	circuit 1	circuit 2	stock
C	4	0	16
R	2	2	12
T	2	4	20
profit	2	3	

$x \to$ circuit 1

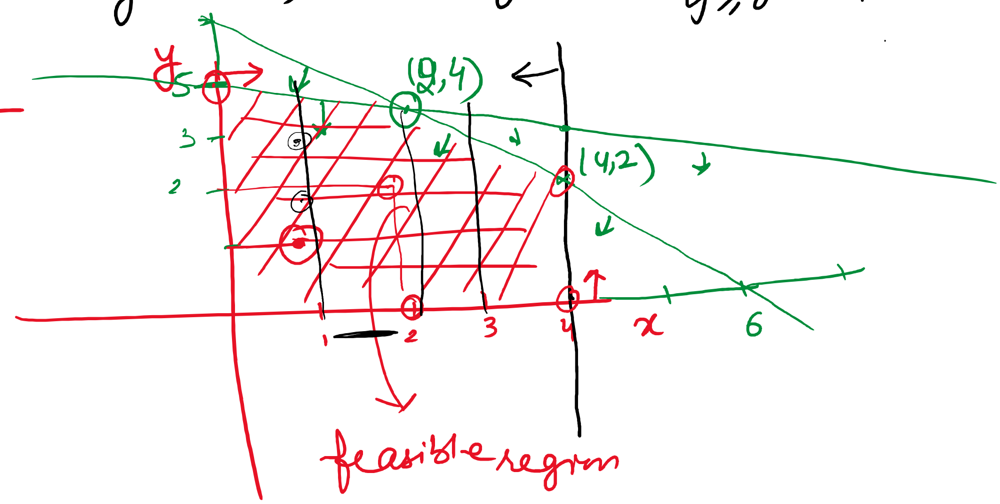
$y \to$ circuit 2

$$\boxed{\max f(x,y) = 2x+3y}$$

$x \le 4$, $x+y \le 6$, $x+2y \le 10$

$$4x \le 16, \quad 2x+2y \le 12, \quad 2x+4y \le 20, \quad x \ge 0, \ y \ge 0$$

Graphical method



$x \le 4$

$x+y \le 6$

$x+2y \le 10$

$x \ge 0$

$y \ge 0$

$y = 4$

* The optimal solution of a L.P.P lies at the corner of a feasible region.

corners of feasible region	(0,0)	(0,5)	(4,0)	(2,4)	(4,2)
$f(x,y)$	0	15	8	16	14

2 circuit 1

4 circuit 2

$\text{Max } f(x,y) = 2x+3y \longrightarrow$ optimal solution (x_0, y_0)

$f(x_0,y_0) \ge f(x,y) \ \forall \ (x,y) \in$ feasible region.

$$\mathbb{R}^n = \{ \underbrace{(x_1, x_2, \ldots, x_n)} \mid x_i \in \mathbb{R} \}$$

$\mathbb{R} \longrightarrow$ vectors.

$\mathbb{R}^n = \{ (x_1, x_2, \ldots, x_n) \mid \ldots \}$ → vectors.

→ vector space over $\mathbb{R}$

$*$ $(x_1, x_2, \ldots, x_n) + (y_1, y_2, \ldots, y_n)$
$= (x_1 + y_1, \ldots, x_n + y_n)$ } vector addition

$\mathbb{R}^2 = \{ (x, y) \mid x, y \in \mathbb{R} \}$

vector space ← $\mathbb{R}^3 = \{ \underbrace{(x, y, z)}_{\text{vector}} \mid x, y, z \in \mathbb{R} \}$

$\mathbb{R}^2, \mathbb{R}^3, \mathbb{R}^n$ vectors tuple.

Scalar Multiplication

$$\mathbb{R} \times \mathbb{R}^3 \longrightarrow \mathbb{R}^3$$

$$\lambda (x, y, z) = (\lambda x, \lambda y, \lambda z)$$

$\mathbb{R} \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$, $\lambda (x_1, \ldots, x_n) = (\lambda x_1, \lambda x_2, \ldots, \lambda x_n)$

$$2 \cdot (2, 3, 5) = (4, 6, 10)$$

$$(1, 9, 2) + (2, 3, 5) = (3, 12, 7)$$

Linear Combination :-

$\mathbb{R}^2 = \{ \boxed{(x, y)} \mid x, y \in \mathbb{R} \}$ → Linear combination of $(1,0)$ and $(0,1)$.

