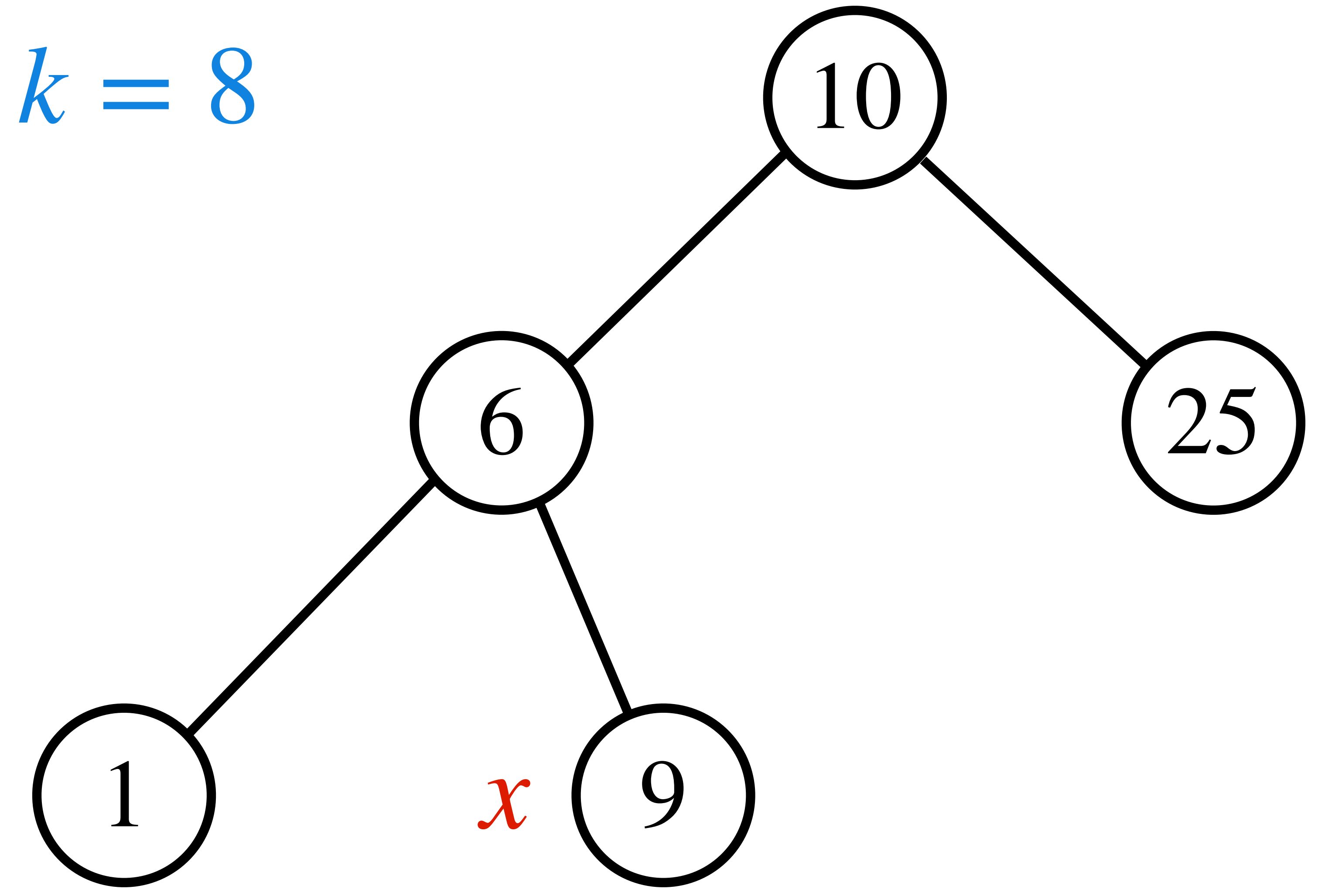

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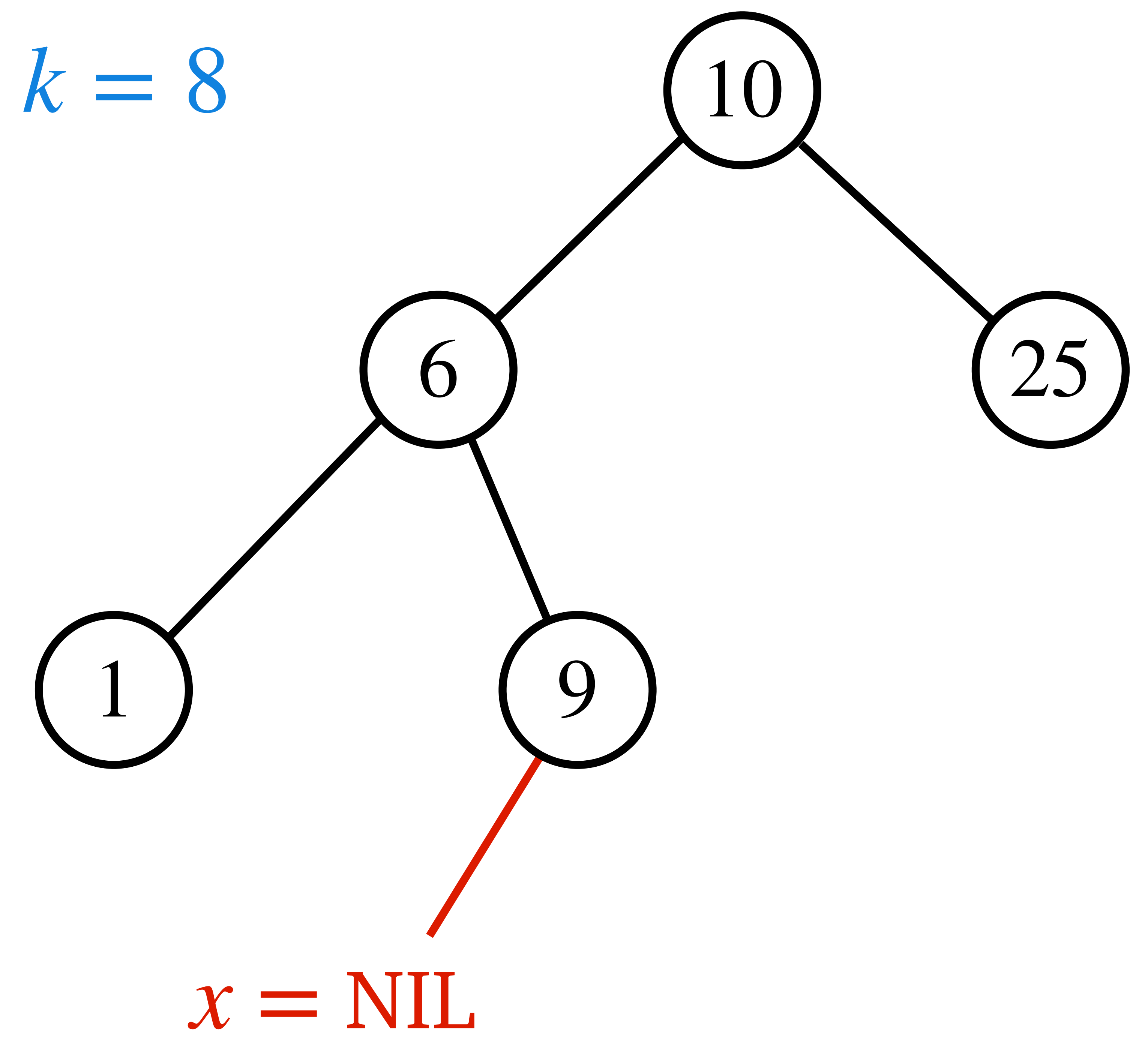

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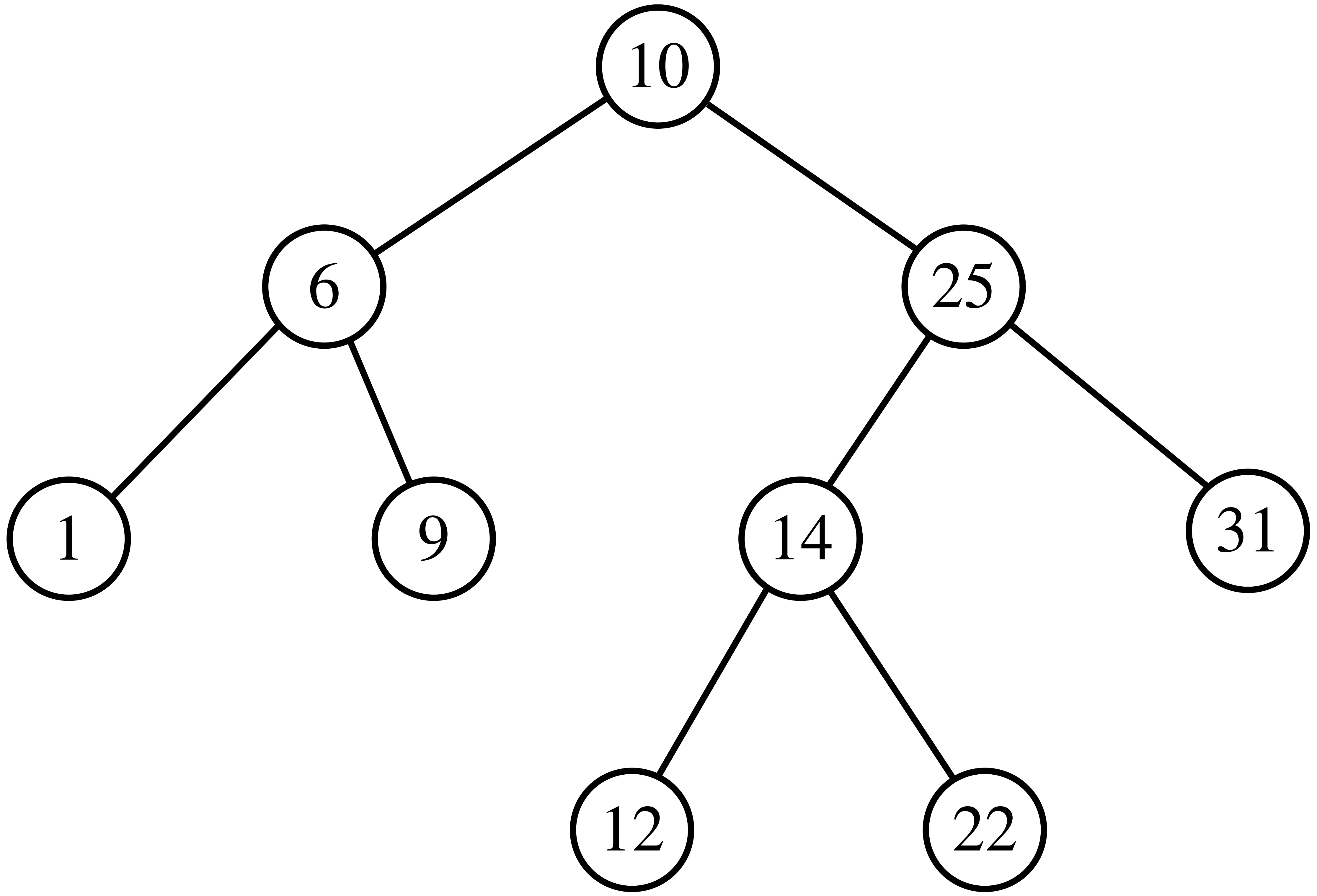

