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ARPA BLOG TEAMS SUBMISSIONS GROUPS CONTESTS PROBLEMSETTING

Arpa's blog

[Tutorial] Sack (dsu on tree)

By Arpa, history, 4 years ago, , ,

Changes are available in history section.

Most of the people know about dsu but what is the "dsu on tree"?

In Iran, we call this technique "Guni" (the word means "sack" in English), instead of "dsu on

I will explain it and post ends with several problems in CF that can be solved by this technique.

What is the dsu on tree?

With dsu on tree we can answer queries of this type:

How many vertices in the subtree of vertex ${f v}$ has some property in ${\cal O}(nlogn)$ time (for all of the queries)?

For example:

Given a tree, every vertex has color. Query is how many vertices in subtree of vertex v are colored with color c?

Let's see how we can solve this problem and similar problems.

First, we have to calculate the size of the subtree of every vertice. It can be done with simple dfs:

int sz[maxn]; void getsz(int v, int p){ sz[v] = 1; // every vertex has itself in its subtree for(auto u : g[v]) **if**(u != p){ getsz(u, v); sz[v] += sz[u]; // add size of child u to its parent(v) } }

Now we have the size of the subtree of vertex v in sz[v].

The naive method for solving that problem is this code(that works in O(N ^ 2) time)

int cnt[maxn]; void add(int v, int p, int x){ cnt[col[v]] += x; for(auto u: g[v]) **if**(u != p) add(u, v, x) void dfs(int v, int p){ **add**(v, p, 1);

→ Pay attention

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06:36:28

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```
//now cnt[c] is the number of vertices in subtree of vertex v that has
color c. You can answer the queries easily.
    add(v, p, -1);
    for(auto u : g[v])
        if(u != p)
            dfs(u, v);
}
Now, how to improve it? There are several styles of coding for this technique.
1. easy to code but O(n \log^2 n).
map<int, int> *cnt[maxn];
void dfs(int v, int p){
   int mx = -1, bigChild = -1;
    for(auto u : g[v])
       if(u != p){
           dfs(u, v);
           if(sz[u] > mx)
               mx = sz[u], bigChild = u;
       }
    if(bigChild != -1)
        cnt[v] = cnt[bigChild];
    else
        cnt[v] = new map<int, int> ();
    (*cnt[v])[ col[v] ] ++;
    for(auto u : g[v])
       if(u != p && u != bigChild){
           for(auto x : *cnt[u])
               (*cnt[v])[x.first] += x.second;
    //now (*cnt[v])[c] is the number of vertices in subtree of vertex v that
has color c. You can answer the queries easily.
}
2. easy to code and O(n \log n).
vector<int> *vec[maxn];
int cnt[maxn];
void dfs(int v, int p, bool keep){
    int mx = -1, bigChild = -1;
    for(auto u : g[v])
       if(u != p && sz[u] > mx)
```

```
mx = sz[u], bigChild = u;
    for(auto u : g[v])
       if(u != p && u != bigChild)
           dfs(u, v, 0);
    if(bigChild != -1)
        dfs(bigChild, v, 1), vec[v] = vec[bigChild];
    else
        vec[v] = new vector<int> ();
    vec[v]->push_back(v);
    cnt[ col[v] ]++;
    for(auto u : g[v])
       if(u != p && u != bigChild)
           for(auto x : *vec[u]){
               cnt[ col[x] ]++;
               vec[v] -> push_back(x);
    //now (*cnt[v])[c] is the number of vertices in subtree of vertex v that
has color c. You can answer the queries easily.
    // note that in this step *vec[v] contains all of the subtree of vertex v.
    if(keep == 0)
        for(auto u : *vec[v])
```

```
retrograd → A (possibly simpler) algorithm
for closest pair problem
\textbf{lexiyvv} \rightarrow \underline{something\ wrong\ with}
organizations ranks @
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Analysis 💭
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functions (C
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Unexpected end of file - token expected
(test case 76)
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\textbf{OsmanliTokadi} \rightarrow \underline{\text{how to use mine problems}}
which prepared in polygon in GYM? 🔊
beginner_610 → Stuck in a problem. ©
pikmike → Educational Codeforces Round 86
[Rated for Div. 2] 📡
\textbf{prashant.2k03} \rightarrow \underline{input:input.txt\ and}
output:output.txt 40
\textbf{OsmanliTokadi} \rightarrow \underline{Preparing\ for\ olymiads}
Contest-2 ©
vovuh → Codeforces Round #642 (Div. 3)
Halit → Preparing for Olympiads Contest#2
 49
```

 $\underline{\mathsf{Detailed}} \to$

```
cnt[ col[u] ]--;
```

}

3. heavy-light decomposition style $O(n \log n)$.

