RNS INSTITUTE OF TECHNOLOGY, BENGALURU - 98
DEPARTMENT OF MATHEMATICS

18CS36 - DISCRETE MATHEMATICAL STRUCTURES

Assignment - II

Submission Last Date: 28.11.2020

Q. No	Questions
1.	Establish each of the following by Mathematical Induction (a) $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$ (b) $\sum_{i=1}^{n} 2^{i-1} = 2^n - 1$ (c) $\sum_{i=1}^{n} i(2^i) = 2 + (n-1)2^{n+1}$ (d) $\sum_{i=1}^{n} i(i!) = (n+1)! - 1$
2.	Prove by Mathematical Induction $n! \geq 2^{n-1}$ for all integers $n \geq 1$.
3.	Prove that every positive integer $n \geq 24$ can be written as a sum of 5's and/or 7's.
4.	How many license plates can be made using (i) Either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits? (ii) Either two uppercase English letters followed by four digits or two digits followed by four uppercase English letters? (iii) Either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits?
5.	A student has three books on C++ and four books on Java. In how many ways can he arrange three books on a shelf (i) If there are no restrictions? (ii) If the languages should alternate? (iii) If all the C++ books must be next to each other? (iv) If all the C++ books must be next to each other and all the Java books must be next to each other?
6.	Find the number of permutations of the letters of the word MASSASAUGA. In how many of these, all four A's together? How many of them begin with S?
7.	How many numbers greater than a million can be formed using the digits 0, 3, 4, 4, 5, 5, 5?
8.	Find the number of arrangements of all the letters in TALLAHASSEE. How many of these arrangements have no adjacent A's?

9.	A woman has 11 close relatives and she wishes to invite 5 of them to dinner. In how many ways can she invite them in the following situations: (i) There is no restriction on the choice. (ii) Two particular persons will not attend separately. (iii) Two particular persons will not attend together.
10.	A certain question paper contains three parts A, B, C with four questions in part A, five questions in part B and six questions in part C. It is required to answer 7 questions selecting at least two questions from each part. In how many ways can a student select his seven questions for answering?
11.	In how many ways can we distribute 7 apples and 6 oranges among 4 children so that each child gets at least 1 apple?
12.	Find the number of positive solutions of the equation $x_1 + x_2 + x_3 = 17$
13.	Find the number of integer solutions of $x_1 + x_2 + x_3 + x_4 + x_5 = 30$, where $x_1 \geq 2, x_2 \geq 3, x_3 \geq 4, x_4 \geq 2, x_5 \geq 0$.
14.	Determine the coefficient of (i) $x\ z^2$ in the expansion of $(2x - y - z)^4$ (ii) $x^2y^2z^3$ in the expansion of $(3x - 2y - 4z)^7$ (iii) $x^1\ y^4$ in the expansion of $(2x^3 - 3xy^2 + z^2)^6$ (iv) $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$.
15.	Define cartesian product of two sets. For any three sets A, B, C, prove the following: (1) $A \times (B \cup C) = (A \times B) \cup (A \times C)$, (2) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (3) $(A \cup B) \times C = (A \times C) \cup (B \times C)$, (4) $(A \cap B) \times C = (A \times C) \cap (B \times C)$, (5) $A \times (B - C) = (A \times B) - (A \times C)$.
16.	Draw the directed graph of the relation R = {(1, 1), (1, 3), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (4, 1)} on the set {1, 2, 3, 4} and write its matrix. Also find in-degree and out-degree of each vertex.
17.	Define an equivalence class. Write the equivalence classes of the equivalence relation R = {(0, 0), (1, 1), (1, 2), (2, 1), (2, 2), (3, 3)} on the set A = {0, 1, 2, 3}.
18.	Define POSET. Let A = {*a*, *b*, *c*}, B = P (A) where P (A) is the power set of A. Let R be is a subset relation on B. Show that (B, R) is a POSET and draw its Hasse diagram. Is it a Lattice?
19.	Let A = {3, 5, 9, 15, 24, 45} and R be a divisibility relation on A. Draw the Hasse diagram for the POSET (A, R) and determine least, greatest, minimal and maximal elements. Also find LUB and GLB of B = {9, 15}.
20.	Let R be an equivalence relation on set A and a, b $\in$ A then prove the following are equivalent: (i) a $\in$ [a] (ii) a R b iff [a] = [b] (iii) If [a] $\cap$ [b] $\neq \phi$ then [a] = [b].

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Multiple choice questions

1. **In how many ways can three different coins be placed in two different purses?**

   (a) 6 (b) 3 (c) 8 (d) 4

2. **How many 6 digit numbers can one make using the digits 1, 3, 3, 7, 7, 8 ?**

   (a) 6! (b) 4! (c) 360 (d) 720

3. **How many bytes contain exactly two 1's?**

   (a) 8 (b) 56 (c) 28 (d) 14

4. **In how many ways can we distribute 7 apples and 6 oranges among 4 children so that each child gets at least 1 apple?**

   (a) 104 (b) $7C_4 \times 6C_4$ (c) $7C_3 \times 6C_3$ (d) $6C_3 \times 9C_6$

5. **Find the value of** $\binom{1}{5, 3, 2, 2}$.

   (a) 210 (b) $\frac{1\ !}{5!\ 3!\ 2!\ 2!}$ (c) 420 (d) Impossible

6. **It is required to seat 5 men and 4 women in a row so that the women occupy the even places. How many such arrangements are possible?**

   (a) 5! × 4! (b) 5! × 4 (c) 9! (d) None.

7. **If A = {$x \in$ N : $x \leq 3$} and B = {$x \in$ W, $x < 2$}, then A × B =**

   a) {(0, 1), (1, 1), (0, 2), (1, 2), (0, 3), (1, 3)} b) {(1, 0), (1, 1), (2, 0), (2, 1), (3, 0), (3, 1)}

   c) {(1, 1), (2, 0), (2, 1), (3, 0), (3, 1)} d) {(1, 0), (1, 1), (0, 2), (1, 2), (3, 0), (3, 1)}

8. **If A = {x: $x^2 - 5x + 6 = 0$}, B = {2, 4}, C = {4, 5} then A × (B ∩ C) is**

   a) {(2, 4), (3, 4)} b) {(4, 2), (4, 3)}

   c) {{2, 4), (3, 4), (4, 4)} d) {(2, 2), (3, 3), (4, 4), (5, 5)}

9. **Let A = {1, 2, 3}. Then the relation R = {(2, 3)} in A is**

   a) symmetric only b) transitive only c) symmetric and transitive only d) none of these

10. **Let A = {1, 2, 3, 4} and R = {(2, 2), (3, 3), (4, 4), (1, 2)} be a relation on A. Then A is**

    a) reflexive b) symmetric c) transitive d) none of these

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