ASSIGNMENT-II DMS

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Q1 Establish each of the following by PMI

(i) $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$

A: First check for $n=1$

$\sum_{i=1}^{1} \frac{1}{i(1+1)} = \frac{1}{1+1}$

$\Rightarrow \frac{1}{1(1+1)} = \frac{1}{2}$

$\Rightarrow \frac{1}{2} = \frac{1}{2}$

LHS = RHS

$\therefore$ P(1) is true

Let P(K) be true

$\Rightarrow \sum_{i=1}^{K} \frac{1}{i(i+1)} = \frac{K}{K+1}$

To prove $P(K+1) \Rightarrow \sum_{i=1}^{K+1} \frac{1}{i(i+1)} = \frac{K+1}{K+2}$

Consider LHS

$\sum_{i=1}^{K+1} \frac{1}{i(i+1)} = \sum_{i=1}^{K} \frac{1}{K(K+1)} + \frac{1}{(K+1)(K+2)}$

$= P(K) + \frac{1}{(K+1)(K+2)}$

$= \frac{K}{(K+1)} + \frac{1}{(K+1)(K+2)}$

$= \frac{K(K+2)+1}{(K+1)(K+2)}$

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$$\Rightarrow \frac{(K^2+2K+1)}{(K+1)(K+2)}$$

$$= \frac{(K+1)^2}{(K+1)(K+2)}$$

$$= \frac{(K+1)}{(K+2)}$$

$= RHS$

hence $P(K+1)$ is also true

$\therefore P(n)$ is true for all values of n

(ii) $P(n) = \sum_{i=1}^{n} 2^{i-1} = 2^n - 1$

let $n = 1$

$$P(1) \Rightarrow \sum_{i=1}^{1} 2^{i-1} = 2^1 - 1 = 2 - 1 = 1$$

$$\Rightarrow 2^{1-1} = 1$$

$$\Rightarrow 2^0 = 1$$

$$\Rightarrow 1 = 1$$

$LHS = RHS$

$\therefore P(1)$ is true

let $P(K)$ be true

$$\sum_{i=1}^{K} 2^{i-1} = 2^K - 1 \text{ is true}$$

To prove

$$P(K+1) \Rightarrow \sum_{i=1}^{K+1} 2^{i-1} = 2^{K+1} - 1$$

$$LHS = \sum_{i=1}^{K+1} 2^{i-1} = \sum_{i=1}^{K} 2^{i-1} + 2^K$$

$$= P(K) + 2^K$$

$$= 2^{K} - 1 + 2^K$$

$$= 2 \cdot 2^K - 1 = 2^{K+1} - 1 = RHS$$

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$\therefore P(K+1)$ is true

Hence $P(n)$ is true for all $n>0$

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(iii) $P(n) = \sum_{i=1}^{n} i(2^i) = 2+(n-1)2^{n+1}$

Ans for $n=1$

$P(1) \Rightarrow \sum_{i=1}^{1} i(2^i) = 2+(1-1)2^{1+1}$

$\Rightarrow 1(2^1) = 2+0(2)^2$

$\Rightarrow 2 = 2$

LHS = RHS

P(1) is true

let P(K) is true

$\Rightarrow \sum_{i=1}^{K} i(2^i) = 2+(K-1)2^{K+1}$

To prove P(K+1)

