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TRANSFORM CALCULUS, FOURIER SERIES AND NUMERICAL TECHNIQUES

18MAT31

ASSIGNMENT - 2

Q1. Obtain the Fourier Series of the function $f(x) = x - x^2$ in $-\pi \leq x \leq \pi$ and hence deduce that

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$$

$\Rightarrow$ Given $f(x) = x - x^2$

$$f(-x) = -x - (-x)^2 = -x - x^2 \neq f(x)$$
$$= -(x + x^2) \neq -f(x)$$

$\therefore$ $f(x)$ is neither even nor odd.

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx = \frac{1}{\pi}\int_{-\pi}^{\pi}(x - x^2)\cdot dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{-\pi}^{\pi} = \frac{1}{\pi}\left\{\left[\frac{\pi^2}{2} + -\frac{\pi^3}{3}\right] - \left[\frac{\pi^2}{2} + \frac{\pi^3}{3}\right]\right\}$$

$$= \frac{1}{\pi}\left[\cancel{\frac{\pi^2}{2}} - \frac{\pi^3}{3} - \cancel{\frac{\pi^2}{2}} - \frac{\pi^3}{3}\right] = -\frac{2\pi^{\cancel{3}2}}{3\cancel{\pi}} = -\frac{2\pi^2}{3}.$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\cdot dx = \frac{1}{\pi}\int_{-\pi}^{\pi}\underset{u}{(x - x^2)}\underset{v}{\cos nx}\,dx$$

$$a_n = \frac{1}{\pi}\left[(x-x^2)\cancel{\left[\frac{\sin nx}{n}\right]}^{0} - (1-2x)\left[-\frac{\cos nx}{n^2}\right] - 2\cancel{\left[\frac{-\sin nx}{n^3}\right]}^{0}\right]_{-\pi}^{\pi}$$

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[when x is substituted with π or $-\pi$]

$$= \frac{1}{\pi}\left[(1-2x)\frac{\cos nx}{n^2}\right]_{-\pi}^{\pi} = \frac{1}{\pi}\left[(1-2\pi)\frac{\cos n\pi}{n^2} - (1+2\pi)\frac{\cos n\pi}{n^2}\right]$$

$$= \frac{1}{\pi n^2}\left[\cancel{\cos n\pi} - 2\pi\cos n\pi - \cancel{\cos n\pi} - 2\pi\cos n\pi\right]$$

$$= \frac{1}{\cancel{\pi} n^2} \times -4\cancel{\pi}\cos n\pi = \frac{4(-1)^{n+1}}{n^2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} \underset{u}{(x-x^2)}\ \underset{v}{\sin nx}\cdot dx$$

$$b_n = \frac{1}{\pi}\left[(x-x^2)\left[\frac{-\cos nx}{n}\right] - (1-2x)\left[-\frac{\sin nx}{n^2}\right] - 2\left[\frac{\cos nx}{n^3}\right]\right]_{-\pi}^{\pi}$$

$$= \frac{1}{\pi}\left[-\frac{2\cos nx}{n^3} + -\frac{(x-x^2)\cos nx}{n}\right]_{-\pi}^{\pi}$$

$$= \frac{-1}{\pi}\left\{\left[\frac{2(-1)^n}{n^3} + \frac{(\pi-\pi^2)\cos n\pi}{n}\right] - \left[\frac{2(-1)^n}{n^3} + \frac{(-\pi-\pi^2)\cos n\pi}{n}\right]\right\}$$

$$= \frac{-1}{\pi}\left[\cancel{\frac{2(-1)^n}{n^3}} + \frac{\pi\cos n\pi}{n} - \cancel{\frac{\pi^2\cos n\pi}{n}} - \cancel{\frac{2(-1)^n}{n^3}} + \frac{\pi\cos n\pi}{n} + \cancel{\frac{\pi^2\cos n\pi}{n}}\right]$$

$$= \frac{-1}{\cancel{\pi}}\left[\frac{2\cancel{\pi}(-1)^n}{n}\right] = \frac{2(-1)^{n+1}}{n}$$

Fourier series is,

$$x - x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos nx + \sum_{n=1}^{\infty} b_n\sin nx$$

$$x - x^2 = -\frac{\pi^2}{3} + \sum_{n=1}^{\infty}\frac{4(-1)^{n+1}}{n^2}\cos nx + \sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n}\sin nx$$

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Put $x = 0$, we get

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$$0 + 0 = \frac{-\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^{n+1}}{n^2} \times 1 + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \times 0$$

$$\frac{\pi^2}{3} = 4\left[1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots\right]$$

$$\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$$

Q2. Obtain the Fourier series of the function $f(x) = |x|$ in $(-\pi, \pi)$ and hence deduce that

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

$\Rightarrow$ Given $f(x) = |x|$

$$f(-x) = |-x| = |x| = f(x)$$

$\therefore$ $f(x)$ is even function and hence $b_n = 0$.

