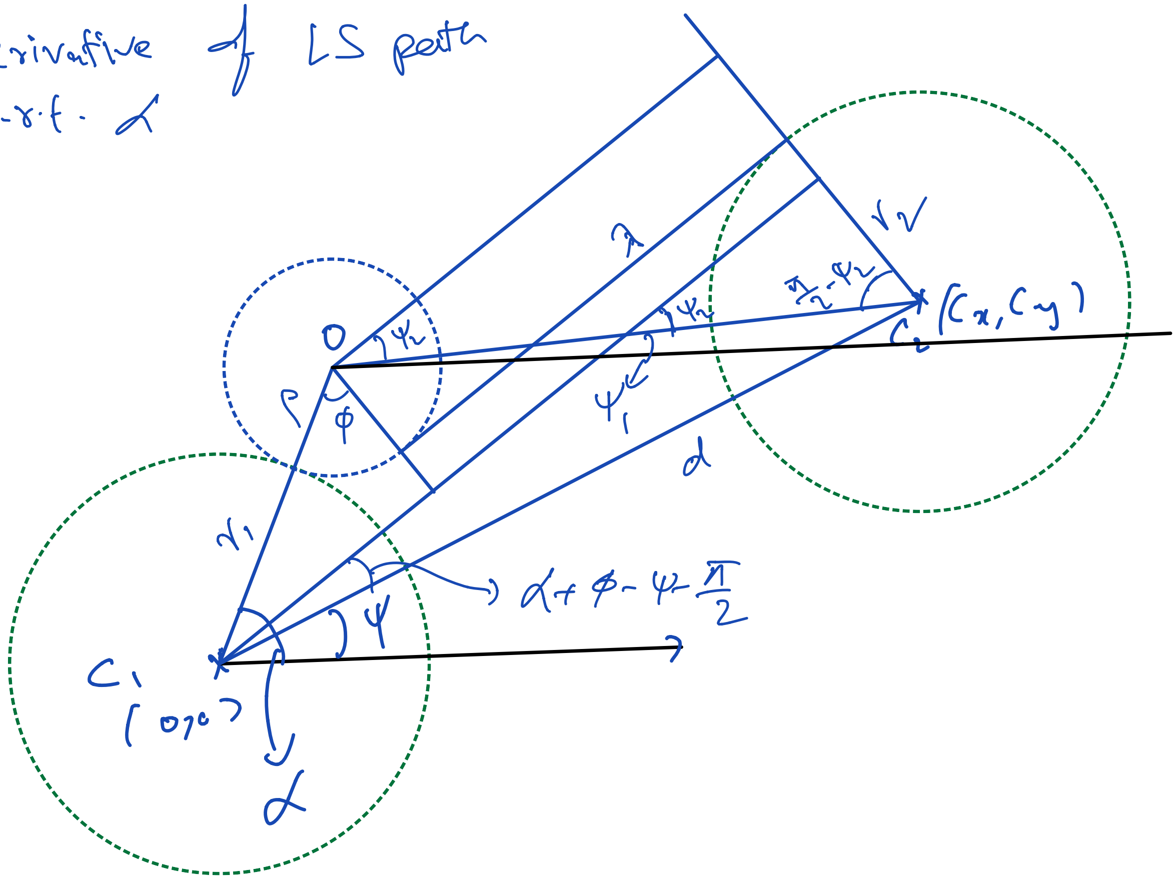
Derivative of LS path w.r.t. $\alpha$




$$\left.\begin{array}{l} \alpha-\frac{\pi}{2}+\phi=\psi_1+\psi_2 \Rightarrow \alpha-\psi_1-\psi_2=\frac{\pi}{2}-\phi \\ \lambda+(r_1+\rho)\sin\phi-d\sin(\alpha+\phi-\psi)=0 \end{array}\right\} \text{useful later}$$

$$l_y = c_y - (r_1+\rho)\sin\alpha$$

$$l_x = c_x - (r_1+\rho)\cos\alpha$$

$$d_{oc_2} = \sqrt{l_x^2 + l_y^2}$$

$$\psi_2 = \sin^{-1}\left(\frac{r_2+\rho}{d_{oc_2}}\right) \qquad \psi_1 = \tan^{-1}\left(\frac{l_y}{l_x}\right)$$