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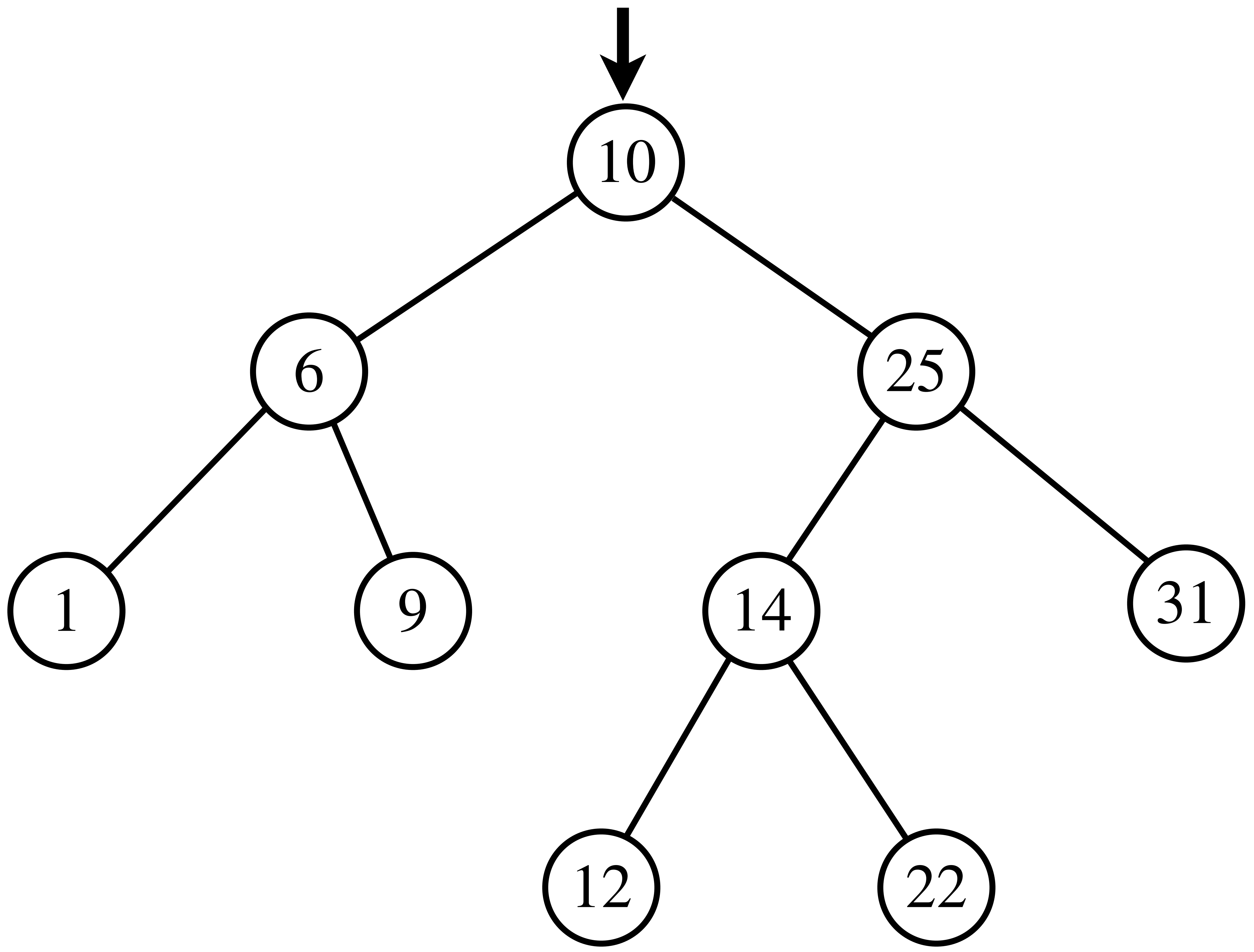

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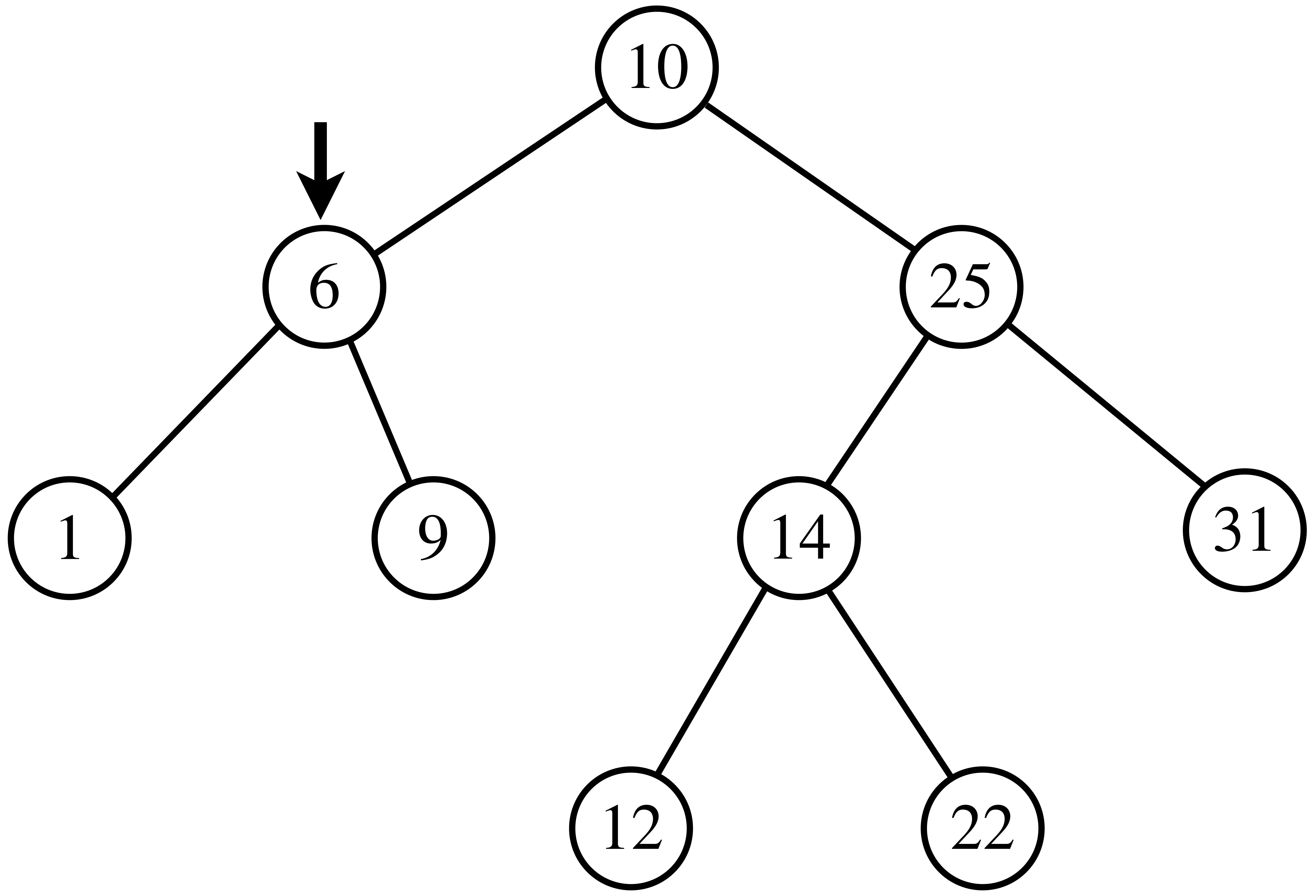

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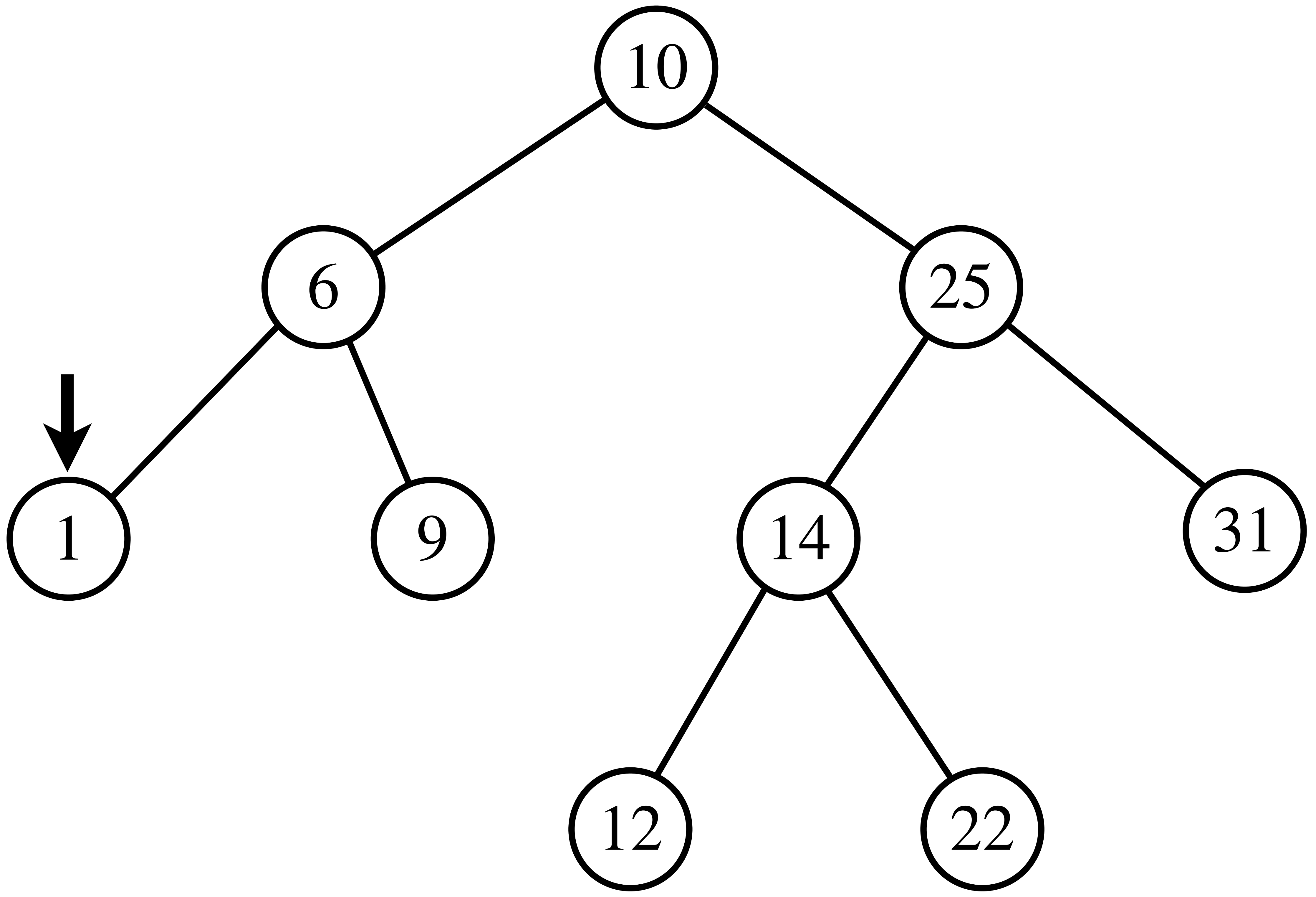