$$\boxed{(2, 5)} = 2(1, 0) + 5(0, 1)$$
$$= (2, 0) + (0, 5) = (2, 5)$$

$$(x, y) = x(1, 0) + y(0, 1)$$
$$= (x, 0) + (0, y) = (x, y)$$

$\mathbb{R}^n = \{ \underbrace{(x_1, x_2, \ldots, x_n)}_{} \mid x_i \in \mathbb{R} \}$, $X_1 = x_1$

$X_1, X_2, \ldots X_n \in \mathbb{R}^n$

$\mathbb{R}^n = \{(x_1, x_2, \ldots, x_n)\}$

Let $X_1, X_2, -, X_k \in \mathbb{R}^n \longrightarrow (\text{———})$

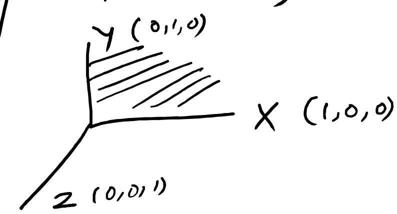
$(1,0 \text{ —— } 0) \quad (1,1,0 \text{ —— } 0)$

$$\boxed{\text{Linear combination of } X_1, X_2, -, X_k \text{ is } \lambda_1 X_1 + \lambda_2 X_2 + \text{——} + \lambda_k X_k}$$

$\mathbb{R}^3 \quad \{ \underline{(1,0,0)}, (0,1,0) \} \longrightarrow$ L.C. of two vectors in $\mathbb{R}^3$.

$$\{ \underline{\lambda_1 (1,0,0) + \lambda_2 (0,1,0)} \,\|\, \lambda_1, \lambda_2 \in \mathbb{R} \}$$

$= \mathbb{R}^3$??



$\mathbb{R}^3 \quad \underline{\{(1,0,0), (0,1,0), (0,0,1)\}}$ → unique

$(x,y,z) = x(1,0,0) + y(0,1,0) + z(0,0,1)$

$(x,y,z) =$

$\boxed{\text{Basis}}$

$\mathbb{R}^2 \quad \{(1,0), (0,1)\}$

(x, y)

$= x(1,0) + y(0,1)$

$\{(1,0), (1,1)\} \longrightarrow$ Basis

$(x,y) = \boxed{(x-y)}(1,0) + \boxed{y}(1,1)$

$x \quad 0 \quad y$ — 2 ways

$\{(1,0), (1,1), (0,1)\} \longrightarrow$ Not a Basis

$x-y \quad y \quad 0$

$\boxed{(x,y)} = x(1,0) + 0(1,1) + y(0,1)$

$(x,y) = (x-y)(1,0) + y(1,1) + 0(0,1)$

→ Expression is Not unique.

Is (x,y) L.C. of $(2,0)$ and $(0,2)$?? → Basis

$$\underline{(x,y)} = \frac{x}{2}(2,0) + \frac{y}{2}(0,2)$$

$(x,y) = $

$(x,y) = (x-y)(1,0) + y(1,1) + 0(0,1) \longrightarrow$ Expression is NOT unique.

$$(2,5) = 2(1,0) + 5(0,1)$$

Basis :- A subset S of $\mathbb{R}^n$ is called Basis of $\mathbb{R}^n$ if every vector of $\mathbb{R}^n$ can be expressed as L.C. of vectors in S in unique way.

$(1,0) \to$ Not a Basis of $\mathbb{R}^2$

$(0,y) \neq \lambda(1,0)$

$\{(1,0), (1,1), (0,1)\}$

$$(2,5) = 2(1,0) + 0(1,1) + 5(0,1)$$

$$(2,5) = -3(1,0) + 5(1,1) + 0(0,1)$$

$\{(1,0), (0,1)\}$, $\{(2,0), (0,2)\}$, $\{(1,0), (1,1)\}$

Basis is Not Unique

$\mathbb{R}^2$ $\mathbb{R}^3$

Number of vectors in Basis of $\mathbb{R}^2$ is always 2.

$\downarrow$

Dimension of V.S.

Dimension of $\mathbb{R}^n = n$

Basis of $\mathbb{R}^n$

$= \{(1,0,-,0), (0,1,0-,0), (0,0,1,0-0), \text{——} \text{——} (0,0,-,0,1)\}$

$(x_1, x_2, -, x_n)$

$= x_1(1, 0 - 0) + x_2(0, 1, - 0) + \text{———} + x_n(0, 0, -, 0, 1).$