$$\lambda^2 = l_x^2 + l_y^2 - (r_2+\rho)^2$$

$$2\lambda\lambda' = 2 l_x l_x' + 2 l_y l_y'$$

$$\lambda' = \frac{1}{\lambda}(r_1+e)\left[\sin\alpha\, l_x - l_y \cos\alpha\right]$$

$$= \frac{1}{\lambda}(r_1+\rho)\left[c_x \sin\alpha - c_y \cos\alpha\right]$$

$$\boxed{\lambda' = \frac{1}{\lambda}(r_1+\rho)\, d \sin(\alpha-\psi)}$$

$$\alpha - \frac{\pi}{2} + \phi = \psi_1 + \psi_2$$

$$\phi' = \psi_1' + \psi_2' - 1$$

$$\psi_1' = \frac{l_x^2}{l_x^2+l_y^2}\,\frac{-l_y l_x' + l_x l_y'}{l_x^2}$$

$$= -\frac{(r_1+e)(l_y \sin\alpha + l_x \cos\alpha)}{l_x^2+l_y^2}$$

$$= -\frac{r_1+\rho}{doc_2}\left(\sin\alpha \sin\psi_1 + \cos\alpha \cos\psi_1\right)$$

$$= -\frac{r_1+\rho}{doc_2} \cos(\alpha - \psi_1)$$

Note that $\quad doc_2 = \sqrt{l_x^2 + l_y^2}$

$$\psi_2' = -\frac{1}{\cos\psi_2} \frac{(\rho + r_2)}{doc_2^2} doc_2'$$

$$= -\frac{1}{\cos\psi_2} \frac{(\rho+r_2)(r_1+\rho)(l_x \sin\alpha - l_y \cos\alpha)}{doc_2^3}$$

$$= -\frac{1}{\cos\psi_2} \frac{(r_1+\rho)(r_2+\rho)}{doc_2^2} \cdot \sin(\alpha - \psi_1)$$

$$= -\frac{1}{\cos\psi_2} \frac{r_1+\rho}{doc_2} \sin\psi_2 \sin(\alpha - \psi_1)$$

$$\phi' = \psi_1' + \psi_2' - 1$$

$$= \frac{(r_1+\rho)}{d\alpha_2}\left[-\cos(\alpha-\psi_1) - \frac{\sin\psi_2}{\cos\psi_2}\sin(\alpha-\psi_1)\right] - 1$$

$$= -\frac{r_1+\rho}{\lambda}\,\frac{\cos(\alpha-\psi_1-\psi_2)}{\quad} - 1$$

$$L'$$

$$= \lambda' + \rho\phi' = \frac{r_1+\rho}{\lambda}\left[d\sin(\alpha-\psi) - \rho\cos(\underbrace{\alpha-\psi_1-\psi_2}_{\frac{\pi}{2}-\phi})\right] - \rho$$

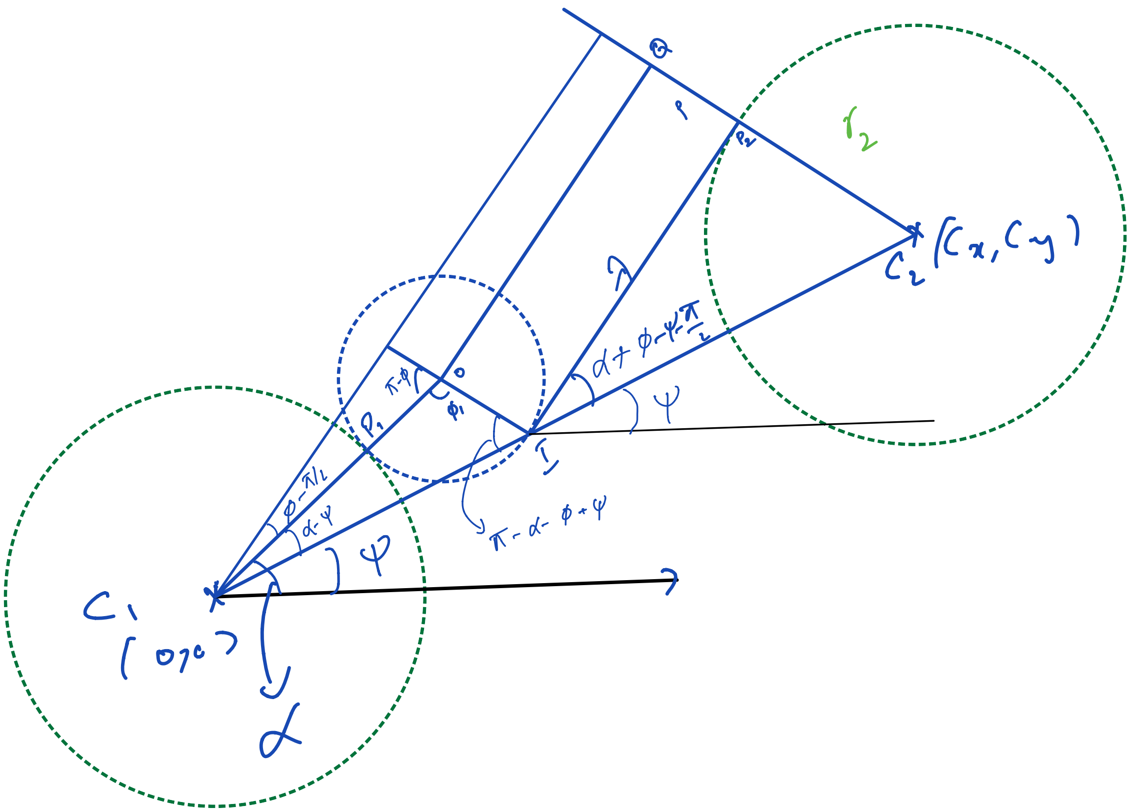
$$= \frac{r_1+\rho}{\lambda}\left[d\sin(\alpha-\psi) - \rho\sin\phi\right] - \rho$$