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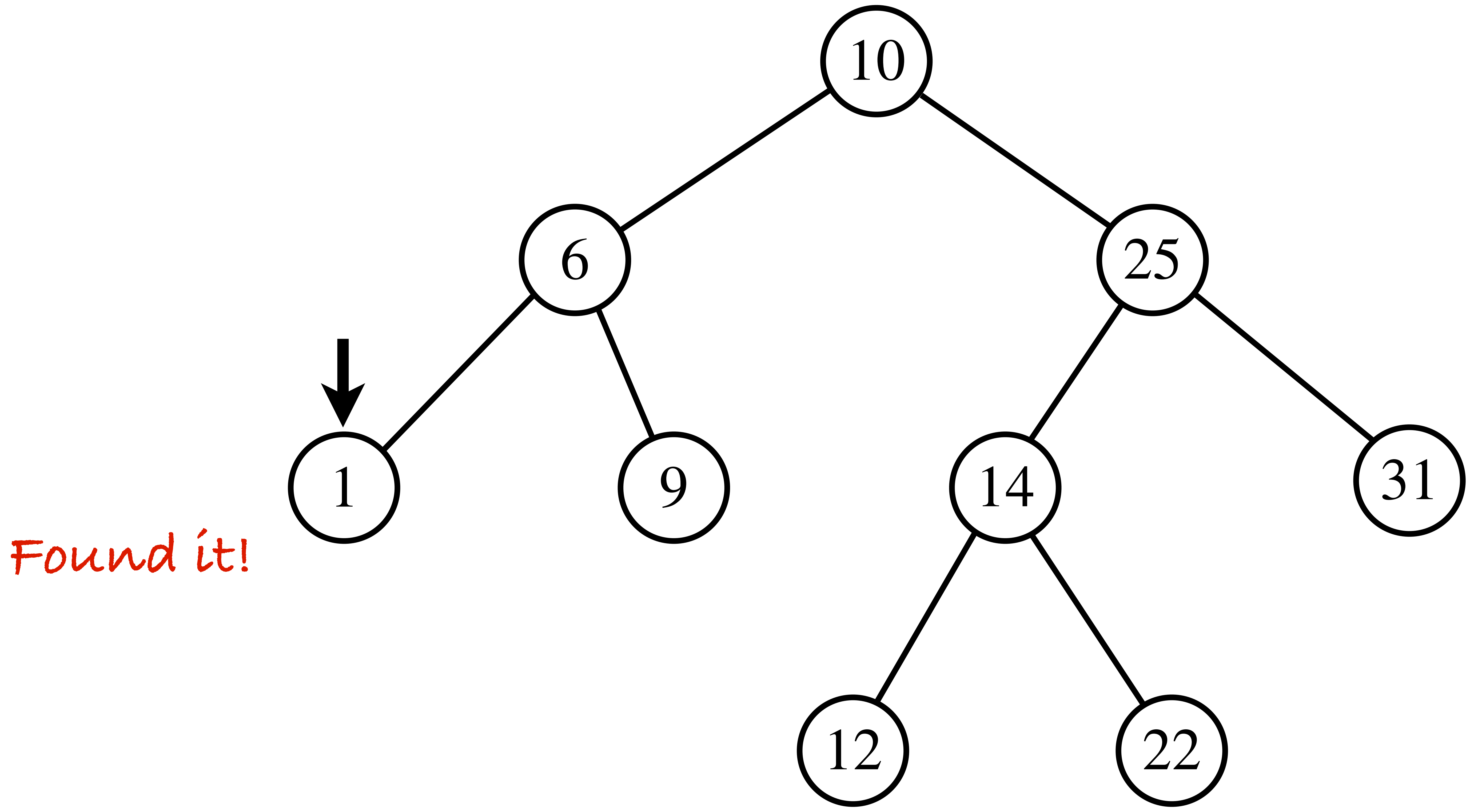

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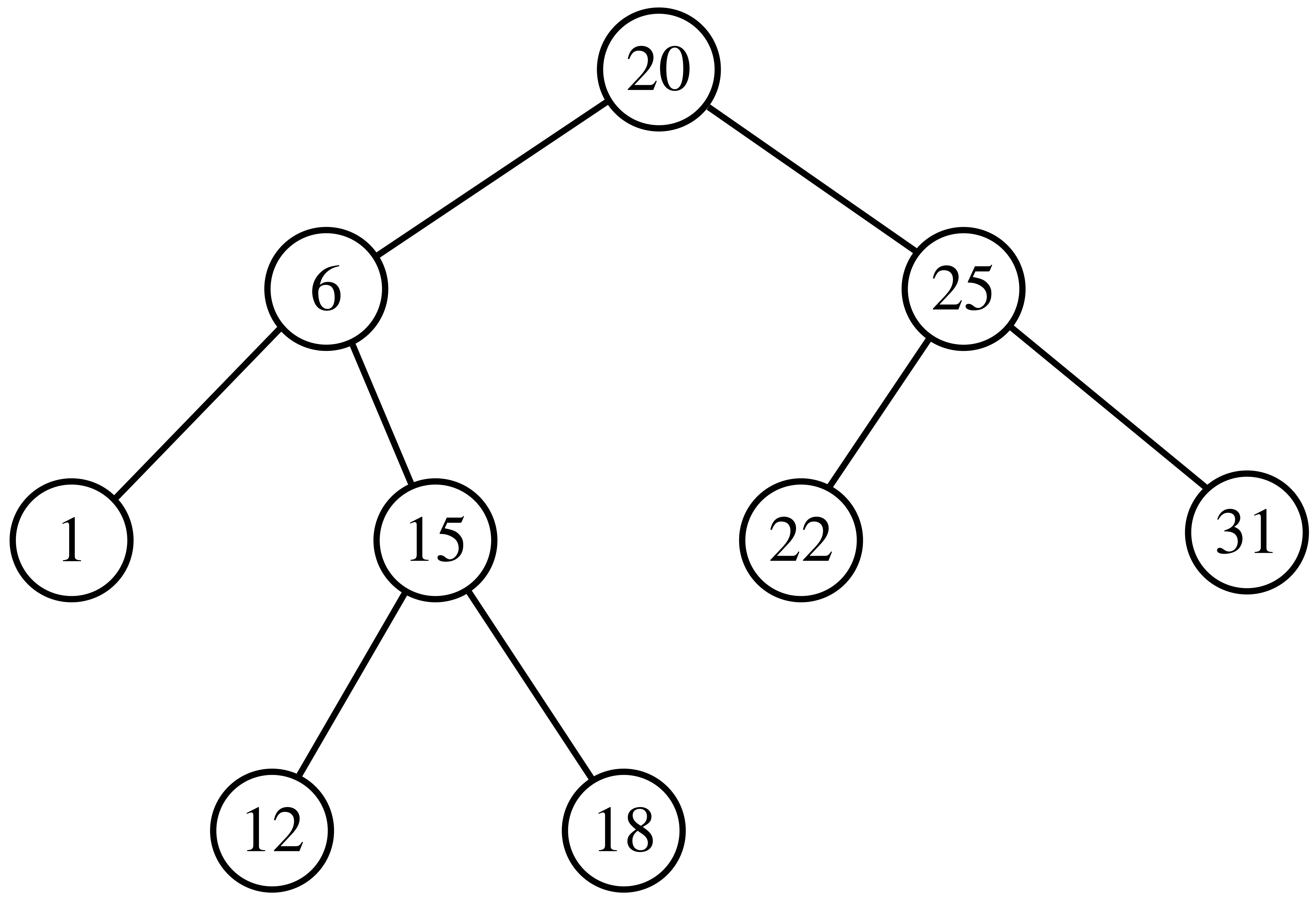

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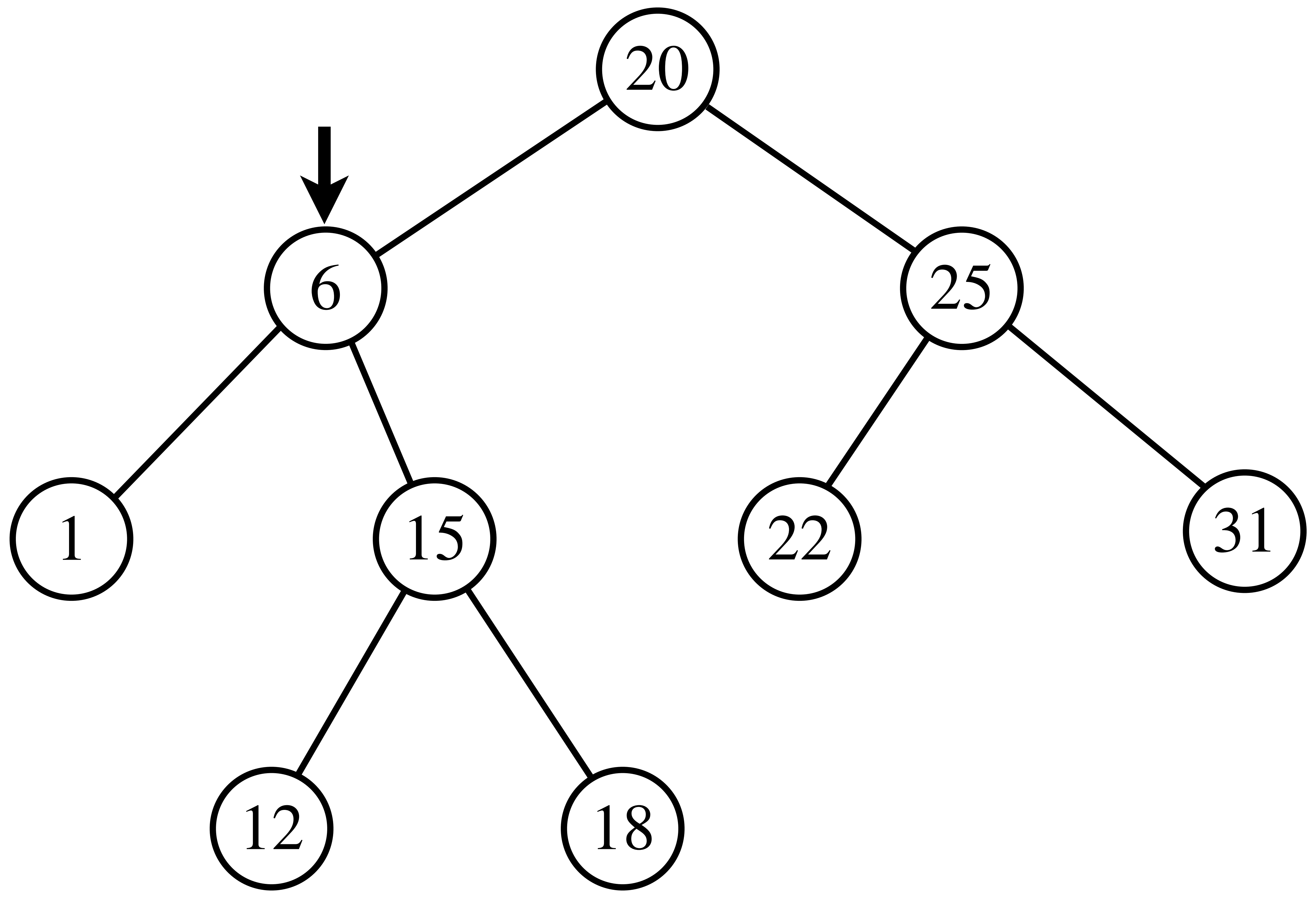

