

1. Write a function called `euler_phi(n)` that calculates Euler's Totient Function, $\phi(n)$. This function counts the number of integers up to n that are coprime with n (i.e., numbers k for which $\text{gcd}(n, k) = 1$).

```
n = int(input('Enter a positive integer :'))
```

```
def gcd(a, n):
```

```
    if a == 0:
```

```
        return n
```

```
    return gcd(n % a, a)
```

```
def etf(n):
```

```
    b = 1
```

```
    for a in range(2, n):
```

```
        if gcd(a, n) == 1:
```

```
            b += 1
```

```
    return b
```

```
print(f'number of integers up to {n} that are coprime with {n} are {etf(n)}')
```

2. Write a function called `mobius(n)` that calculates the Möbius function, $\mu(n)$. The function is defined as: $\mu(n) = 1$ if n is a square-free positive integer with an even number of prime factors. $\mu(n) = -1$ if n is a square-free positive integer with an odd number of prime factors. $\mu(n) = 0$ if n has a squared prime factor

```
def mobius(n):
```

```
    if n <= 0:
```

```
        return 0
```

```
    if n == 1:
```

```
        return 1
```

```
    prime_factors_count = 0
```

```
    p = 2
```

```
    while p * p <= n:
```

```
        if n % p == 0:
```

```
            prime_factors_count += 1
```

```
            n //= p
```

```
        if n % p == 0:
```

```
            return 0
```

```
        p += 1
```

```
    if n > 1:
```

```

    prime_factors_count += 1
if prime_factors_count % 2 == 0:
    return 1
else:
    return -1

```

3. Write a function called `divisor_sum(n)` that calculates the sum of all positive divisors of `n` (including 1 and `n` itself). This is often denoted by $\sigma(n)$.

```

n = int(input('Enter a positive integer: '))
def divisor_sum(n):
    if n <= 0:
        return 0
    total_sum = 0
    for i in range(1, n + 1):
        if n % i == 0:
            total_sum += i
    return total_sum
print(f"The sum of the divisors of {n} is {divisor_sum(n)}")

```

4. Write a function called `prime_pi(n)` that approximates the prime-counting function, $\pi(n)$. This function returns the number of prime numbers less than or equal to `n`.

```

def is_prime(n):
    if n <= 1:
        return False
    for i in range(2, int(n**0.5) + 1):
        if n % i == 0:
            return False
    return True

def prime_pi(n):
    if n < 2:
        return 0
    count = 0
    for number in range(2, n + 1):
        if is_prime(number):
            count += 1
    return count

print(f"Number of prime numbers <= {n} are {prime_pi(n)}")

```

5. Write a function called `legendre_symbol(a, p)` that calculates the Legendre symbol (a/p) , which is a useful function in quadratic reciprocity. It is defined for an odd prime `p` and an integer `a` not divisible by `p` as: $(a/p) = 1$ if `a` is a quadratic residue modulo `p` (i.e., there exists an integer `x` such that $x^2 = a \pmod{p}$). $(a/p) = -1$ if `a` is a quadratic non-residue modulo `p`.

```

def main(a, p):

```

```

m = 0

y = 0

x = 0

if a % p != 0:

    if p % 2 != 0:

        for i in range(1, p + 1):

            if p % i == 0:

                m += 1

        if m == 2:

            y = a % p

            for i in range(1, y + 1):

                if i * i == y:

                    x = 1

                    break

            else:

                x = -1

        if x == 1:

            print("a/p=1")

        else:

            print("a/p= -1")

```

6. Write a function factorial(n) that calculates the factorial of a non-negative integer n (n!).

```

x=int(input( 'Enter a non-negative ihTEGER' )

def factorial(x):

    f = 1

    if x < 0:

        print('wrong input')

    else:

        for i in range(1, x + 1):

            f *= i

```

```
print( 'FACTORIAL:' , f)
factorial(x)
```

7. Write a function `is_palindrome(n)` that checks if a number reads the same forwards and backwards.

```
n = int(input('Enter an integer: '))
```

```
def palindrome(n):
```

```
    k = n
```

```
    l = 0
```

```
    while k != 0:
```

```
        j = k % 10
```

```
        l = l * 10 + j
```

```
        k = k // 10
```

```
    if l == n:
```

```
        print("palindrome")
```

```
    else:
```

```
        print("not palindrome")
```

```
palindrome(n)
```