```
int cnt[maxn];
bool big[maxn];
void add(int v, int p, int x){
    cnt[ col[v] ] += x;
    for(auto u: g[v])
        if(u != p && !big[u])
            add(u, v, x)
void dfs(int v, int p, bool keep){
    int mx = -1, bigChild = -1;
    for(auto u : g[v])
       if(u != p \&\& sz[u] > mx)
          mx = sz[u], bigChild = u;
    for(auto u : g[v])
        if(u != p && u != bigChild)
            dfs(u, v, 0); // run a dfs on small childs and clear them from cnt
    if(bigChild != -1)
        dfs(bigChild, v, 1), big[bigChild] = 1; // bigChild marked as big and
not cleared from cnt
    add(v, p, 1);
    //now cnt[c] is the number of vertices in subtree of vertex v that has
color c. You can answer the queries easily.
    if(bigChild != -1)
        big[bigChild] = 0;
    if(keep == 0)
        add(v, p, -1);
}
```

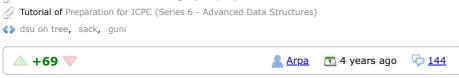
4. My invented style $O(n \log n)$.

This implementation for "Dsu on tree" technique is new and invented by me. This implementation is easier to code than others. Let st[v] dfs starting time of vertex v, ft[v] be it's finishing time and ver[time] is the vertex which it's starting time is equal to time.

```
int cnt[maxn];
void dfs(int v, int p, bool keep){
    int mx = -1, bigChild = -1;
    for(auto u : g[v])
       if(u != p \&\& sz[u] > mx)
          mx = sz[u], bigChild = u;
    for(auto u : g[v])
        if(u != p && u != bigChild)
            dfs(u, v, 0); // run a dfs on small childs and clear them from cnt
    if(bigChild != -1)
        dfs(bigChild, v, 1); // bigChild marked as big and not cleared from
cnt
    for(auto u : g[v])
        if(u != p && u != bigChild)
            for(int p = st[u]; p < ft[u]; p++)</pre>
                cnt[ col[ ver[p] ] ]++;
    cnt[ col[v] ]++;
    //now cnt[c] is the number of vertices in subtree of vertex v that has
color c. You can answer the queries easily.
    if(keep == 0)
        for(int p = st[v]; p < ft[v]; p++)</pre>
            cnt[ col[ ver[p] ] ]--;
}
```

But why it is $O(n \log n)$? You know that why dsu has $O(q \log n)$ time (for q queries); the code uses the same method. Merge smaller to greater.

If you have heard heavy-light decomposition you will see that function add will go light edges only, because of this, code works in $O(n \log n)$ time. Any problems of this type can be solved with same dfs function and just differs in add function. Hmmm, this is what you want, problems that can be solved with this technique: (List is sorted by difficulty and my code for each problem is given, my codes has heavylight style) 600E - Lomsat gelral: heavy-light decomposition style: Link, easy style: Link. I think this is the easiest problem of this technique in CF and it's good to start coding with this problem. 570D - Tree Requests: 17961189 Thanks to Sorasorasora; this problem is also good for start Squ507 (SGU is unavailable, read the problem statements here) This problem is also good for the start. HackerEarth, The Grass Type This problem is also good for start (See bhishma's comment 246E - Blood Cousins Return: 15409328 208E - Blood Cousins: 16897324 IOI 2011, Race (See SaSaSaS's comment below). 291E - Tree-String Problem : See **bhargav104**'s comment below. 1009F - Dominant Indices: 40332812 Arpa-Style. Thanks to baymaxx. 343D - Water Tree: 15063078 Note that problem is not easy and my code doesn't use this technique (dsu on tree), but AmirAz 's solution to this problem uses this technique : 14904379. 375D - Tree and Queries: 15449102 Again note that problem is not easy:)). 716E - Digit Tree: 20776957 A hard problem. Also can be solved with centroid decomposition. 741D - Arpa's letter-marked tree and Mehrdad's Dokhtar-kosh paths: 22796438 A hard problem. You must be very familiar with Dsu on tree to solve it. For Persian users, there is another problem in Shaazzz contest round #4 (season 2016-2017) problem 3 that is a very hard problem with this technique. If you have another problem with this tag, give me to complete the list :)). And after all, special thanks from amd who taught me this technique. Announcement of EIO 18-19 Suvelaager (Kompsik) ▲ +69 ▼ Arpa 📆 4 vears ago **144**





Write comment?



4 years ago, <u>#</u> | A +4





4 years ago, # $^{\sim}$ | بدك تضل تثمنيك عكل بوستات الخرا \rightarrow Reply

<u></u> -6 ▼

gotosleep



11 months ago, # $^{\wedge}$ | unlike this comment

← Rev. 3 **0**

shubhankarsharma00



What does the variable "keep" denote ?

→ <u>Reply</u>



4 years ago, # \wedge | \leftarrow Rev. 4 \wedge +3

The way I understand HLD here is basically if a child is the big child, we don't want to recompute answer for it to reduce computation. So we just store the answer for it in the cnt array already so that it's parent doesn't need to re-dfs this subtree. keep denotes whether or not this child is that big child. Please correct me if I'm wrong. :)

 \rightarrow Reply

4 years ago, <u>#</u> ^ |

← Rev. 3 **△ 0** ▼

Look at last two lines:

```
if(keep == 0)
add(v, p, -1);
```



It means that if keep == false after dfs clear v s subtree information from cnt . And if keep == true , don't clear v s subtree information from cnt . In other word if keep == true after calling dfs , for each u from subtree of vertice v , col[u] is in cnt (cnt[col[u]]++).

And **NibNalin** is right. keep is true if and only if v is biggest child of it's parent.

→ <u>Reply</u>

4 years ago, # ^ |

△ 0 ▼

 $\mbox{\ensuremath{\mbox{Hi}}}$ Arpa, thanks a ton for this awe some post. I have really learnt lot from it.



I do have one question though. What is the advantage of having keep=false? If that part is kept as it is, without clearing, doesnt the computation become faster? Can you please help clearing this doubt?

 $\rightarrow \underline{\mathsf{Reply}}$

4 years ago, # ^ |



Consider vertex v has two children, q and p. If you call dfs for both of them with keep = true, they will mixed up their information, and queries will be incorrectly answered.

→ <u>Reply</u>

Hi, Thanks.

4 years ago, # ^ | **0 v**



I guess this method is only viable if DP cannot be used? (i.e. Too many states to memoize)

3 years ago, # \wedge \vdash Rev. 2

Lance_HAOH

 \rightarrow Reply

4 years ago, # ^ |

A +34

A 0 V

Observations to understand the complexity:

- 1. The dfs function visits each node exactly once.
- 2. The problem might seem with the add function. You might think that it is making the algorithm n^2. Note that in the add function, we only go down from a vertex to its children if the edge connecting the vertex to the child is a light edge.



flatline

You can think of it in this way, each vertex v will be visited by a call to the add function for any ancestor of v that is connected to a light edge. Since there are at most log(n) light edges going up from any vertex to the root, each vertex will be visited at most log(n) times.

So the algorithm is: Say you are at a vertex v, first you find the bigchild, then you run dfs on small childs, passing the value of keep as v. Why? So they are cleared from cnt. Then you run a dfs on bigchild, and you do not clear it from cnt. Now, cnt stores the results for all vertices in the subtree of bigchild (since we cleared for small childs and didn't do so for the bigchild), so we call the function to "add" the information of children of current vertex that are connected to it via a light edge. Now we are ready to compute the answer

→ <u>Reply</u>



gogateiii

3 years ago, # \wedge | \leftarrow Rev. 3

As you said "add" function goes down using only light edges, Don't these two lines if(bigChild != -1)
big[bigChild] = 0; of heavy light decomposition implementation would affect it as if you call "add" after all dfs are done and returned to the root then we only have one heavy edge marked that of root itself others are zero so as "add" goes below it traverses whole tree. Help me here.





gogateiiit

3 years ago, # \triangle | Got it.







gogatojijt

For those who had same doubt as I had: First lets understand why it is wrong to remove it, consider you are at particular node(let it be called A) in recursion, above line is not there, you have 3 children one of them is big child(3rd) while others are normal so you traversed inside 1st and came back to A then you traversed inside 2nd child if you do not have above line then while going inside this children you would have all answer for big children of 1st child which would mess your answer. Now let's understand why complexity is O(n(log(n)):

to measure how many times add function visits the ever y node.

Note 2: For first add called at a node: A node will visited by add only through its ancestors which are connected by light edges so n nodes log(n) light edges above it this gives us O(n(log(n)))

Note 3: For second add called at a node: Now somebody may protest that that after above mentioned line(big[bigChild]=0) we are unmarking heavy edge and also calling add after that which may mess up complexity as it travels every node below it which is O(n) but keep==0 condition ensures that for each node there atmost log(n) nodes above in ancestor which have keep=0 function is called.which again gives O(n(log(n)).

Giving us finally O(n(log(n))) complexity. Follow this link to understand heavy light decomposition's properties:

https://blog.anudeep2011.com/heavy-light-decomposition/

 \rightarrow Reply

```
4 years ago, # |
```

In second easy with O(n*lg n)

Why if(keep==false) we delete only vertex from main vector



but we don't delete vertex from cnt which we changed here:

```
4 years ago, <u>#</u> <u>^</u> |
```

← Rev. 2 **△ 0** ▼

<u></u> 0 🔻

There was a mistake in writing. I'm sorry.

Thanks for reporting this problem.

code should be:

```
if(u != p && u != bigChild){
    for(auto x : *vec[u])
        cnt[ col[x] ]++;
}
```

Auron

Instead of:

```
if(u != p && u != bigChild){
    for(auto u : *vec[u])
        cnt[ col[u] ]++;
}
```

I have edited that.

