Lecture 5

Hashing (contd.), Binary Search Trees

Independent Uniform Hashing

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Suppose every element of the dynamic set has a distinct key from the universe

$U = \{0, 1, \ldots, n-1\}$

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What is the expected length of a chain now?

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Sample Space : $(1,1)$

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$$\begin{array}{rcccc} \text{Sample Space}: & (1,1) & \ldots & (2,3) \\ & \downarrow & & \downarrow \\ X: & 2 & \ldots & \end{array}$$

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$$\begin{array}{rcccc} \text{Sample Space} & : & (1,1) & \dots & (2,3) \\ & & \downarrow & & \downarrow \\ X & : & 2 & \dots & 5 \end{array}$$

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Sample Space :	(1,1)	…	(2,3)	(1,4)	…	(5,2)	…	(6,6)
	↓		↓	↓		↓		↓
X :	2	…	5	5	…	7	…	12

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$$\textbf{Ex}(X) = 2.(1/36) + 3.(2/36) + \ldots\ldots\ldots + 12.(1/36)$$

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 & \downarrow & & \downarrow & \downarrow & & \downarrow & & \downarrow \\
X: & 2 & \dots & 5 & 5 & \dots & 7 & \dots & 12
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$$\textbf{Ex}(X) = 2.(1/36) + 3.(2/36) + \dots\dots\dots + 12.(1/36) = 7$$

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$$\mathbf{Ex}(X) = 2.(1/36) + 3.(2/36) + \ldots\ldots\ldots + 12.(1/36) = 7$$

Linearity of Expectation: $\mathbf{Ex}[X_1 + X_2 + \ldots + X_n] = \mathbf{Ex}[X_1] + \mathbf{Ex}[X_2] + \ldots + \mathbf{Ex}[X_n]$

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$$T_j = X_0 + X_1 + \ldots + X_{n-1}$$

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$$\textbf{Ex}[T_j] \ = \ \textbf{Ex}[X_0 + X_1 + \ldots + X_{n-1}]$$

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Search's Time Complexity: $O(1)$, when $n = m$.

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  - If "second-choice" location is already filled, then place it in "third-choice" location if it's free.

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- Number of elements cannot exceed the size of hash table.

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$$h : U \times \{0,1,\ldots,m-1\} \to \{0,1,\ldots,m-1\}$$

Such that for all the keys k:

Probe sequence $\langle h(k,0), h(k,1), \ldots, h(k,m-1) \rangle$ is a permutation of $\langle 0,1,\ldots,m-1 \rangle$