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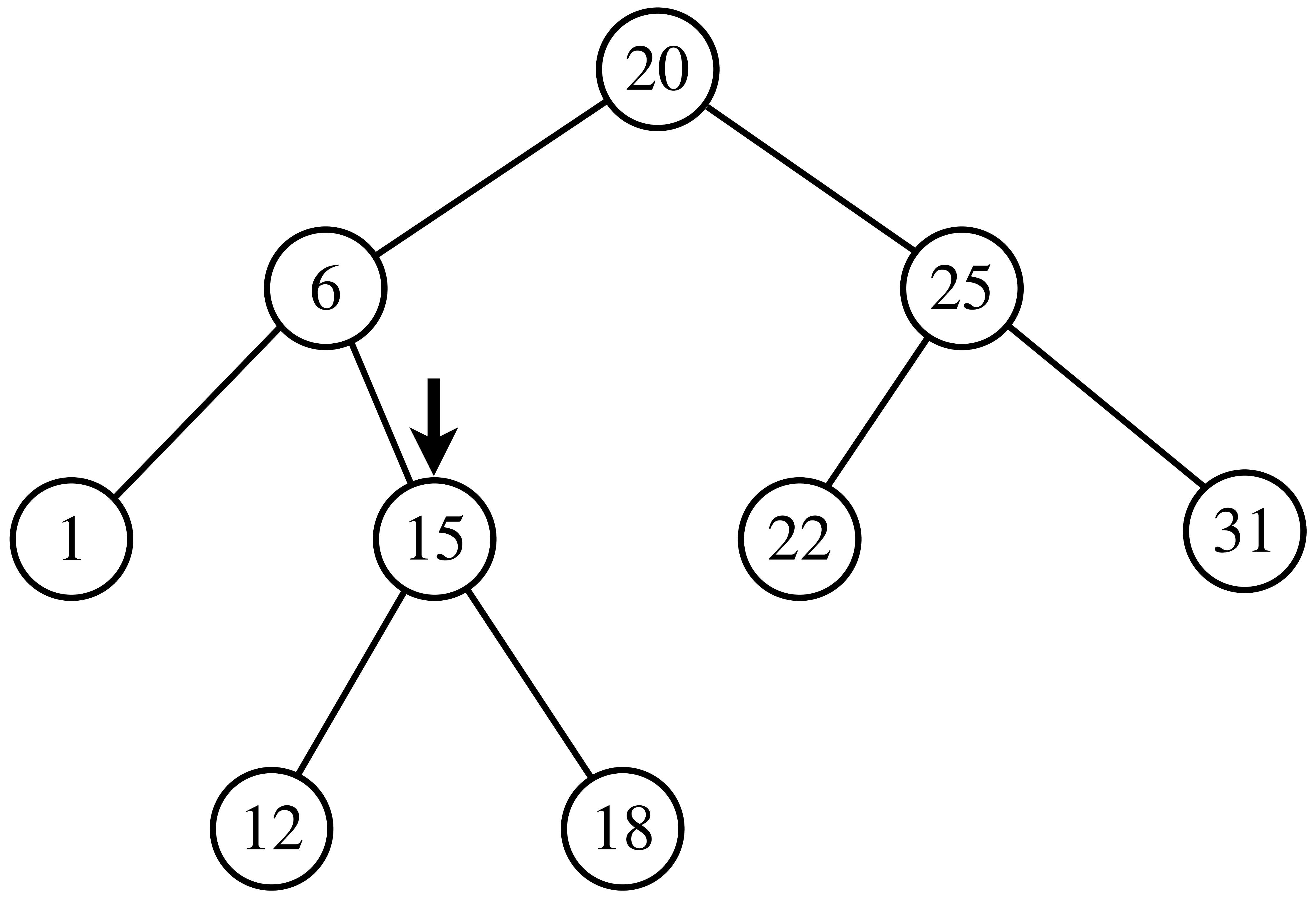

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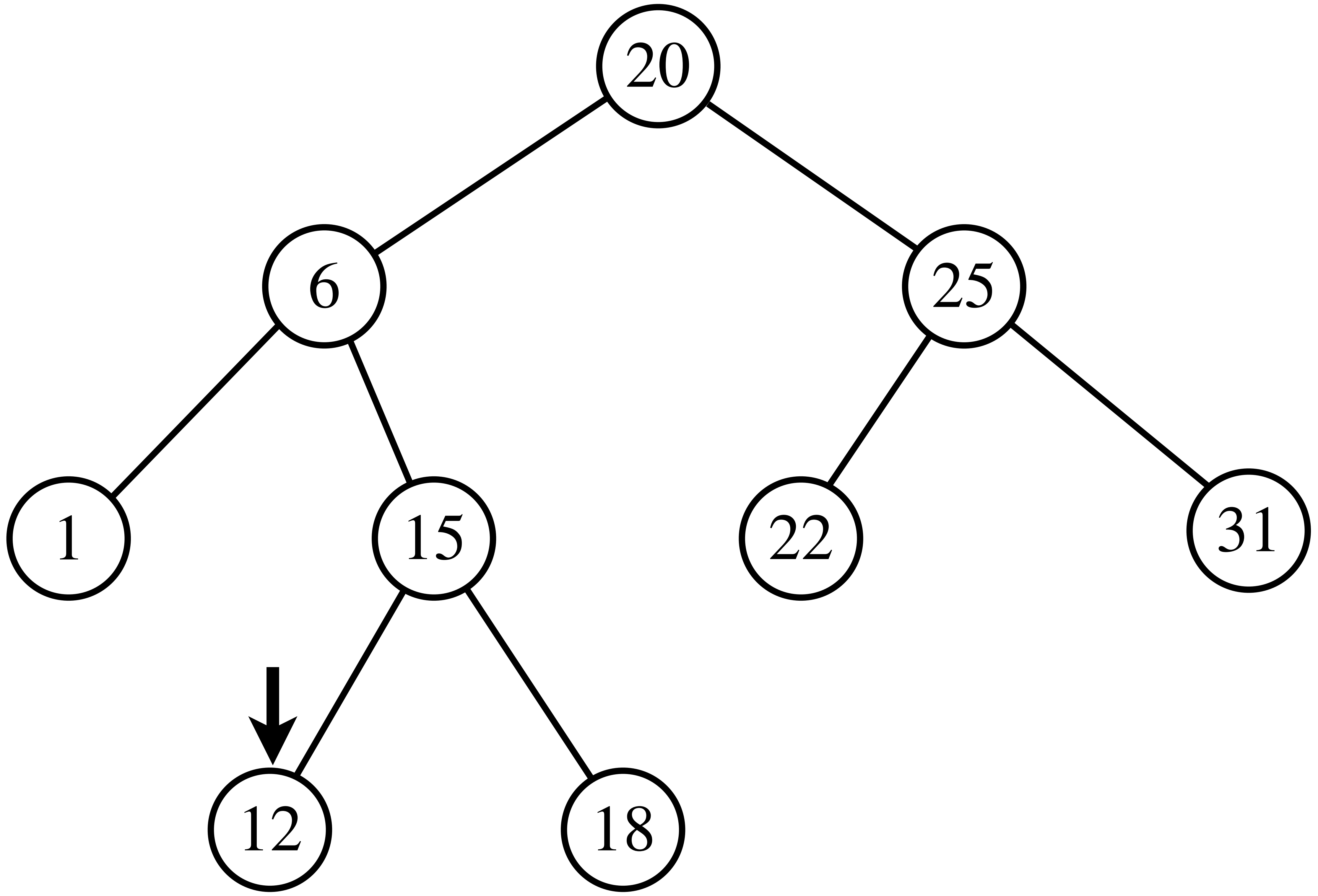

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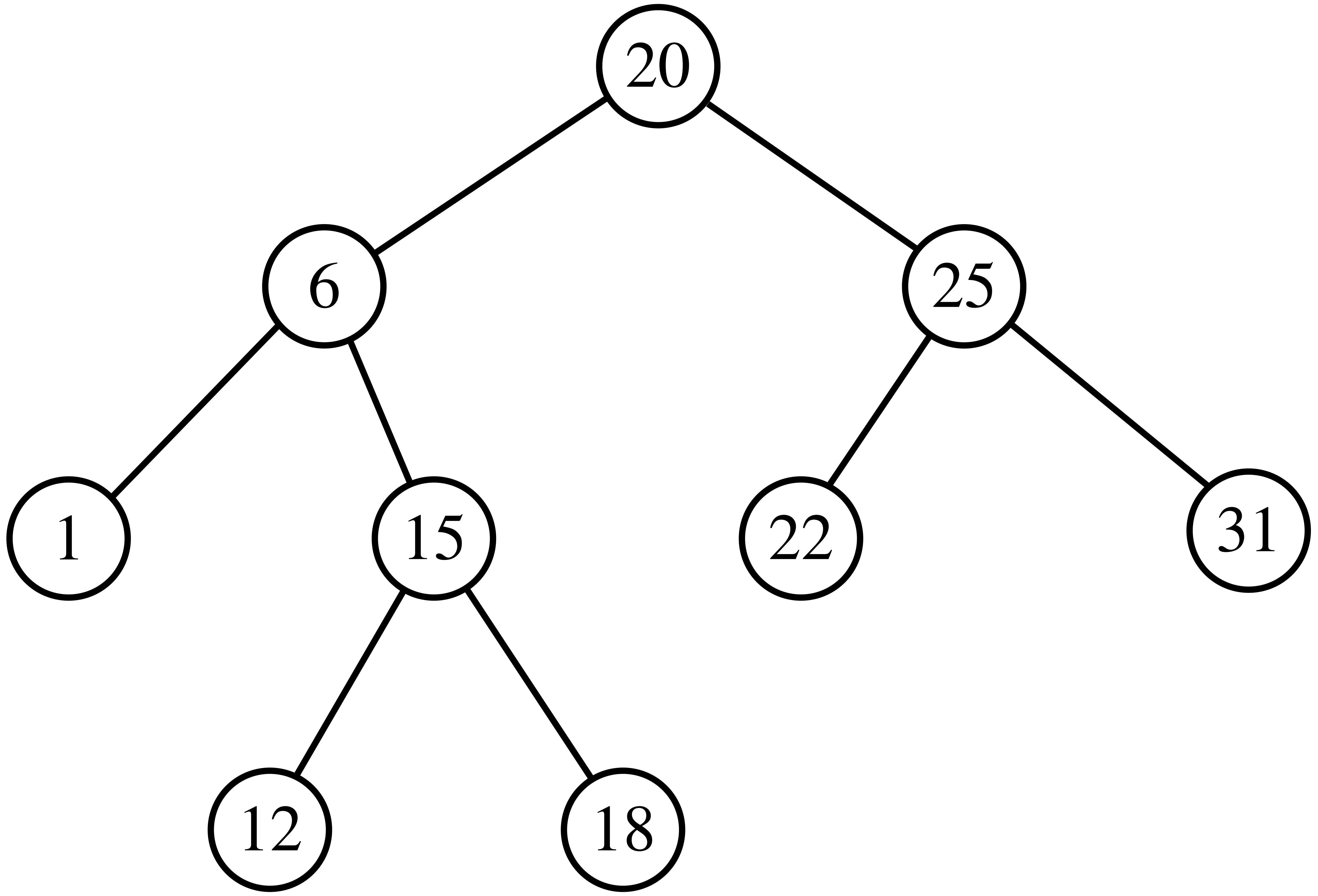