$$a_0 = \frac{2}{\pi} \int_0^{\pi} f(x)dx = \frac{2}{\pi} \int_0^{\pi} |x| \cdot dx$$

$$= \frac{2}{\pi} \int_0^{\pi} x \cdot dx = \frac{\cancel{2}}{\pi}\left[\frac{x^2}{\cancel{2}}\right] = \frac{1}{\cancel{\pi}} \times \pi^{\cancel{2}} = \pi$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \cdot dx = \frac{2}{\pi} \int_0^{\pi} |x| \cos nx \, dx$$

$$= \frac{2}{\pi} \int_0^{\pi} \underset{u}{x} \underset{v}{\cos nx} \cdot dx$$

$$= \frac{2}{\pi}\left[(x)\left(\frac{\sin nx}{n}\right) - 1\left(\frac{-\cos nx}{n^2}\right)\right]_0^{\pi} = \frac{2}{\pi}\left[\frac{\cos nx}{n^2}\right]_0^{\pi}$$

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$$a_n = \frac{2}{\pi n^2}\left[\cos n\pi - \cos 0\right]$$

$$a_n = \frac{2}{\pi n^2}\left[(-1)^n - 1\right]$$

Fourier Series is given by,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$$

$$|x| = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi n^2}\left[(-1)^n - 1\right]\cos nx$$

Put $x = 0$, $0 = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\left[\frac{(-1)^n - 1}{n^2} \times 1\right]$

$$-\frac{\pi}{2} = \frac{2}{\pi}\left[\frac{-2}{1^2} + 0 - \frac{2}{3^2} + 0 - \frac{2}{5^2} + \cdots\right]$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

Q3. Obtain the Fourier series of the function

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

$\Rightarrow$ Given $\phi_1(x) = -\pi$, $\phi_2(x) = x$

$$\phi_1(-x) = -\pi \neq \phi_2(x)$$

$\therefore$ $f(x)$ is neither even nor odd.

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi\, dx + \int_{0}^{\pi} x\cdot dx\right]$$

$$= \frac{1}{\pi}\left[-\pi(x)_{-\pi}^{0} + \left(\frac{x^2}{2}\right)_0^{\pi}\right]$$

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$$= \frac{1}{\pi}\left\{-\pi(0+\pi) + \left(\frac{\pi^2}{2} - 0\right)\right\} = \frac{1}{\pi}\left\{-\pi^2 + \frac{\pi^2}{2}\right\}$$

$$a_0 = \frac{1}{\pi} \times \frac{-\pi^2}{2} = -\frac{\pi}{2}.$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\, dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi \cos nx \cdot dx + \int_{0}^{\pi} \underset{u}{x}\, \underset{v}{\cos nx} \cdot dx\right]$$

$$= \frac{1}{\pi}\left[-\pi\left(\frac{\sin nx}{n}\right)_{-\pi}^{0} + x\left(\frac{\sin nx}{n}\right) - 1\left(\frac{-\cos nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{\cos nx}{n^2}\right]_0^{\pi} = \frac{1}{\pi n^2}\left[\cos n\pi - \cos 0\right]$$

$$a_n = \frac{1}{n^2\pi}\left[(-1)^n - 1\right].$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\, dx = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\pi \sin nx \cdot dx + \int_{0}^{\pi} \underset{u}{x}\, \underset{v}{\sin nx} \cdot dx\right]$$

$$= \frac{1}{\pi}\left[-\pi\left(\frac{-\cos nx}{n}\right)_{-\pi}^{0} + x\left(\frac{-\cos nx}{n}\right) - 1\left(\frac{-\sin nx}{n^2}\right)\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\left(\frac{\pi}{n}\cos nx\right)_{-\pi}^{0} - \left(\frac{x\cos nx}{n}\right)_0^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}(\cos 0 - \cos n\pi) - \left(\frac{\pi \cos n\pi}{n} - 0\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n}\left[1-(-1)^n\right] - \frac{\pi}{n}[-1]^n\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi}{n} - \frac{\pi}{n}(-1)^n - \frac{\pi}{n}(-1)^n\right] = \frac{1}{n}\left[1 - 2(-1)^n\right].$$

Fourier Series is

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$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \frac{-\pi}{4} + \sum_{n=1}^{\infty} \frac{1}{n^2\pi^1}\left((-1)^n - 1\right)\cos nx + \sum_{n=1}^{\infty} \frac{1}{n}\left[1 - 2(-1)^n\right]\sin nx$$

$$f(x) = \frac{-\pi}{4} + \frac{1}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^n - 1}{n^2}\cos nx + \sum_{n=1}^{\infty} \frac{1-2(-1)^n}{n}\sin nx$$

Put $x = 0$.

At $x = 0$, $f(x)$ is discontinuous, then

$$\frac{f(0^+) + f(0^-)}{2} = \frac{-\pi}{4} + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n^2}\cos 0 + 0$$

$$\frac{-\pi + 0}{2} = \frac{-\pi}{4} + \frac{1}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n^2}$$

$$-\frac{\pi}{2} + \frac{\pi}{4} = \frac{1}{\pi}\left[\frac{-2}{1^2} + \frac{-2}{3^2} + \frac{-2}{5^2} + \dots\right]$$

$$\frac{-\pi}{4} = \frac{-2}{\pi}\left[\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots\right]$$

$$\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$$

Q4. Obtain the Fourier series of the function $f(x) = \frac{\pi - x}{2}$ in $[0, 2\pi]$ and hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

$\Rightarrow$ Given $f(x) = \frac{\pi - x}{2} = \frac{1}{2}(\pi - x)$

$$f(2\pi - x) = \frac{1}{2}\left(\pi - (2\pi - x)\right) = \frac{1}{2}(\pi - 2\pi + x)$$

$$= \frac{1}{2}(-\pi + x) = \frac{-1}{2}(\pi - x) = -f(x)$$

$\therefore$ $f(x)$ is a odd function.

