

1. Write a function called `euler_phi(n)` that calculates Euler's Totient Function, $\phi(n)$. This function counts the number of integers up to n that are coprime with n (i.e., numbers k for which $\text{gcd}(n, k) = 1$).

```
n = int(input('Enter a positive integer :'))  
  
def gcd(a, n):  
  
    if a == 0:  
  
        return n  
  
    return gcd(n % a, a)  
  
def etf(n):  
  
    b = 1  
  
    for a in range(2, n):  
  
        if gcd(a, n) == 1:  
  
            b += 1  
  
    return b  
  
print(f'number of integers up to {n} that are coprime with {n} are {etf(n)}')
```

2. Write a function called `mobius(n)` that calculates the Möbius function, $\mu(n)$. The function is defined as: $\mu(n) = 1$ if n is a square-free positive integer with an even number of prime factors. $\mu(n) = -1$ if n is a square-free positive integer with an odd number of prime factors. $\mu(n) = 0$ if n has a squared prime factor

```
def mobius(n):  
    if n <= 0:  
        return 0  
    if n == 1:  
        return 1  
    prime_factors_count = 0  
    p = 2  
    while p * p <= n:  
        if n % p == 0:  
            prime_factors_count += 1  
            n //= p  
        if n % p == 0:  
            return 0  
        p += 1  
    if n > 1:
```

```

    prime_factors_count += 1
    if prime_factors_count % 2 == 0:
        return 1
    else:
        return -1

```

3. Write a function called divisor_sum(n) that calculates the sum of all positive divisors of n (including 1 and n itself). This is often denoted by $\sigma(n)$.

```

n = int(input('Enter a positive integer: '))
def divisor_sum(n):
    if n <= 0:
        return 0
    total_sum = 0
    for i in range(1, n + 1):
        if n % i == 0:
            total_sum += i
    return total_sum
print(f"The sum of the divisors of {n} is {divisor_sum(n)}")

```

4. Write a function called prime_pi(n) that approximates the prime-counting function, $\pi(n)$. This function returns the number of prime numbers less than or equal to n

```

def is_prime(n):
    if n <= 1:
        return False
    for i in range(2, int(n**0.5) + 1):
        if n % i == 0:
            return False
    return True

```

```

def prime_pi(n):
    if n < 2:
        return 0
    count = 0
    for number in range(2, n + 1):
        if is_prime(number):
            count += 1
    return count

```

```
print(f"Number of prime numbers <= {n} are {prime_pi(n)}")
```

5. Write a function called legendre_symbol(a, p) that calculates the Legendre symbol (a/p) , which is a useful function in quadratic reciprocity. It is defined for an odd prime p and an integer a not divisible by p as: $(a/p) = 1$ if a is a quadratic residue modulo p (i.e., there exists an integer x such that $x^2 \equiv a \pmod{p}$). $(a/p) = -1$ if a is a quadratic non-residue modulo p.

```
def main(a, p):
```

```

m = 0
y = 0
x = 0
if a % p != 0:
    if p % 2 != 0:
        for i in range(1, p + 1):
            if p % i == 0:
                m += 1
        if m == 2:
            y = a % p
        for i in range(1, y + 1):
            if i * i == y:
                x = 1
                break
        else:
            x = -1
    if x == 1:
        print("a/p=1")
    else:
        print("a/p= -1")

```

6. Write a function factorial(n) that calculates the factorial of a non-negative integer n ($n!$).

```

x=int(input( 'Enter a non-negative integer' ) )
def factorial(x):
    f = 1
    if x < 0:
        print('wrong input')
    else:
        for i in range(1, x + 1):
            f *= i

```

```
print( 'FACTORIAL:' , f)
factorial(x)
```

7. Write a function `is_palindrome(n)` that checks if a number
reads the same forwards and backwards.

```
n = int(input('Enter an integer: '))
```

```
def palindrome(n):
```

```
    k = n
```

```
    l = 0
```

```
    while k != 0:
```

```
        j = k % 10
```

```
        l = l * 10 + j
```

```
        k = k // 10
```

```
    if l == n:
```

```
        print("palindrome")
```

```
    else:
```

```
        print("not palindrome")
```

```
palindrome(n)
```

