Formation Control

August 1, 2020

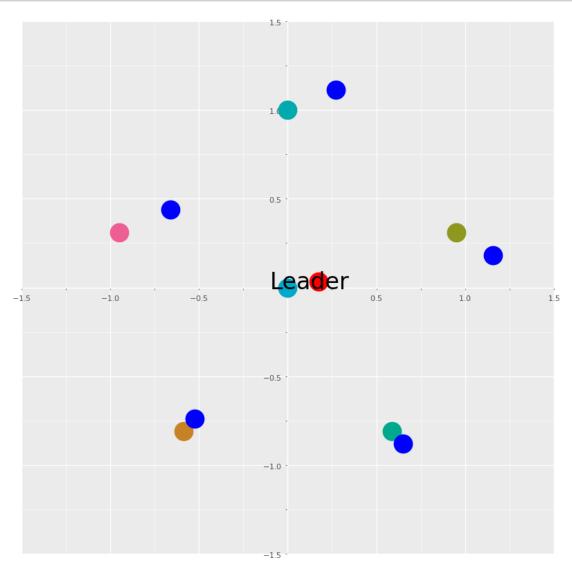
[1]: using DifferentialEquations, Plots; pyplot();

```
using LinearAlgebra
      theme(:ggplot2)
[2]: n = 6;
     kv = 0.05;
      Adj = [0 1 0 0 0 1;
              1 0 1 0 0 1;
              0 1 0 1 0 1;
              0 0 1 0 1 1;
              0 0 0 1 0 1;
              1 1 1 1 1 0];
      d = zeros(n,n);
      c1 = cos(2*pi/5);
      c2 = cos(pi/5);
      s1 = sin(2*pi/5);
      s2 = sin(4*pi/5);
      x_{coor} = [0; -s1; -s2; s2; s1; 0];
      y_{coor} = [1; c1; -c2; -c2; c1; 0];
      z_coor = [0.0; 0.0; 0.0; 0.0; 0.0; 0.0];
[3]: for ii = 1:n
           for jj = 1:n
                d[ii,jj] = 
       \Rightarrow \operatorname{sqrt}((\operatorname{x\_coor}[ii] - \operatorname{x\_coor}[jj])^2 + (\operatorname{y\_coor}[ii] - \operatorname{y\_coor}[jj])^2 + (\operatorname{z\_coor}[ii] - \operatorname{z\_coor}[jj])^2)
      end
[4]: ub = 0.5;
                                                  # Upper bound for random ini. condition
      1b = -0.5;
                                                  # Lower bound for random ini. condition
      tfinal = 10;
                                                  # Simulation ending time assume always
                                                  # starts at 0
                                                  # ODE step
     h = 1e-2;
```

```
[5]: mutable struct para
     n
     kv
     Adj
     d
     end
[]:
[6]: p=para(n,kv,Adj,d)
[6]: para(6, 0.05, [0 1 ... 0 1; 1 0 ... 0 1; ... ; 0 0 ... 0 1; 1 1 ... 1 0], [0.0
     1.1755705045849463 ... 1.1755705045849463 1.0; 1.1755705045849463 0.0 ...
     0.0 0.9999999999999; 1.0 0.999999999999 ... 0.999999999999 0.0])
[7]: q_0 = [0.3147]
                    -0.5377 -0.8093
                                                 1.4082
                                                         -0.3581;
                                       1.0527
                                                 0.2944
                                                          -0.0782;
            1.4058
                     0.4414
                             -0.7621
                                       -1.1514
                0
                          0
                                                               0];
                                   0
                                             0
                                                      0
[8]: q_0 = reshape(q_0,(1,:));
[9]: time_span = (0.0, 10.0);
[10]: function f(du,u,p,t)
         n = p.n
         kv = p.kv
         Adj = p.Adj
         d = p.d
         u = reshape(u,(3,:))
         z = zeros(2*n-3,1)
         R = zeros(2*n-3,3*n)
         e = zeros(n,n)
         ord = 1
         for i=1:n-1
             for j=i+1:n
                e[i,j] = sqrt((u[:,i]-u[:,j])'*(u[:,i]-u[:,j]))-d[i,j]
                 # print("i =",i,"j=",j,"\n")
                if Adj[i,j] == 1
                    z[ord] = e[i,j]*(e[i,j]+2*d[i,j]);
                    R[ord,3*i-2:3*i] = (u[:,i]-u[:,j])';
                    R[ord,3*j-2:3*j] = (u[:,j]-u[:,i])';
                    ord = ord+1;
