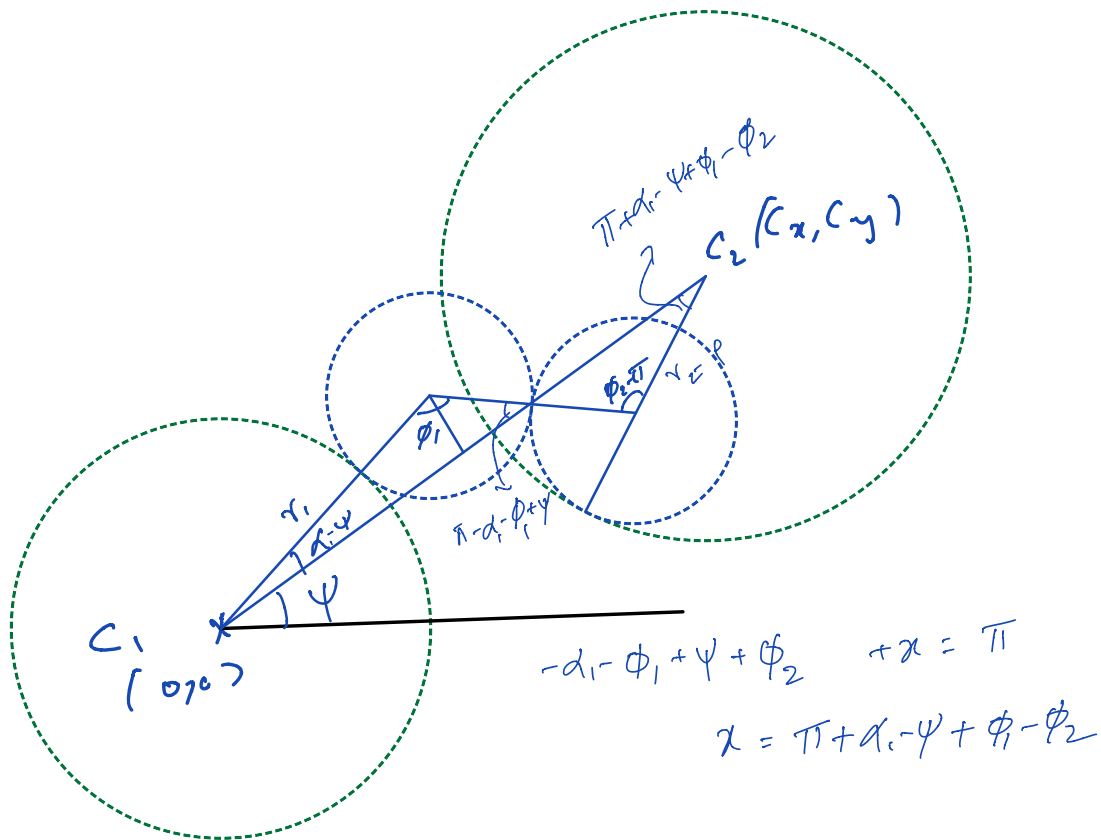


Computation of local minimum LR



$$(r_1 + r) \sin(\alpha_1 - \psi) = r \sin(\alpha_1 + \phi_1 - \psi)$$

$$= - (r_2 - r) \sin(\underbrace{\alpha_1 - \psi + \phi_1 - \phi_2}_{\alpha_2 - \psi})$$

$$(r_1 + r) \cos(\alpha_1 - \psi) - 2r \cos(\alpha_1 + \phi_1 - \psi) - (r_2 - r) \cos(\alpha_1 - \psi + \phi_1 - \phi_2) = d$$

$$\eta := \cos(\alpha_1 - \psi + \phi_1)$$

$$\xi := \cos(\alpha_2 - \psi)$$

$$\alpha_1 + \phi_1 - \phi_2 = \alpha_2$$

$$\begin{aligned} \rho^2(1-\eta^2) &= (r_2-\rho)^2(1-\xi^2) \\ &= (r_2-\rho)^2 - (r_2-\rho)^2\xi^2 \end{aligned}$$