8. Write a function `mean_of_digits(n)` that returns the average of all digits in a number.

```
n= int(input("enter an integer "))
```

```
def mean_of_digits(n):
```

```
    c = 0
```

```
    m = 0
```

```
    k = n
```

```

while k != 0:
    j = k % 10
    m += j
    c += 1
    k //= 10 # integer division

```

```

avg = m / c
print(avg)

```

mean_of_digits(n)

9. Write a function digital_root(n) that repeatedly sums the digits of a number until a single digit is obtained.

```

n=int(input("enter an integer "))
def digital_root(n):
    while n >= 10: # keep going until n is a single digit
        s = 0
        while n > 0:
            s += n % 10
            n //= 10
        n = s
    return n

```

digital_root(n)

10. Write a function is_abundant(n) that returns True if the sum of proper divisors of n is greater than n.

```

n=int(input("enter an integer "))
def is_abundant(n):
    k = n

```

```
s = 0

for i in range(1, k):

    if k % i == 0:

        s += i
```

```
if s > n:

    print("true")

else:

    print("false")
```

```
is_abundant(n)
```

11. Write a function `is_deficient(n)` that returns True if the sum of proper divisors of `n` is less than `n`.

```
n = int(input("Enter a positive integer: "))

def is_deficient(n):

    s = 1

    i = 2

    while i * i <= n:

        if n % i == 0:

            s += i

            if i * i != n:

                s += n // i

        i += 1

    return s < n

result = is_deficient(n)

print(result)
```

12. Write a function for harshad number is_harshad(n) that checks if a number is divisible by the sum of its digits.

```
def is_harshad(n):

    n = int(n)
    original_n = n
    sum_of_digits = 0
    while n > 0:
        digit = n % 10
        sum_of_digits += digit
        n //= 10

    if (original_n % sum_of_digits == 0):
        print( original_n , "is divisible by the sum of its digits")
    else:
        print( original_n , "is not divisible by the sum of its digits")

num = int(input("Enter a number: "))

is_harshad(num)
```

13. Write a function is_automorphic(n) that checks if a number's square ends with the number itself.

```
def is_automorphic(n):

    sq= n**2
    n2=n
    a=0
    while (n!=0):
        a=a+1
        n=n/10
```

```

last=sq%a
if( last==n2):
    print( n2, "is an automorphic number")
else:
    print( n2, "is not an automorphic number")

num=int(input("enter a number "))

is_automorphic(num)

```

14. Write a function `is_pronic(n)` that checks if a number is the product of two consecutive integers.

```

def is_pronic(n):
    a=0
    for i in range
        (1,n): if(
            i*(i+1)==n):
                a=1
                break
    else:
        a=0

    return a

num= int(input("enter a number "))
if (is_pronic(num)==1):
    print( num , "is a pronic number")
elif (is_pronic(num)==0):
    print( num , "is not a pronic number")

```


15. Write a function `prime_factors(n)` that returns the list of prime factors of a number.

```
n = int(input("Enter a positive integer: "))

def prime_factors(n):
    factors = []

    while n % 2 == 0:
        factors += [2]
        n //= 2

    i = 3
    while i * i <= n:
        while n % i == 0:
            factors += [i]
            n //= i
        i += 2

    if n > 2:
        factors += [n]

    return factors

if n <= 1:
    print([])
else:
    print(prime_factors(n))
```

16. Write a function `count_distinct_prime_factors(n)` that returns how many unique prime factors a number has.

```
def count_distinct_prime_factorial(n):  
    if n < 2:  
        return 0 # 0 and 1 have no prime factors  
  
    count = 0  
    i = 2  
    while i * i <= n:  
        if n % i == 0:  
            count += 1  
            while n % i == 0:  
                n //= i  
            i += 1  
    if n > 1:  
        count += 1 # n itself is a prime number  
  
    return count  
  
print(count_distinct_prime_factorial(60))
```