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$a_0 = 0$ and $a_n = 0$.

$$b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin nx \cdot dx = \frac{\cancel{2}}{\pi}\int_0^{\pi}\left(\frac{\pi - x}{\cancel{2}}\right)_u \sin x\,dx_v$$

$$= \frac{1}{\pi}\left[(\pi - x)\left[\frac{-\cos nx}{n}\right] - (-1)\left[\frac{-\sin nx}{n^2}\right]\right]_0^{\pi} \text{ [Using Bernouli's Rule]}$$

$$= \frac{-1}{n\pi}\left[(\pi - x)\cos nx\right]_0^{\pi} = \frac{-1}{n\pi}\left[0 - \pi\cos 0\right]$$

$$= \frac{\cancel{\pi}}{n\cancel{\pi}} \quad [\because \cos 0 = 1]$$

$$b_n = \frac{1}{n}$$

Fourier Series is,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$\frac{\pi - x}{2} = 0 + 0 + \sum_{n=1}^{\infty}\frac{1}{n}\sin nx$$

Put $x = \pi/2$, $\frac{\pi - \pi/2}{2} = \sum_{n=1}^{\infty}\frac{1}{n}\sin n\frac{\pi}{2}$

$$\frac{\pi - \pi/2}{2} = \frac{1}{1}\sin\frac{\pi}{2} + \frac{1}{2}\sin 2\frac{\pi}{2} + \frac{1}{3}\sin\frac{3\pi}{2} + \frac{1}{4}\sin\frac{4\pi}{2} + \frac{1}{5}\sin\frac{5\pi}{2} + \dots$$

$$\frac{\pi/2}{2} = 1 + 0 - \frac{1}{3} + 0 + \frac{1}{5} + \dots$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} + \dots$$

Q5. Find the Fourier series expansion of the function

$$f(x) = \begin{cases} \pi x, & 0 < x < 1 \\ \pi(2 - x), & 1 < x < 2 \end{cases}$$

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$\Rightarrow \quad \phi(x) = \pi x \;,\; \psi(x) = \pi(2-x)$, Given $l=1$.

$\psi(2l-x) = \phi(2-x) = \pi(2-(2-x)) = \pi x = \phi(x)$

$\therefore f(x)$ is even function.

$b_n = 0.$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx = \frac{2}{1}\int_0^1 \pi x\,dx = \cancel{2}\left[\frac{\pi x^2}{\cancel{2}}\right]_0^1 = \pi.$$

$$a_n = \frac{2}{1}\int_0^1 \pi x \cos n\pi x\,dx = 2\pi\left[\frac{x\sin n\pi x}{n\pi} - 1\left(\frac{-\cos n\pi x}{n^2\pi^2}\right)\right]_0^1$$

$$a_n = \frac{2\pi}{n^2\pi^2}\left[\cos n\pi - 1\right] = \frac{2}{n^2\pi}\left[(-1)^n - 1\right]$$

Fourier series is given by,

$$f(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi}\left((-1)^n - 1\right)\cos n\pi x$$

Put $x = 2$,

$$0 = \frac{\pi}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n^2} \times 1$$

$$\Rightarrow -\frac{\pi}{2} = \frac{2}{\pi}\left[\frac{-2}{1^2} - \frac{2}{3^2} - \frac{2}{5^2} \cdots\right]$$

$$\Rightarrow \frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \infty.$$

Q6. Obtain the Fourier series of the function

$$f(x) = \begin{cases} 2-x, & 0<x<4 \\ x-6, & 4<x<8 \end{cases}$$

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$

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$\Rightarrow \quad 2l = 8 \Rightarrow l = 4.$

$\phi_1(x) = 2 - x \qquad \phi_2(x) = x - 6$

$\phi_1(8-x) = 2 - 8 + x = x - 6 = \phi_2(x)$

$\therefore f(x)$ is an even function.

$b_n = 0$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx = \frac{1}{2}\int_0^4 (2-x)\cdot dx = \frac{1}{2}\left[2x - \frac{x^2}{2}\right]_0^4$$

$$= \frac{1}{2}\left[8 - 8 - 0 + 0\right] = \underline{\underline{0}}.$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{4}\,dx = \frac{1}{2}\int_0^4 (2-x)\cos\frac{n\pi x}{4}\,dx$$

$$= \frac{1}{2}\left[2\,\frac{\sin n\pi x/4}{n\pi x/4} + \left(\frac{\cos n\pi x/4}{n^2\pi^2/16}\right)\right]_0^4$$

$$= \frac{1}{2}\left[\frac{16(-1)^n}{n^2\pi^2} - \frac{16}{n^2\pi^2}\right]$$

$$= \frac{8}{n^2\pi^2}\underline{\underline{\left((-1)^n - 1\right)}}.$$

Fourier series is,

$$f(x) = \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n}\cos\frac{n\pi x}{2} + 0 + 0$$

at $x = 4$, $f(x) = 1$

$$1 = \frac{8}{\pi^2}\left[\frac{1}{1^2} + 0 + \frac{1}{3^2} + 0 + \frac{1}{5^2} + \dots\right]$$

$$\underline{\underline{\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots}}.$$

Q7. Obtain the half range Fourier Cosine series for the function $f(x) = \sin x$ in $[0, \pi]$.