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OA-INSERT(T, x)**:**

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OA-INSERT(T, x)**:**

$i = 0$

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    $q = h(x\,.\,key, i)$

    if $T[q] == NIL$

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while i ≠ m
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    q = h(x.key, i)
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    else
        i = i + 1
while i ≠ m
error "Hash table overflow"
```

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    $q = h(k, i)$

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while T[q] ≠ NIL and i ≠ m
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    if $T[q] \,.\, key == x \,.\, key$

        $T[q] = NIL$

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        else
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    while T[q] ≠ NIL and i ≠ m
    return NIL
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  if $T[q].key == x.key$
    $T[q] = NIL$
    return
  else
    $i = i + 1$
while $T[q] \neq NIL$ **and** $i \neq m$
return NIL

Flaw: Deletion like this can affect the search or deletion of previously inserted element.

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$$h(k, i) = (h'(k) + i) \,\%\, m$$

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$h(k, i) = (h'(k) + i) \,\%\, m$, where $h'(k)$ is an ordinary hash function.

Double Hashing:

$h(k, i) = (h_1(k) + ih_2(k)) \,\%\, m$

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Linear Probing:

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To hit all the slots

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What is a Binary Search Tree?

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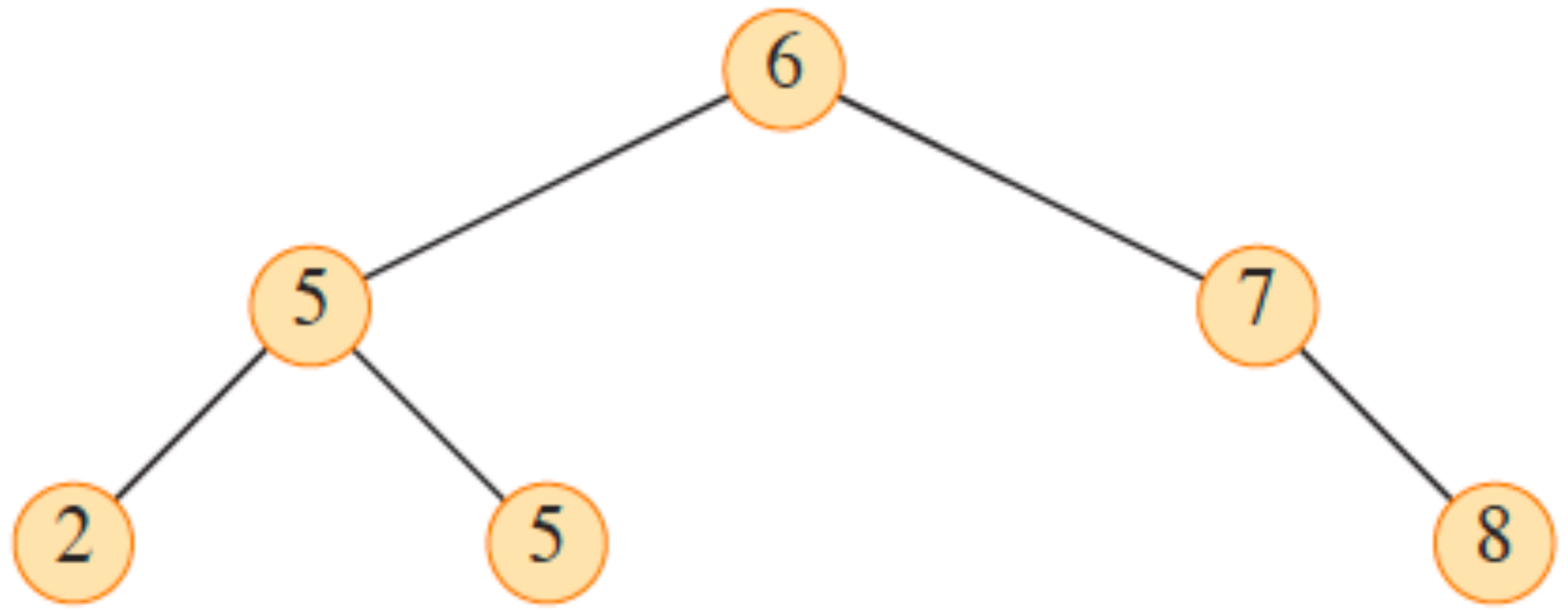

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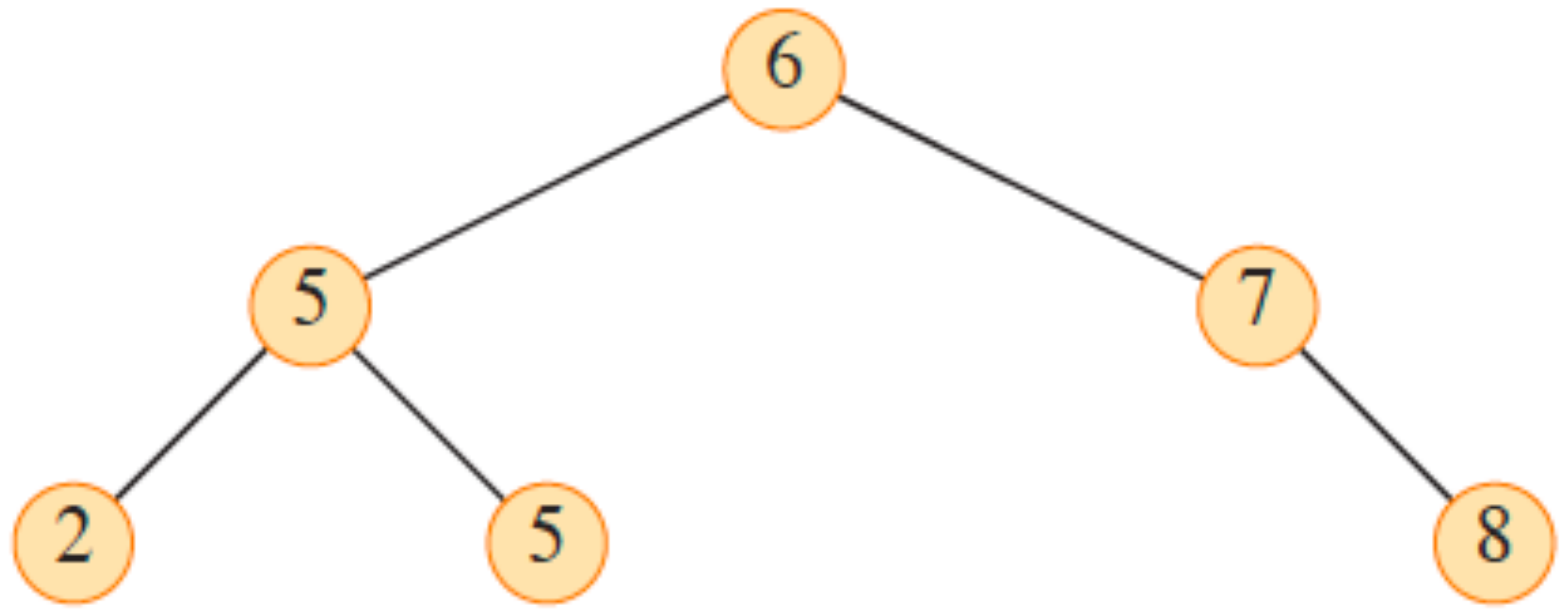