17. Write a function `is_prime_power(n)` that checks if a number can be expressed as p^k where p is prime and $k \geq 1$.

```
import time  
  
import os  
  
import psutil  
  
def memory_usage():  
    process = psutil.Process(os.getpid())  
    return process.memory_info().rss  
  
def is_prime_power(n):
```

```

if n < 2:
    return False

for p in range(2, int(n**0.5)+1):
    # check if p is prime
    is_prime = True
    for i in range(2, int(p**0.5)+1):
        if p % i == 0:
            is_prime = False
            break
    if not is_prime:
        continue
    # try different k values
    k = 1
    value = p
    while value < n:
        value *= p
        k += 1
    if value == n:
        return True
    return False

start_time = time.time()

n = int(input("Enter a number to check if it's a prime power: "))

if is_prime_power(n):
    print(f"{n} is a prime power.")
else:
    print(f"{n} is not a prime power.")

print(f"memory usage :{memory_usage()} bytes")

end_time = time.time()

print(f"\nExecution Time: {end_time - start_time:.6f} seconds")

```

18. Write a function `is_mersenne_prime(p)` that checks if $2^p - 1$ is a prime number (given that p is prime).

```
def memory_usage():  
    process = psutil.Process(os.getpid())  
    return process.memory_info().rss
```

```
def is_mersenne_prime(p):
```

```
    a=2 ** p - 1
```

```
    b=0
```

```
    c=0
```

```
    if p < 2:
```

```
        return 0
```

```
    for i in range(2,p):
```

```
        if p%i==0:
```

```
            b=1
```

```
            break
```

```
    else:
```

```
        b=0
```

```
    for j in range(2,a):
```

```
        if a%i==0:
```

```
            c=1
```

```
            break
```

```
        else:
```

```
            c=0
```

```
    if b==0 and c==0:
```

```
        return True
```

```
    else:
```

```
        return False
```

```

start_time = time.time()

p = int(input("Enter a prime number to check if it's a Mersenne prime: "))

if is_mersenne_prime(p):

    print(f"2^{p} - 1 is a Mersenne prime.")

else:

    print(f"2^{p} - 1 is not a Mersenne prime.")


print(f"memory usage :{memory_usage()} bytes")

end_time = time.time()

print(f"\nExecution Time: {end_time - start_time:.6f} seconds")

```

19. Write a function `twin_primes(limit)` that generates all twin prime pairs up to a given limit.

```

import time

import sys

start_time = time.time()

def twin_prime(limit):

    def is_prime(n):

        if n < 2:

            return False

        for i in range(2, int(n ** 0.5) + 1):

            if n % i == 0:

                return False

        return True

    twin_primes = []

    for num in range(2, limit - 1):

        if is_prime(num) and is_prime(num + 2):

            twin_primes.append((num, num + 2))

    return twin_primes

```

```

limit = 130

end_time = time.time()

time_taken = end_time-start_time

print(f"The twin pairs are:{twin_prime(130)}")

print("time taken to run this code:",time_taken,"seconds")

print("memory usage:",sys.getsizeof(twin_prime(limit)),"KB")

# Output: [(3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73)]

```

20. Write a function Number of Divisors

(d(n)) count_divisors(n) that returns how many positive divisors a number has.

```

import time

import os

import psutil


def memory_usage():

    process = psutil.Process(os.getpid())

    return process.memory_info().rss


def count_divisors(n):

    count = 0

    for i in range(1, int(n**0.5)+1):

        if n % i == 0:

            if i * i == n:

                count += 1

            else:

                count += 2

    return count


start_time = time.time()

n = int(input("Enter a number to count its divisors: "))

```

```
print(f"Number of divisors of {n}: {count_divisors(n)}")

print(f"memory usage :{memory_usage()} bytes")

end_time = time.time()

print(f"\nExecution Time: {end_time - start_time:.6f} seconds")
```

21. Write a function aliquot_sum(n) that returns the sum of all proper divisors of n (divisors less than n).

```
def aliquot_sum(n):
    s = 0
    for i in range(1, n):
        if (n % i == 0):
            s = s + i
    return s

n = int(input("Enter a number: "))
print(f"The aliquot sum of {n} is: {aliquot_sum(n)}")
```

22. Write a function are_amicable(a, b) that checks if two numbers are amicable (sum of proper divisors of a equals b and vice versa).

```
def are_amicable(a,b):
    s1 = 0
    for i in range(1, a):
        if (a % i == 0):
            s1 += i

    s2 = 0
    for j in range(1, b):
        if (b % j == 0):
            s2 += j

    if (s1 == b and s2 == a):
        print("numbers are amicable")
    else:
        print("numbers are not amicable")

a = int(input("Enter a number: "))
b = int(input("Enter another number: "))
are_amicable(a, b)
```