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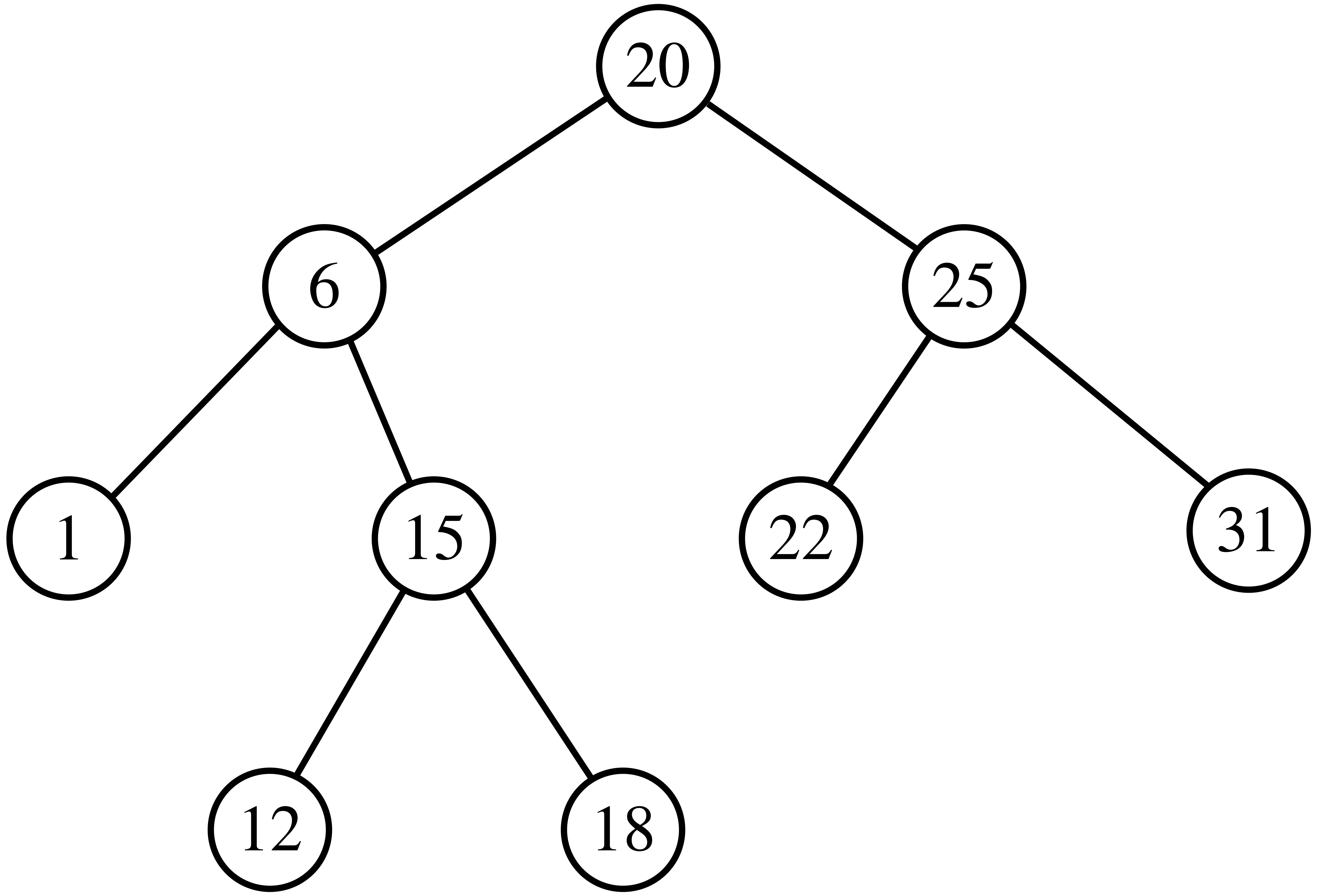

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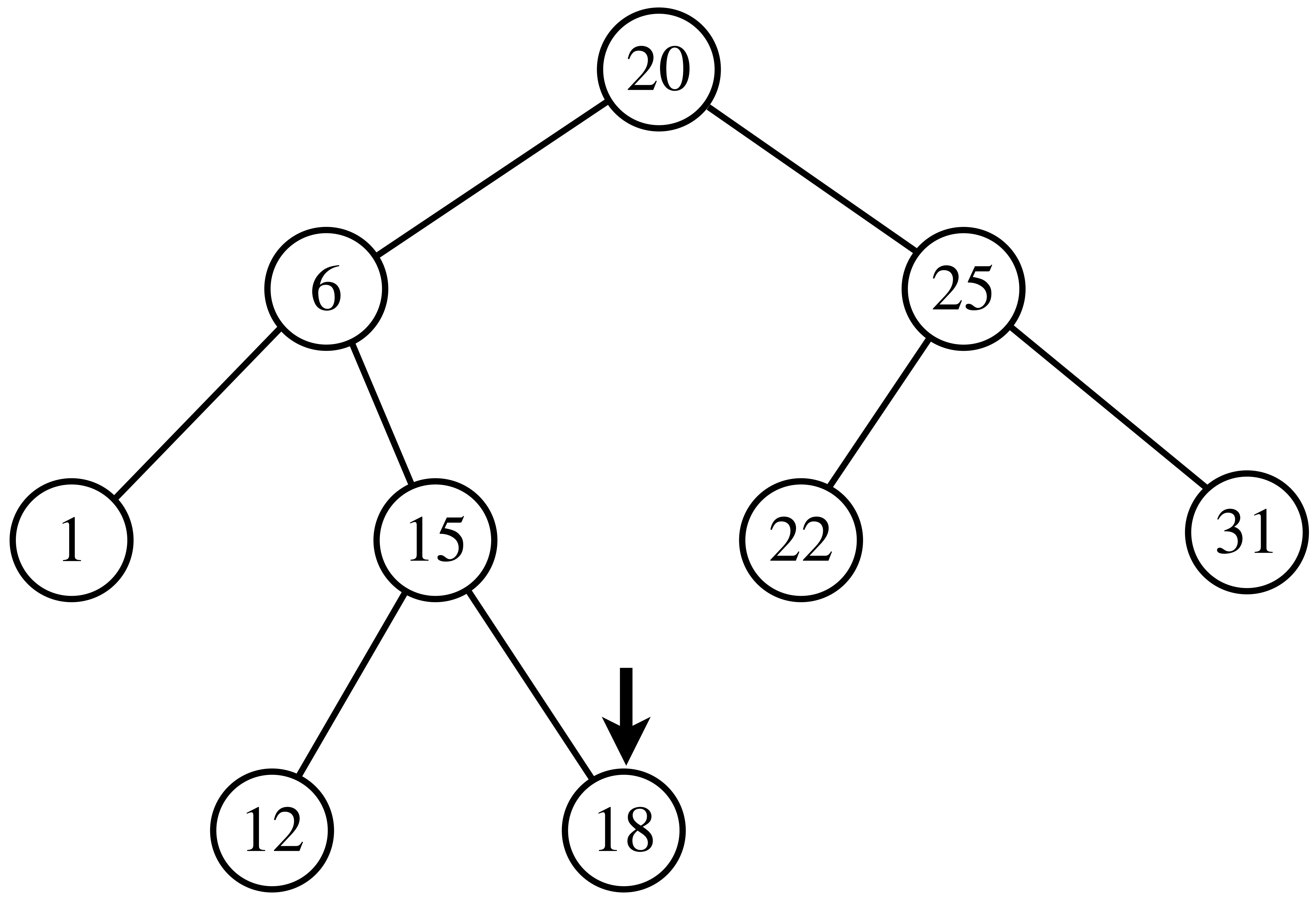