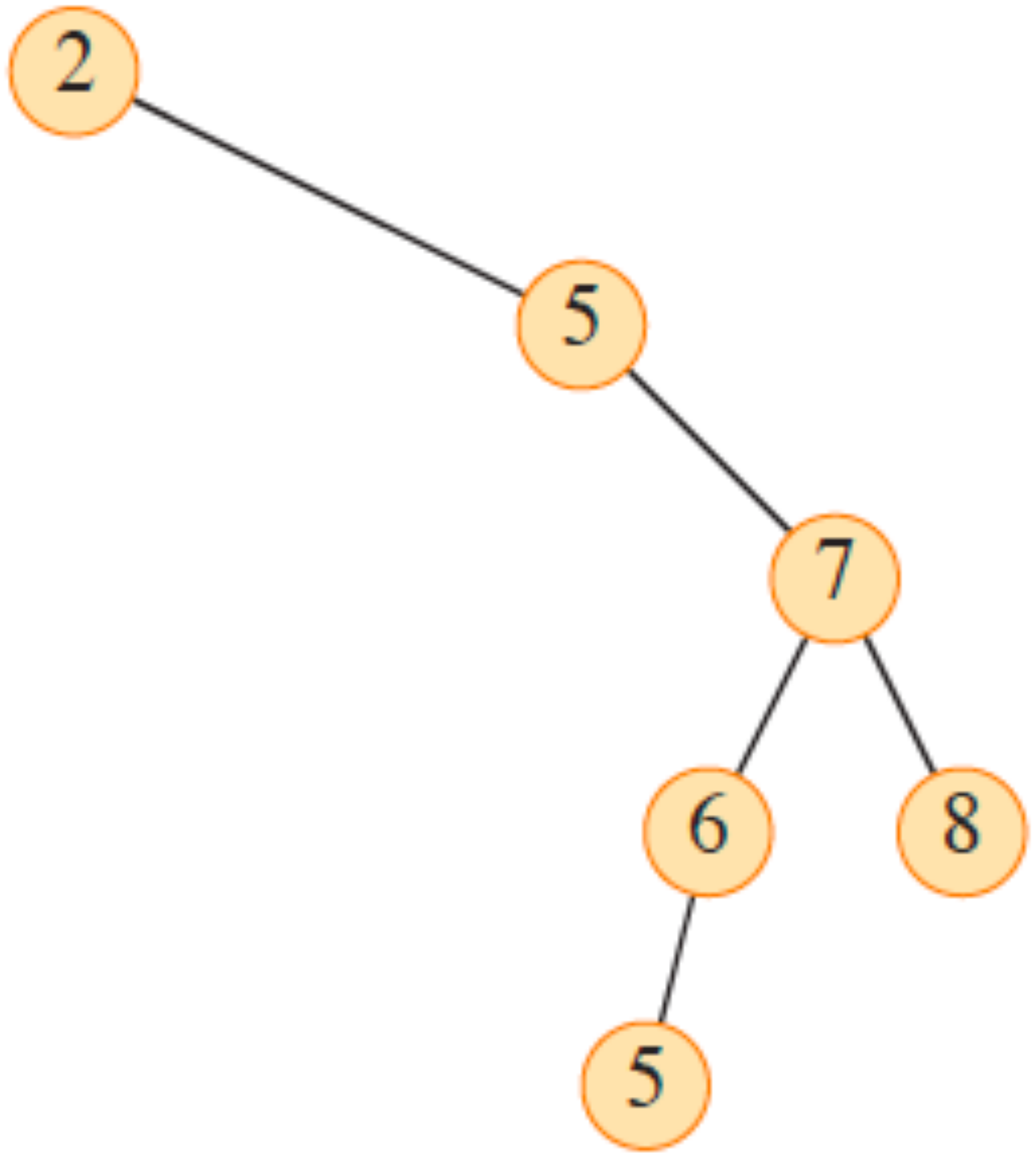

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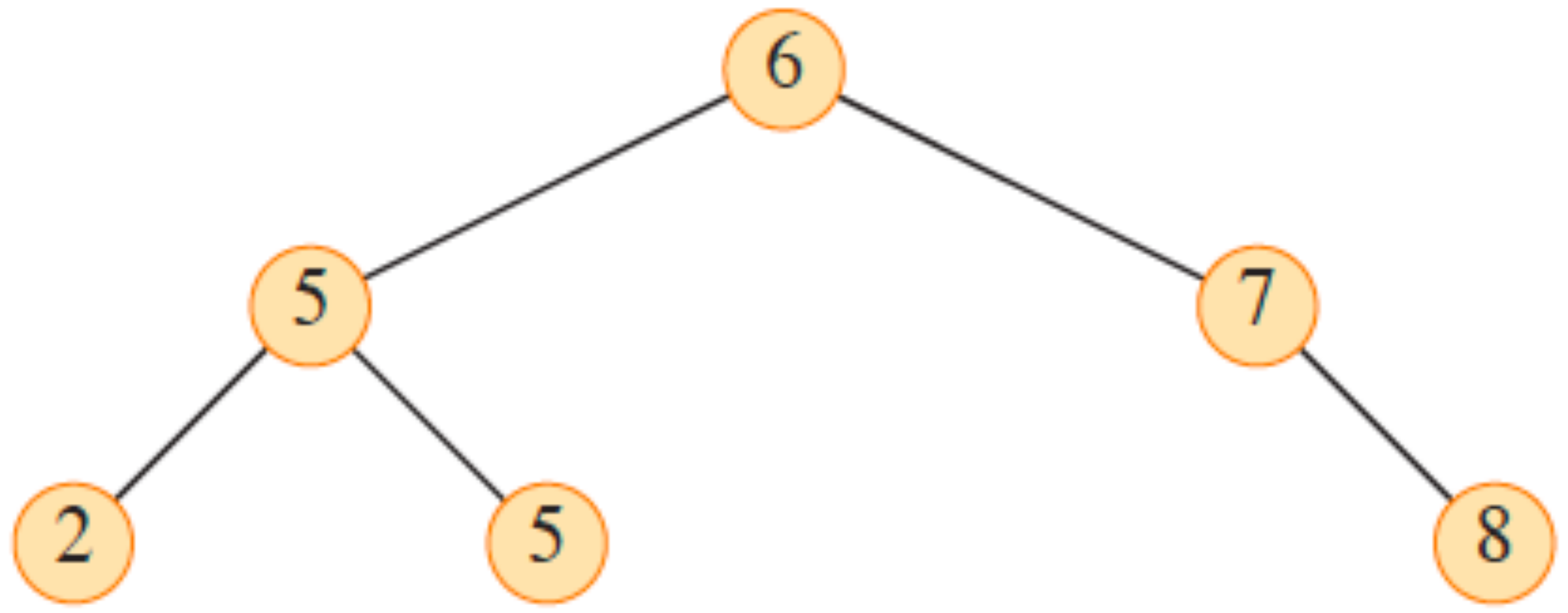

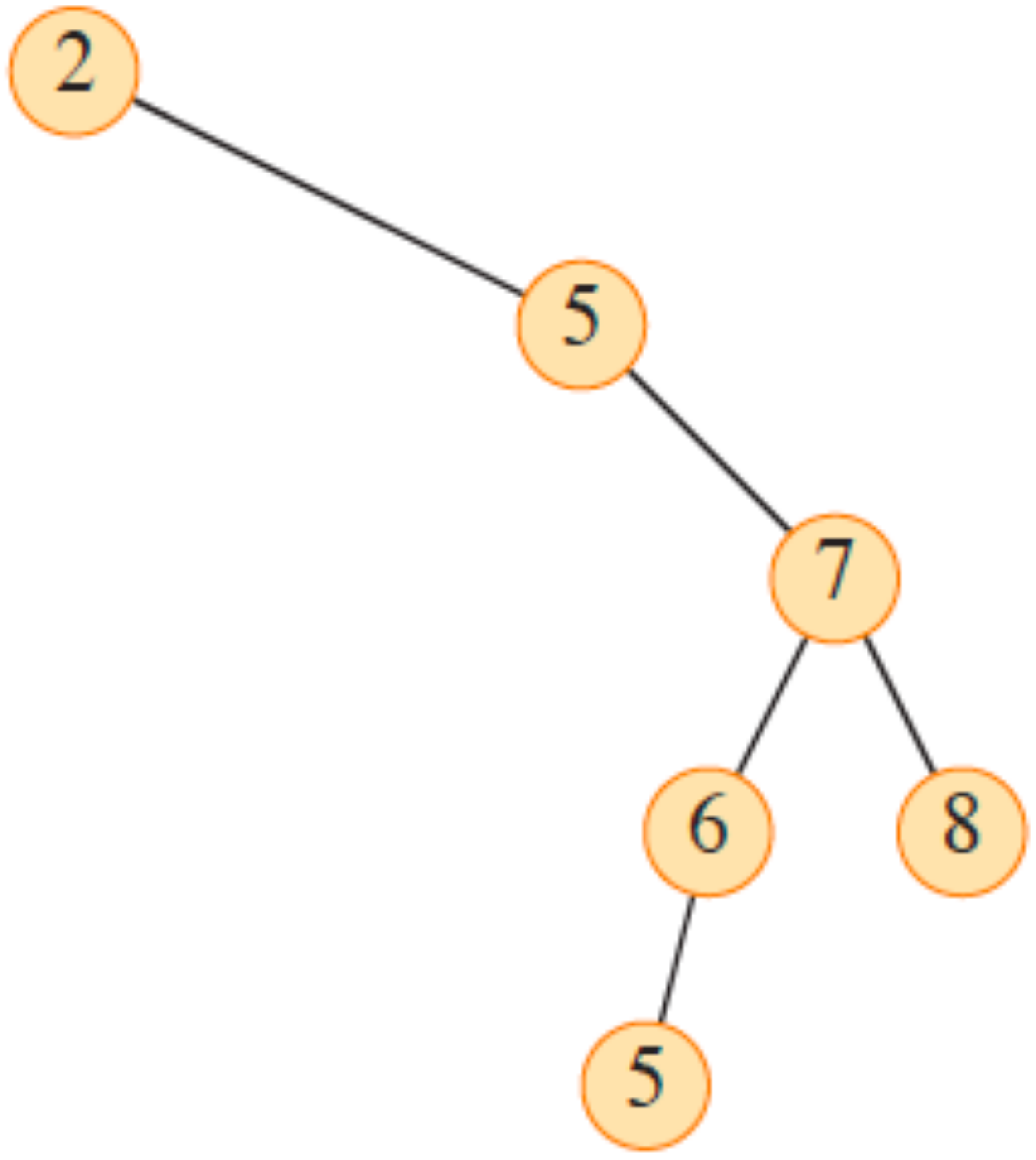

Source: CLRS

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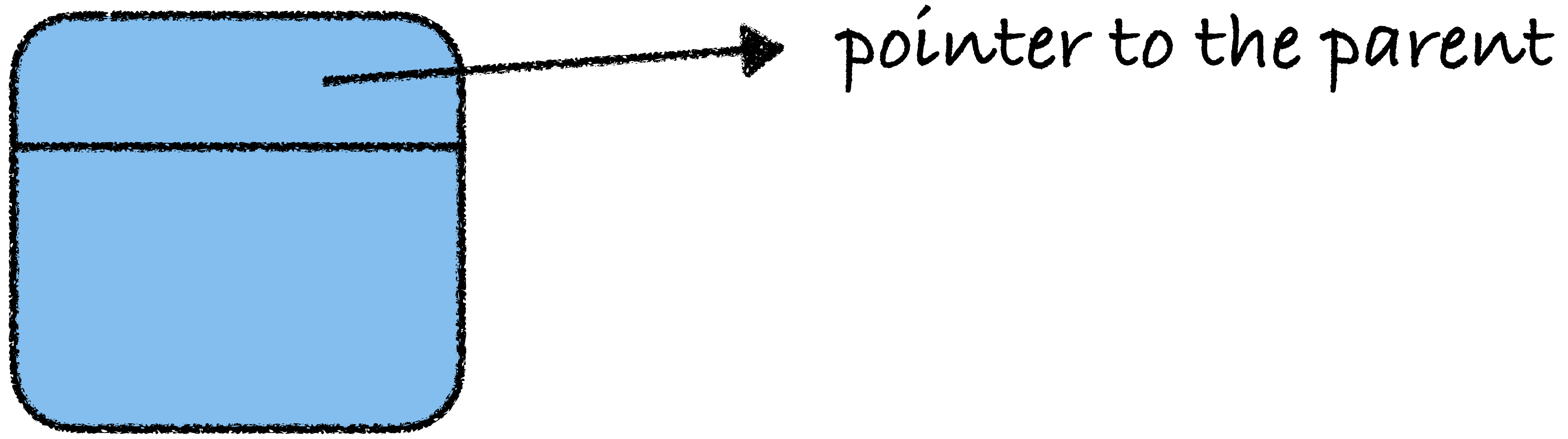

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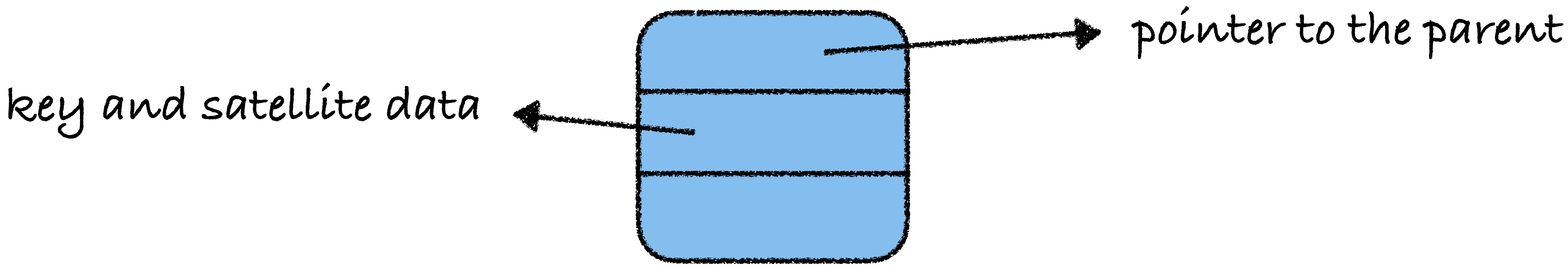

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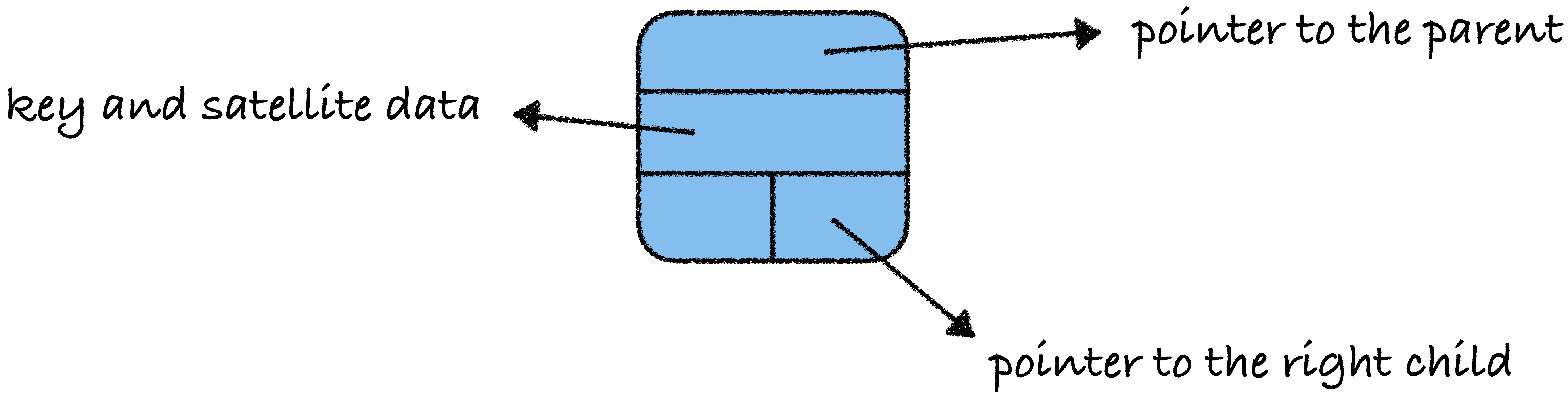