23. Write a function `multiplicative_persistence(n)` that counts how many steps until a number's digits multiply to a single digit.

```
def multiplicative_persistence(n):
    c = 0
    while n >= 10:
        p = 1
        temp_n = n
        while temp_n > 0:
            digit = temp_n % 10
            p *= digit
            temp_n //= 10
        n = p
        c += 1
    return c
```

```
n = int(input("Enter a number: "))
print(multiplicative_persistence(n))
```

24. Write a function `is_highly_composite(n)` that checks if a number has more divisors than any smaller number.

```
def count_divisors(num):
    if num <= 0:
        return 0
    if num == 1:
        return 1

    count = 0
    i = 1
    while i * i <= num:
        if num % i == 0:
            if i * i == num:
                count += 1
            else:
                count += 2
        i += 1
    return count
```

```
def is_highly_composite(n):
    if n <= 0:
        return False
    if n == 1:
```



```

        return True

divisors_of_n = count_divisors(n)

for i in range(1, n):
    divisors_of_i = count_divisors(i)
    if divisors_of_i >= divisors_of_n:
        return False

return True

number = int(input("Enter a positive integer: "))

if is_highly_composite(number):
    print(f"\n{n{number}} IS highly composite.")
else:
    print(f"\n{n{number}} is NOT highly composite.")

if number > 0:
    print(f"Divisors: {count_divisors(number)}")

```

25. Write a function for ModularExponentiation `mod_exp(base, exponent, modulus)` that efficiently calculates $(base^{exponent}) \% modulus$. prime factors of a number.

```

def mod_exp(base, exponent, modulus):

    if modulus == 1:
        return 0

    result = 1
    base %= modulus

    while exponent > 0:
        if exponent % 2 == 1:
            result = (result * base) % modulus

        base = (base * base) % modulus
        exponent //= 2

    return result

b = int(input("Enter the base: "))
e = int(input("Enter the exponent: "))
m = int(input("Enter the modulus: "))

```

```

if e < 0:
    print("This function does not handle negative exponents.")
else:
    final_result = mod_exp(b, e, m)
    print(f"\nResult: ({b}^{e}) mod {m} = {final_result}")

```

26. Write a function Modular Multiplicative

Inverse mod_inverse(a, m) that finds the number x such

that $(a * x) \equiv 1 \pmod{m}$.

```
import time
```

```
import sys
```

```
import resource
```

```
def mod_inverse(a, m):
```

```
    def extended_gcd(a, b):
```

```
        if a == 0:
```

```
            return b, 0, 1
```

```
        gcd, x1, y1 = extended_gcd(b % a, a)
```

```
        x = y1 - (b // a) * x1
```

```
        y = x1
```

```
        return gcd, x, y
```

```
    gcd, x, _ = extended_gcd(a % m, m)
```

```
    if gcd != 1:
```

```
        return None
```

```
    return (x % m + m) % m
```

```
# Test
```

```
start_time = time.time()
```

```
result = mod_inverse(3, 26)
```

```
end_time = time.time()
```

```
runtime = end_time - start_time
```

```
mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024 # bytes

print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")
```

27. Write a function chinese Remainder Theorem

Solver crt(remainders, moduli) that solves a system of congruences $x \equiv r_i \pmod{m_i}$.

```
import time

import sys

import resource

from math import gcd


def crt(remainders, moduli):

    if len(remainders) != len(moduli):

        return None

    prod = 1

    for m in moduli:

        prod *= m

    result = 0

    for rem, mod in zip(remainders, moduli):

        p = prod // mod

        result += rem * extended_inverse(p, mod) * p

    return result % prod


def extended_inverse(a, m):

    m0, x0, x1 = m, 1, 0

    if m == 1:

        return 0

    while a > 1:

        q = a // m

        m, a = a % m, m
```

```

    x0, x1 = x1 - q * x0, x0

    return x1 + m0 if x1 < 0 else x1

# Test

start_time = time.time()

result = crt([2, 3], [3, 5])

end_time = time.time()

runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")

