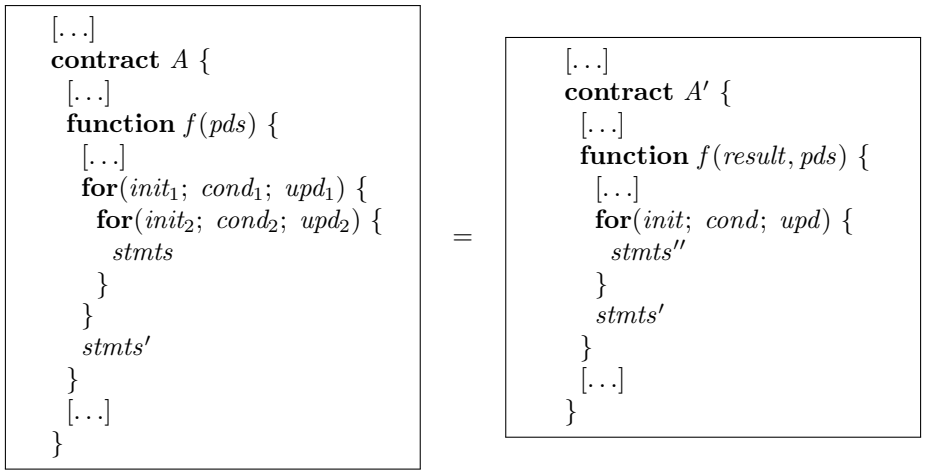
Rule 0.21 ⟨*Avoid Nested Loops*⟩



where

The outer loop has initialization $init_1$, condition $cond_1$, and update upd_1;

The inner loop has initialization $init_2$, condition $cond_2$, and update upd_2;

stmts represents the nested loop body statements;

stmts′ represents statements following the nested loops;

stmts″ represents the optimized single-loop body;

result is a pre-computed parameter or auxiliary data structure used to avoid nesting;

pds are the parameter declarations of function f.

provided

The nested loop computation can be restructured into a single loop using auxiliary data structures (e.g., mappings, arrays);

Alternatively, the result can be pre-computed off-chain and passed as a parameter;

The transformation maintains computational equivalence while reducing complexity from $O(n \times m)$ to $O(n + m)$ or $O(n)$;

When using mappings for lookups, initialization occurs in a separate function or off-chain;

The optimized version produces the same final state as the nested loop version.

Invariant:

Let s_i and s_i' be the initial state of A and A', respectively.

Let s_f and s_f' be the state reached by A and A', respectively, after $A.f()$ and $A'.f()$ are executed from s_i and s_i', respectively.

Then, the coupling invariant is

$$\forall\, s_i, s_i' \,.\, (s_i = s_i') \rightarrow (s_f = s_f')$$