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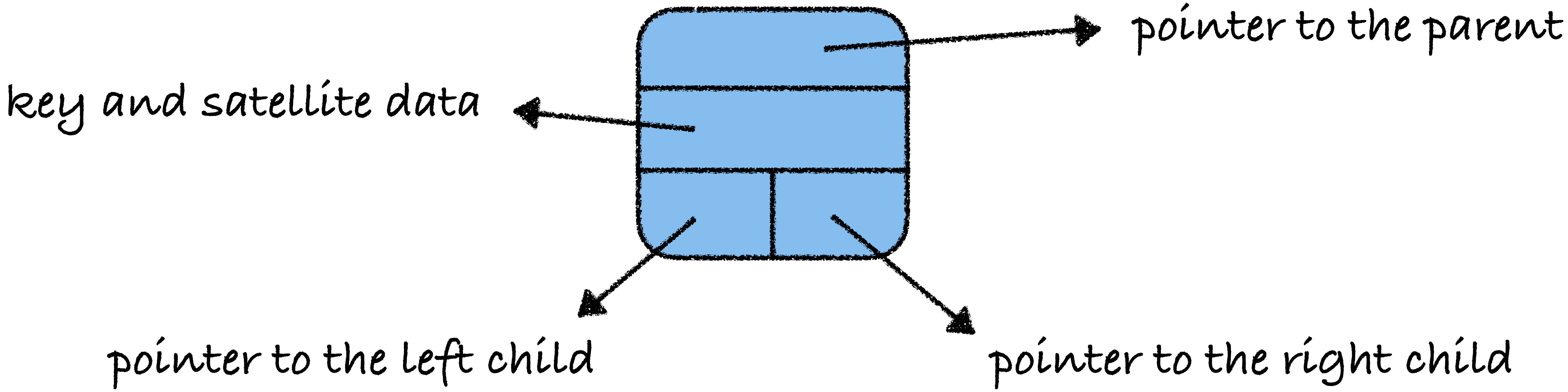

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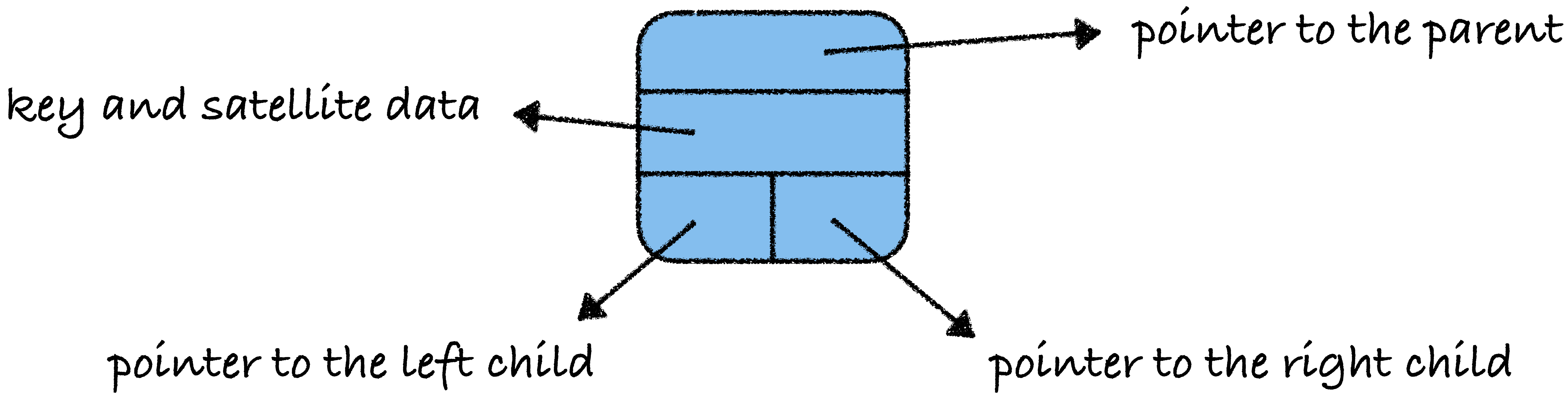

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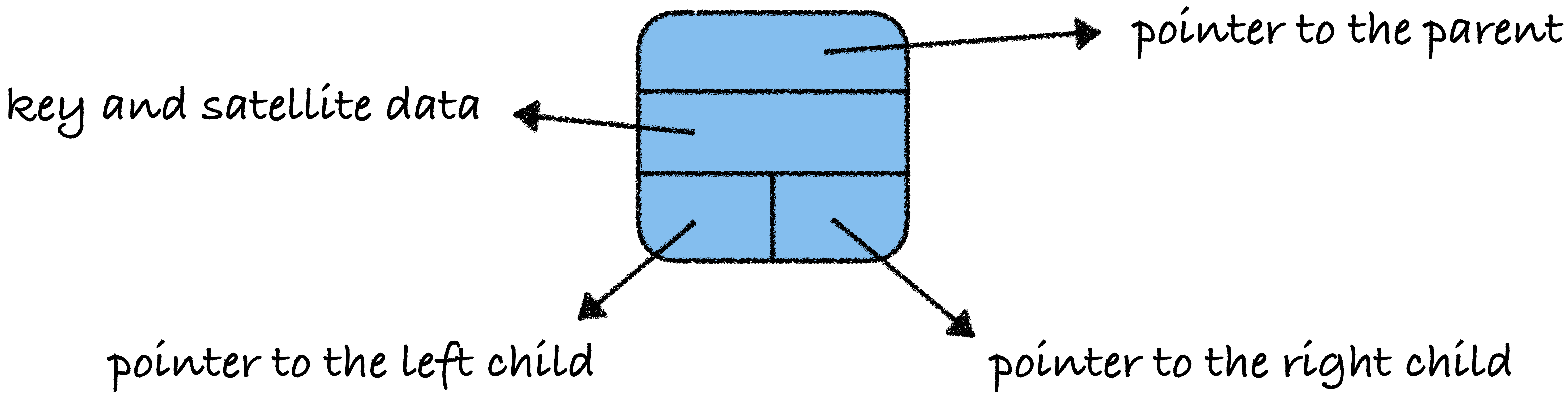

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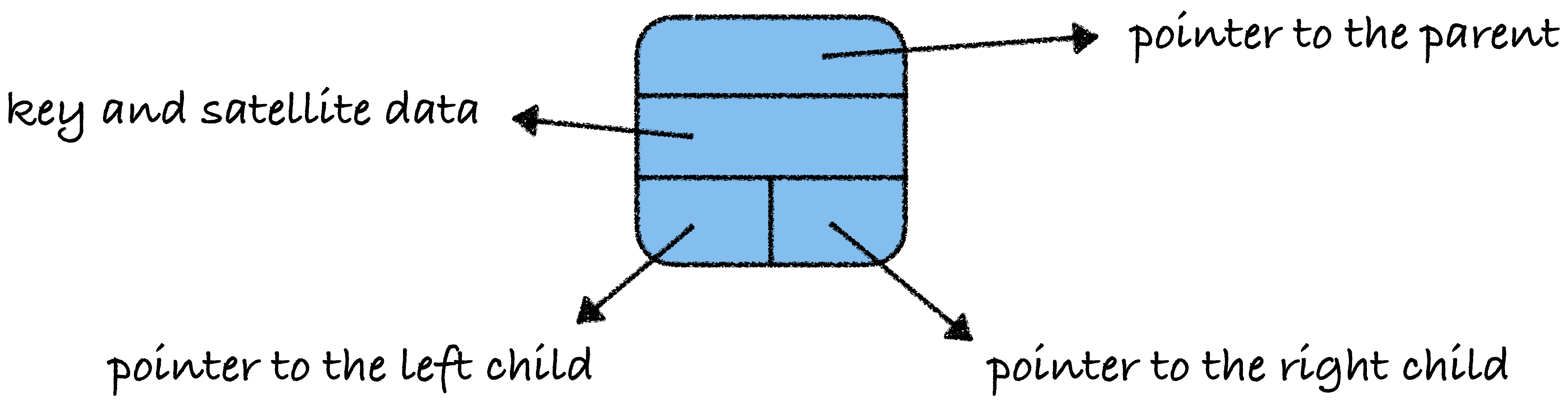

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  Let x be a node in BST and y, z be the nodes in its left, right subtree, respectively.