```

28. Write a function Quadratic Residue

Check `is_quadratic_residue(a, p)` that checks if $x^2 \equiv a \pmod p$ has a solution.

```

import time

import sys

import resource

def is_quadratic_residue(a, p):

    if p == 2:

        return True if a % 2 == 0 or a % 2 == 1 else False

    if a % p == 0:

        return True

    return pow(a, (p - 1) // 2, p) == 1

```

```

# Test

start_time = time.time()

result = is_quadratic_residue(2, 7)

end_time = time.time()

runtime = end_time - start_time

```

```
mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024  
print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")
```

29. Write a function `order_mod(a, n)` that finds the smallest

positive integer k such that $ak \equiv 1 \pmod{n}$.

```
import time
```

```
import sys
```

```
import resource
```

```
def order_mod(a, n):
```

```
    if gcd(a, n) != 1:
```

```
        return None
```

```
    k = 1
```

```
    pow_a = a % n
```

```
    while pow_a != 1:
```

```
        pow_a = (pow_a * a) % n
```

```
        k += 1
```

```
        if k > n: # Cycle detection
```

```
            return None
```

```
    return k
```

```
from math import gcd
```

```
start_time = time.time()
```

```
result = order_mod(2, 7)
```

```
end_time = time.time()
```

```
runtime = end_time - start_time
```

```
mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024
```

```
print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")
```

30. Write a function Fibonacci Prime

Check `is_fibonacci_prime(n)` that checks if a number is both Fibonacci and prime.

```
import time
```

```
import sys
```

```
import resource
```

```
def is_fibonacci(n):
```

```
    if n < 0:
```

```
        return False
```

```
    a, b = 0, 1
```

```
    while b < n:
```

```
        a, b = b, a + b
```

```
    return b == n
```

```
def is_prime(n):
```

```
    if n <= 1:
```

```
        return False
```

```
    if n <= 3:
```

```
        return True
```

```
    if n % 2 == 0 or n % 3 == 0:
```

```
        return False
```

```
    i = 5
```

```
    while i * i <= n:
```

```
        if n % i == 0 or n % (i + 2) == 0:
```

```
            return False
```

```
        i += 6
```

```
    return True
```

```
def is_fibonacci_prime(n):
```

```
return is_fibonacci(n) and is_prime(n)
```

```
start_time = time.time()
```

```
result = is_fibonacci_prime(13)
```

```
end_time = time.time()
```

```
runtime = end_time - start_time
```

```
mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024
```

```
print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")
```

31. Write a function Lucas Numbers

Generator `lucas_sequence(n)` that generates the first `n`

Lucas numbers (similar to Fibonacci but starts with 2,

1).

```
import time
```

```
import sys
```

```
import resource
```

```
def lucas_sequence(n):
```

```
    if n == 0:
```

```
        return [2]
```

```
    if n == 1:
```

```
        return [2, 1]
```

```
    seq = [2, 1]
```

```
    for i in range(2, n + 1):
```

```
        seq.append(seq[-1] + seq[-2])
```

```
    return seq[:n]
```

```
start_time = time.time()
```

```

result = lucas_sequence(10)

end_time = time.time()

runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")

```

32. Write a function for Perfect Powers

Check `is_perfect_power(n)` that checks if a number can be expressed as a^b where $a > 0$ and $b > 1$.

```

import time

import sys

import resource

from math import log, sqrt, ceil

def is_perfect_power(n):

    if n < 1:

        return False

    for b in range(2, int(log(n, 2)) + 1):

        a = round(n ** (1 / b))

        if a ** b == n:

            return True

    return False

```

```

start_time = time.time()

result = is_perfect_power(16)

end_time = time.time()

runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")

```


33. Write a function Collatz Sequence

Length collatz_length(n) that returns the number of steps

for n to reach 1 in the Collatz conjecture.

```
import time
```

```
import sys
```

```
import resource
```

```
def collatz_length(n):
```

```
    if n <= 0:
```

```
        return 0
```

```
    length = 1
```

```
    while n != 1:
```

```
        if n % 2 == 0:
```

```
            n //= 2
```

```
        else:
```

```
            n = 3 * n + 1
```

```
        length += 1
```

```
    return length
```

```
start_time = time.time()
```

```
result = collatz_length(27)
```

```
end_time = time.time()
```

```
runtime = end_time - start_time
```

```
mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024
```

```
print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")
```

