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**Rule 0.15** *⟨Reduce Mathematical Expressions⟩*


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$$\begin{array}{c}
 [\dots] \\
 \mathbf{contract} \ A \ \{ \\
 \quad [\dots] \\
 \quad \mathbf{function} \ f(pds) \ \{ \\
 \quad \quad [\dots] \\
 \quad \quad \text{result} = expr_1; \\
 \quad \quad stmts \\
 \quad \} \\
 \quad [\dots] \\
 \}
 \end{array}
 =
 \begin{array}{c}
 [\dots] \\
 \mathbf{contract} \ A' \ \{ \\
 \quad [\dots] \\
 \quad \mathbf{function} \ f(pds) \ \{ \\
 \quad \quad [\dots] \\
 \quad \quad \text{result} = expr_2; \\
 \quad \quad stmts \\
 \quad \} \\
 \quad [\dots] \\
 \}
 \end{array}$$

**where**

$expr_1$  is a complex mathematical or logical expression in contract  $A$ ;

$expr_2$  is the reduced form of  $expr_1$  with fewer operations;

$result$  is a variable storing the expression result;

$pds$  are the parameter declarations of function  $f$ ;

$stmts$  represents the sequence of statements following the expression.

**provided**

The expressions  $expr_1$  and  $expr_2$  are semantically equivalent (evaluate to the same value);

$expr_2$  requires fewer operations than  $expr_1$ ;

The reduction may apply algebraic rules such as: factoring (e.g.,  $a \cdot x + b \cdot x = (a + b) \cdot x$ ), De Morgan's laws (e.g.,  $\neg x \wedge \neg y \equiv \neg(x \vee y)$ ), distributive properties, or constant folding;

No side effects are introduced or removed by the transformation;

The reduction maintains numerical precision and overflow behavior.

**Invariant:**

Let  $s_i$  and  $s'_i$  be the initial state of  $A$  and  $A'$ , respectively.

Let  $s_f$  and  $s'_f$  be the state reached by  $A$  and  $A'$ , respectively, after  $A.f()$  and  $A'.f()$  are executed from  $s_i$  and  $s'_i$ , respectively.

Then, the coupling invariant is

$$\forall s_i, s'_i . (s_i = s'_i) \rightarrow (s_f = s'_f)$$


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