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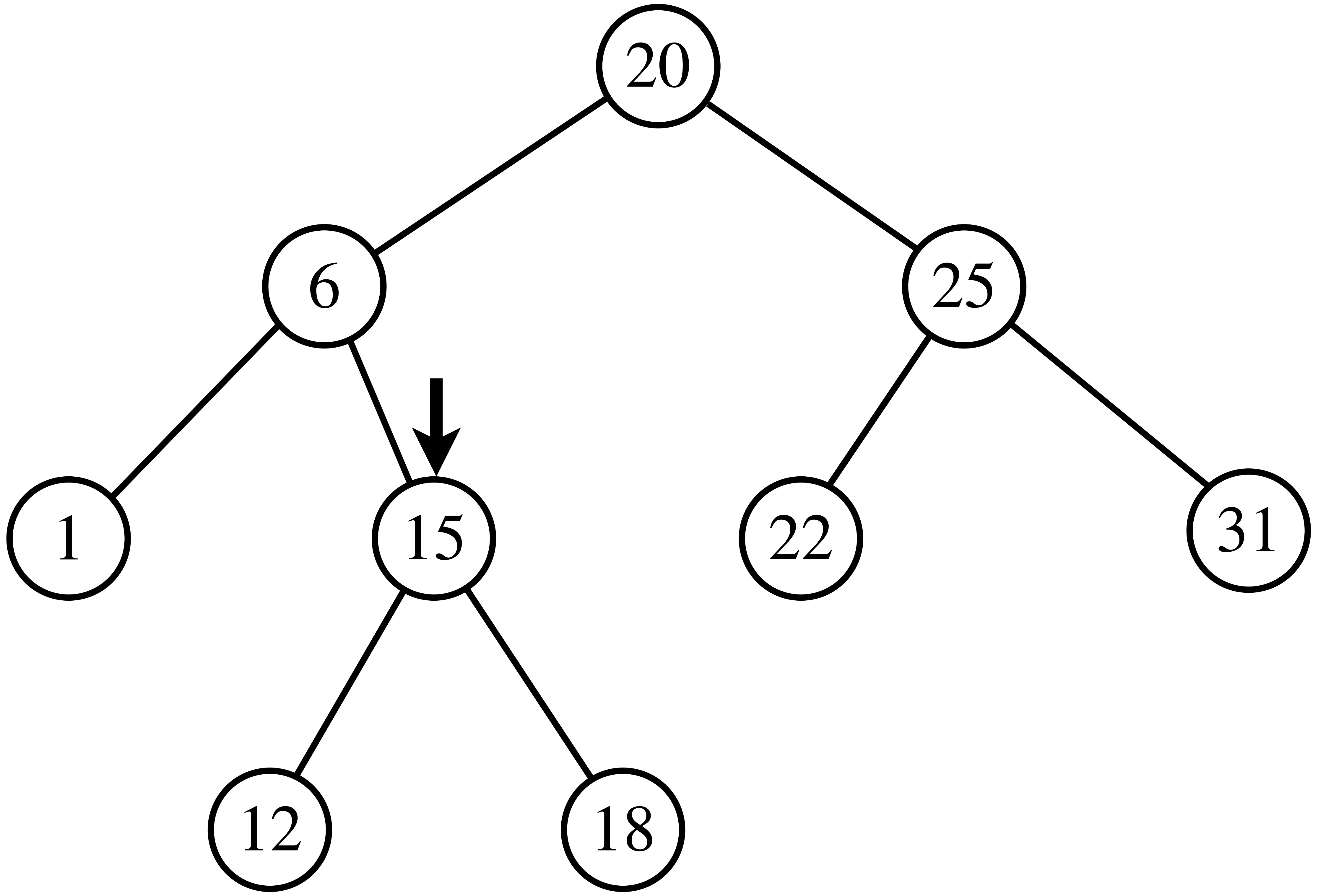

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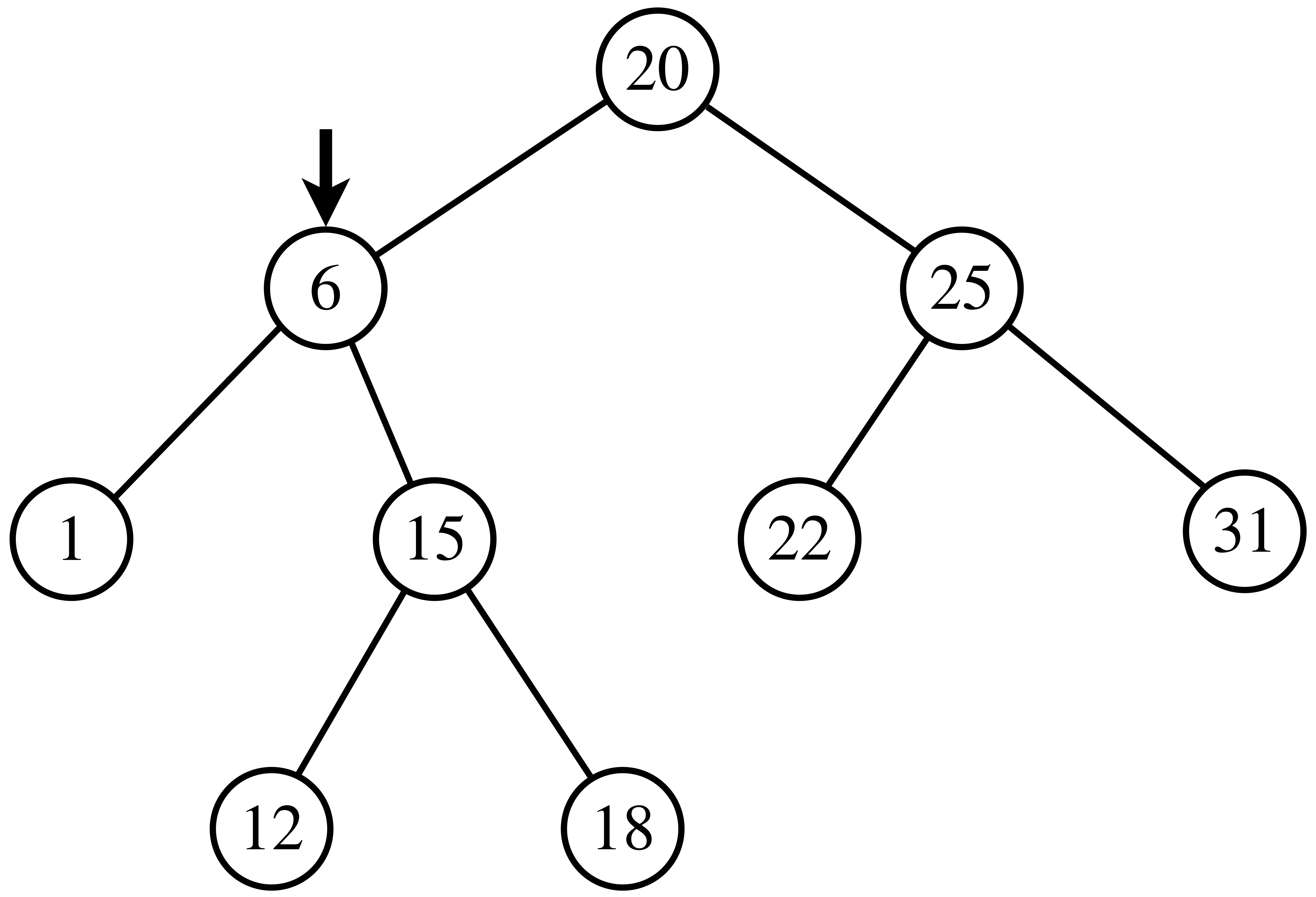

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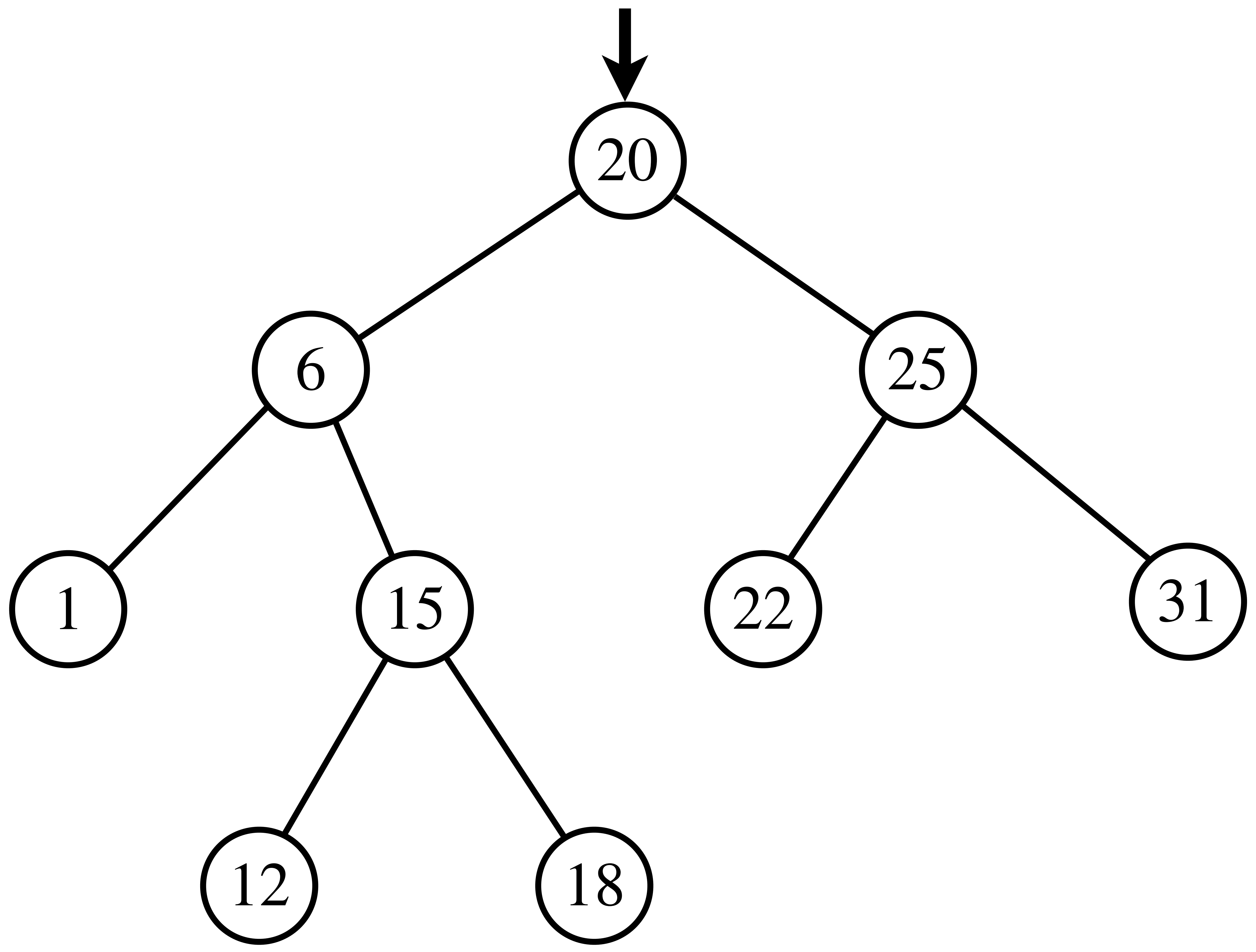