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$\Rightarrow$ $f(x) = \sin x$, $l = \pi$

Half range cosine series is given by,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} \cancel{dx}$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} \sin x \cdot dx = \frac{2}{\pi} \left[-\cos x \right]_0^{\pi} = \frac{2}{\pi} [2]$$

$$a_0 = \frac{4}{\pi}$$

$$a_n = \frac{2}{\pi} \int_0^{\pi} \sin x \cos nx \, dx = \frac{1}{\pi} \int_0^{\pi} \sin (n+1)x \, dx - \sin (n-1)x \, dx$$

$$= \frac{1}{\pi} \left[\frac{\cos (n-1)x}{n-1} - \frac{\cos (n+1)x}{n+1} \right]_0^{\pi}$$

$$= \frac{1}{\pi} \left[\frac{(-1)^{n-1}}{n-1} - \frac{(-1)^{n+1}}{n+1} \right] = \frac{1}{\pi} \left[\frac{-(-1)^n}{n-1} + \frac{(-1)^n}{n+1} \right]$$

$$= \frac{1}{\pi} \left[-2(-1)^n \right]$$

Half range series $\Rightarrow$ (cosine series)

$$f(x) = \frac{1}{\pi} \sum_{n=1}^{\infty} (-2)(-1)^n \cos nx + \frac{4/\pi}{2}$$

$$= \frac{2}{\pi} \left[1 + \sum_{n=1}^{\infty} (-1)^{n+1} \cos nx \right]$$

Q8. Find the Fourier half range sine series of the function $f(x) = 2x - x^2$ in $[0, 3]$.

$\Rightarrow$ Half range sine series is given by,

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi}{l} x$$

$$b_n = \frac{2}{3} \int_0^{3} (2x - x^2) \sin \frac{n\pi}{3} x \cdot dx$$

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$$= \frac{2}{3}\left[2\left[-\frac{3x}{n\pi}\cos\frac{n\pi x}{3} + \frac{9}{n^2\pi^2}\cancel{\sin\frac{n\pi x}{3}}^{0}\right]_0^3 + \left[-\frac{3x^2}{n\pi}\cos\frac{n\pi x}{3} + \frac{18x}{n^2\pi^2}\cancel{\sin\frac{n\pi x}{3}}^{0}\right]_0^3\right]$$

$$= \frac{2}{3}\left[\frac{-18}{n\pi}(-1)^n - \frac{27}{n\pi}(-1)^n\right]$$

$$= -\frac{12}{n\pi}(-1)^n - \frac{18}{n\pi}(-1)^n = \frac{30(-1)^{n+1}}{n\pi}.$$

Half range sine series is ⇒

$$f(x) = \frac{30}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin\frac{n\pi x}{3}.$$

Q9. Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data.

x (deg)	0	45	90	135	180	225	270	315
y	2	1.5	1	0.5	0	0.5	1	1.5

⇒

x	y	$y\cos x$	$y\sin x$
0	2	2	0
45	1.5	1.06	1.06
90	1	0	1
135	0.5	−0.353	+0.353
180	0	0	0
225	0.5	−0.353	−0.353
270	1	0	−1
315	1.5	1.060	−1.060
	$\Sigma y = 8$	$\Sigma = 3.414$	$\Sigma = 0$

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$n = 8$

$a_0 = 2\frac{\Sigma y}{n} = 2 \times \frac{8}{8} = 2$

$a_1 = 2\frac{\Sigma y\cos x}{n} = \frac{2 \times 3.414}{8} = 0.8535$.

$b_1 = 2\frac{\Sigma y\sin x}{n} = \frac{2\times 0}{8} = 0$.

Fourier Series upto first harmonic is

$f(x) = \frac{a_0}{2} + a_1\cos x + b_1 \sin x = 1 + 0.8535\cos x + 0$

$= 1 + 0.8535\cos x$.

Q10. Express y as a Fourier series upto first harmonic given that

x (deg)	0	30	60	90	120	150	180	210	240	270	300	330
y	1.8	1.1	0.3	0.16	1.5	1.3	2.16	1.25	1.3	1.52	1.76	2.0

$\Rightarrow$

x	y	y cos x	y sin x
0	1.8	1.8	0
30	1.1	0.953	0.55
60	0.3	0.15	0.259
90	0.16	0	0.16
120	1.5	−0.75	1.299
150	1.3	−1.125	0.65
180	2.16	−2.16	0
210	1.25	−1.082	−0.625
240	1.3	−0.65	−1.126
270	1.52	0	−1.52
300	1.76	0.88	−1.524
330	2.0	1.732	−1
	$\Sigma y = 16.15$	$\Sigma = -0.252$	$\Sigma = -2.877$

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$n = 12$

$a_0 = 2\frac{\Sigma y}{n} = 2 \times \frac{16.15}{12} = 2.692$

$a_1 = 2\frac{\Sigma y \cos x}{n} = \frac{2 \times -0.252}{12} = -0.042$

$b_1 = 2\frac{\Sigma y \sin x}{n} = \frac{2 \times -2.877}{12} = -0.4795$

Fourier series upto first harmonic is

$f(x) = \frac{a_0}{2} + a_1 \cos x + b_1 \sin x = 1.346 - 0.042\cos x - 0.4795 \sin x$

Q11. The following table gives the variations of periodic current over a period

t (sec)	0	T/6	T/3	T/2	2T/3	5T/6	T
A (amp)	1.98	1.3	1.05	1.3	-0.88	-0.25	1.98

Show that there is a constant part of 0.75 A in the current A and obtain the amplitude of the first harmonic.

