25 January 2025 16:00

Fibonacci search method

- Decide number iterations.

Fibonacci sequence.

$F_1 = 1, \quad F_2 = 1, \quad F_n = F_{n-1} + F_{n-2}$

$F_3 = F_1 + F_2 = 2$

$F_4 = F_3 + F_2 = 1+2=3$

$F_5 = F_3 + F_4 = 3+2=5$

$F_6 = F_4 + F_5 = 8$

$F_7 = 5+8 = 13$

1, 1, 2, 3, 5, 8, 13, 21, ---

fibonacci sequence.

$\boxed{\begin{array}{l} d = g(b-a) \\ x_1 = a+d \end{array}}$

* Relation b/w fibonacci search method and Golden method

$$\lim_{n\to\infty} \frac{F_{n-1}}{F_n} = \boxed{0.618} = \underline{\underline{g}}$$

$n=2$: $\frac{F_1}{F_2} = 1$, $n=3$: $\frac{F_2}{F_3} = \frac{1}{2}$, $n=4$: $\frac{F_3}{F_4} = \frac{2}{3}$, $n=5$: $\frac{F_4}{F_5} = \frac{3}{5}$, $n=6$: $\frac{F_5}{F_6} = \frac{5}{8}$

$n=7$: $\frac{F_6}{F_7} = \frac{8}{13} = 0.615$, $n=8$: $\frac{F_7}{F_8} = \frac{13}{21} = 0.61$

New interval of search $[x_1, b] = [a_2, b_2]$

$$x_2 = b_2 - \frac{F_{n-2}}{F_{n-1}}(b_2 - a_2)$$

$$x_2 = b - \frac{F_{n-2}}{F_{n-1}}(b - x_1)$$

$$\left(\quad \frac{F_{n-1}}{} (b-a) \right)$$

$$x_2 = \qquad F_{n-1}$$

$$x_2 = b - \frac{F_{n-2}}{F_{n-1}}\left(b - \left(b - \frac{F_{n-1}}{F_n}(b-a)\right)\right)$$

$$x_2 = b - \frac{F_{n-2}}{F_{n-1}}\left(\cancel{b} - \cancel{b} + \frac{F_{n-1}}{F_n}(b-a)\right)$$

$$x_2 = b - \frac{F_{n-2}}{\cancel{F_{n-1}}} \times \frac{\cancel{F_{n-1}}}{F_n}(b-a)$$

$$F_n = F_{n-1} + F_{n-2}$$
$$F_{n-2} = F_n - F_{n-1}$$

$$x_2 = b - \frac{F_{n-2}}{F_n}(b-a)$$

$$x_2 = b - \frac{(F_n - F_{n-1})}{F_n}(b-a)$$

$$x_2 = b - \left(1 - \frac{F_{n-1}}{F_n}\right)(b-a) =$$

$$= \cancel{b} - (\cancel{b} - a) + \frac{F_{n-1}}{F_n}(b-a)$$

$$= a + \frac{F_{n-1}}{F_n}(b-a) = y_1$$

Fibonacci search method

→ Unimodal function with single minima in interval of search.

$\min \varphi(x) \rightarrow$ objective

$\ell \rightarrow$ allowable length of uncertainity

$[a_1, b_1] \rightarrow$ Intial interval of search.

such that $F_n > |b_1 - a_1|$

single interval of $[a_1, b_1]$

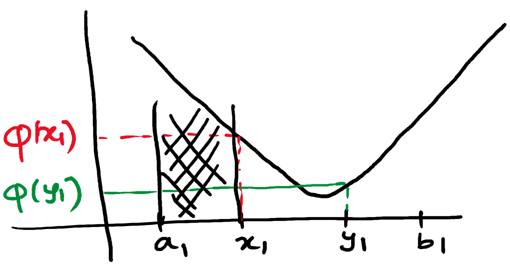
① Choose F_n such that $F_n > \left|\dfrac{b_1 - a_1}{\ell}\right|$

② $$x_1 = b_1 - \frac{F_{n-1}}{F_n}(b_1 - a_1) \quad \text{and} \quad y_1 = a_1 + \frac{F_{n-1}}{F_n}(b_1 - a_1)$$

③ Compute $\varphi(x_1)$ and $\varphi(y_1)$.