8. Write a function `mean_of_digits(n)` that returns the average
of all digits in a number.

```
n= int(input("enter an integer "))
```

```
def mean_of_digits(n):
```

```
    c = 0
```

```
    m = 0
```

```
    k = n
```

```
while k != 0:  
    j = k % 10  
    m += j  
    c += 1  
    k /= 10 # integer division
```

```
avg = m / c  
print(avg)
```

```
mean_of_digits(n)
```

9. Write a function digital_root(n) that repeatedly sums the digits of a number until a single digit is obtained.

```
n=int(input("enter an integer "))  
def digital_root(n):  
    while n >= 10: # keep going until n is a single digit  
        s = 0  
        while n > 0:  
            s += n % 10  
            n /= 10  
        n = s  
    return n
```

```
digital_root(n)
```

10. Write a function is_abundant(n) that returns True if the sum of proper divisors of n is greater than n.

```
n=int(input("enter an integer "))  
def is_abundant(n):  
    k = n
```

```
s = 0  
for i in range(1, k):
```

```
    if k % i == 0:
```

```
        s += i
```

```
if s > n:
```

```
    print("true")
```

```
else:
```

```
    print("false")
```

```
is_abundant(n)
```

11. Write a function `is_deficient(n)` that returns True if the

sum of proper divisors of `n` is less than `n`.

```
n = int(input("Enter a positive integer: "))
```

```
def is_deficient(n):
```

```
    s = 1
```

```
    i = 2
```

```
    while i * i <= n:
```

```
        if n % i == 0:
```

```
            s += i
```

```
            if i * i != n:
```

```
                s += n // i
```

```
    i += 1
```

```
return s < n
```

```
result = is_deficient(n)
```

```
print(result)
```

12. Write a function for harshad number is_harshad(n) that

checks if a number is divisible by the sum of its digits.

```
def is_harshad(n):

    n = int(n)
    original_n = n
    sum_of_digits = 0
    while n > 0:
        digit = n % 10
        sum_of_digits += digit
        n /= 10

    if (original_n % sum_of_digits == 0):
        print( original_n , "is divisible by the sum of its digits")
    else:
        print( original_n , "is not divisible by the sum of its digits")
```

```
num = int(input("Enter a number: "))
```

```
is_harshad(num)
```

13. Write a function is_automorphic(n) that checks if a

number's square ends with the number itself.

```
def is_automorphic(n):

    sq= n**2
    n2=n
    a=0
    while (n!=0):
        a=a+1
        n=n/10
```

```

last=sq%a
if( last==n2):
    print( n2, "is an automorphic number")
else:
    print( n2, "is not an automorphic number")

num=int(input("enter a number "))

is_automorphic(num)

```

14. Write a function `is_pronic(n)` that checks if a number is
the product of two consecutive integers.

```

def is_pronic(n):
    a=0
    for i in range
        (1,n): if(
            i*(i+1)==n):
                a=1
                break
    else:
        a=0

    return a

```

```

num= int(input("enter a number "))
if (is_pronic(num)==1):
    print( num , "is a pronic number")
elif (is_pronic(num)==0):
    print( num , "is not a pronic number")

```

15. Write a function prime_factors(n) that returns the list of prime factors of a number.

```
n = int(input("Enter a positive integer: "))
```

```
def prime_factors(n):
```

```
    factors = []
```

```
    while n % 2 == 0:
```

```
        factors += [2]
```

```
        n /= 2
```

```
i = 3
```

```
while i * i <= n:
```

```
    while n % i == 0:
```

```
        factors += [i]
```

```
        n /= i
```

```
    i += 2
```

```
if n > 2:
```

```
    factors += [n]
```

```
return factors
```

```
if n <= 1:
```

```
    print([])
```

```
else:
```

```
    print(prime_factors(n))
```

16. Write a function `count_distinct_prime_factors(n)` that returns how many unique prime factors a number has.

```
def count_distinct_prime_factorial(n):

    if n < 2:

        return 0 # 0 and 1 have no prime factors

    count = 0

    i = 2

    while i * i <= n:

        if n % i == 0:

            count += 1

            while n % i == 0:

                n //= i

            i += 1

        if n > 1:

            count += 1 # n itself is a prime number

    return count

print(count_distinct_prime_factorial(60))
```

