



where

- arr* is a dynamic array of type $T[]$ in contract A ;
- map* is a mapping from **uint** to T in contract A' ;
- size* is a counter tracking the number of elements in the mapping;
- T is the element type of the array and mapping values;
- val* is a value of type T being added;
- idx* is an index used to access elements;
- max_uint256 represents $2^{256} - 1$.

provided

- The contract does not require iterating over all elements frequently;
- Element access is primarily done by index/key rather than sequential iteration;
- The mapping provides sufficient functionality for the use case;
- A separate *size* counter is maintained to track the number of elements;
- Array operations like **push** are replaced with direct mapping assignments and size increments;
- Bounds checking uses *size* instead of *arr.length*;
- Both implementations include overflow protection to prevent wraparound in A and arithmetic overflow in A' ;
- The overflow check can be implemented using **require**, custom errors, or any equivalent validation mechanism that reverts on overflow.

Invariant:

- Let s_i and s'_i be the initial state of A and A' , respectively.
- Let s_f and s'_f be the state reached by A and A' , respectively, after $A.f()$ and $A'.f()$ are executed from s_i and s'_i , respectively.
- Then, the coupling invariant is

$$\forall s_i, s'_i. (s_i = s'_i) \rightarrow (s_f = s'_f)$$