```
\rightarrow \underline{\mathsf{Repl}}_{\mathsf{y}}
```

```
4 years ago, <u>#</u> |
```

Thanks!

→ Reply



← Rev. 3 ▲ **+12** ▼

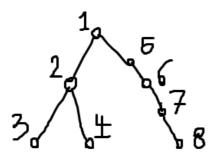


I can't understand why the second code is correct...

carri understand why the second code is com-

Consider this example:

4 years ago, <u>#</u> |



We wanna calculate the cnt for Vertex 8. These are the steps:

Going to Vertex 1

Going to Vertex 2 by keep=0

Going to Vertex 3 by keep=0,Vec[3]={3}

Going to Vertex 4 by keep=1,Vec[4]={4},Cnt[color[4]]=1

Going back to Vertex 2, Vec[2]={2,4}, Cnt[color[4]]=0, Cnt[color[3]]=1

And then when we go to Vertices 5,6,7,8 still Cnt[color[3]]=1.

Also sorry if I did the steps wrong...

UPD Thank you for editing the blog. My problem fixed.

4 years ago, <u>#</u> |





Great post. If you explained the idea before showing the code, it would be better to understand. Also commenting the variables meaning in the code would be of great help.

It would be good to mention that most solutions will answer the queries offline, which may be a problem sometime (maybe someone didn't notice this lol).



gabrielsimoes

Also, it would be nice to post hints about the solutions.

Proving explicitly why it is nlogn would be good too (ie. as each node's subtree set gets merged into one set of size equal or greater, and the base set has size 1 and the last set has size n, then we take log n steps to go from 1 to n. Summarizing, each node gets merged logn times, so the total complexity is O(nlogn)).

Here's my solution to 343D, maybe it will be of help to someone: 18958875. A lot easier to code than the one in the tutorial.

→ <u>Reply</u>





Thanks for your suggestions first!



I proved that it is O(nlogn): You know that why dsu has O(q log n) time (for g queries); the code uses same method. Merge smaller to greater.

And about your code (18958875), anyone has a different opinion!



4 years ago, # ^ |



Thanks for the reply!

Yeah, you did prove. People who remember DSU's proof will most likely understand. I stated a more extensive proof would be better thinking about people who don't exactly know the proof. Don't take me wrong, but they may get a little confused reading this proof.

I mentioned my code exactly herause everyone has a

→ Reply



Sorry this might be a stupid question to bring up, but why is the complexity of the heavy-light decomposition style one in O(nlogn)?

In the case where each node has at most two children: Denote the root node of the tree as u, which is of size s. The child of u connected to the lighter edge is of size at most $\frac{s}{2}$. So the total number of nodes on which we run the "add" function would be at most $\frac{s}{2} + \frac{s}{4} + \cdots = s$. So I don't understand where the log(n) factor comes from.



The online tutorial for HLD says a new chain is built when we arrive at a child node via a lighter edge, where each chain is stored as a segment tree, and so I can see there is indeed a O(logn) factor involved.

Regardless can you perhaps elaborate a little bit more on the time complexity of the dsu structure? Thank you!

→ Reply

4 years ago, <u>#</u> <u>↑</u> |



Hi!

The online tutorial for HLD says a new chain is built when we arrive at a child node via a lighter edge, where each chain is stored as a segment tree, and so I can see there is indeed a O(logn) factor involved.



As you know, if you use segment tree in heavy-light decomposition, each query time complexity will be $O(\log^2(n))$. Because in each query you will go $O(\log(n))$ chains and in each chain it will spend $O(\log(n))$ time.

Now, I will proof that "heavy-light decomposition style implementation" of "dsu on tree" is O(n.log(n)):

Consider a complete binary tree with n vertices. In dfs function you will run another dfs function in child (T(n/2) * 2) and you will call add function and it will spend O(n/2) time. So, $T(n) = n/2 + 2 * T(n/2) = O(n.log(n)) \rightarrow \text{Reply}$





You know that why dsu has $O(q \log n)$ time (for q queries); the code uses same method. Merge smaller to areater.



Pardon me , but I don't follow. Which dsu are you talking about? The one with inverse-Ackermann function?

→ Reply



3 years ago, <u>#</u> <u>^</u> |

A 0 V

No. Dsu with size compare. Like this:

```
Arpa

return par[x] == x ? x : find(par[x]);

}

void merge(int v, int u){

v = find(v), u = find(u);

if(v == u) return;

if(size[v] < size[u]) swap(v, u);

par[u] = v;

size[v] += size[u];

}

→ Reply
```

4 years ago, # | ← Rev. 2 **△ 0** ▼



In easy to code but O(nlog^ 2), I cant't understand why do we store the size of subtrees of vertices in array sz and use it as the criteria for choosing the big child, I think we should store in the array "sz" the number of distinct colors in the subtree of any node v, because that is what we actually iterate on when transferring the map from v to u, why is this wrong?

 $\rightarrow Reply$





Hi

It isn't wrong! Both of your method and mine have the same worst case. But your average is better.

 \rightarrow Reply



14 months ago, # ^ |

<u></u> 0 🔻

Is child's subtree-size working as a heuristic for the number-of-distinct-colors in that child?

→ <u>Reply</u>



algo.experiments

4 years ago, # |

← Rev. 2

0

about answering queries OFFLINE right during the dfs. After the dfs has finished the cnt[v] will no longer be a valid map for vertices that were chosen as bigChild.

Ahh, thanks gabrielsimoes, for anyone struggling to understand: n*log^2n is

→ Reply