                 end
             end
```

```
end
          control = -kv * R' * z
          control = reshape(control,(1,:))
          du .= control;
      end
[10]: f (generic function with 1 method)
[11]: prob = ODEProblem(f,q_0_vec,time_span,p)
      sol = solve(prob, saveat=0.1)
      pos = sol.u
[11]: 101-element Array{Array{Float64,2},1}:
       [0.3147 1.4058 ... -0.0782 0.0]
       [0.30842678862813994 1.3927913813211976 ... -0.07396898554134113 0.0]
       [0.3029691184556908 1.3808556808014574 ... -0.06989686960240833 0.0]
       [0.298187972192451 1.3698443227076347 ... -0.06599350183135236 0.0]
       [0.29397463968446186 1.3596368709775513 ... -0.0622610071964926 0.0]
       [0.2902439278499775 1.3501354194689228 ... -0.058696483611583486 0.0]
       [0.2869278996711181 1.3412592031974386 ... -0.05529427568477855 0.0]
       [0.2839704352583543 1.3329397160479293 ... -0.05204755913995581 0.0]
       [0.28132519709257536 1.3251188697478264 ... -0.04894887653593608 0.0]
       [0.27895416438603665 1.3177475523368491 ... -0.0459903776506315 0.0]
       [0.2768254345385491 1.310783526502298 ... -0.043164323069599 0.0]
       [0.2749117208779738 1.304189886489155 ... -0.04046323671931769 0.0]
       [0.27318923570603754 \ 1.297933919486112 \ \dots \ -0.03788001424643709 \ 0.0]
       [0.2690329264362398 1.1251755227342017 ... 0.029343561065221856 0.0]
       [0.2692192825138863 1.1242442992409636 ... 0.02957257443909716 0.0]
       [0.26940375270888706 1.1233253217608257 ... 0.029795273053418003 0.0]
       [0.26958632038981695 1.1224183359520394 ... 0.030011816671178262 0.0]
       [0.26976696731919364 1.121523079792877 ... 0.030222386835033997 0.0]
       [0.2699456793827502 1.120639311203193 ... 0.030427131752582215 0.0]
       [0.27012244493306486 1.119766795475741 ... 0.0306261951652648 0.0]
       [0.2702972547286862 1.1189053049511206 ... 0.030819717019344592 0.0]
       [0.2704701019341333 1.1180546190177771 ... 0.03100783346590539 0.0]
       [0.27064098211989585 1.117214524112002 ... 0.031190676860851956 0.0]
       [0.27080989326243393 1.1163848137179326 ... 0.03136837576491001 0.0]
       [0.27097683574417825 1.1155652883675529 ... 0.031541054943626225 0.0]
[13]: anim = @animate for i in 1:length(pos)
          plt = scatter(5,xlim=(-1.5,1.5),ylim=(-1.5,1.5), c=:red, aspect_ratio =_{\sqcup}
       →1,legend=false, framestyle=:origin)
          pst = pst=reshape(pos[i],(3,6))
          scatter!(x_coor',y_coor', markersize=20,aspect_ratio = 1 )
```

```
scatter!(plt,pst[1,1:5],pst[2,1:5] ,markersize=20,c=:blue, legend=false)
scatter!(plt,[pst[1,6]],[pst[2,6]] ,markersize=20,c=:red,legend=false)
annotate!([(pst[1,6]-0.05, pst[2,6], text("Leader",24))])
scatter!(size=(800,800))
end
```



```
[13]: Animation("/tmp/jl_tUEQdb", ["000001.png", "000002.png", "000003.png", "000004.png", "000005.png", "000006.png", "000007.png", "000008.png", "000009.png", "000010.png" ... "000092.png", "000093.png", "000094.png", "000095.png", "000096.png", "000097.png", "000098.png", "000099.png", "000100.png", "000101.png"])
```

```
[14]: gif(anim,fps=10)
```