⇒ Omit last value [repeated]. $2l = T \Rightarrow l = T/2$

x	$\theta = \frac{\pi}{T/2} x$	$\cos\theta$	$\sin\theta$	$y\cos\theta$	$y\sin\theta$	y
0	0	1	0	1.98	0	1.98
T/6	60	0.5	0.866	0.65	1.1258	1.3
T/3	120	-0.5	0.866	-0.525	0.9093	1.05
T/2	180	-1	0	-1.3	0	1.3
2T/3	240	-0.5	-0.866	0.44	0.7621	-0.88
5T/6	300	0.5	-0.866	-0.125	0.2165	-0.25
~~T~~				$\Sigma y\cos\theta = 1.12$	$\Sigma y \sin\theta = 3.0137$	$\Sigma y = 4.5$

$n = 6$

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$$a_0 = \frac{2\Sigma y}{n} = \frac{2 \times 4.5}{6} = 1.5$$

$$a_1 = \frac{2\Sigma y\cos\theta}{n} = \frac{1.12}{3} = 0.3733$$

$$b_1 = \frac{2\Sigma y\sin\theta}{n} = 1.0046$$

Fourier Series upto first harmonic,

$$y = \frac{a_0}{2} + a_1\cos\theta + b_1\sin\theta$$

$$= \frac{1.5}{2} + 0.3733\cos\theta + 1.0046\sin\theta$$

Constant part $= a_0/2 = 1.5/2 = 0.75$.

Amplitude of first harmonic $= \sqrt{a_1^2 + b_1^2} = \sqrt{0.3733^2 + 1.0046^2}$

$= 1.0717$.

Q12. Obtain the constant term and the coefficients of the first sine and cosine terms in the Fourier expansion of y from the following data.

x	0	1	2	3	4	5
y	9	18	24	28	26	20

$\Rightarrow 2l = 6 \Rightarrow l = 3.$

x	$\theta = \frac{\pi x}{3}$	y	$y\cos\theta$	$y\sin\theta$
0	0	9	9	0
1	60	18	9	15.588
2	120	24	-12	20.784
3	180	28	-28	0
4	240	26	-13	-22.516
5	300	20	10	-17.32
		$\Sigma y = 125$	$\Sigma = -25$	$\Sigma = -3.464$

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$n = 6.$

$$a_0 = \frac{2\Sigma y}{n} = \frac{2\times 125}{6} = 41.67$$

$$a_1 = \frac{2\Sigma y\cos x}{n} = \frac{2\times -25}{6} = -8.33$$

$$b_1 = \frac{2\Sigma y\sin x}{n} = \frac{2\times -3.464}{6} = -1.155$$

Fourier series upto first harmonic is,

$$y = \frac{a_0}{2} + a_1\cos\theta + b_1\sin\theta + \ldots$$

$$y = 20.835 - 8.33\cos\theta - 1.155\sin\theta.$$

Q13. Using the Taylor's series method solve $\frac{dy}{dx} = x^2+y^2$, $y(0)=1$ at the point $x=0.2, 0.3$. consider upto 4^{th} degree term.

$\Rightarrow$ $x_0 = 0, y_0 = 1$

$y' = x^2 + y^2$ $\qquad$ $y'(0) = 1$

$y'' = 2x + 2y\cdot y'$ $\qquad$ $y''(0) = 0 + 2(1)(1) = 2$

$y''' = 2 + 2(yy'' + y'y')$ $\qquad$ $y'''(0) = 2 + 2(1)(2) + 2(1)^2$

$= 2 + 2yy'' + 2y'^2$ $\qquad$ $= 2+4+2 = 8$

$y^{iv} = 0 + 2(y'y'' + yy''')$ $\qquad$ $y^{iv}(0) = 6(1)(2) + 2(1)(8)$

$+ 2\cdot 2y'y''$ $\qquad$ $= 12 + 16$

$= 6y'y'' + 2yy'''$ $\qquad$ $= 28$

Taylor's series,

$$y(x) = y(x_0) + (x-x_0)y'(x_0) + \frac{(x-x_0)^2}{2!}y''(x_0) + \frac{(x-x_0)^3}{3!}y'''(x_0) + \frac{(x-x_0)^4}{4!}y^{iv}(x_0) + \ldots$$

$$y(x) = 1 + x(1) + \frac{x^2}{2}(2) + \frac{x^3}{6}(8) + \frac{x^4}{24}(28) + \ldots$$

$y(x) = 1 + x + x^2 + \frac{4}{3}x^3 + \frac{7}{6}x^4 + \ldots$

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Put $x = 0.2$ and $x = 0.3$,

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$$y(0.2) = 1 + 0.2 + (0.2)^2 + \frac{4}{3}(0.2)^3 + \frac{7}{6}(0.2)^4$$

$$= 1.2525.$$

$$y(0.3) = 1 + 0.3 + (0.3)^2 + \frac{4}{3}(0.3)^3 + \frac{7}{6}(0.3)^4$$

$$= 1.43545.$$

Q14. Using Runge-Kutta method solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$ by taking the step length as 0.2.