```
4 years ago, # | A +8
```



http://codeforces.com/problemset/problem/291/E 291E - Tree-String Problem

Arpa This problem can also be done by dsu on trees. Calculate hash value for each suffix value of target string. Then for each suffix of an edge if it is a valid prefix of the target string we would just need the frequency of the hash value of the remaining suffix of the target string in its subtree which can be maintained by this technique. The case when the entire string occurs in an edge can be dealt with separately.

→ Reply



4 years ago, # ^ |

▲ +5 ¥

Thanks added to list, but it can be solved very easier : 19827525, just use KMP.

→ Reply



3 years ago, <u>#</u> <u>↑</u> |

<u></u> 0 🔻

Just use hashes:) http://codeforces.com/contest/291/submission/29431526

 \rightarrow Reply



Hello Arpa,

Mooncrater

The technique which you've used in this question, for the KMP solution, in which you've created the nxt 2D array, is it

called something? Is it some recognized idea or did vou just

canca sometiming: is it some recognized idea or did you just came with it on the go?

Basically, you removed the inner while loop of KMP for matching strings. Instead of jumping multiple times to check whether the current characters match, you created a DP, using which we can do that in a single jump. That saves a lot of time, as we would have to do that for each string in the tree, while matching.

So, is this some already defined method?

→ Reply





Hello.

Do you know automata? I'm building automata from the string.

→ Reply



3 months ago, # ^ | I do know automata. But damn. That's some beautiful automata you got there.

△ 0 ▼

GG.





Please send me a code of the solution with this techinque)



3 months ago, $\frac{\#}{}$

Here is bhargav104's solution

EDIT: Sorry. It doesn't use DSU on trees. Rather, it uses KMP. \rightarrow Reply



3 years ago, <u>#</u> |

← Rev. 3 A +13 V

← Rev. 3 **△ 0** ▼

Actually, in China, we call this method as "Heuristic Merge" which always merge the smaller to the bigger. Not hard to understand each vertex will be visited in O(log n) times because when we visited a vertex, then the size of tree which the vertex is in doubled.

 \rightarrow Reply

▲ +1 ▼ 3 years ago, <u>#</u> |

Hey Arpa,



In your my invented style I'm unable to understand that why in third loop are you not checking for u not being parent of v. Why are you only checking for just u not being the big child.

Thanks in Advance



<u></u> 0 🔻 3 years ago, <u>#</u> <u>↑</u> |

Sorry, fixed. It's because I've copied my code from 741D - Arpa's lettermarked tree and Mehrdad's Dokhtar-kosh paths, input for this problem was a rooted tree.

→ <u>Reply</u>



A +2 V 3 years ago, <u>#</u> ^ | Thanks a lot,

Also, I tilling there is one more mistage. Tou here, added col[v] to the array. Am I missing something. Thanks in advance.

→ <u>Reply</u>



3 years ago, <u>#</u> <u>↑</u> |

△ 0 ▼

You are right, I'm very thankful to you. I was careless while coping the code from polygon.



In the easy to code O(nlogn) method vec[v] stores all the vertices in the the subtree rooted at v . How will this fit into memory if we are not deleting the vec of child after merging it with the parent

 \rightarrow Reply



3 years ago, <u>#</u> <u>^</u> ∣

▲ +1 Ψ



Used memory is always less than or equal to time complexity, so when time complexity is $\mathcal{O}(n \cdot \log n)$, used memory is less than or equal to $\mathcal{O}(n \cdot \log n)$. In this case, used memory is $\mathcal{O}(n \cdot \log n)$. Although if you delete useless vec's the memory become $\mathcal{O}(n)$.

→ <u>Reply</u>



3 years ago, <u>#</u> <u>↑</u> |

△ 0 ▼

Thanks for the reply . I think this problem can also be solved using your approach. (The Grass Type HackerEarth)

→ Reply



3 years ago, # ^ | ← Rev. 3

I'll add this problem to the post if I find it related, I'm thankful anyway.



Edit: Note that this is not my approach, but I'm the first man who publishes a tutorial about this (not sure), Sack has been used in INOI, IOI and ACM several times, so it isn't a new thing, invented be

Edit: Added.

→ Reply



3 years ago, <u>#</u> <u>^</u> |

△ 0 ▼

Can you mention probelms from the IOI that are solved with sack?

SaYami

→ Reply



3 years ago, R<u>#</u>v. <u>2</u> | ▲ **+6** ▼ I'll add one of them tonight.

Edit: Added.

→ <u>Reply</u>



3 years ago, # **^0**

Wow, I didn't think of solving it with sack. Thx

 $\rightarrow \underline{\mathsf{Reply}}$



3 years ago, <u>#</u> |

Hi Arpa, I can not understand, why is this approach called dsu on tree? This approach has a nice trick to reduce complexity by saving data about "big child". I can't see any special similarity with general dsu approach. In general dsu

tutorial there is no "merge" function. Am I missing something?

Also I see that, in your 600E's solution 14554536 you used merge functon. I can't understand, could you please explain that code?

 \rightarrow Reply



3 years ago, # \triangle |