17. Write a function `is_prime_power(n)` that checks if a number can be expressed as p^k where p is prime and $k \geq 1$.

```
import time

import os

import psutil

def memory_usage():

    process = psutil.Process(os.getpid())

    return process.memory_info().rss

def is_prime_power(n):
```

```
if n < 2:
    return False

for p in range(2, int(n**0.5)+1):
    # check if p is prime
    is_prime = True

    for i in range(2, int(p**0.5)+1):
        if p % i == 0:
            is_prime = False
            break

    if not is_prime:
        continue

    # try different k values
    k = 1
    value = p

    while value < n:
        value *= p
        k += 1

    if value == n:
        return True

return False

start_time = time.time()

n = int(input("Enter a number to check if it's a prime power: "))

if is_prime_power(n):
    print(f"{n} is a prime power.")
else:
    print(f"{n} is not a prime power.")

print(f"memory usage :{memory_usage()} bytes")
end_time = time.time()
print(f"\nExecution Time: {end_time - start_time:.6f} seconds")
```

18. Write a function `is_mersenne_prime(p)` that checks if $2^p - 1$

is a prime number (given that p is prime).

```
def memory_usage():

    process = psutil.Process(os.getpid())

    return process.memory_info().rss

def is_mersenne_prime(p):

    a=2 ** p - 1

    b=0

    c=0

    if p < 2:

        return 0

    for i in range(2,p):

        if p%i==0:

            b=1

            break

        else:

            b=0

    for j in range(2,a):

        if a%j==0:

            c=1

            break

        else:

            c=0

    if b==0 and c==0:

        return True

    else:

        return False
```

```

start_time = time.time()

p = int(input("Enter a prime number to check if it's a Mersenne prime:"))

if is_mersenne_prime(p):

    print(f"2^{p} - 1 is a Mersenne prime.")

else:

    print(f"2^{p} - 1 is not a Mersenne prime.")

print(f"memory usage :{memory_usage()} bytes")

end_time = time.time()

print(f"\nExecution Time: {end_time - start_time:.6f} seconds")

```

19. Write a function twin_primes(limit) that generates all twin prime pairs up to a given limit.

```

import time

import sys

start_time = time.time()

def twin_prime(limit):

    def is_prime(n):

        if n < 2:

            return False

        for i in range(2, int(n ** 0.5) + 1):

            if n % i == 0:

                return False

        return True

    twin_primes = []

    for num in range(2, limit - 1):

        if is_prime(num) and is_prime(num + 2):

            twin_primes.append((num, num + 2))

    return twin_primes

```

```

limit = 130

end_time = time.time()

time_taken = end_time-start_time

print(f"The twin pairs are:{twin_prime(130)}")

print("time taken to run this code:",time_taken,"seconds")

print("memory usage:",sys.getsizeof(twin_prime(limit)),"KB")

# Output: [(3, 5), (5, 7), (11, 13), (17, 19), (29, 31), (41, 43), (59, 61), (71, 73)]

```

20. Write a function Number of Divisors

(d(n)) count_divisors(n) that returns how many positive divisors a number has.

```
import time
```

```
import os
```

```
import psutil
```

```

def memory_usage():

    process = psutil.Process(os.getpid())

    return process.memory_info().rss

def count_divisors(n):

    count = 0

    for i in range(1, int(n**0.5)+1):

        if n % i == 0:

            if i * i == n:

                count += 1

            else:

                count += 2

    return count

start_time = time.time()

n = int(input("Enter a number to count its divisors: "))

```

```

print(f"Number of divisors of {n}: {count_divisors(n)}")
print(f"memory usage :{memory_usage()} bytes")
end_time = time.time()
print(f"\nExecution Time: {end_time - start_time:.6f} seconds")

```

21. Write a function aliquot_sum(n) that returns the sum of all

proper divisors of n (divisors less than n).

```

def aliquot_sum(n):
    s = 0
    for i in range(1, n):
        if (n % i == 0):
            s = s + i
    return s

```

n = int(input("Enter a number: "))

print(f"The aliquot sum of {n} is: {aliquot_sum(n)}")

22. Write a function are_amicable(a, b) that checks if two

numbers are amicable (sum of proper divisors of a

equals b and vice versa).

```

def are_amicable(a,b):
    s1 = 0
    for i in range(1, a):
        if (a % i == 0):
            s1 += i

    s2 = 0
    for j in range(1, b):
        if (b % j == 0):
            s2 += j

    if (s1 == b and s2 == a):
        print("numbers are amicable")
    else:
        print("numbers are not amicable")