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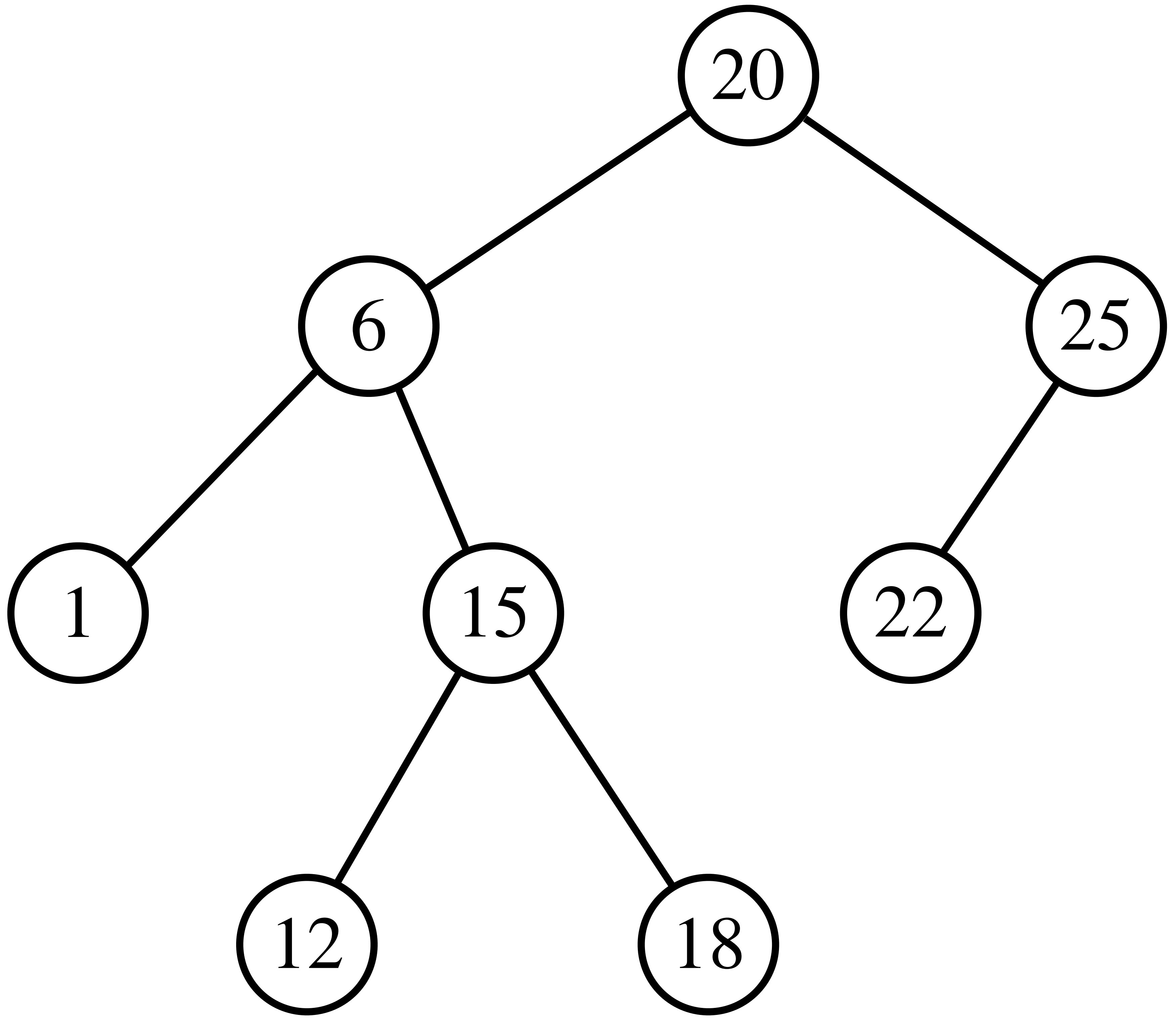

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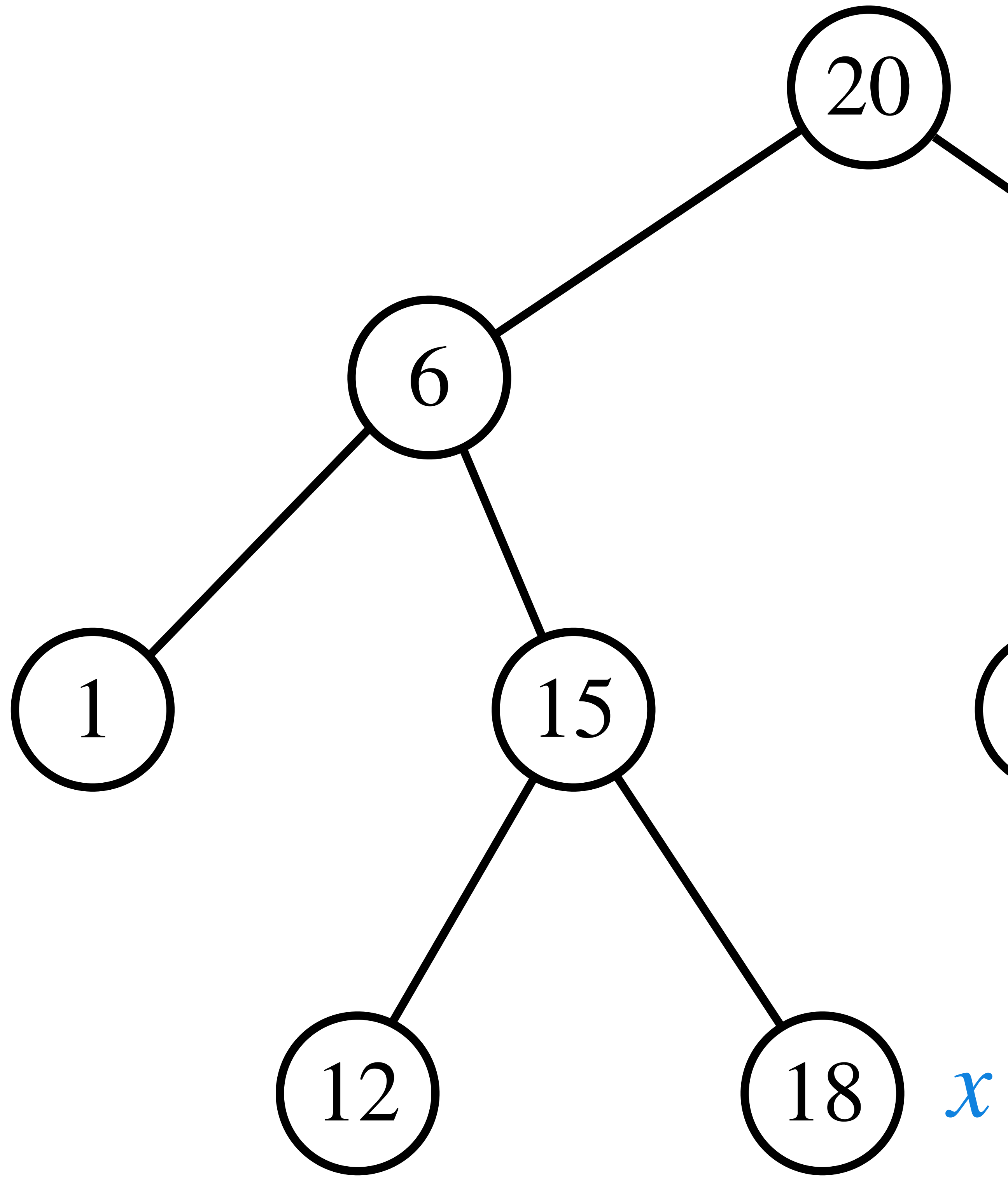

Querying a BST: Successor

Goal: Given a node x of a BST find its successor.

Algorithm: Call **Tree-Successor**(x) to find x's successor in T.

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Tree-Successor(x):
1.  if $x.right \neq$ NIL
2.      return Tree-Minimum($x.right$)
3.  else
4.      $y = x.p$
5.      while $y \neq$ NIL and $x \neq y.left$
6.          $x = y, y = y.p$
7.      return $y$
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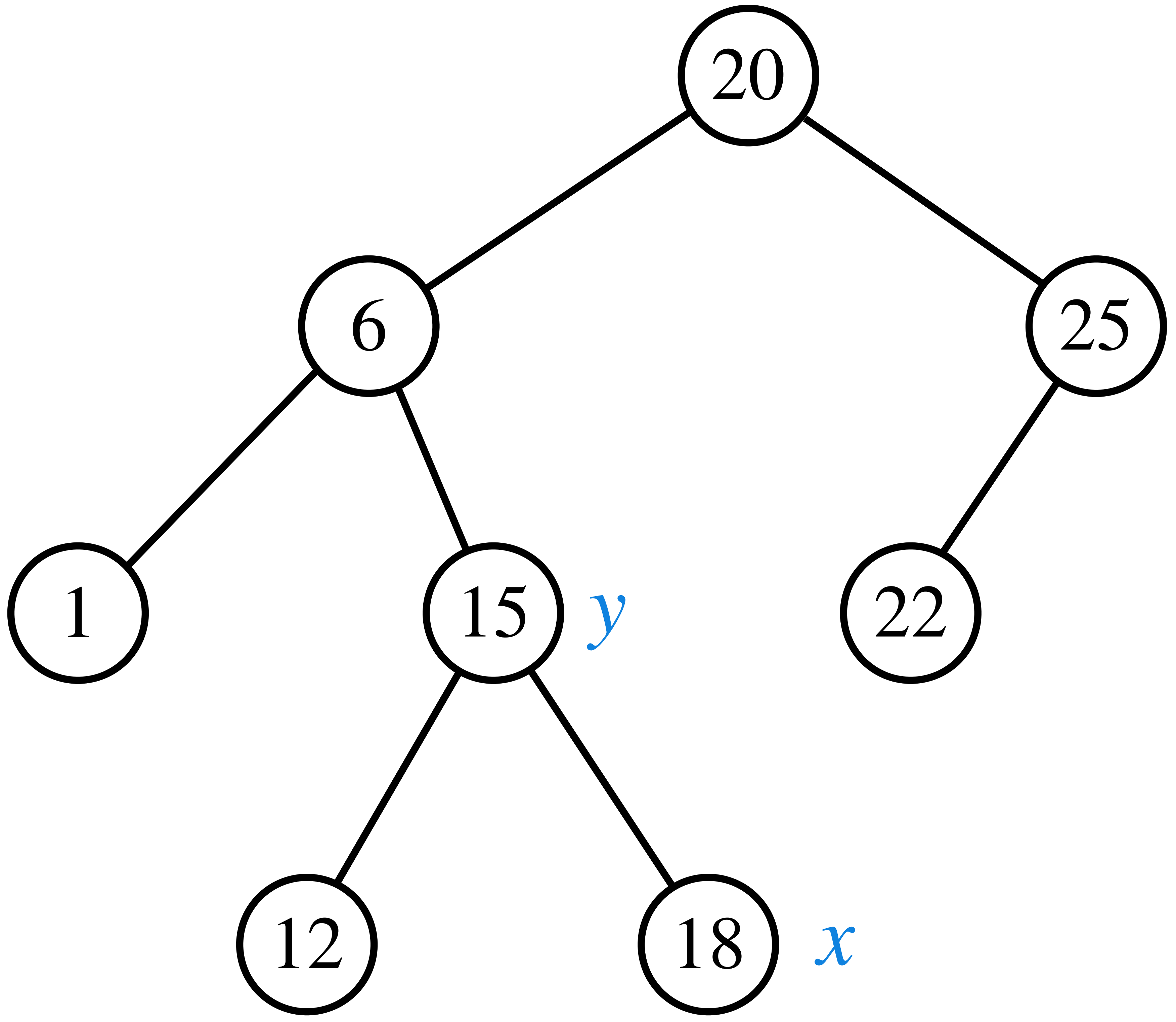