$\Rightarrow$ Here $x_0 = 0$, $y_0 = 1$, $x_1 = 0.2$, $h = 0.2$

$$f(x, y) = \frac{y^2 - x^2}{y^2 + x^2}$$

$$K_1 = hf(x_0, y_0) = 0.2 f(0, 1) = 0.2\left[\frac{1^2 - 0^2}{1^2 + 0^2}\right] = 0.2.$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$$

$$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right) = 0.2 f(0.1, 1.1)$$

$$= 0.2\left[\frac{1.1^2 - 0.1^2}{1.1^2 + 0.1^2}\right] = 0.1967.$$

$$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$$

$$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.1967}{2}\right) = 0.2 f(0.1, 0.0983)$$

$$= 0.2\left[\frac{0.0983^2 - 0.1^2}{0.0983^2 + 0.1^2}\right] = 0.1967$$

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$$K_4 = hf(x_0 + h, y_0 + K_3)$$
$$= 0.2 f(0 + 0.2, 1 + 0.1967)$$
$$= 0.2 f(0.2, 1.1967)$$
$$= 0.2 \left[\frac{1.1967^2 - 0.2^2}{1.1967^2 + 0.2^2} \right] = \underline{\underline{0.1891}}.$$

$$K = \frac{1}{6} [K_1 + 2K_2 + 2K_3 + K_4]$$
$$= \frac{1}{6} [0.2 + 2(0.1967) + 2(0.1964) + 0.1891]$$
$$= \underline{\underline{0.1959}}.$$

$$y_1 = y_0 + K = 1 + 0.1959 = \underline{\underline{1.1959}}.$$

Q15. Using Runge-Kutta method solve $\frac{dy}{dx} = y(x+y)$ with $y(0) = 1$ at $x = 0.2$ by taking the step length as 0.2.

$\Rightarrow$ $x_0 = 0$, $y_0 = 1$, $x_1 = 0.2$, $h = 0.2$

$$f(x, y) = y(x + y)$$

$$K_1 = hf(x_0, y_0) = 0.2 f(0, 1) = 0.2 \times 1(0+1) = \underline{\underline{0.2}}.$$

$$K_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$$
$$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right) = 0.2 f(0.1, 1.1)$$
$$= 0.2 \times 1.1 (1.2) = \underline{\underline{0.264}}.$$

$$K_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$$
$$= 0.2 f\left(0 + \frac{0.2}{2}, 1 + \frac{0.264}{2}\right) = 0.2 f(0.1, 1.132)$$
$$= 0.2 \times 1.132 (1.232) = \underline{\underline{0.278}}.$$

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$K_4 = hf(x_0+h, y_0+K_3)$
$= 0.2 f(0+0.2, 1+0.278)$
$= 0.2 f(0.2, 1.278)$
$= 0.2 \times 1.278 (1.478) = 0.377$.

$y(0.2) = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$
$= 1 + \frac{1}{6}[0.2 + 2(0.264) + 2(0.278) + 0.377]$
$= 1 + 0.2768$
$= 1.2768$.

Q16. Using the modified Eulers method solve the IVP $\frac{dy}{dx} = x+y^2, y(0)=1$ at $x=0.2$ by taking $h=0.1$. Carry out 2 iterations at each step.

$\Rightarrow$ Given $x_0 = 0, y_0 = 1, h = 0.1$
$f(x,y) = x+y^2$, $x_1 = x_0 + h = 0+0.1 = 0.1$
By Eulers method, $y_1^{(E)} = y_0 + hf(x_0, y_0)$
$y_1^{(E)} = 1 + 0.1 f(0,1)$
$y_1^{(E)} = 1 + 0.1(0+1^2) = 1+0.1 = 1.1$

By using modified Eulers formula,
$y_1^{(1)} = y_0 + \frac{h}{2}\{f(x_0,y_0) + f(x_1, y_1^{(E)})\}$

$y_1^{(1)} = y_0 + \frac{0.1}{2}[1 + (0.1 + (1.1)^2)]$
$= 1 + (0.05 \times 2.31) = 1.115$.

$y_1^{(2)} = y_0 + \frac{h}{2}\{f(x_0, y_0) + f(x_1, y_1^{(1)})\}$

$$y_1^{(2)} = 1 + \frac{0.1}{2}\left[1 + 0.1 + (1.115)^2\right]$$

$$= 1.117.$$

$\therefore \quad y(0.1) = 1.117.$

II Stage : Let $x_0 = 0.1$, $y_0 = 1.117$

$x_1 = x_0 + h = 0.2$

$f(x, y) = x + y^2$

$f(x_0, y_0) = 0.1 + (1.117)^2 = 1.348$

By Euler's formula, we have

$$y_1^{(0)} = y_0 + hf(x_0, y_0)$$

$$y_1^{(0)} = 1.117 + (0.1)(1.348) = 1.252$$

By modified Euler's formula,

$$y_1^{(1)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(0)})\right]$$

$$= 1.117 + \frac{0.1}{2}\left[1.348 + 0.2 + (1.252)^2\right]$$

$$= 1.273.$$

$$y_1^{(2)} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_1, y_1^{(1)})\right]$$

$$= 1.117 + \frac{0.1}{2}\left[1.348 + 0.2 + (1.273)^2\right]$$

$$= 1.275.$$

$\therefore \quad y(0.2) = 1.275.$

Q17. Using the modified Euler's method solve the IVP $\frac{dy}{dx} = x + y^2$, $y(0) = 1$ at $x = 1$ in steps of 0.5.