$$= \frac{(r_1+\rho)d\sin(\alpha-\psi)}{\lambda} - \frac{\rho}{\lambda}\left[(r_1+\rho)\sin\phi + \lambda\right]$$

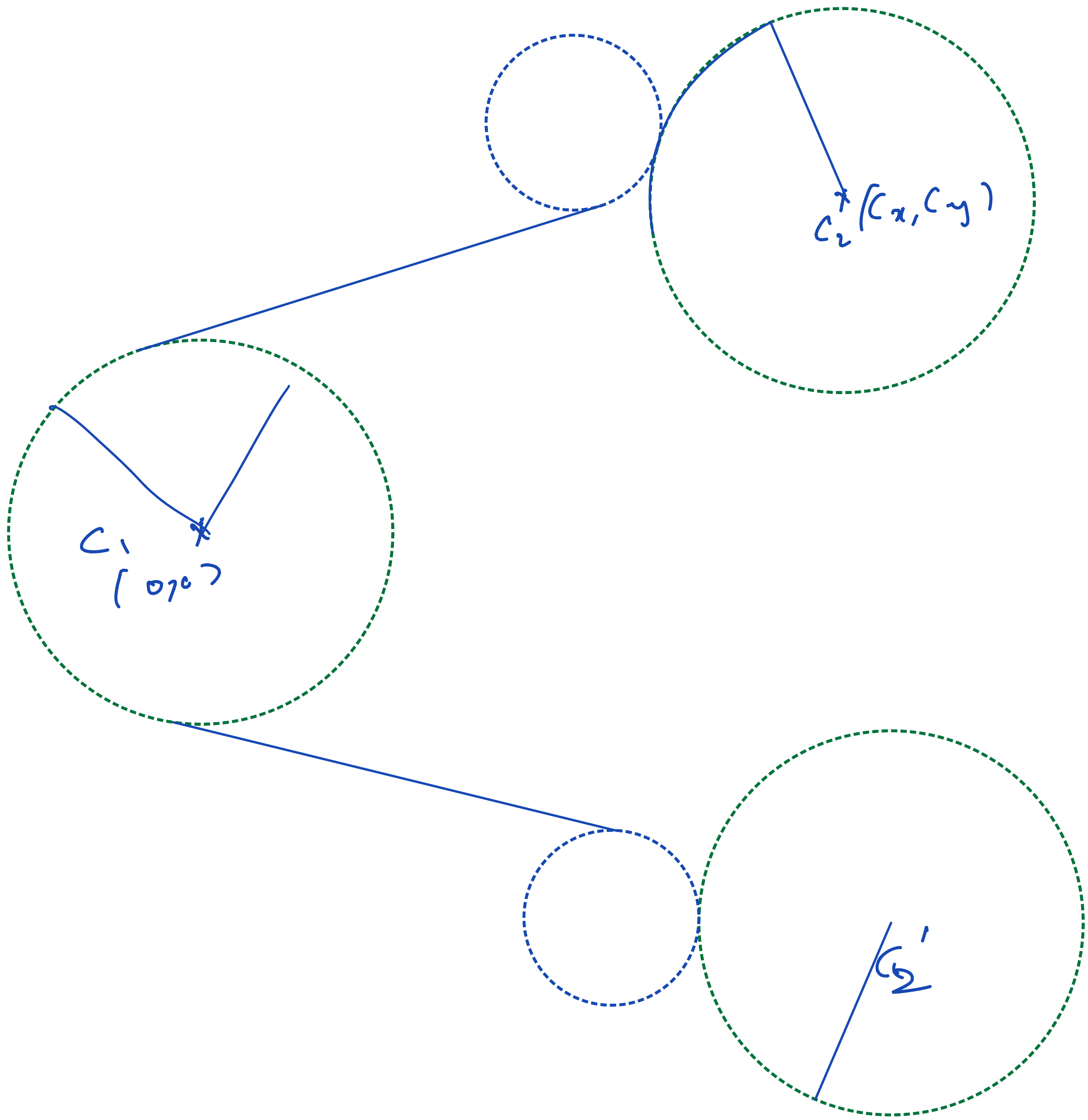
$$= \frac{(r_1+\rho)}{\lambda}d\sin(\alpha-\psi) - \frac{\rho d}{\lambda}\sin(\alpha+\phi-\psi)$$