  Then, $y.key \leq x.key \leq z.key$.

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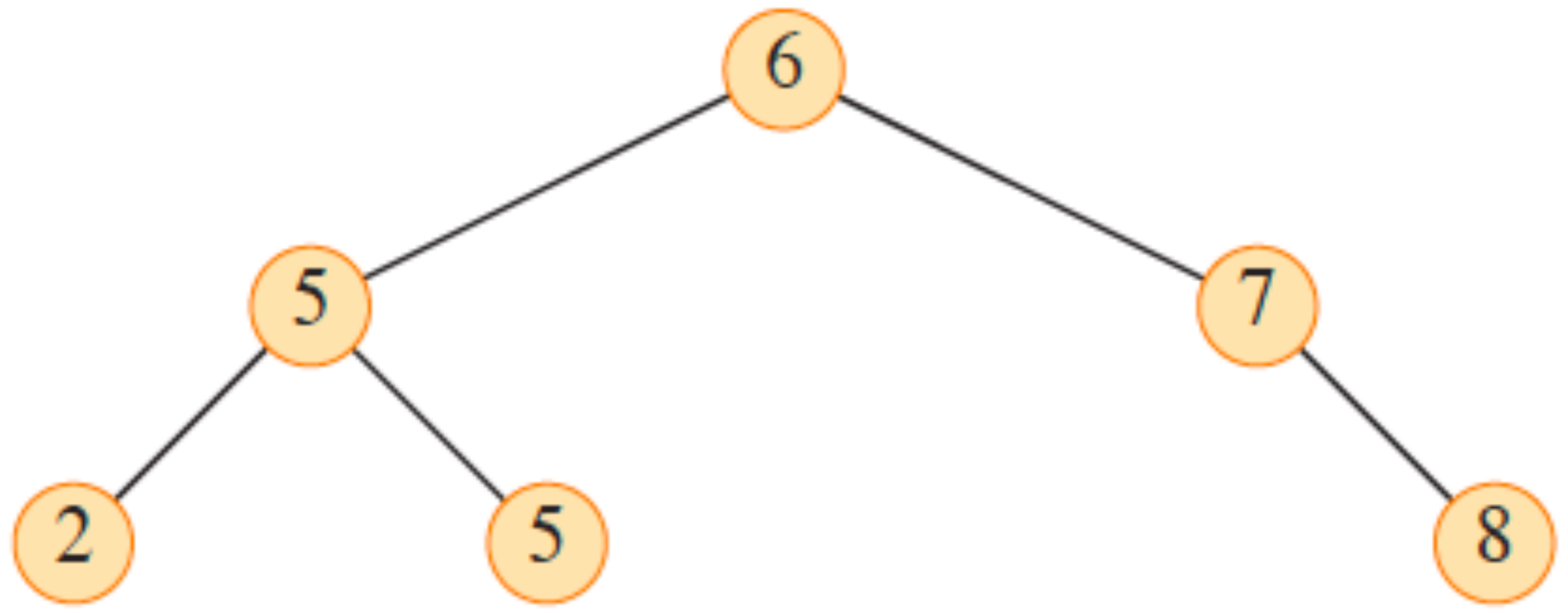

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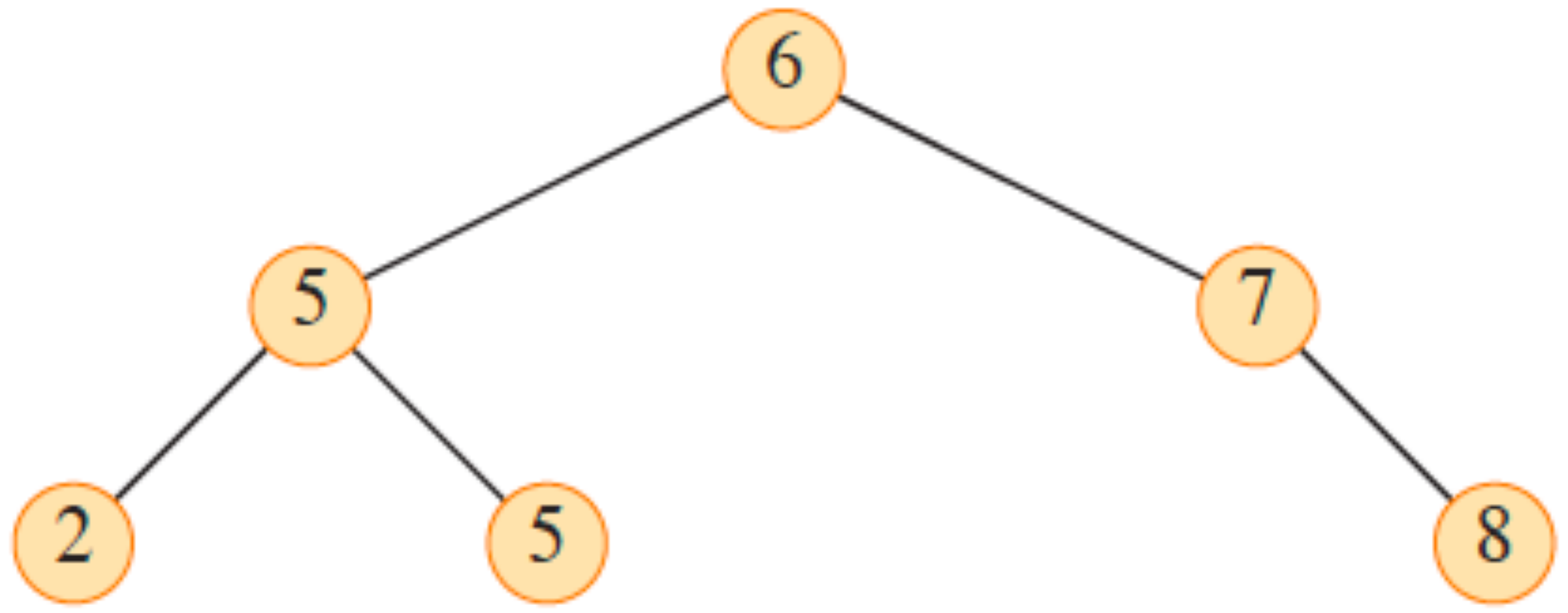

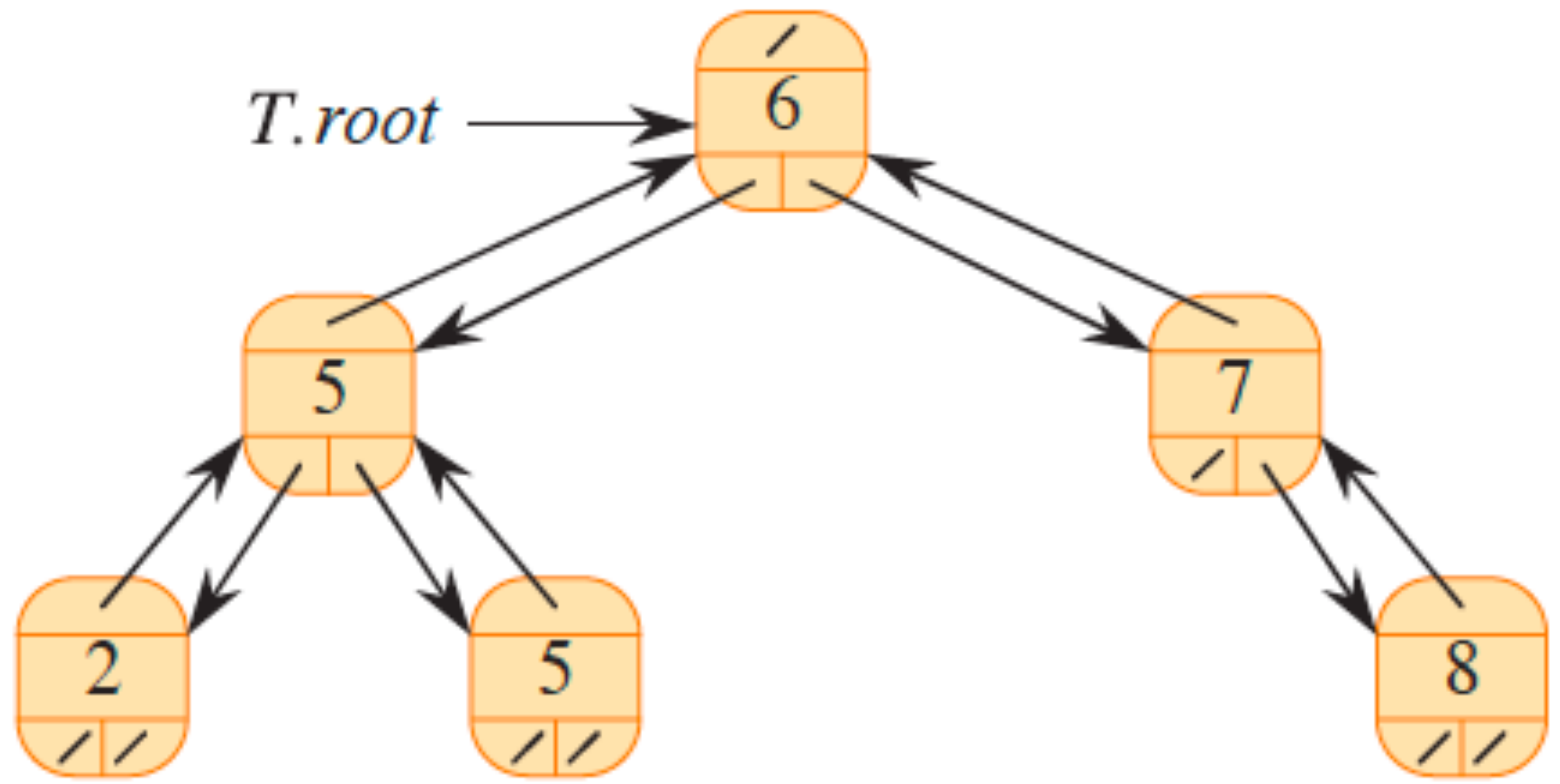

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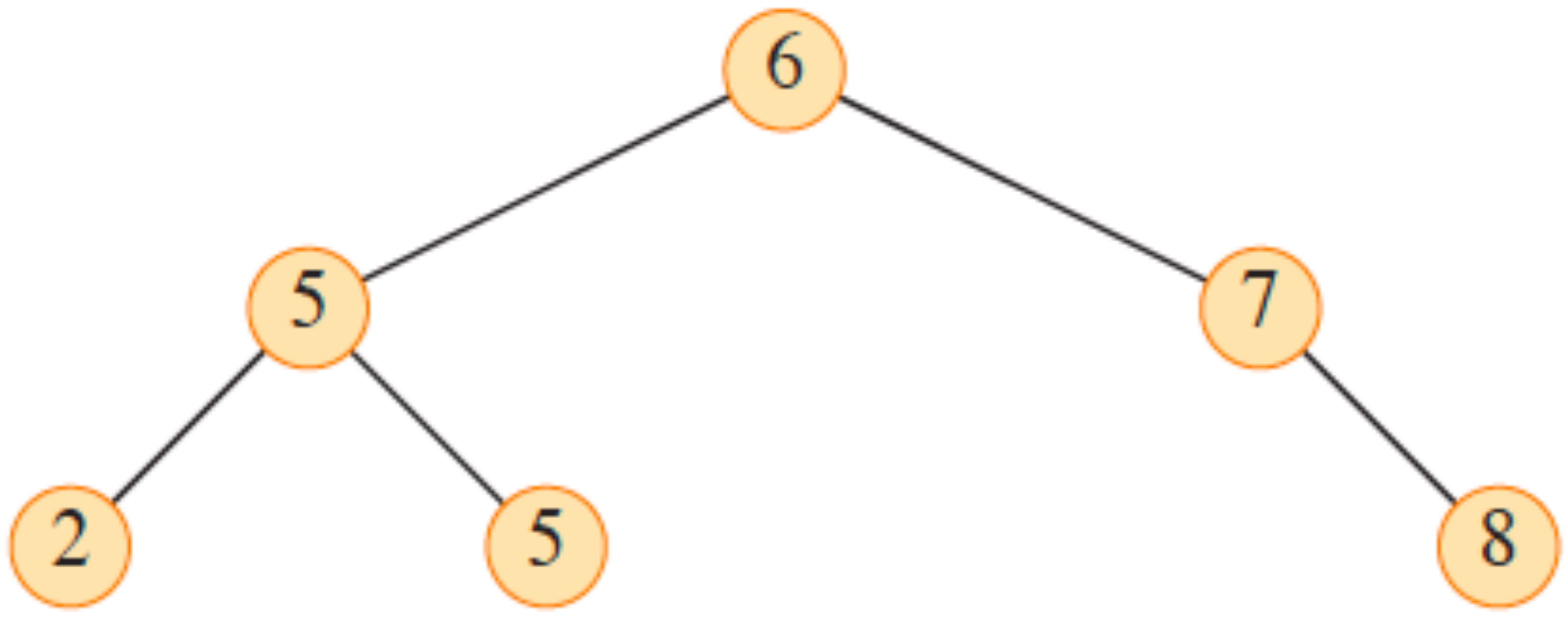

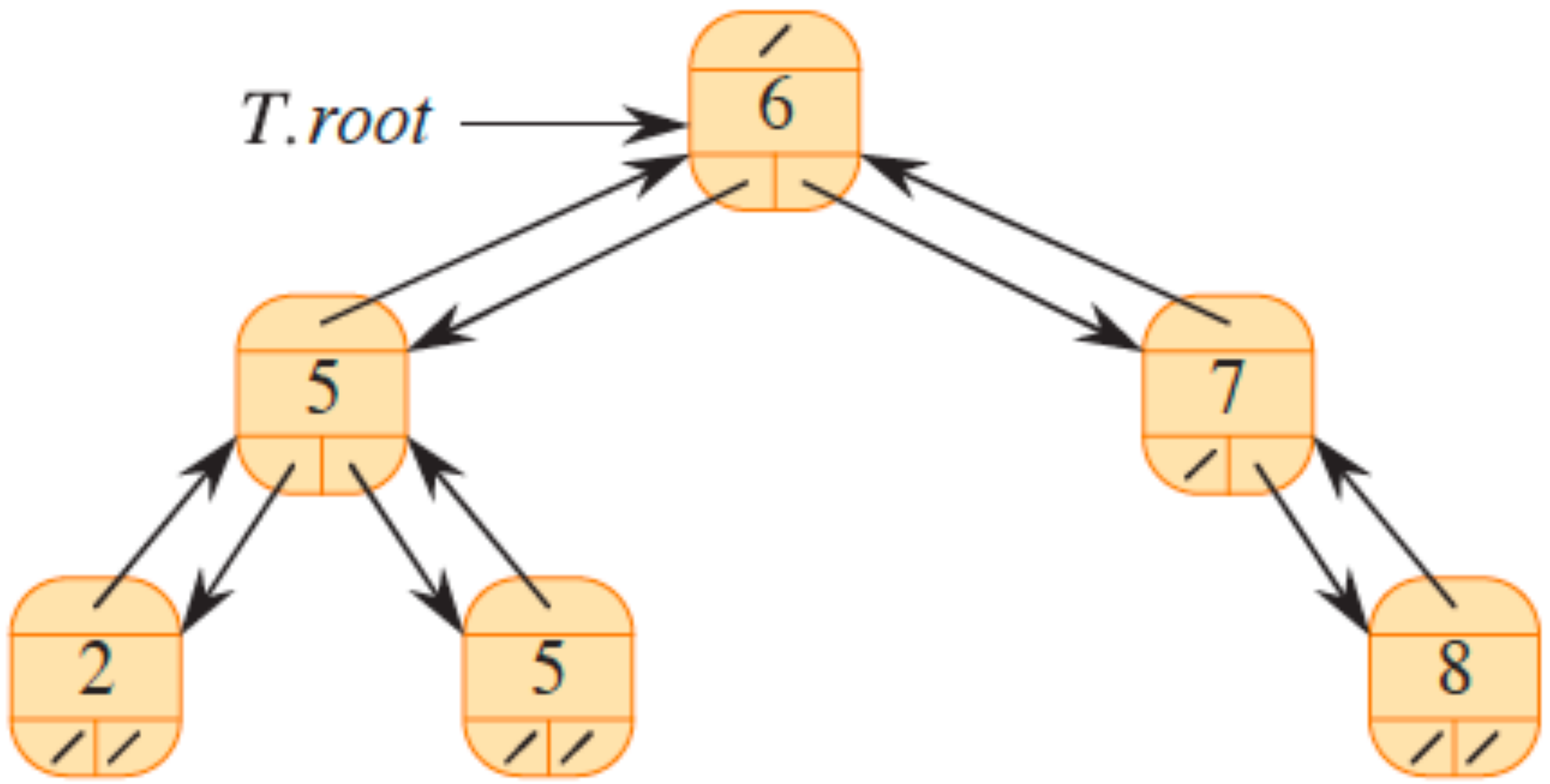

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