In fact we are merging information of small children with big child. Think more.

Arpa

In that code, mrg function merges information in u into v .

 \rightarrow Repl

3 years ago, # | **+1**

Hi Arpa! Thanks for making this tutorial.



beAwesome fo

I just want to make sure my understanding is correct: this merging smaller maps into larger ones takes logarithmic time because when a vertex is merged, the new map it is in at least twice its size. Hence, merging can only happen log(n) times for each of the n vertices, leading to a total runtime of O(nlogn)?

Thanks!

→ <u>Reply</u>



3 years ago, # $^{\wedge}$ | Yes, but note that if you use map, it's $O(n \cdot \log^2 n)$.

 \rightarrow Reply

// (the some
f= (sof() == true) {
 sof().step();
 belowsome();
}

beAwesome

3 years ago, # <u>^</u> |

If you use unordered_map, does it become $O(n \cdot log n)$, then? $\longrightarrow \underbrace{\text{Reply}}_{}$



3 years ago, # \wedge |

query. But

Unordered_map is theoretically O(n) per query. But you can suppose that it's O(1) per query in code.

 $\rightarrow \underline{\mathsf{Reply}}$



3 years ago, $\frac{\#}{}$

→ Renly



surajghosh

3 years ago, <u>#</u> |

<u></u> 0 🔻

A 0 W

Why do we need to iterate through the children of v after add(v,p,-1) in the naive approach?

→ <u>Reply</u>



3 years ago, <u>#</u> <u>↑</u> |

This 758E.Read this comment on how to use it.

△ 0 ▼

A +8 V

dfs() solves the problem for all the nodes, not just one. So, after you've gotten the answer for v, it will calculate the answer for its children.

 $\rightarrow Re$

3 years ago, <u>#</u> |



101 Hack 47 Summing Tree was solved using this technique by satyaki3794 Submission

 $\rightarrow \underline{\mathsf{Reply}}$



3 years ago, <u>#</u> |

A +3 V

also 778C - Peterson Polyglot is solvable with a similar tecnique: is that DSU on tree?

lukecavabarrett

- Donly



3 years ago, <u>#</u> <u>^</u> ∣ yes

A +3 V



3 years ago, <u>#</u> |

△ +6 ▼

Can anyone give me a link to "Shaazzz contest round #4 (season 2016-2017) problem 3" or tell me where can I find it? Thanks.

→ Reply



3 years ago, <u>#</u> <u>↑</u> |

▲ +5 ▼

It's a Persian contest. → <u>Reply</u>



3 years ago, <u>#</u> <u>^</u> |

▲ +6 ▼

Can you tell me where can I find it? I searched for it just now but didn't get it.

→ <u>Reply</u>









▲ +6 ▼

→ Reply



3 years ago, <u>#</u> |

△ 0 ▼

I can AC easily Problem 375D by the 3th way ,but WA by the 4th way.... WA on the test 4.. why..

 $\rightarrow Reply$



3 years ago, # |

▲ 0 ▼

APIO 2016 Fireworks uses this, but is a much harder problem.

 \rightarrow Reply



3 years ago, <u>#</u> |



Arpa, in the **Easy to code but** $O(nlog^2 n)$ section code you have written a subtree of vertex v that has color c. You can answer the queries vertices in subtree of vertex v that has color c. You can answer the queries easily. . Will (*cnt)[c] changed with (*cnt[v])[c] ? → Reply

3 years ago, # ^ |

▲ 0 ▼

Hi. Thanks. Edited. \rightarrow Reply

3 years ago, <u>#</u> |

△ 0 ▼



You can solve APIO 2012 Dispatching with this technique too.

Trath



3 years ago, <u>#</u> |

▲ 0 ▼

IOI 2011 — Race What is the idea of DSII on tree for this problem? I know of a

 \rightarrow Reply



10 months ago, # _^ |

▲ +8 ▼

keep map<distance from root, depth> for all subtrees. By merging them from smaller to larger, you can get O(nlog^2n) solution.

KwanghyunOn



2 years ago, <u>#</u> | ← Rev. 2 914E - Palindromes in a Tree can solve with sack too, Arpa:)

ps: for this problem centroid decompose works too ... :)

 \rightarrow Reply



▲ 0 ▼ 2 years ago, <u>#</u> |

can anyone please explain how to solve 716E using this technique? → Reply

2 years ago, # |

▲ +5 ▼

In the contest 600, no one can view other's submissions except his own. That's why no can see your submissions for 600E - Lomsat gelral except you.

So, please give alternating link of your solutions for the first problem in the list, 600E - Lomsat gelral

→ Reply



2 years ago, # ^ | **△** 0 ▼

Thanks for your feedback. Here it is: Link.

→ <u>Reply</u>



shahidul_brur

△ 0 ▼

→ Reply

2 years ago, # ^ |

Thank you!

2 years ago, # |

← Rev. 2 **△** 0 ▼

Another problem which can be solved by this technique: Coloring Tree Its easier than 600E - Lomsat gelral.

One more: 932F - Escape Through Leaf

 \rightarrow Reply

2 years ago, <u>#</u> |

← Rev. 2

△ 0 ▼



lzoi_hhn

Hi, I would like to ask you about the code in heavy-light decomposition style. Why the bigChild is cleared before clearing the subtree at the end of the code? From my perspective, in add function bigChild will be visited and the array "big" doesn't make sense.Can you explain it in detail? Thanks a lot.

 \rightarrow Reply



2 years ago, # |

▲ -7 ▼

No good explanation! Only code! Worst tutorial.

 $\rightarrow \underline{\mathsf{Reply}}$



2 years ago, <u>#</u> ^ |

A 0 V

If you have a doubt why not ask in the comments rather than whining about how bad the tutorial is.

 $\rightarrow \underline{\mathsf{Reply}}$

