Golden Section method-

→ function in one variable

→ in $[a,b]$, ∃ only one minima.

Let $f(x)$ be a unimodal function in $[a,b]$

(i) $d = 0.618 \times (b-a)$

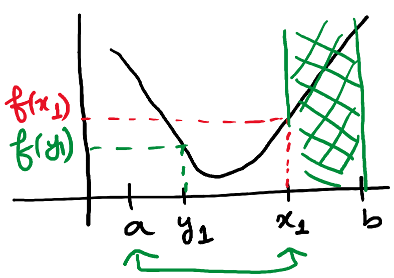
(ii) $x_1 = a+d$

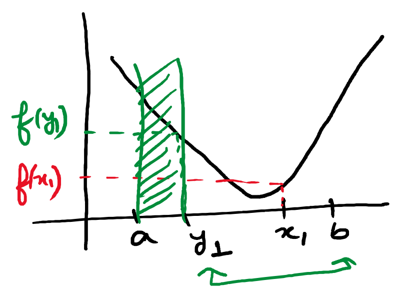
(iii) $y_1 = b-d$

⊛ (iv) $f(x_1)$ and $f(y_1)$

$f(x_1) > f(y_1)$ → New interval of search is $[a, x_1]$

$f(x_1) < f(y_1)$ → New interval of search is $[y_1, b]$





Second Iteration

Interval of search $[a, x_1] = [a_2, b_2]$

$d = 0.618 \times (b_2 - a_2)$

$x_2 = y_1 \, (= a_2 + d)$

$y_2 = b_2 - d$

Compute $f(x_2)$ and $f(y_2)$

$f(x_2) = f(y_1)$

Second Iteration

Interval of search $[y_1, b] = [a_2, b_2]$

$d = 0.618 \times (b_2 - a_2)$

$x_2 = a_2 + d$

$y_2 = x_1 \, (= b_2 - d)$

Compute $f(x_2)$ and $f(y_2) = f(x_1)$

* Repeat from the step ⊛ onwards.

* Where to stop iterations :- Let allowable length of error interval is l, then stop the iterations if length of interval of search is

less than l.

Example :- $f(x) = x^2 - 6x + 15$

Find $\min f(x)$ using Golden section method in interval $[0, 10]$.