If $\varphi(x_1) > \varphi(y_1)$

Then, new interval of search is $[x_1, b_1]$



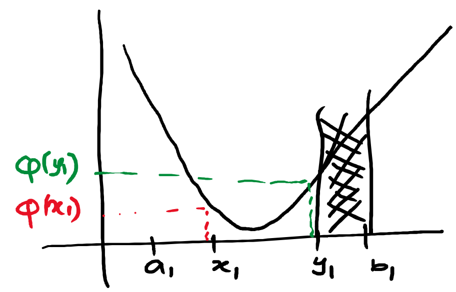
$$[x_1, b_1] = [a_2, b_2]$$

$$x_2 = b_2 - \frac{F_{n-2}}{F_{n-1}}(b_2 - a_2) \; (= y_1)$$

$$y_2 = a_2 + \frac{F_{n-2}}{F_{n-1}}(b_2 - a_2)$$

If $\varphi(x_1) < \varphi(y_1)$

Then, new interval of search is $[a_1, y_1]$



$$[a_1, y_1] = [a_2, b_2]$$

$$x_2 = b_2 - \frac{F_{n-2}}{F_{n-1}}(b_2 - a_2)$$

$$y_2 = a_2 + \frac{F_{n-2}}{F_{n-1}}(b_2 - a_2) \; (= x_1)$$

* Notice that at k'th iteration

$$x_k = b_k - \frac{F_{n-k}}{F_{n-k+1}}(b_k - a_k)$$

$$y_k = a_k + \frac{F_{n-k}}{F_{n-k+1}}(b_k - a_k)$$

* Notice that at $(n-2)^{th}$ iteration

$$x_{n-2} = y_{n-2}$$

So No new evaluation is possible in this ... perform, new evaluation at

So No new evaluation ... at iteration, we perform, new evaluation at

interval of search $\ni x_{n-1} + \varepsilon$, for suitably chosen ε.

Question :- Min (x^2+2x) using fibonacci search method.

- Intial interval of search is $[-3, 5] = [a, b]$
- Allowable length of uncertainity is $\underline{\underline{1}} = l$

Solution :- $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5$, $F_6 = 8$, $F_7 = 13$, $F_8 = 21$, ---

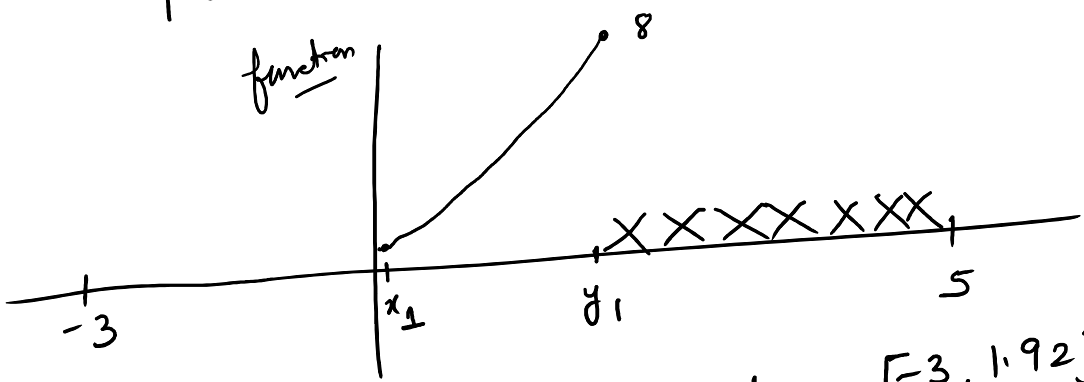
$$\left|\frac{b-a}{l}\right| = \left|\frac{5-(-3)}{1}\right| = 8 < F_7 = 13$$