```

```

a = int(input("Enter a number: "))
b = int(input("Enter another number: "))
are_amicable(a, b)

```

23. Write a function multiplicative_persistence(n) that counts how many steps until a number's digits multiply to a single digit.

```
def multiplicative_persistence(n):
    c = 0
    while n >= 10:
        p = 1
        temp_n = n
        while temp_n > 0:
            digit = temp_n % 10
            p *= digit
            temp_n /= 10
        n = p
        c += 1
    return c

n = int(input("Enter a number: "))
print(multiplicative_persistence(n))
```

24. Write a function is_highly_composite(n) that checks if a number has more divisors than any smaller number.

```
def count_divisors(num):
    if num <= 0:
        return 0
    if num == 1:
        return 1

    count = 0
    i = 1
    while i * i <= num:
        if num % i == 0:
            if i * i == num:
                count += 1
            else:
                count += 2
        i += 1
    return count
```

```
def is_highly_composite(n):
    if n <= 0:
        return False
    if n == 1:
```

```

        return True

    divisors_of_n = count_divisors(n)

    for i in range(1, n):
        divisors_of_i = count_divisors(i)
        if divisors_of_i >= divisors_of_n:
            return False

    return True

number = int(input("Enter a positive integer: "))

if is_highly_composite(number):
    print(f"\n{number} IS highly composite.")
else:
    print(f"\n{number} is NOT highly composite.")

if number > 0:
    print(f"Divisors: {count_divisors(number)}")

```

25. Write a function for ModularExponentiation mod_exp(base, exponent, modulus) that efficiently calculates $(base^{exponent}) \% modulus$.

```

def mod_exp(base, exponent, modulus):

    if modulus == 1:
        return 0

    result = 1
    base %= modulus

    while exponent > 0:
        if exponent % 2 == 1:
            result = (result * base) % modulus

        base = (base * base) % modulus
        exponent //= 2

    return result

b = int(input("Enter the base: "))
e = int(input("Enter the exponent: "))
m = int(input("Enter the modulus: "))

```

```

if e < 0:
    print("This function does not handle negative exponents.")
else:
    final_result = mod_exp(b, e, m)
    print(f"\nResult: {b}^{e} mod {m} = {final_result}")

```

26. Write a function Modular Multiplicative

Inverse mod_inverse(a, m) that finds the number x such

that $(a * x) \equiv 1 \pmod{m}$.

```
import time
```

```
import sys
```

```
import resource
```

```
def mod_inverse(a, m):
```

```
    def extended_gcd(a, b):
```

```
        if a == 0:
```

```
            return b, 0, 1
```

```
        gcd, x1, y1 = extended_gcd(b % a, a)
```

```
        x = y1 - (b // a) * x1
```

```
        y = x1
```

```
        return gcd, x, y
```

```
    gcd, x, _ = extended_gcd(a % m, m)
```

```
    if gcd != 1:
```

```
        return None
```

```
    return (x % m + m) % m
```

```
# Test
```

```
start_time = time.time()
```

```
result = mod_inverse(3, 26)
```

```
end_time = time.time()
```

```
runtime = end_time - start_time
```

```
mem_usage = resource.getusage(resource.RUSAGE_SELF).ru_maxrss * 1024 # bytes
print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")
```

27. Write a function chinese Remainder Theorem

Solver crt(remainders, moduli) that solves a system of congruences $x \equiv r_i \pmod{m_i}$.

```
import time
import sys
import resource
from math import gcd
```

```
def crt(remainders, moduli):
    if len(remainders) != len(moduli):
        return None
    prod = 1
    for m in moduli:
        prod *= m
    result = 0
    for rem, mod in zip(remainders, moduli):
        p = prod // mod
        result += rem * extended_inverse(p, mod) * p
    return result % prod
```

```
def extended_inverse(a, m):
    m0, x0, x1 = m, 1, 0
    if m == 1:
        return 0
    while a > 1:
        q = a // m
        m, a = a % m, m
        x0, x1 = x1 - q * x0, x0
```

```

x0, x1 = x1 - q * x0, x0

return x1 + m0 if x1 < 0 else x1

# Test

start_time = time.time()

result = crt([2, 3], [3, 5])

end_time = time.time()

runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")