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$\Rightarrow$ Given $x_0 = 0, y_0 = 1, h = 0.5$

$f(x,y) = x + y^2$, $x_1 = x_0 + h = 0 + 0.5 = 0.5$

By Euler's method, $y_1^{(E)} = y_0 + hf(x_0, y_0)$

$$y_1^{(E)} = 1 + 0.5 f(0,1) = 1 + 0.5(0+1^2) = 1.5.$$

By using modified Euler's formula,

$$y_1^{(1)} = y_0 + \frac{h}{2}\left\{f(x_0,y_0) + f(x_1, y_1^{(E)})\right\}$$

$$y_1^{(1)} = 1 + \frac{0.5}{2}\left[(0+1^2) + (0.5 + (1.5)^2)\right]$$

$$= 1.9375.$$

$$y_1^{(2)} = 1 + \frac{0.5}{2}\left[1 + (0.5 + (1.9375)^2)\right]$$

$$= 2.3135.$$

$\therefore y(0.5) = 2.3135.$

II stage : $x_0 = 0.5, y_0 = 2.3135$

$x_1 = x_0 + h = 0.5 + 0.5 = 1.$

$f(x_0, y_0) = 0.5 + (2.3135)^2 = 5.8522.$

By Euler's method, $y_1^{(E)} = y_0 + hf(x_0, y_0) = 2.3135 + 0.5(5.8522)$
$= 5.2396.$

By using modified Euler's formula,

$$y_1^{(1)} = y_0 + \frac{h}{2}\left\{f(x_0,y_0) + f(x_1, y_1^{(E)})\right\}$$

$$= 2.3135 + \frac{0.5}{2}\left[5.8522 + 1 + (5.2396)^2\right]$$

$$= 10.8899.$$

$$y_1^{(2)} = 2.3135 + \frac{0.5}{2}\left[5.8522 + 1 + (10.8899)^2\right]$$

$$= 33.674.$$

$\therefore y(1) = 33.674.$

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Q18. Using Milne's predictor-corrector method, find y when $x = 0.4$ given that $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$, $y(0.3) = 2.090$. Apply the corrector formula twice.

⇒ Here $h = 0.1$, $f(x,y) = 2e^x - y$.

x	y	$f(x,y) = 2e^x - y$
$x_0 = 0$	$y_0 = 2$	$f_0 = f(0,2) = 2e^0 - 2 = 0$
$x_1 = 0.1$	$y_1 = 2.01$	$f_1 = f(0.1, 2.01) = 2e^{0.1} - 2.01 = 0.2003$
$x_2 = 0.2$	$y_2 = 2.04$	$f_2 = f(0.2, 2.04) = 2e^{0.2} - 2.04 = 0.4028$
$x_3 = 0.3$	$y_3 = 2.09$	$f_3 = f(0.3, 2.09) = 2e^{0.3} - 2.09 = 0.6097$
$x_4 = 0.4$	$y_4 = ?$	

Predict y_4 using milne's predictor formula,

$$y_4^{(P)} = y_0 + \frac{4h}{3}(2f_1 - f_2 + 2f_3)$$

$$= 2 + \frac{4(0.1)}{3}\left[2(0.2003) - 0.4028 + 2(0.6097)\right]$$

$$= 2.16229$$

We find $f_4^{(P)} = f(x_4, y_4^{(P)}) = f(0.4, 2.16229)$

$$= 2e^{0.4} - 2.16229 = 0.82135.$$

Now correct y_4 by using milne's corrector formula,

$$y_4^{(C)} = y_2 + \frac{h}{3}\left[f_2 + 4f_3 + f_4^{(P)}\right]$$

$$= 2.04 + \frac{0.1}{3}\left[0.4028 + 4(0.6097) + 0.82135\right]$$

$$= 2.16209.$$

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To correct y_4 further, take $y_4^{(P)} = 2.16209$

$f_4^{(P)} = f(x_4, y_4^{(P)}) = f(0.4, 2.16209)$

$= 2e^{0.4} - 2.16209 = 0.82155$.

$y_4^{(c)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4^{(P)}]$

$= 2.04 + \frac{0.1}{3}[0.4028 + 4[0.6097] + 0.82155]$

$= 2.16211$.

Q19. Using Milne's predictor-corrector method find y when $x = 1.4$ given that $\frac{dy}{dx} = x^2 + \frac{y}{2}$, $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$. Apply the corrector formula twice.