$\Rightarrow \sum_{i=1}^{K+1} i(2^i) = 2+K2^{K+2}$

Consider LHS

$$\sum_{i=1}^{K+1} i(2^i) = \sum_{i=1}^{K} i(2^i) + (K+1)(2^{K+1})$$

$$= 2+(K-1)2^{K+1} + (K+1)(2^{K+1})$$

$$= 2+K2^{K+1} - \cancel{2^{K+1}} + K2^{K+1} + \cancel{2^{K+1}}$$

$$= 2 + K2^{K+1} + K2^{K+1}$$

$$= 2+K[2 \cdot 2^{K+1}]$$

$$= 2+K2^{K+2}$$

$$= RHS$$

$\therefore P(K+1)$ is true

Hence $P(n)$ is true for all $n>0$

(iv) $\sum_{i=1}^{n} i(i!) = (n+1)! - 1$

... times + (5+5+...5 times) + 15

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let n = 1

$P(1) \Rightarrow \sum_{i=1}^{1} i(i!) = (1+1)! - 1$

$\Rightarrow 1(1!) = 2! - 1 = 2 - 1$

$\Rightarrow 1 = 1$

$\therefore P(1)$ is true

Assume $P(K)$ is true

$\Rightarrow \sum_{i=1}^{k} i(i!) = (k+1)! - 1$

To prove

$P(K+1) \Rightarrow \sum_{i=1}^{k+1} i(i!) = (k+2)! - 1$

Consider LHS

$\sum_{i=1}^{K+1} i(i!) = \sum_{i=1}^{k} i(i!) + (k+1)(k+1)!$

$= (k+1)! - 1 + (k+1)(k+1)!$

$= (k+1)!(k+1+1) - 1$

$= k+2(k+1)! - 1$

$= (k+2)! - 1$

$=$ RHS

$\therefore P(k+1)$ is true

hence $P(n)$ is true for all $n > 0$

Q2 Prove by PMI $n! \geq 2^{n-1}$ for all $n \geq 1$

Ans $P(n) = n! \geq 2^{n-1}$

$P(1) \Rightarrow 1! \geq 2^{1-1}$

$\Rightarrow 1 \geq 2^0$.

$\Rightarrow 1 \geq 1$

$\therefore P(1)$ is true

assume P(K) is true

$P(K) \Rightarrow K! \geq 2^{K-1}$

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To prove

$P(K+1) \Rightarrow (K+1)! \geq 2^{K}$

~~Case~~

$\cancel{(K+1)(K!) \geq 2 \cdot 2^{K}}$

$2^{K} \leq 2 \times 2^{K-1}$

$\Rightarrow 2^{K} \leq 2 \times K!$ $\quad (P(K))$

$2^{K} \leq (K+1) \times K!$ $\quad (\text{as } 2 \leq K+1)$

$2^{K} \leq (K+1)!$

$\Rightarrow (K+1)! \geq 2^{K}$

$= RHS$

hence proved

$\therefore P(n)$ is true for all $n \geq 1$

Q3 Prove that every positive integer $n \geq 24$ can be written as sum of 5's and/or 7's

Ans Let $P(n) = n$ can be written as sum of 5's and/or 7's

when $n \geq 24$

when $n = 24$

$P(24) = 24 = 2 \times 5 + 2 \times 7$ which is true

$\therefore P(24)$ is true

Assume $P(K)$ is true

$\therefore K =$ sum of 5's and/or 7's

To show $P(K+1)$ is true

$K+1 = (7+7+\cdots r \text{ times}) + (5+5+\cdots s \text{ times}) + 1$

$K+1 = (7+7+\cdots (r-2) \text{ times} + (5+5+\cdots s \text{ times}) + 7+7+1$

$K+1 = (7+7+\cdots (r-2) \text{ times} + (5+5+\cdots s \text{ times}) + 15$

$k+1 =$ [illegible]

$=$ Sum of 5's and 7's

$\therefore P(k+1)$ is true

$\therefore P(n)$ is true for all $n \geq 24$ by PMI

Q4 How many license plates can be made using

(i) Either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits

Ans $10 \times 10 \times 10 \times 26 \times 26 \times 26 + 26 \times 26 \times 26 \times 10 \times 10 \times 10$

$= \cancel{25,625,808} = 35152000$

(ii) Either two uppercase letters followed by 4 digits or 2 digits followed by 4 uppercase letters

Ans $10 \times 10 \times 26 \times 26 \times 26 \times 26 + 26 \times 26 \times 26 \times 26 \times 10 \times 10$

$= \cancel{74,030,112}$ 52457600

(iii) Either three uppercase letters followed by three digits or four uppercase followed by two digits

Ans $26 \times 26 \times 26 \times 10 \times 10 \times 10 + 26 \times 26 \times 26 \times 26 \times 10 \times 10$

$= \cancel{49,827,960}$ 63273600 ways

Q5 A student has three books on C++ and four books on Java. In how many ways can be arrange three books on a shelf (i) if there are no restrictions (ii) If the languages should alternate? (iii) If all the C++ books must be next to each other? (iv) If all the C++ books must be next to each other and all the java books must be next to each other?