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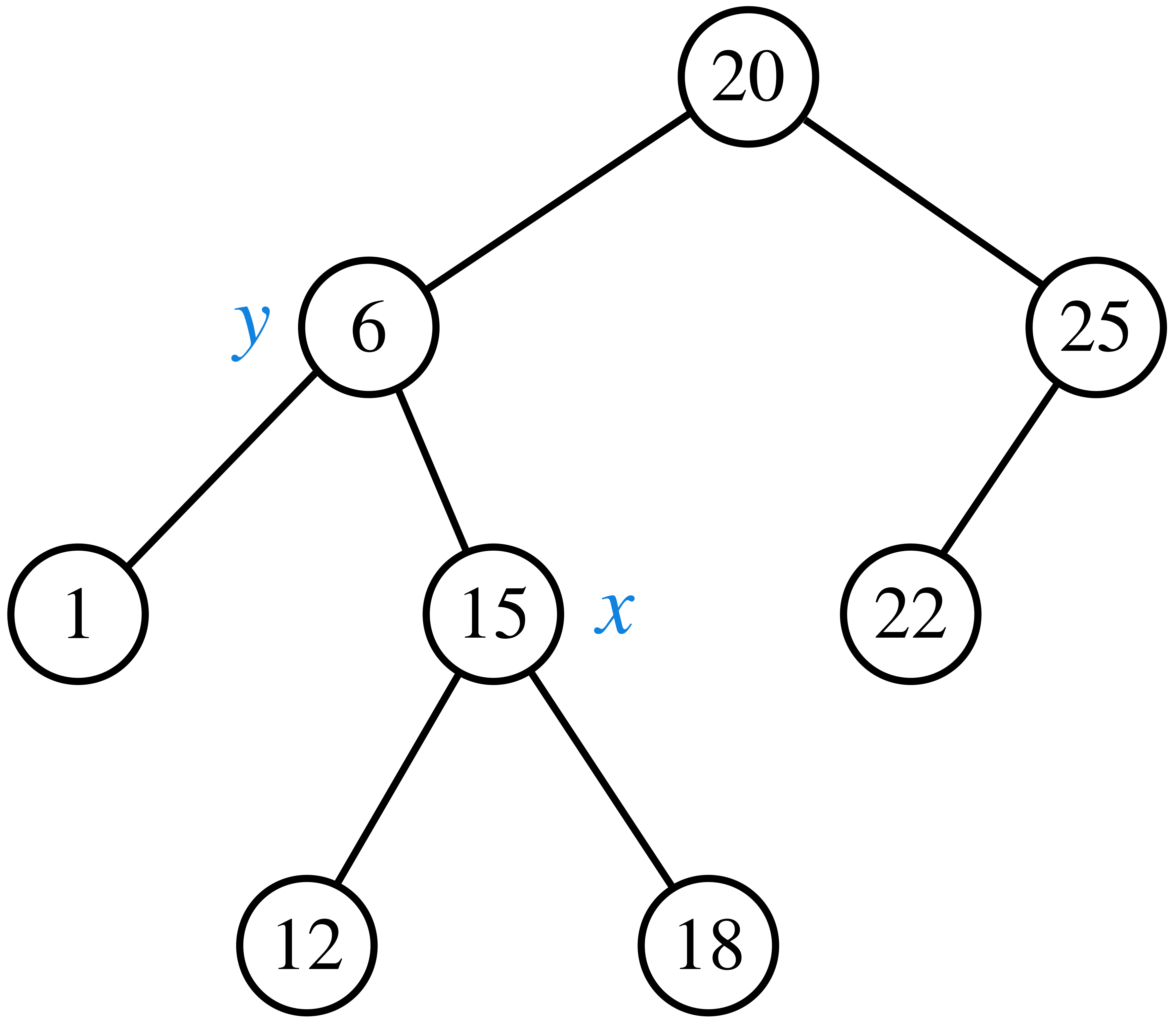

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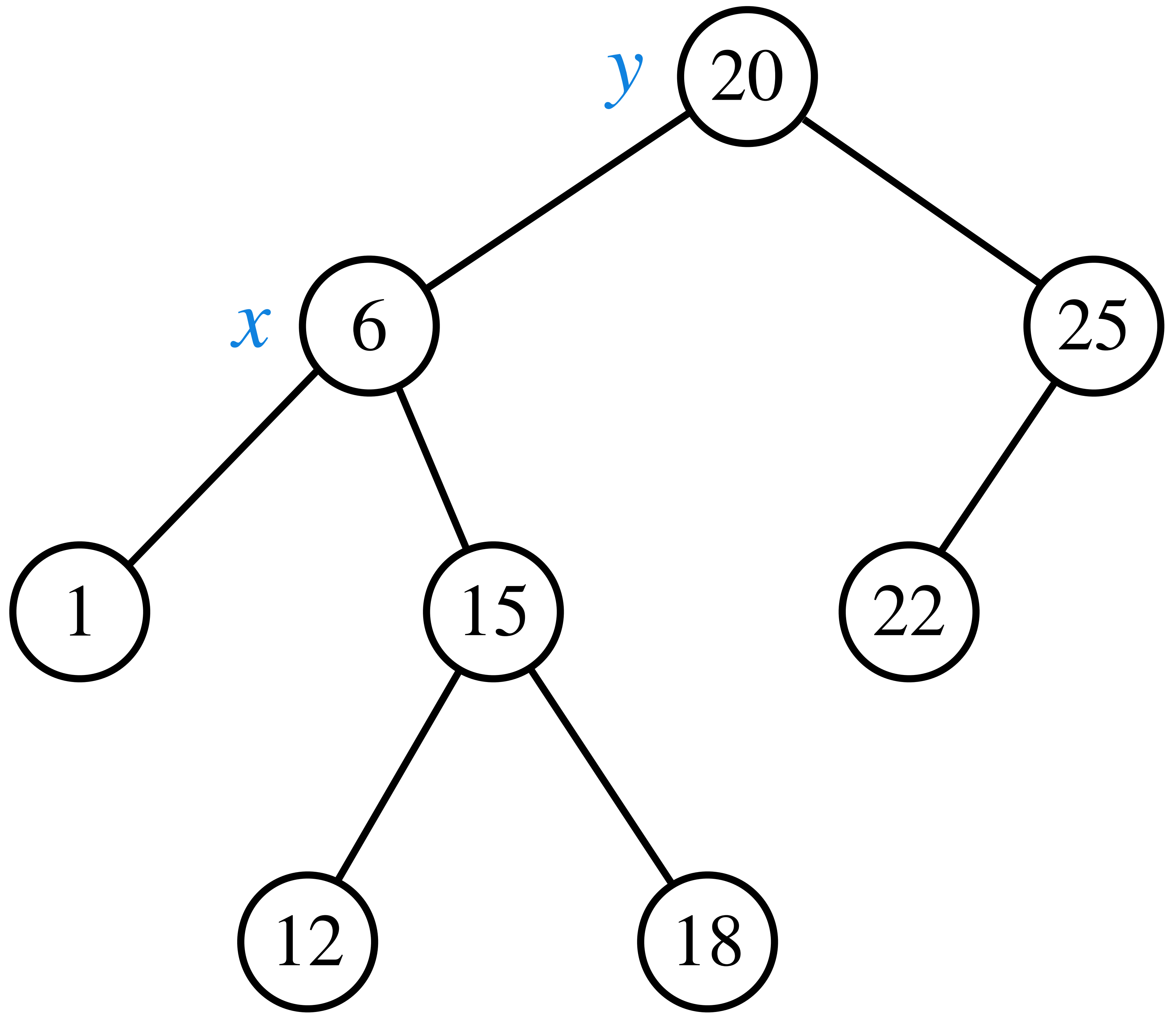

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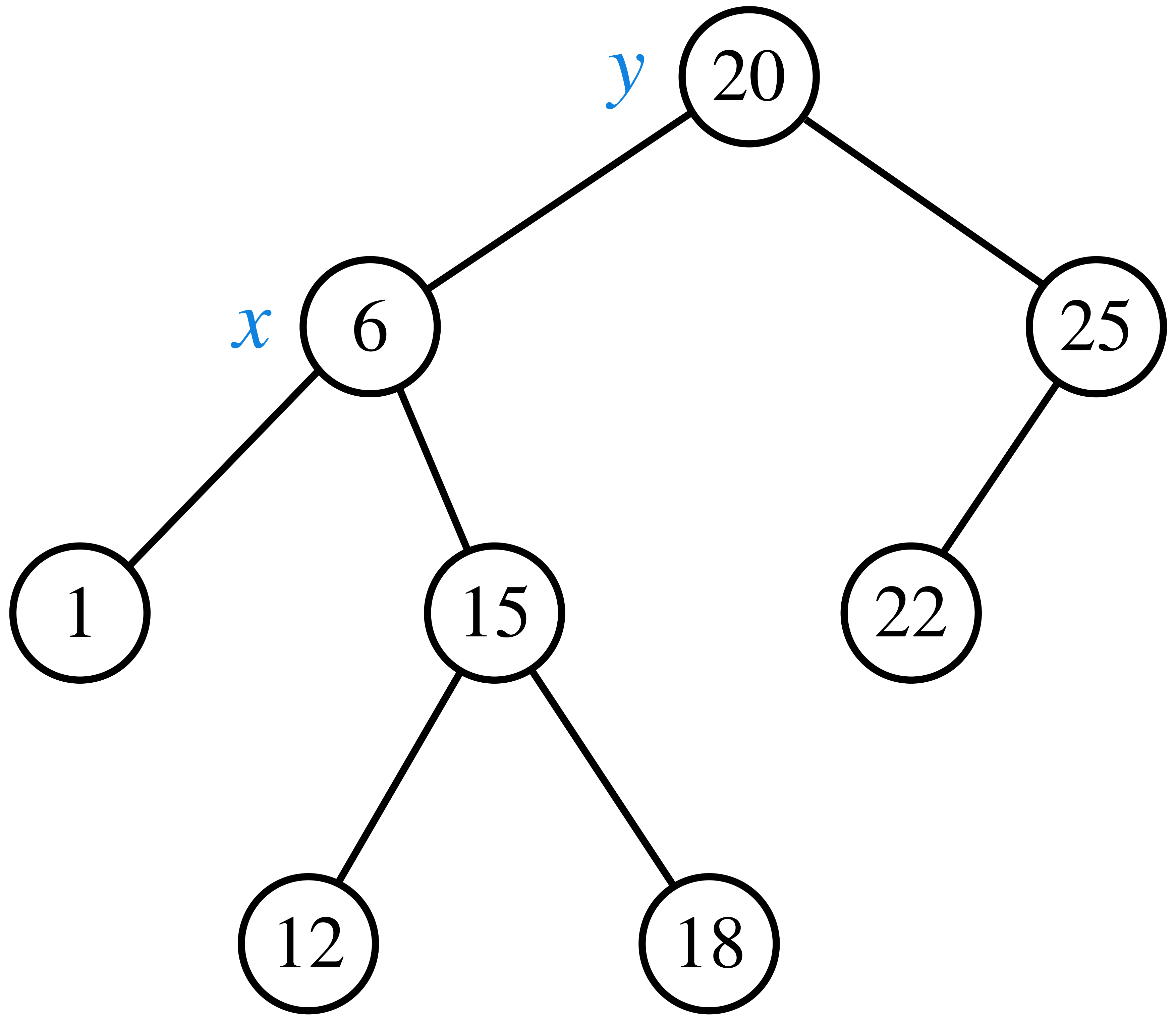

Runtime: $O(h)$, where h = height of T.