$$x_1 = b_1 - \frac{F_6}{F_7}(b_1 - a_1) = 5 - \frac{8}{13}(8) = 0.077$$

$$y_1 = a_1 + \frac{F_6}{F_7}(b_1 - a_1) = -3 + \frac{8}{13}(8) = 1.92$$

$$\varphi(x_1) = x_1^2 + 2x_1 \qquad \varphi(y_1) = y_1^2 + 2y_1$$

$$\varphi(0.077) = 0.159 \qquad \varphi(1.92) = 7.52$$



New interval of search $[-3, 1.92] = [a_2, b_2]$

$$x_2 = b_2 - \frac{F_5}{F_6}(b_2 - a_2) = 1.92 - \frac{5}{8}(1.92 + 3)$$

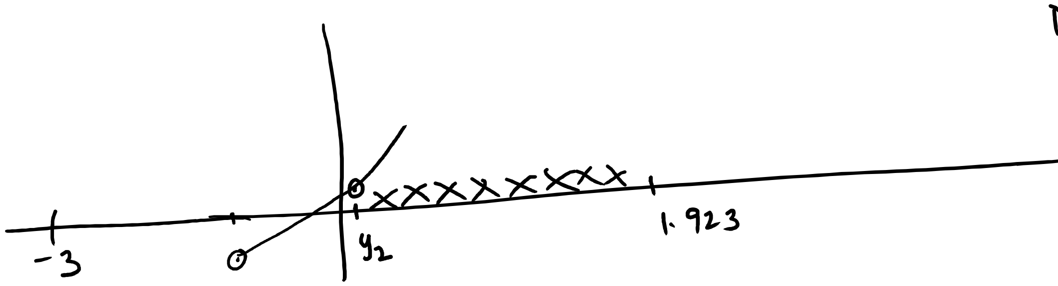
$$= -1.1538$$

$x_2 = b_2 \quad \overline{F_6} \; (b_2 - a) \qquad 8$

$= -1.1538$

$y_2 = x_1 = 0.077 \quad , \quad \varphi(y_2) = 0.1597$

$\varphi(x_2) = -0.9763$



New interval of search $[-3, 0.077]$

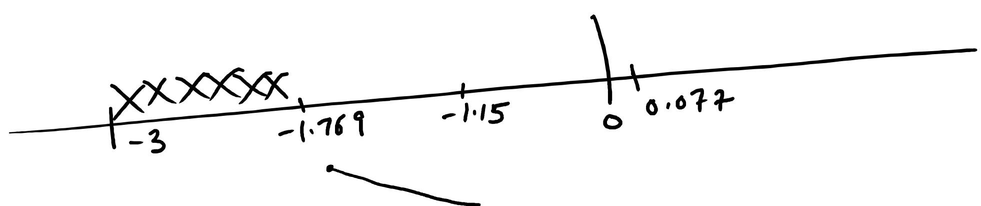
Third Iteration :- $[a_3, b_3] = [-3, 0.077]$

$$x_3 = b_3 - \frac{F_4}{F_5}(b_3 - a_3) = -1.769$$

$\varphi(x_3) = -0.408$

$\varphi(x_2) = \varphi(y_3) = -0.9763$

$-1.15 = x_2 = y_3$



New interval of search $[-1.769, 0.077]$

$$x_k = b_k - \frac{f_{n-k}}{f_{n-k+1}}(b_k - a_k)$$

$$y_k = a_k + \frac{f_{n-k}}{f_{n-k+1}}(b_k - a_k)$$

$\boxed{n-1}$

$$y_k = a_k + \frac{F_{n-k}}{F_{n-k+1}}(b_k - a_k)$$

$\boxed{n-1}$

$k = n-2$

$$x_{n-2} = b_{n-2} - \frac{F_2}{F_3}(b_{n-2} - a_{n-2})$$

$$y_{n-2} = a_{n-2} + \frac{F_2}{F_3}(b_{n-2} - a_{n-2})$$

$F_2 = 1, \quad F_3 = 2$

n

$\boxed{n-1}$

$$\checkmark\; x_{n-2} = b - \frac{1}{2}(b-a) = \frac{b+a}{2} \checkmark$$

$$\parallel$$

$$\checkmark\; y_{n-2} = a + \frac{1}{2}(b-a) = \frac{b+a}{2} \checkmark = \underline{x_{n-3}}$$

at this we choose small ε, suitably

$$f(x_{n-2} + \varepsilon)$$