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(i) ${}^7P_3 = \frac{7!}{4!} = 7\times6\times5 = 210$ ways

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(ii) Case 1

C++, Java, C++

$= 3\times4\times\cancel{3}2$
$= \cancel{36}24$

Case 2

Java, C++, Java

$= 4\times3\times3$
$= 36$

Total ways $= 24+36 = 60$

(iii) 3 books can be arranged in 3 ways

$\therefore$ 3! ways $= 6$ ways

(iv) Consider the three C++ books as 1 unit and 4 java books as 1 unit no of arranging 1 set of C++ is 3! and 1 set of Java = 2! and both sets can be placed in 2 ways

$\therefore$ required no. of ways $= 2\times4!\times3!$
$= 2\times24\times6$
$= 288$

06 Find all the no. of permutations of the letters of the word MASSASAUGA. In how many of these all 4 A's are together how many begin with S?

Ans (i) 10 alphabets is repeated 3 times and A4

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(i) ∴ total permutations $= \frac{10!}{3! \times 4!} = 25,200$

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(ii) Consider 4 A's together as one unit now we have 7 units in which S is repeated thrice

∴ total permutations $= \frac{7!}{3!} = 840$

(iii) First alphabet is fixed as S, Now 9 alphabets remaining, S repeated twice, A 4 times

∴ no. of permutations $= \frac{9!}{2! \times 4!} = 7560$

Q7 How many numbers greater than a million can be formed using digits 0, 3, 4, 4, 5, 5, 5

A) Given digits are 0, 3, 4, 4, 5, 5, 5

we have to form no.s > a million

This can be done by $\frac{7!}{2!3!} = 360$

∴ 360 no. are > million formed by these digits

Q8 Find the no. of arrangements of all the letters in TALLAHASEE. How many of these arrangements have no adjacent A's

A) no. of letters in the word is 11 of which 3 are A's, 2 each are L's, S's, E's and 1 each are T and H

∴ no of arrangements $= \frac{11!}{3!2!2!2!} = 8,31,600$

Now ignore 3 A's remaining 8 can be arranged in $\frac{8!}{3!\,4!\,1!}$ [illegible]

There are 9 possible positions of 4 A's chosen by

$$= \binom{9}{3} = 84 \text{ ways}$$

∴ Total no = 560×84

Q7 A woman has 11 close relatives and she wishes to invite 5 of them to dinner in how many ways can she invite if

(i) no restriction on choice

Ans As no restrict choice is selected in

$$^{11}C_5 = \frac{11!}{6!5!} = \frac{11\times\cancel{10}\times\cancel{9}\times\cancel{8}\times7}{\cancel{5}\times\cancel{4}\times\cancel{3}\times\cancel{2}} = 462 \text{ ways}$$

(ii) Two particular persons will not attend separately

Ans If both of them are invited then three more relatives are to be selected from the remaining 9 relatives

This can be done in

$$C(9,5) = \frac{9!}{5!4!} = 126 \text{ ways}$$

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$\therefore$ The total no. of ways in which the invitees can be selected in this case is

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$84 + 126 = 210$ ways

(iii) Two particular persons will not attend together

Ans Since two particular persons will not attend together $\therefore$ only one of them

no. of ways P_1 can be invited and P_2 not invited is

$$C(9,4) = \frac{9!}{5!\,4!} = 126 \text{ ways}$$

Similarly no. of ways choosing invitees with P_1 not invited and P_2 invited is 126

If both P_1 and P_2 are not invited no. of ways

$$C(9,5) = \frac{9!}{5!\,4!} = 126 \text{ ways}$$

$\therefore$ total no. of ways $= 126 + 126 + 126 = 378$

Q10 A certain question paper contains three parts A, B, C with 4 questions in part A, five questions in part B and six questions in part C. It is required to answer 7 questions selecting at least two questions from each part. In how many ways can a student select his seven questions for answering.