$$= \frac{d}{d\lambda}\left[(r_1+\rho)\sin(\alpha-\varphi) - \rho\sin(\alpha+\phi-\varphi)\right] = 0$$

This is true when inflexion point $I$ is collinear with $\overline{C_1C_2}$.




$$(r_1+\rho)\sin(\alpha-\varphi) - \rho\sin(\alpha+\phi-\varphi) = 0$$



$C_2 (C_x, C_y)$
$C_1$
$(0,0)$
$C_2'$

Compute $\alpha$ @ min LS

$$(r_1+\rho)\sin(\alpha-\psi) - \rho\sin(\alpha+\phi-\psi) = 0$$

$$\underbrace{(r_1+\rho)\cos(\alpha-\psi) - \rho\cos(\alpha+\phi-\psi)}_{C_1 I} = d + \underbrace{\frac{r_2}{\cos(\alpha+\phi-\psi)}}_{-I C_2}$$

Squaring and adding the above two identities

$$\gamma := \alpha+\phi-\psi$$

$$(r_1+\rho)^2 = \rho^2\sin^2(\gamma) + \left(d + \frac{r_2}{\cos\gamma} + \rho\cos\gamma\right)^2$$

$$(r_1+\rho)^2 = \rho^2\sin^2\gamma + d^2 + \frac{r_2^2}{\cos^2\gamma} + \rho^2\cos^2\gamma + \frac{2r_2 d}{\cos\gamma}$$

$$+ 2\rho d\cos\gamma + 2r_2\rho$$

$$(r_1+\rho)^2\cos^2\rho = (\rho^2 - \rho^2\cos^2\gamma)\cos^2\gamma + d^2\cos^2\gamma + r_2^2$$

$$+\rho^2\cos^4\gamma + 2r_2 d\cos\gamma + 2\rho d\cos^3\gamma + 2r_2\rho\cos^2\gamma$$

$$\eta := \cos\gamma$$

$$\rho^2\eta^2 - \cancel{\rho^2\eta^4} + d^2\eta^2 + r_2^2 + \cancel{\rho^2\eta^4} + 2r_2 d\eta + 2\rho d\eta^3$$

$$+ \underbrace{\left(2r_2\rho - (r_1+\rho)^2\right)}\eta^2 = 0$$

$$2\rho d\eta^3 + \eta^2\underbrace{\left(\rho^2+d^2+2r_2\rho-(r_1+\rho)^2\right)}_{d^2-r_1^2+2r_2\rho-2r_1\rho} + 2r_2 d\eta + r_2^2 = 0$$

Find $\eta$

$\cos(\alpha+\phi-\psi)$ is known.

$$(r_1+\rho)\cos(\alpha-\psi) = \rho\eta + d + \frac{r_2}{\eta}$$

$$\boxed{\alpha = \cos^{-1}\left(\frac{\rho\eta^2+d\eta+r_2}{\eta(r_1+\rho)}\right)+\psi}$$

$$\alpha_1+\phi-\psi = \cos^{-1}(\eta)$$

$$\alpha_2 = \alpha_1+\phi = \cos^{-1}(\eta)+\psi$$