```

28. Write a function Quadratic Residue

Check `is_quadratic_residue(a, p)` that checks if $x^2 \equiv a \pmod{p}$

has a solution.

```
import time
```

```
import sys
```

```
import resource
```

```
def is_quadratic_residue(a, p):
```

```
    if p == 2:
```

```
        return True if a % 2 == 0 or a % 2 == 1 else False
```

```
    if a % p == 0:
```

```
        return True
```

```
    return pow(a, (p - 1) // 2, p) == 1
```

```
# Test
```

```
start_time = time.time()
```

```
result = is_quadratic_residue(2, 7)
```

```
end_time = time.time()
```

```
runtime = end_time - start_time
```

```
mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024
print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")
```

29. Write a function `order_mod(a, n)` that finds the smallest positive integer k such that $ak \equiv 1 \pmod{n}$.

```
import time
```

```
import sys
```

```
import resource
```

```
def order_mod(a, n):
```

```
    if gcd(a, n) != 1:
```

```
        return None
```

```
    k = 1
```

```
    pow_a = a % n
```

```
    while pow_a != 1:
```

```
        pow_a = (pow_a * a) % n
```

```
        k += 1
```

```
        if k > n: # Cycle detection
```

```
            return None
```

```
    return k
```

```
from math import gcd
```

```
start_time = time.time()
```

```
result = order_mod(2, 7)
```

```
end_time = time.time()
```

```
runtime = end_time - start_time
```

```
mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024
```

```
print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")
```

30. Write a function Fibonacci Prime

Check `is_fibonacci_prime(n)` that checks if a number is both Fibonacci and prime.

```
import time
```

```
import sys
```

```
import resource
```

```
def is_fibonacci(n):
```

```
    if n < 0:
```

```
        return False
```

```
    a, b = 0, 1
```

```
    while b < n:
```

```
        a, b = b, a + b
```

```
    return b == n
```

```
def is_prime(n):
```

```
    if n <= 1:
```

```
        return False
```

```
    if n <= 3:
```

```
        return True
```

```
    if n % 2 == 0 or n % 3 == 0:
```

```
        return False
```

```
i = 5
```

```
while i * i <= n:
```

```
    if n % i == 0 or n % (i + 2) == 0:
```

```
        return False
```

```
i += 6
```

```
return True
```

```
def is_fibonacci_prime(n):
```

```

        return is_fibonacci(n) and is_prime(n)

start_time = time.time()

result = is_fibonacci_prime(13)

end_time = time.time()

runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")

```

31. Write a function Lucas Numbers

Generator lucas_sequence(n) that generates the first n

Lucas numbers (similar to Fibonacci but starts with 2,
1).

```
import time
```

```
import sys
```

```
import resource
```

```
def lucas_sequence(n):
```

```
    if n == 0:
```

```
        return [2]
```

```
    if n == 1:
```

```
        return [2, 1]
```

```
    seq = [2, 1]
```

```
    for i in range(2, n + 1):
```

```
        seq.append(seq[-1] + seq[-2])
```

```
    return seq[:n]
```

```
start_time = time.time()
```

```
result = lucas_sequence(10)

end_time = time.time()

runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")
```

32. Write a function for Perfect Powers

Check `is_perfect_power(n)` that checks if a number can be
expressed as a^b where $a > 0$ and $b > 1$.

```
import time

import sys

import resource

from math import log, sqrt, ceil
```

```
def is_perfect_power(n):

    if n < 1:

        return False

    for b in range(2, int(log(n, 2)) + 1):

        a = round(n ** (1 / b))

        if a ** b == n:

            return True

    return False
```

```
start_time = time.time()

result = is_perfect_power(16)

end_time = time.time()

runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")
```

33. Write a function Collatz Sequence

Length collatz_length(n) that returns the number of steps

for n to reach 1 in the Collatz conjecture.

```
import time
```

```
import sys
```

```
import resource
```

```
def collatz_length(n):
```

```
    if n <= 0:
```

```
        return 0
```

```
    length = 1
```

```
    while n != 1:
```

```
        if n % 2 == 0:
```

```
            n /= 2
```

```
        else:
```

```
            n = 3 * n + 1
```

```
        length += 1
```

```
    return length
```

```
start_time = time.time()
```

```
result = collatz_length(27)
```

```
end_time = time.time()
```

```
runtime = end_time - start_time
```

```
mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024
```

```
print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")
```