Ans

Part A	Part B	Part C	No. of ways	
2	2	3	$^4C_2 \times {^5C_2} \times {^6C_3}$	$= 1200$
2	3	2	$^4C_2 \times {^5C_3} \times {^6C_2}$	$= 900$
3	2	2	$^4C_3 \times {^5C_2} \times {^6C_2}$	$= 600$

$\therefore$ Total ways $= 1200 + 900 + 600 = 2700$ ways

Q11 In how many ways we can distribute 7 apples 6 oranges among 4 children so that each child gets at least 1 apple

Ans Distribute 1 apple to each child. Now 3 apples and 6 oranges to 4 children

- No of ways of distributing 3 apple to 4 children

$$= \binom{n+r-1}{r} = \binom{6}{3} = 20$$

- No of ways of distributing 6 oranges to 4 children

$$= \binom{n+r-1}{r} = \binom{9}{6} = 84$$

- By product rule required no $= 20 \times 84 = 1680$ ways

Q.12 Find the no of positive solutions of the equations $x_1 + x_2 + x_3 = 17$

Ans $x_1 + x_2 + x_3 = 0$

$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

$y_1 = x_1 - 0, \quad y_2 = x_2 - 0, \quad y_3 = x_3 - 0,$

$x_1 + x_2 + x_3 = 17$

$\Rightarrow y_1 + y_2 + y_3 = 17$

where $y_1, y_2, y_3 \geq 0$

$n = 3$

$r = 17$

$\therefore$ required no $= \binom{n+r-1}{r} = \binom{3+17-1}{17} = \binom{19}{17}$

$= \frac{19 \times 18}{2} = 19 \times 9 = 171$ positive solutions

Q13 Find the no of integer solutions of $x_1 + x_2 + x_3 + x_4 + x_5 = 30$
where $x_1 \geq 2, x_2 \geq 3, x_3 \geq 4, x_4 \geq 2, x_5 \geq 0$

Ans $y_1 = x_1 - 2$ $\quad y_2 = x_2 - 3$ $\quad y_3 = x_3 - 4$

$\Rightarrow x_1 = y_1 + 2$ $\quad \Rightarrow x_2 = y_2 + 3$ $\quad \Rightarrow x_3 = y_3 + 4$

$y_4 = x_4 - 2$ $\quad y_5 = x_5 - 0$

$\Rightarrow x_4 = y_4 + 2$ $\quad \Rightarrow x_5 = y_5$

$x_1 + x_2 + x_3 + x_4 + x_5 = 30$

$\Rightarrow y_1 + 2 + y_2 + 3 + y_3 + 4 + y_4 + 2 + y_5 = 30$

$\Rightarrow y_1 + y_2 + y_3 + y_4 + y_5 = 30 - 11$

$\Rightarrow y_1 + y_2 + y_3 + y_4 + y_5 = 19$

$\Rightarrow n = 5 \cdot r = 19$

$\therefore$ no of solutions $= \binom{5+19-1}{19} = \binom{23}{19}$

$$= \frac{23 \times \cancel{22}^{11} \times \cancel{21}^{7} \times \cancel{20}^{5}}{\cancel{4}_1 \times \cancel{3}_1 \times \cancel{2}_1}$$

$= 18,975$

Q14 Determine coefficient of

(i) xz^2 in expansion of $(2x - y - z)^4$

Ans here $n = 4$

$n_1 = 1$

$n_2 = 0$

$n_3 = 2$

coefficient $= \binom{4}{1\ 0\ 2} (2)^1 = \frac{4!}{1! \times 2!} \times 2 = 24$

(ii) $x^2y^2z^3$ in the expansion of $(3x-2y-4z)^7$

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Ans $n = 7$

$n_1 = 2$

$n_2 = 2$

$n_3 = 3$

∴ Coefficient is given by $\binom{7}{2\ 2\ 3}(3)^2(-2)^2(-4)^3$

$= \dfrac{7!}{2! \times 2! \times 3!} \times \overset{3}{\cancel{9}} \times \cancel{4} \times \overset{32}{\cancel{-64}}$

