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1a)  $\exists x P(x)$ , where  $P(x)$  is the statement " $x < 2$ "

-->  $0 < 2$ , hence proving the statement True

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1b)  $\forall x P(x)$ , where  $P(x)$  is the statement " $x < 2$ "

-->  $2 \geq 2$ , hence proving the statement False

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1c)  $\exists x (P(x) \vee Q(x))$  where  $P(x)$  is the statement " $x < 2$ " and where  $Q(x)$  is the statement " $x > 7$ "

-->  $0 < 2$  or  $0 > 5$ , hence proving the statement True

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1d)  $\forall x (P(x) \vee Q(x))$  where  $P(x)$  is the statement " $x < 2$ " and where  $Q(x)$  is the statement " $x > 7$ "

-->  $2 < (x = 2) < 7$ , hence proving the statement False

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1e) Prove De Morgan's Law for the Existential Quantifier where  $P(x)$  is the statement " $x < 5$ "

prove  $\neg \exists x (x < 5) \equiv \forall x (x \geq 5)$

First proving: not(for some  $P(x)$ , the negation

-->  $0 < 5$  proves the statement is False and the negation of that is True

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Proving: for all (not( $P(x)$ ))

--> not( $5 \geq 5$ ), proving True

Since the negation and  $\forall x (x \geq 5)$  proven True the statement is True

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1f) Prove De Morgan's Law for the Universal Quantifier where  $P(x)$  is the statement " $x < 5$ "

prove  $\neg \forall x (x < 5) \equiv \exists x (x \geq 5)$

First proving: not(for all  $P(x)$ , the negation

-->  $0 < 5$  proves the statement is False and the negation of that is True

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Proving: for some (not( $P(x)$ ))

--> not( $5 \geq 5$ ), proving True

Since the negation and  $\exists x (x \geq 5)$  are proven True the overall statement is True

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2a)  $\forall x \forall y P(x, y)$

--> False For  $(x, y) = (1, 1)$ ,  $xy = 1$  proves the statement wrong  
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2b)  $\forall x \exists y P(x, y)$

True

-->  $(x, y) = (0, 0)$  satisfies the problem statement  $xy = 0$

-->  $(x, y) = (1, 0)$  satisfies the problem statement  $xy = 0$

-->  $(x, y) = (2, 0)$  satisfies the problem statement  $xy = 0$

-->  $(x, y) = (3, 0)$  satisfies the problem statement  $xy = 0$

-->  $(x, y) = (4, 0)$  satisfies the problem statement  $xy = 0$

-->  $(x, y) = (5, 0)$  satisfies the problem statement  $xy = 0$

-->  $(x, y) = (6, 0)$  satisfies the problem statement  $xy = 0$

-->  $(x, y) = (7, 0)$  satisfies the problem statement  $xy = 0$

-->  $(x, y) = (8, 0)$  satisfies the problem statement  $xy = 0$

-->  $(x, y) = (9, 0)$  satisfies the problem statement  $xy = 0$

-->  $(x, y) = (10, 0)$  satisfies the problem statement  $xy = 0$   
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2c)  $\exists x \forall y P(x, y)$

--> True, for  $x = 0$ , the proposition is true for all values of  $y$   
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2d)  $\exists x \exists y P(x, y)$

--> True: for  $(x, y) = (0, 0)$ ,  $xy = 0$  proves the statement true

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