Solution :-

① $d = 0.62 \times (10 - 0) = 6.2$

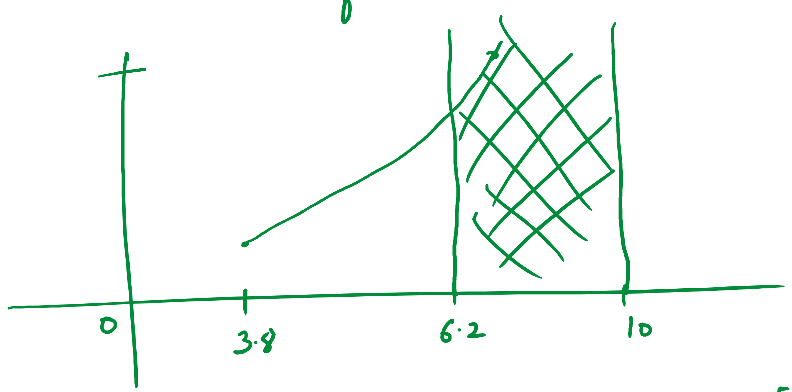
② $x_1 = a + d = 6.2$

③ $y_1 = b - d = 10 - 6.2 = 3.8$

$f(x_1) = f(6.2) = (6.2)^2 - 6 \times 6.2 + 15 = 16.24$

$f(y_1) = f(3.8) = (3.8)^2 - 6 \times 3.8 + 15 = 6.64$

$f(x_1) > f(y_1)$



New interval of search $[0, 6.2]$

second Iteration :- $[0, 6.2] = [a_2, b_2]$

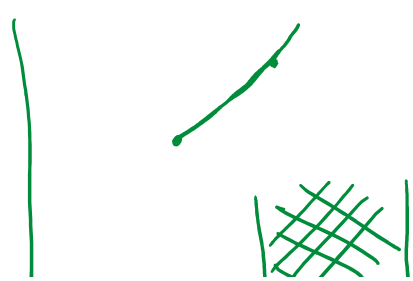
$d = 0.62 \times 6.2 = 3.8$

$x_2 = 0 + 3.8 = 3.8 = y_1$

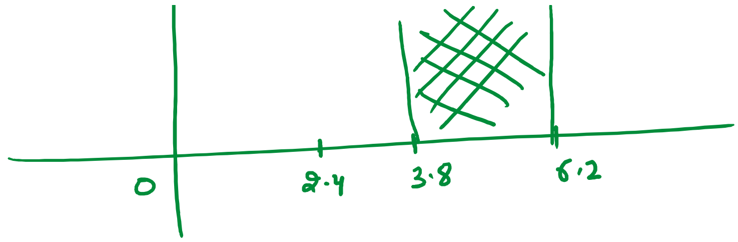
$y_2 = 6.2 - 3.8 = 2.4$

$f(x_2) = f(3.8) = 6.64$ $\quad f(y_2) = f(2.4) = 6.39$

$f(x_2) > f(y_2)$



New interval of search $[0, 3.8]$



$[0, 3.8]$

Third Iteration :-

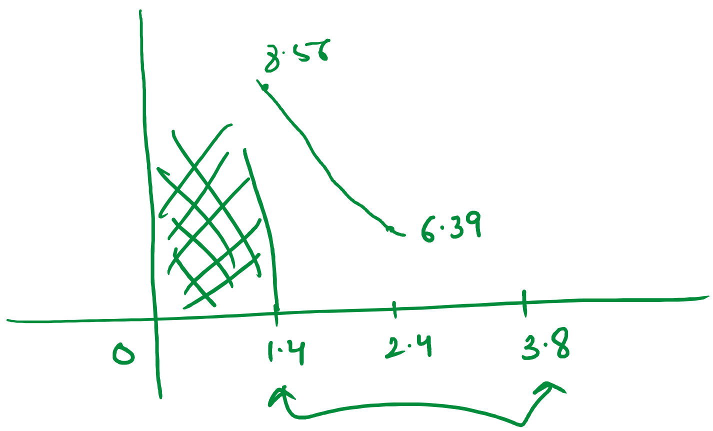
$[a_3, b_3] = [0, 3.8]$

$d = 0.62 \times 3.8 = 2.4$

$x_3 = a_3 + d = 2.4 = y_2$

$y_3 = b_3 - d = 3.8 - 2.4 = 1.4$

$f(x_3) = f(y_2) = 6.39$ $f(y_3) = f(1.4) = 8.56$



New interval of search $[1.4, 3.8]$ is $\underline{\underline{2.4}}$

- $f(x) = x^2 - 6x + 15$

$$\frac{d f(x)}{dx} = 2x - 6$$

$2x - 6 = 0$

$2x = 6$

$\boxed{x = 3}$ → minima

$$\frac{d^2 f(x)}{dx^2} = 2 > 0$$

$f(3) = 9 - 18 + 15 = 6$

- Fibonacci search method

  - Decide number iterations.

- Decide number iterations.

Fibonacci sequence.

$F_1 = 1, \quad F_2 = 1, \quad F_n = F_{n-1} + F_{n-2}$

$1, 1, 2, 3, 5, 8, 13, --$
$21, --$

$F_3 = F_1 + F_2 = 2$

$F_4 = F_3 + F_2 = 1+2=3$

$F_5 = F_3 + F_4 = 3+2=5$

fibonacci sequence.

$F_6 = F_4 + F_5 = 8$

$F_7 = 5+8 = 13$

$\boxed{\begin{array}{l} d = g(b-a) \\ x_1 = a+d \end{array}}$

* Relation b/w fibonacci search method and Golden method

$$\lim_{n\to\infty} \frac{F_{n-1}}{F_n} = \boxed{0.618} = g$$

n=2 $\frac{F_1}{F_2} = 1$, n=3 $\frac{F_2}{F_3} = \frac{1}{2}$ n=4 $\frac{F_3}{F_4} = \frac{2}{3}$ n=5 $\frac{F_4}{F_5} = \frac{3}{5}$ n=6. $\frac{F_5}{F_6} = \frac{5}{8}$

n=7 $\frac{F_6}{F_7} = \frac{8}{13} = 0.615$ $\frac{F_7}{F_8} = \frac{13}{21} = 0.61$ n=8

New iterval of search $[x_1, b] = [a_2, b_2]$

$$x_2 = b_2 - \frac{F_{n-2}}{F_{n-1}}(b_2 - a_2)$$

$$x_2 = b - \frac{F_{n-2}}{F_{n-1}}(b - x_1)$$

$$x_2 = b - \frac{F_{n-2}}{F_{n-1}}\left(b - \left(b - \frac{F_{n-1}}{F_n}(b-a)\right)\right.$$

$$x_2 = b - \frac{F_{n-2}}{F_{n-1}}\left(\cancel{b} - \cancel{b} + \frac{F_{n-1}}{F_n}(b-a)\right)$$

$$x_2 = b - \frac{F_{n-2}}{\cancel{F_{n-1}}} \times \frac{\cancel{F_{n-1}}}{F_n}(b-a)$$

$F_n = F_{n-1} + F_{n-2}$

$F_{n-2} = F_n - F_{n-1}$

$$x_2 = b - \frac{F_{n-2}}{F_n}(b-a)$$

$$x_2 = b - \frac{(F_n - F_{n-1})}{F_n}(b-a)$$

$$x_2 = b - \left(1 - \frac{F_{n-1}}{F_n}\right)(b-a) =$$

$$= \cancel{b} - (\cancel{b} - a) + \frac{F_{n-1}}{F_n}(b-a)$$

$$= a + \frac{F_{n-1}}{F_n}(b-a) = y_1$$