$= -483,840$

(iii) $x^{11}y^4$ in the expansion of $(2x^3-3xy^2+z^2)^6$

Ans $n = 6$

$n_1 = 3$

$n_2 = 2$

Coefficient is given by $\binom{6}{3\ 2} \times (2)^3 \times (-3)^2$

$= \dfrac{6 \times 5 \times \cancel{4} \times \cancel{3!}}{\cancel{2!}} \times 8 \times 9 = 4320$

(iv) $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$

Ans $n = 16$

$n_1 = 2$

$n_2 = 3$

$n_3 = 2$

$n_4 = 5$

∴ Coefficient is given by $\binom{16}{2\ 3\ 2\ 5}$

$$\frac{16!}{2!\times 3!\times 2!\times 5!} = \frac{16\times 15\times 14\times 13\times 12\times 11\times 10\times 9\times 8\times 7\times 6}{2\times 3\times 2\times 2}$$

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$= 16\times 15\times 7\times 13\times 4\times 11\times 5\times 9\times 4\times 7\times 6$

$= 7264857600$

Q15 Define cartesian product of two sets. For any three sets A, B, C. prove the following

(i) $A\times(B\cup C) = (A\times B)\cup(A\times C)$

Ans Let A and B be any two sets. Then the set of all ordered pairs (x, y) where $x\in A$ and $y\in B$ is called the Cartesian product of A and B and is denoted by $A\times B$

Let $(x,y)\in A\times(B\cup C)$

$\Leftrightarrow x\in A$ and $y\in(B\cup C)$

$\Leftrightarrow x\in A$ and $y\in(B\cup C)$

$\Leftrightarrow x\in A$ and $(y\in B$ or $y\in C)$

$(x\in A$ and $y\in B)$ or $(x\in A$ and $y\in C)$

$\Leftrightarrow (x,y)\in A\times B$ or $(x,y)\in A\times C$

$\Leftrightarrow (x,y)\in(A\times B)\cup(A\times C)$

$\therefore A\times(B\cup C) = (A\times B)\cup(A\times C)$

(ii) $A\times(B\cap C) = (A\times B)\cap(A\times C)$

Ans Let $(x,y)\in A\times(B\cap C)$

$\Leftrightarrow x\in A$ and $y\in(B\cap C)$

$\Leftrightarrow x\in A$ and $(y\in B$ and $y\in C)$

$\Leftrightarrow (x\in A$ and $y\in B)$ and $(x\in A$ and $y\in C)$

$\Leftrightarrow (x,y)\in A\times B$ and $(x,y)\in A\times C$

$\Leftrightarrow (x,y)\in(A\times B)\cap(A\times C)$

$\therefore A\times(B\cap C) = (A\times B)\cap(A\times C)$

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(iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