34. Write a function Polygonal Numbers polygonal_number(s,

n) that returns the n-th s-gonal number.

```

import time
import sys
import resource

def polygonal_number(s, n):
    if s < 3 or n < 1:
        return None
    return n * (n - 1) * (s - 2) // 2 + n

start_time = time.time()
result = polygonal_number(5, 3) # Pentagon
end_time = time.time()
runtime = end_time - start_time
mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024
print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")

```

35. Write a function Carmichael Number

Check `is_carmichael(n)` that checks if a composite number

`n` satisfies $a^{n-1} \equiv 1 \pmod{n}$ for all a coprime to n .

```

import time
import sys
import resource
from math import gcd

def is_carmichael(n):
    if n <= 1 or is_prime(n):
        return False
    for a in range(2, n):
        if gcd(a, n) == 1 and pow(a, n - 1, n) != 1:

```

```

        return False

    return True


def is_prime(n):
    if n <= 1:
        return False

    if n <= 3:
        return True

    if n % 2 == 0 or n % 3 == 0:
        return False

    i = 5

    while i * i <= n:
        if n % i == 0 or n % (i + 2) == 0:
            return False

        i += 6

    return True

```

```

start_time = time.time()

result = is_carmichael(561)

end_time = time.time()

runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")

```

36. Implement the probabilistic Miller-Rabin

test is_prime_miller_rabin(n, k) with k rounds.

```

import time

import sys

import resource

```

```
from random import randrange

def is_prime_miller_rabin(n, k=40):

    if n <= 1:
        return False

    if n <= 3:
        return True

    if n % 2 == 0:
        return False

    r, s = 0, n - 1

    while s % 2 == 0:
        r += 1
        s //= 2

    for _ in range(k):
        a = randrange(2, n - 1)

        x = pow(a, s, n)

        if x == 1 or x == n - 1:
            continue

        for _ in range(r - 1):
            x = pow(x, 2, n)

        if x == n - 1:
            break

    else:
        return False

    return True

start_time = time.time()

result = is_prime_miller_rabin(13)

end_time = time.time()
```

```
runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")
```

37. Implement pollard_rho(n) for integer factorization using

Pollard's rho algorithm.

```
import time

import sys

import resource

from math import gcd

from random import randint
```

```
def pollard_rho(n):
```

```
    if n % 2 == 0:
        return 2
```

```
    x = randint(1, n - 1)
```

```
    y = x
```

```
    c = randint(1, n - 1)
```

```
    d = 1
```

```
    while d == 1:
```

```
        x = (x * x + c) % n
```

```
        y = (y * y + c) % n
```

```
        y = (y * y + c) % n
```

```
        d = gcd(abs(x - y), n)
```

```
    if d == n:
```

```
        return None
```

```
    return d
```

```
start_time = time.time()
```

```

result = pollard_rho(315)

end_time = time.time()

runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")

```

38. Write a function `zeta_approx(s, terms)` that approximates the Riemann zeta function $\zeta(s)$ using the first 'terms' of the series.

```

import time

import sys

import resource


def zeta_approx(s, terms=1000):

    if s <= 1:

        return float('inf')

    total = 0.0

    for k in range(1, terms + 1):

        total += 1 / (k ** s)

    return total

```

```

start_time = time.time()

result = zeta_approx(2, 1000)

end_time = time.time()

runtime = end_time - start_time

mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss * 1024

print(f"Result: {result:.6f}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes")

```

39. Write a function Partition Function

`p(n)` `partition_function(n)` that calculates the number of distinct ways to write `n` as a sum of positive integers.

```
import time
import sys
import resource

def partition_function(n):
    if n < 0:
        return 0
    p = [0] * (n + 1)
    p[0] = 1
    for i in range(1, n + 1):
        j = 1
        while True:
            pent = j * (3 * j - 1) // 2
            if pent > i:
                break
            sign = 1 if (j % 2 == 1) else -1
            p[i] += sign * p[i - pent]
            pent_neg = (-j) * (3 * (-j) - 1) // 2
            if pent_neg <= i:
                sign = 1 if (j % 2 == 0) else -1
                p[i] += sign * p[i - pent_neg]
            j += 1
    return p[n]
```

```
start_time = time.time()
result = partition_function(10)
end_time = time.time()
runtime = end_time - start_time
mem_usage = resource.getrusage(resource.RUSAGE_SELF).ru_maxrss *
1024
print(f'Result: {result}, Runtime: {runtime:.6f}s, Memory: {mem_usage} bytes')
```