34. Write a function Polygonal Numbers polygonal_number(s,

n) that returns the n-th s-gonal number.

```

import time

import sys

import resource

def polygonal_number(s, n):

    if s < 3 or n < 1:

        return None

    return n * (n - 1) * (s - 2) // 2 + n


start_time = time.time()

result = polygonal_number(5, 3) # Pentagon

end_time = time.time()

runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")

```

35. Write a function Carmichael Number

Check `is_carmichael(n)` that checks if a composite number

`n` satisfies $a^{n-1} \equiv 1 \pmod n$ for all `a` coprime to `n`.

```

import time

import sys

import resource

from math import gcd

def is_carmichael(n):

    if n <= 1 or is_prime(n):

        return False

    for a in range(2, n):

        if gcd(a, n) == 1 and pow(a, n - 1, n) != 1:

```

```

        return False

    return True

def is_prime(n):

    if n <= 1:

        return False

    if n <= 3:

        return True

    if n % 2 == 0 or n % 3 == 0:

        return False

    i = 5

    while i * i <= n:

        if n % i == 0 or n % (i + 2) == 0:

            return False

        i += 6

    return True


start_time = time.time()

result = is_carmichael(561)

end_time = time.time()

runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")

```

36. Implement the probabilistic Miller-Rabin

test `is_prime_miller_rabin(n, k)` with `k` rounds.

```

import time

import sys

import resource

```

```
from random import randrange
```

```
def is_prime_miller_rabin(n, k=40):
```

```
    if n <= 1:
```

```
        return False
```

```
    if n <= 3:
```

```
        return True
```

```
    if n % 2 == 0:
```

```
        return False
```

```
    r, s = 0, n - 1
```

```
    while s % 2 == 0:
```

```
        r += 1
```

```
        s //= 2
```

```
    for _ in range(k):
```

```
        a = randrange(2, n - 1)
```

```
        x = pow(a, s, n)
```

```
        if x == 1 or x == n - 1:
```

```
            continue
```

```
        for _ in range(r - 1):
```

```
            x = pow(x, 2, n)
```

```
            if x == n - 1:
```

```
                break
```

```
        else:
```

```
            return False
```

```
    return True
```

```
start_time = time.time()
```

```
result = is_prime_miller_rabin(13)
```

```
end_time = time.time()
```

```
runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")
```

37. Implement pollard_rho(n) for integer factorization using

Pollard's rho algorithm.

```
import time

import sys

import resource

from math import gcd

from random import randint
```

```
def pollard_rho(n):

    if n % 2 == 0:

        return 2

    x = randint(1, n - 1)

    y = x

    c = randint(1, n - 1)

    d = 1

    while d == 1:

        x = (x * x + c) % n

        y = (y * y + c) % n

        y = (y * y + c) % n

        d = gcd(abs(x - y), n)

    if d == n:

        return None

    return d
```

```
start_time = time.time()
```

```

result = pollard_rho(315)

end_time = time.time()

runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")

```

38. Write a function `zeta_approx(s, terms)` that approximates the Riemann zeta function $\zeta(s)$ using the first 'terms' of the series.

```

import time

import sys

import resource

```

```

def zeta_approx(s, terms=1000):

    if s <= 1:

        return float('inf')

    total = 0.0

    for k in range(1, terms + 1):

        total += 1 / (k ** s)

    return total

```

```

start_time = time.time()

result = zeta_approx(2, 1000)

end_time = time.time()

runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result:.6f}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")

```

39. Write a function Partition Function

$p(n)$ partition_function(n) that calculates the number of distinct ways to write n as a sum of positive integers.

```
import time
import sys
import resource

def partition_function(n):
    if n < 0:
        return 0
    p = [0] * (n + 1)
    p[0] = 1
    for i in range(1, n + 1):
        j = 1
        while True:
            pent = j * (3 * j - 1) // 2
            if pent > i:
                break
            sign = 1 if (j % 2 == 1) else -1
            p[i] += sign * p[i - pent]
            pent_neg = (-j) * (3 * (-j) - 1) // 2
            if pent_neg <= i:
                sign = 1 if (j % 2 == 0) else -1
                p[i] += sign * p[i - pent_neg]
            j += 1
    return p[n]

start_time = time.time()
result = partition_function(10)
end_time = time.time()
runtime = end_time - start_time
mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss *
1024
print(f'Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes')
```