Ans Let $(x, y) \in (A \cup B) \times C$

$\Leftrightarrow x \in (A \cup B)$ and $y \in C$

$\Leftrightarrow (x \in A$ or $x \in B)$ and $y \in C$

$\Leftrightarrow (x \in A$ and $y \in C)$ or $(x \in B$ and $y \in C)$

$\Leftrightarrow (x, y) \in A \times C$ or $(x, y) \in B \times C$

$\Leftrightarrow (x, y) \in (A \times C) \cup (B \times C)$

$\therefore (A \cup B) \times C = (A \times C) \cup (B \times C)$

(iv) $(A \cap B) \times C = (A \times C) \cap (B \times C)$

Ans Let $(x, y) \in (A \cap B) \times C$

$\Leftrightarrow x \in (A \cap B)$ and $y \in C$

$\Leftrightarrow (x \in A$ and $x \in B)$ and $y \in C$

$\Leftrightarrow (x \in A$ and $y \in C)$ and $(x \in B$ and $y \in C)$

$\Leftrightarrow (x, y) \in A \times C$ and $(x, y) \in B \times C$

$\Leftrightarrow (x, y) \in (A \times C) \cap (B \times C)$

$\therefore (A \cap B) \times C = (A \times C) \cap (B \times C)$

(v) $A \times (B - C) = (A \times B) - (A \times C)$

Ans Let $(x, y) \in A \times (B - C)$

$\Leftrightarrow x \in A$ and $y \in (B - C)$

$\Leftrightarrow x \in A$ and $(y \in B$ and $y \notin C)$

$\Leftrightarrow (x \in A$ and $y \in B)$ and $(x \in A$ and $y \notin C)$

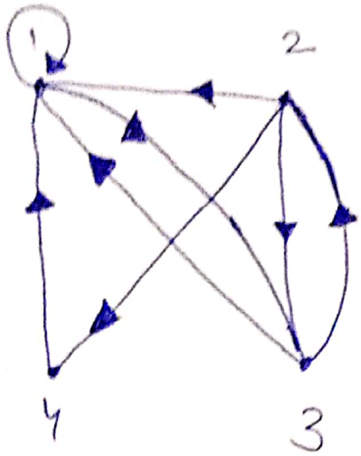
$\Leftrightarrow (x, y) \in A \times B$ and $(x, y) \notin A \times C$

$\Leftrightarrow (x, y) \in (A \times B) - (A \times C)$

$A \times (B - C) = (A \times B) - (A \times C)$

Q16 Draw the directed graph of the relation $R = \{(1,1), (1,3), (2,1), (2,3), (2,4), (3,1), (3,2), (4,1)\}$ on the set $\{1,2,3,4\}$ and write the matrix. Also find in degree and out degree of each vertex.

Ans The relational matrix is $M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$



Vertices	In-degree	out-degree
1	4	2
2	1	3
3	2	2
4	1	1

Q17 Define an equivalance class. Write the equivalence classes of the equivalence relation $R = \{(0,0), (1,1), (1,2), (2,1), (2,2), (3,3)\}$ on the set $A = \{0,1,2,3\}$

Ans Let R be an equivalence relation on a set A and $a \in A$. Then the set of all those elements x of A which are related to a by R is called the equivalence class of w.r.t 'R'. The equivalence class of a is denoted by $R(a)$ or $[a]$

i.e $R(a)$ or $[a] = \{x \in A \mid (x,a) \in R\}$

$R(0)$ or $[0] = \{(0,0)\} = \{0\}$

$R(1)$ or $[1] = \{(1,1), (2,1)\} = \{1,2\}$

$R(2)$ or $[2] = \{(1,2), (2,2\} = \{1,2\}$

$R(3)$ or $[3] = \{(2,3), (3,3\} = \{2,3\}$

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Q19 Define POSET; let $A = \{a, b, c\}$, $B = P(A)$ where $P(A)$ is the power set of A. Let R be is a subset relation on B. Show that (B, R) is a POSET and draw its Hasse diagram. Is it a lattice? IRNATSON ADITYA D edings

Ans Let A be a non-empty set. Suppose there exist non-empty subsets $A_1, A_2, \ldots, A_k$ of A such that