$\Rightarrow$ Here $h = 0.1$, $f(x, y) = x^2 + \frac{y}{2}$

x	y	$f(x,y) = x^2 + \frac{y}{2}$
$x_0 = 1$	$y_0 = 2$	$f_0 = f(1, 2) = 1^2 + \frac{2}{2} = 1+1 = 2$.
$x_1 = 1.1$	$y_1 = 2.2156$	$f_1 = f(1.1, 2.2156) = (1.1)^2 + \frac{2.2156}{2} = 2.3178$.
$x_2 = 1.2$	$y_2 = 2.4649$	$f_2 = f(1.2, 2.4649) = (1.2)^2 + \frac{2.4649}{2} = 2.6724$.
$x_3 = 1.3$	$y_3 = 2.7514$	$f_3 = f(1.3, 2.7514) = (1.3)^2 + \frac{(2.7514)}{2} = 3.0657$.

Predict y_4 using Milne's predictor formula,

$y_4^{(P)} = y_0 + \frac{4h}{3}[2f_1 - f_2 + 2f_3]$

$= 2 + \frac{4(0.1)}{3}[2(2.3178) - 2.6724 + 2(3.0657)]$

$= 3.07927$.

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Find $f_4^{(P)} = f(x_4, y_4^{(P)}) = f(1.4, 3.07927)$

$= (1.4)^2 + \frac{3.07927}{2} = 3.4996$.

Correct the value y_4 by Milne's Corrector formula,

$$y_4^{(c)} = y_2 + \frac{h}{3}\left[f_2 + 4f_3 + f_4^{(P)}\right]$$

$$= 2.4649 + \frac{0.1}{3}\left[2.67244 + 4(3.0657) + 3.4996\right]$$

$$= 3.0777.$$

To correct the value of y_4, take

$$y_4^{(P)} = 3.0777.$$

$$f_4^{(P)} = f(x_4, y_4^{(P)}) = f(1.4, 3.0777) = 3.4988$$

$$y_4^{(c)} = y_2 + \frac{h}{3}\left[f_2 + 4f_3 + f_4\right]$$

$$= 2.4649 + \frac{0.1}{3}\left[2.67245 + 4(3.0657) + 3.4988\right]$$

$$= 3.0794.$$

$\therefore y_4(1.4) = 3.0794$.

Q20. Using Milne's predictor-corrector method find y when $x = 1.4$ given that $\frac{dy}{dx} = x^2(1+y)$, $y(1) = 1$, $y(1.1) = 1.233$, $y(1.2) = 1.548$, $y(1.3) = 1.979$. Apply the corrector formula twice.

$\Rightarrow$ Here $h = 0.1$

$$f(x, y) = x^2 + x^2 y$$

x	y	$f(x,y) = x^2 + x^2 y$
$x_0 = 1$	$y_0 = 1$	$f_0 = f(1,1) = 1^2 + 1^2(1) = 2$.
$x_1 = 1.1$	$y_1 = 1.233$	$f_1 = f(1.1, 1.233) = (1.1)^2 + [(1.1)^2 \times 1.233] = 2.7019$
$x_2 = 1.2$	$y_2 = 1.548$	$f_2 = f(1.2, 1.548) = (1.2)^2 + [(1.2)^2 \times 1.548] = 3.6691$.
$x_3 = 1.3$	$y_3 = 1.979$	$f_3 = f(1.3, 1.979) = (1.3)^2 + [(1.3)^2 \times 1.979] = 5.0345$.

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Predict y_4 using Milne's predictor formula,

$$y_4^{(P)} = y_0 + \frac{4h}{3}[2f_1 - f_2 + 2f_3]$$

$$= 1 + \frac{4 \times 0.1}{3}[2 \times 2.7019 - 3.6691 + 2(5.0345)]$$

$$= 2.5738.$$

Find $f_4^{(P)} = f(x_4, y_4^{(P)}) = f(1.4, 2.5738) = 7.0046$.

correct the value y_4 by Milne's corrector formula,

$$y_4^{(C)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4^{(P)}]$$

$$= 1.548 + \frac{0.1}{3}[3.6691 + 4(5.0345) + 2.5738]$$

$$= 2.4274.$$

To correct y_4 further, take $y_4^{(P)} = 2.4274$

$$f_4^{(P)} = f(x_4, y_4^{(P)}) = f(1.4, 2.4274) = 6.7177.$$

$$y_4^{(C)} = y_2 + \frac{h}{3}[f_2 + 4f_3 + f_4^{(P)}]$$

$$= 1.548 + \frac{0.1}{3}[3.6691 + 4(5.0345) + 6.7177]$$

$$= 2.5655.$$

$\therefore\ y(1.4) = 2.5655$.

MCQ's

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1. b> Sine.

2. a> 0.

3. a> Cosine.

4. c> $-\pi/2$.

5. a> $\int_0^2 x^2 \cos\frac{n\pi x}{2}\,dx$.

6. c> Origin.

7. a> $\frac{a_n}{2}$.

8. c> not defined.

9. a> $f(-x) = -f(x)$.

10. b> $2l$.

11. a> $\frac{1}{2}\left[f(c-0) + f(c+0)\right]$.

12. c> π.

13. c> $\pi^2/8$.

14. b> First harmonic.

15. a> 0.

16. a> $f(x) = \frac{1}{(\sqrt{2\pi})^2}\int_{-\infty}^{\infty} F(s)\, e^{-isx} \cdot dx$.

17. a> $s^2/2$.

18. b> $\frac{1}{s^2+1}$.

19. c> $\pi/2$.

20. b> Fourier Transform.