```
2 years ago, <u>#</u> |
                                                                                             ▲ +5 ▼
                  Nice tutorial!
EXPIRED
                  and also happy Nowruz:)
  M_H_H_7
                  \rightarrow Reply
                            2 years ago, <u>#</u> ^ |
                                                                             ← Rev. 2 △ 0 ▼
                            Could you please explain 2no. style ? [Upd : Got it ]
                            \rightarrow Reply
         Ehsan_sShuvo
                                      23 months ago, # ^ |
                                                                               ← Rev. 3
                                                                                              △ 0 ▼
                                      In 2nd style Arpa can you please explain?..when we do
                                      cnt[u]=cnt[heavy child] does this happen in O(1)?
                                      → Reply
                                                23 months ago, \# \wedge | \leftarrow Rev. 2 \wedge \bullet
                                                if yes -> how? else what the benefit of doing this?
                                                upd[got it]
                                                \rightarrow Reply
                                                                                              ▲ 0 ▼
                  2 years ago, <u>#</u> |
                  If i do my code using 2nd approach, shouldn't my problem be static? And for
                  query operation, we will do offline query, won't that?
                  → Reply
Ehsan_sShuvo
                                                                                             ▲ +8 ▼
                  2 years ago, # |
                  For those who still confused about the time complexity, I found this explanation by
                  radoslav11 helps a lot.
   Azurey
                                                                                              △ 0 ▼
                  2 years ago, <u>#</u> |
                  What is col[v]?
                   \rightarrow Reply
                                                                                              <u></u> 0 🔻
                            2 years ago, <u>#</u> ^ |
                            Color of the v-th vertex
                  2 years ago, <u>#</u> |
                  I loved the way you used the Euler tour sequence to iterate through the subtrees.
                  My 2 cents is a way to use C++ to implement this nicely. The main idea is to use
                  a structure, that keeps pointers instead of st[v] and ft[v], and to give it
                  a begin() and end() member function, so that you can iterate through it with
                  for(int u : T[i])
                       cout << u << " is in the subtree of " << i << "\n";
                  To actually solve the problem, maybe in a Heavy-Light style, I kept also for each
    _Na2Th
                  vertices a pointer to the leaf of its heavy-path, so that I could only change the
                  answer in the leaf.
                  My full implementation of lomsat gelral can be found here
                  → <u>Reply</u>
                                                                                              △ 0 ▼
                  23 months ago, # |
                  Hi, Arpa, I used this technique to solve 600E - Lomsat gelral, its a very neat
                  technique.. but wont this get a MLE? I am getting a MLE with the O(nlog^2n0
```

yaksha

method. I saw your solution and I see its the same as mine, but mine details

→ Reply









Solutions for Educational Codeforces Round 2 is private, nobody can't see your code. Perhaps you do not use pointers, dynamic memory allocation, and memory cleanup. In this technique it is forbidden to copy huge containers.

dmkozyrev

I solved this problem with Euler tour on tree and Mo algorithm in $O(n \cdot \sqrt{n})$ time and O(n) memory. Code.

→ Renly



23 months ago, # |



Great blog but I am not able to understand the logic behind the O(nlogn) solution it would be a great help if anyone can explain it.

→ Reply





I have a question for style 3 i.e HLD style.

I'm not so sure what's happening in there, I have 2 assumptions. Both of them are wrong, so it would great if someone could point out the mistake and explain what's right.

We are in root.



The_Wolfpacl

- 1. We first go down the light edges of root and when we finish with them, we clear the cnt array, so there is absolutely nothing left in it and then we proceed to the heavy child of the root and then we just update cnt array. Now we go back to the light edges of the root and (here's the problem) as the cnt array only contains information for the heavy child of the root, we must go through EVERY vertex in subtrees of light children of the root. If we don't go to the heavy children in the subtrees (as it proposes in tutorial?), then the answer is wrong, as we didn't count them (remember that we cleared the cnt array).
- We first go down the light edges of the root, but this time, for every heavy child, we keep the information.But then as we proceed to the heavy child of the root, the array cnt won't be empty and the answer for heavy child will be incorrect.
- → <u>Reply</u>





Arpa

Consider you entered vertex v, let it's heavy child h. First, go to all child of v except h and calculate the answer for them and each time clear the cnt array. Then go to h and calculate the answer and don't clear the cnt array. Then traverse all of the subtree of v except subtree of h, and add them to cnt.

 $\rightarrow \underline{\mathsf{Reply}}$



22 months ago, # ^ |

I got it, thanks for the quick reply!

+13

→ Reply





A +18 V



Another problem can be solved with this technique. **Arpa** please add this one in the list.

http://codeforces.com/contest/1009/problem/F (Dominant Indices)

→ Reply



22 months ago, <u>#</u> |

△ 0 ▼

 \rightarrow Reply



22 months ago, # ^ | Can't view your submission

A +8 V

 \rightarrow Reply



22 months ago, # ^ | **▲** +5 ▼

Firstly, thank you for reaching out to help:). I got my mistake.

→ Reply



22 months ago, # |





I'm continuously getting TLE in test case 30 in 570 : D : Tree Request (The second one given in the blog practice problem) while implementing through DSU similar to what is given here. Submission Id: 40780911, can somebody make a look over this and provide hint to optimize it.

→ <u>Reply</u>



16 months ago, # _^ |

<u></u> 0 🔻

My solution passed the time limit after replacing endl with "\n".

m0nk3ydluffy

22 months ago, <u>#</u> |





This blog was a bit code-heavy for me when I first read this. Hence I have tried to simplify the concept a bit more in a more textual fashion in my own tutorial spinoff. I have tried to provide the intuition behind small-to-large merging including small to large on trees, also knows as DSU on trees.

However I. haven't provided any code as the code given by the OP is more than enough.

DSU-on-Tree-Intuition

 \rightarrow Reply

17 months ago, # |





sagarthecoder2

Method -3

WHy here use bigchild??

and why use "keep==0 " ??







sorry but you absolutely wrong about the complexity, i know the trick merge smaller to greater but it works when you don't for(i = 0; i < greater.size(); i++) again, but here in O(nlogn ^ 2) you use cnt[v] = cnt[bigchild], it equal to "for(int i = 0; i < cnt[bigchild].size(); i++) {} ", so it's not O(nlogn 2)

 $\rightarrow Reply$



17 months ago, # ^ |

A 0 V

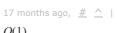
<u></u> 0 🔻

Note that cnt[v] is a pointer. → Reply

17 months ago, # <u>^</u> | you mean that it take O(logn) time?

→ Reply

mra2322001



O(1).



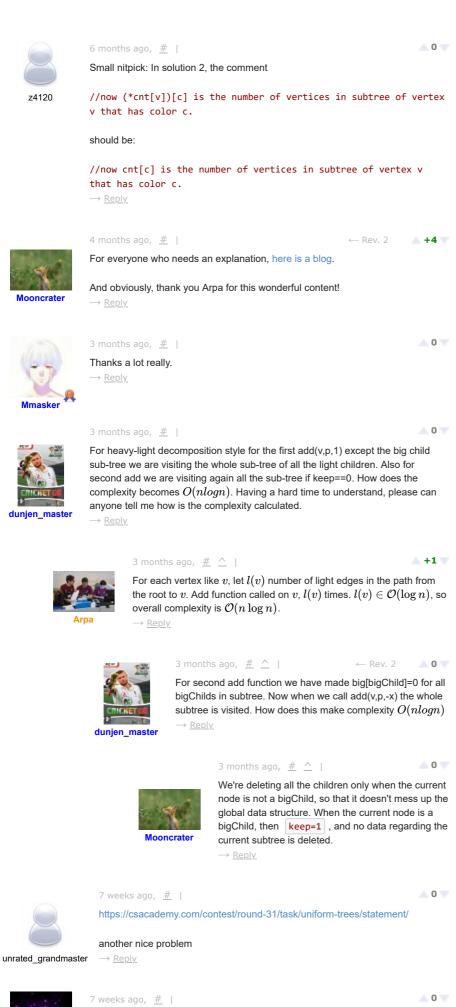
```
void dfs(int v, int p){
    // Subtract cnt[c] for query here, e.g. answer[query] -= cnt[c];