(i) $A = A_1 \cup A_2 \cup \cdots\cdots \cup A_k$ and

(ii) $A_i \cap A_j = \phi$ for $i \neq j$

Then the set $P = \{A_1, A_2, \ldots, A_k\}$ is called a partition of A

Q Power set of $A = \{a, b, c\}$ is given by

$P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

(a) We know that every set is a subset of itself

i.e $\forall x \in P(S) \Rightarrow x \subseteq x$

$\Rightarrow (x, x) \in R$

$\therefore$ R is reflexive

(b) Let xRy and yRx $\forall x, y \in B$

i.e $x \subseteq y$ and $y \subseteq x$

$\Rightarrow x = y$

$\therefore$ R is antisymmetric

(c) Let xRy and yRz

i.e $x \subseteq y$ and $y \subseteq z$

$\Rightarrow x \subseteq z$

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∴ R is transitive

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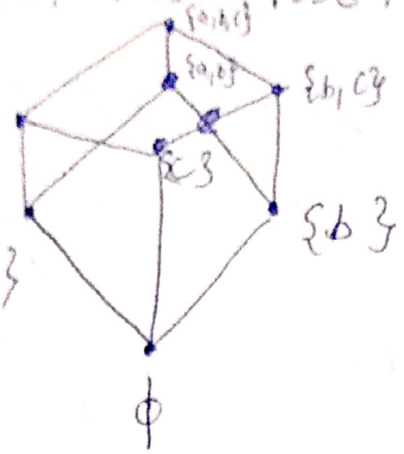
∴ R is partial order

∴ (B, R) is a POSET

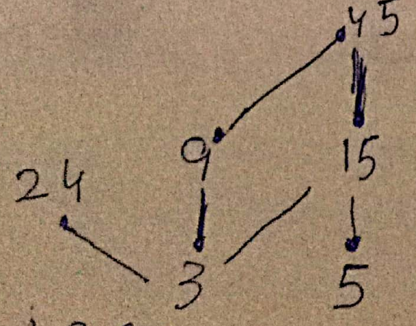
Hasse diagram



∴ It is not lattice as {a} and {b} have no least upper bound

Q19 Let A = {3, 5, 9, 15, 24, 45} and R be divisibility relation on A. Draw the Hasse diagram for the POSET (A, R) and determine least, greatest, minimal and maximal elements also find LUB and GLB of B = {9, 15}

Ans



Maximal elements = {24}, {45}

Minimal elements = {3}, {5}

There is no least element

There is no greatest element

Given B = {9, 15}

LB of 9 = {3}

LB 15 = {3, 5}

LB of B = {3} ∴ GLB = {3}

UB of 9 = {45}

UB of 15 = {45}

∴ UB of B = {45}

LUB = 45

Q20. Let R be an equivalence relation on set A and $a, b \in A$ then prove the following are equivalent

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(i) $a \in [a]$

As $\forall a \in A \Rightarrow (a,a) \in R$ [by reflexive]

$\Rightarrow a \in [a]$

(ii) aRb iff $[a] = [b]$

As let aRb

i.e. $(a,b) \in R$

Case 1:

let $x \in [A]$

$\Rightarrow (x,a) \in R$

now $(x,a),(a,b) \in R \Rightarrow (x,b) \in R$ [transitive]

$\Rightarrow x \in [b]$

$\therefore [a] \subseteq [b]$

Case 2

let $x \in [b]$

$\Rightarrow (x,b) \in R$

now $(x,b),(a,b) \in R$ [by symmetric]

$\Rightarrow (x,a) \in R$ [by transitive]

$\Rightarrow x \in [a]$

$\Rightarrow [b] \subseteq [a]$

$\therefore$ From both case $[a] = [b]$

Conversely let $[a] = [b]$ we have $a \in [a]$, $a \in [b]$ from (i)

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$(a, b) \in R$

aRb

(iii) If $[a] \cap [b] \neq \phi$ then $[a] = [b]$

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Given $[a] \cap [b] \neq \phi$

Let $x \in [a] \cap [b]$

$\Rightarrow x \in [a]$ and $x \in [b]$

$\Rightarrow (x, a) \in R$ and $(x, b) \in R$

$(a, x) \in R$ and $(x, b) \in R$ [by symmetric]

$\Rightarrow \{a, b\} = R$ [by Transitive]

$\Rightarrow aRb$

$\Rightarrow [a] = [b]$

QUIZ

Q1 (b) 3

Q2 (a) 720

Q4 (d) ${}^6C_3 \times {}^9C_6$

Q3 (c) 28

Q5 (d) Impossible

Q6 (a) $5! \times 4!$

Q7 (b) $\{(1,0), (1,1), (2,0), (2,1), (3,0), (3,1)\}$

Q8 (a) $\{(2,4), (3,4)\}$

Q9 (d) none of these

Q10 (c) Transitive

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