Info: Saved animation to
 fn = /home/bbm/Documents/research/FormationControl/tmp.gif
@ Plots /home/bbm/.julia/packages/Plots/LWw1t/src/animation.jl:104

[14]: Plots.AnimatedGif("/home/bbm/Documents/research/FormationControl/tmp.gif")

```
anim = @animate for i in 1:length(pos)

plt = scatter(5,xlim=(-1.5,1.5),ylim=(-1.5,1.5), zlim=(-1.5,1.5),c=:

→red,legend=false, framestyle=:origin)

pst = pst=reshape(pos[i],(3,6))

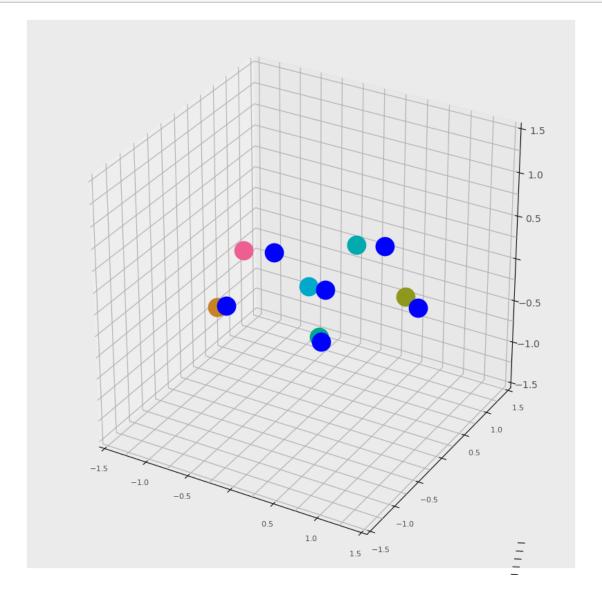
scatter!(x_coor',y_coor', z_coor', markersize=20))

scatter!(plt,pst[1,:],pst[2,:] , pst[3,:],markersize=20,c=:blue,

→legend=false)

scatter!(size=(800,800))

end
```