cnt[col[v]]++;
    for(auto u : g[v])
        if(u != p)
            dfs(u, v);

// Add cnt[c] for query here, e.g. answer[query] += cnt[c];
}

Reply
```

```
14 months ago, # |
                  Can somebody help me understand why my solution 51874074 for problem 246E
 idk,
 google it.
                  - Blood Cousins Return gets WA on test 50? I tried to implement the Heavy Light
                  Decomposition style dsu
                  → <u>Reply</u>
     idLe.
                  13 months ago, <u>#</u> |
                                                                                            △ 0 ▼
                  How to answer online queries using this technique??
                   → <u>Reply</u>
   lil chonk
                                                                                          ▲ +4 ▼
                            13 months ago, <u>#</u> <u>↑</u> |
                            I didn't find till today.
                            → Reply
                  13 months ago, # |
                  why did he declare every time using pointer, can't we simply declare array of map
                  for first approach as map<int,int> cnt[maxx] instead of map<int,int>*cnt[maxx] ?
  nsut_coder
                            13 months ago, # ^ |
                                                                                          A +8 V
                            Due to this line, cnt[v] = cnt[bigChild]. Because we are using pointer, we
                            only pass the reference. In other case, whole map will be copied(time
                            complexity: linear in size of map).
             abhayps
                                      11 months ago, # ^ |
                                                                             ← Rev. 2
                                                                                            A 0 V
                                     Then, will the information for the child be lost when we modify
                                     the parent's map which actually references the child's map... If
                                      so then how to answer queries in case when I need to access
                                     the nodes' map later?
                     Varun_Shah
                                      → Reply
                                                                              ← Rev. 2
                                                                                            △ 0 ▼
                  13 months ago, <u>#</u> |
                  Why is my code getting TLE for problem 600E — Lomsat gelral? I'm using the
                  technique no. 1 from this tutorial.
Underdog_eagle
                  → Reply
                            11 months ago, # ^ |
                                                                            ← Rev. 2
                            your code is working in {\cal O}(N^2) . You can't go through the whole
                            (*cnt[u]) to answer queries.
          dragonfire100
                                                                                            △ 0 ▼
                                      11 months ago, # ^ |
                                     after 2 months!!!
                                      you think he / she, will check this blog again looking for his
                                     answer?
                                                                                         ▲ +16 ▼
                  11 months ago, # |
                  Love this.
                   → Reply
                                                                                          ▲ +1 ▼
                  9 months ago, # |
   CCCgiff
                  Thank you for the article. Now I understand it.
chenyewei_1234
```



Can we know the shortest distance from a vertex using dsu? → Reply

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