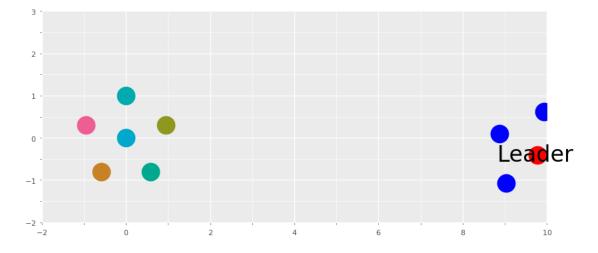


0.1 Formation Acquistion

[20]: Desired_velocity (generic function with 1 method)

```
e = zeros(n,n)
          ord = 1
          for i=1:n-1
              for j=i+1:n
                  e[i,j] = sqrt((u[:,i]-u[:,j])'*(u[:,i]-u[:,j]))-d[i,j]
                   # print("i =",i,"j=",j,"\n")
                  if Adj[i,j] == 1
                       z[ord] = e[i,j]*(e[i,j]+2*d[i,j]);
                       R[ord, 3*i-2:3*i] = (u[:,i]-u[:,j])';
                       R[ord,3*j-2:3*j] = (u[:,j]-u[:,i])';
                       ord = ord+1;
                   end
              end
          end
          vd = Desired_velocity(t,u);
          control = -kv * R' * z + vd
          control = reshape(control,(1,:))
          du .= control;
      end
[21]: form_with_vel (generic function with 1 method)
[22]: prob2= ODEProblem(form_with_vel,q_0_vec,time_span,p)
      sol2 = solve(prob2, saveat=0.1)
      pos2 = sol2.u
[22]: 101-element Array{Array{Float64,2},1}:
       [0.3147 1.4058 ... -0.0782 0.0]
       [0.2585877489032584 1.5498331540580363 ... 0.027768571847526048 0.0]
       [0.20176198507722604 1.6742920620109476 ... 0.13571157083282565 0.0]
       [0.14600504603814507 1.778809385556364 ... 0.2438023528493907 0.0]
       [0.0930044551333009 1.8631214199214459 ... 0.35043446784338017 0.0]
       [0.04435122037772749 1.9270879657693636 ... 0.4541703784909652 0.0]
       [0.0015323942756549048 1.9706979380386478 ... 0.5537196065968468 0.0]
       [-0.03406927945007219 \ 1.9940841016838609 \ ... \ 0.647914799781772 \ 0.0]
       [-0.06118578557013045 1.9975228846933892 ... 0.7357065887535489 0.0]
       [-0.0786695919498487 1.9814335219844719 ... 0.8161620753113588 0.0]
       [-0.08549343959196985 1.9463869239501903 ... 0.8884529169783595 0.0]
       [-0.08076040447401704 1.8930956478380543 ... 0.9518617931276299 0.0]
       [-0.06371040048317184 1.8224072976122991 ... 1.00578627632593 0.0]
       [8.0395471407798 -0.2588396730077755 ... 0.6408812801661242 0.0]
       [8.232754447859941 - 0.40575068470419856 ... 0.5520397645926393 0.0]
       [8.431149895465657 -0.5471753516120196 ... 0.45901533318051674 0.0]
```

```
 \begin{bmatrix} 8.633768678478036 & -0.6816394380453502 & \dots & 0.36275920148259466 & 0.0 \end{bmatrix} \\ [8.839608732905006 & -0.8077848321851198 & \dots & 0.26423375652814324 & 0.0 ] \\ [9.047630969858158 & -0.9243700025242326 & \dots & 0.1644124712612649 & 0.0 ] \\ [9.256759275552744 & -1.0302699978675742 & \dots & 0.06427990454089175 & 0.0 ] \\ [9.465880511307693 & -1.1244764473320115 & \dots & -0.03516829885921875 & 0.0 ] \\ [9.67384451354559 & -1.2060975603463866 & \dots & -0.13292540825047386 & 0.0 ] \\ [9.879498583044935 & -1.274352488135437 & \dots & -0.22798402333990386 & 0.0 ] \\ [10.081865266779777 & -1.3285583091246578 & \dots & -0.3194038861251736 & 0.0 ] \\ [10.279944462165043 & -1.3681905320720333 & \dots & -0.40627411198279956 & 0.0 ] \\ \end{bmatrix}
```



```
[29]: Animation("/tmp/jl_9Tx1wy", ["000001.png", "000002.png", "000003.png", "000004.png", "000005.png", "000006.png", "000007.png", "000008.png", "000009.png", "000010.png" ... "000092.png", "000093.png", "000094.png", "000095.png", "000096.png", "000097.png", "000098.png", "000099.png", "000100.png", "000101.png"])
```

Info: Saved animation to
 fn = /home/bbm/Documents/research/FormationControl/tmp.gif

[31]: gif(anim2,fps=10)

@ Plots /home/bbm/.julia/packages/Plots/LWw1t/src/animation.jl:104

[31]: Plots.AnimatedGif("/home/bbm/Documents/research/FormationControl/tmp.gif")

```
[32]: anim3 = @animate for i in 1:length(pos2)
    plt = scatter(5,xlim=(-2,12),ylim=(-2,3), zlim=(-1.5,1.5),c=:
    →red,legend=false, framestyle=:origin)
    pst = pst=reshape(pos2[i],(3,6))
    scatter!(x_coor',y_coor', z_coor', markersize=20)
    scatter!(plt,pst[1,:],pst[2,:] , pst[3,:],markersize=20,c=:blue,
    →legend=false)
    scatter!(size=(800,800))
end
```