$$\pm \sqrt{-\rho^2(1-\eta^2) + (r_2-\rho)^2} = (r_2-\rho)\xi$$

$$(r_1+\rho)^2 = (r_2-\rho)^2(1-\xi^2) + (d+2\rho\eta+(r_2+\rho)\xi)^2$$

$$\begin{aligned} (r_1+\rho)^2 &= (r_2-\rho)^2 + d^2 + 4\rho^2\eta^2 + 4\rho d\eta \\ &\quad + 4\rho(r_2-\rho)\eta\xi + 2d(r_2-\rho)\xi \end{aligned}$$

$$\begin{aligned} (r_1+\rho)^2 - (r_2-\rho)^2 - d^2 - 4\rho^2\eta^2 - 4\rho d\eta \\ = \pm(4\rho\eta + 2d) \sqrt{r_2^2 - 2r_2\rho + \rho^2\eta^2} \end{aligned}$$

$$\gamma := r_1^2 - r_2^2 + 2\rho(r_1+r_2) - d^2$$

$$\begin{aligned} (\gamma - 4\rho^2\eta^2 - 4\rho d\eta)^2 &= (16\rho^2\eta^2 + 4d^2 + 16\rho d\eta) \\ &\quad \times (r_2^2 - 2r_2\rho + \rho^2\eta^2) \end{aligned}$$

$$\begin{aligned}
& \underline{\gamma^2 + 16p^4\gamma^4 + 16p^2d^2\gamma^2 - 8\gamma p^2\gamma^2 - 8\gamma p d\gamma + 32p^3d\gamma^3} \\
& = \underline{16p^2r_2^2\gamma^2} - \underline{32p^3r_2\gamma^2} + \cancel{16p^4\gamma^4} + \underline{4d^2r_2^2} \\
& \quad - \underline{8pr_2d^2} + \underline{4p^2d^2\gamma^2} + \underline{16pr_2^2d\gamma} - \underline{32p^2r_2d\gamma} \\
& \quad + \underline{16p^3d\gamma^3}
\end{aligned}$$

$$\begin{aligned}
& 16p^3d\gamma^3 + \gamma^2(12p^2d^2 - 8\gamma p^2 - 16p^2r_2^2 + 32p^3r_2) \\
& + \gamma(-8\gamma p d - 16pr_2^2d + 32p^2r_2d) + \gamma^2 \\
& - 4d^2r_2^2 + 8pr_2d^2 = 0
\end{aligned}$$

$$\begin{aligned}
C_2 &= \underline{12p^2d^2} - \underline{16p^2r_2^2} + \underline{32p^3r_2} - 8p^2(\underline{r_1^2 - r_2^2} + \underline{2r_1p} + \underline{2r_2p - d^2}) \\
&= 20p^2d^2 - 8p^2r_2^2 + 16p^3r_2 - 16p^3r_1 - 8p^2r_1^2
\end{aligned}$$

$$\begin{aligned}
C_3 &= \underline{32p^2r_2d} - 16pr_2d - 8pd(\underline{r_1^2 - r_2^2} + \underline{2pr_1} + \underline{2pr_2 - d^2}) \\
&= 16p^2r_2d - 16pr_2d - 8pr_1^2d + 8pr_2^2d - 16p^2r_1d \\
& \quad + 8pd^3
\end{aligned}$$

solve for η

$$(r_1 + p) \sin(\alpha_1 - \psi) = p \sin(\alpha_1 + \phi_1 - \psi)$$

$$= p \sqrt{1 - \eta^2}$$

$$\alpha_1 = \sin^{-1} \left(\pm \frac{p \sqrt{1 - \eta^2}}{r_1 + p} \right) + \psi$$

$$p \sin(\alpha_1 + \phi_1 - \psi) = -(r_2 - p) \sin \left(\underbrace{\alpha_1 - \psi + \phi_1 - \phi_2}_{\alpha_2 - \psi} \right)$$

$$\left(-\frac{p}{r_2 - p} \right) \sqrt{1 - \eta^2} = \sin(\alpha_2 - \psi)$$

$$\alpha_2 = \psi + \pi + \sin^{-1} \left(\frac{p \sqrt{1 - \eta^2}}{r_2 - p} \right)$$

$$\cos^{-1} \eta = \alpha_1 - \psi + \phi_1$$

$$\phi_1 = \cos^{-1} \eta + \psi - \alpha_1$$

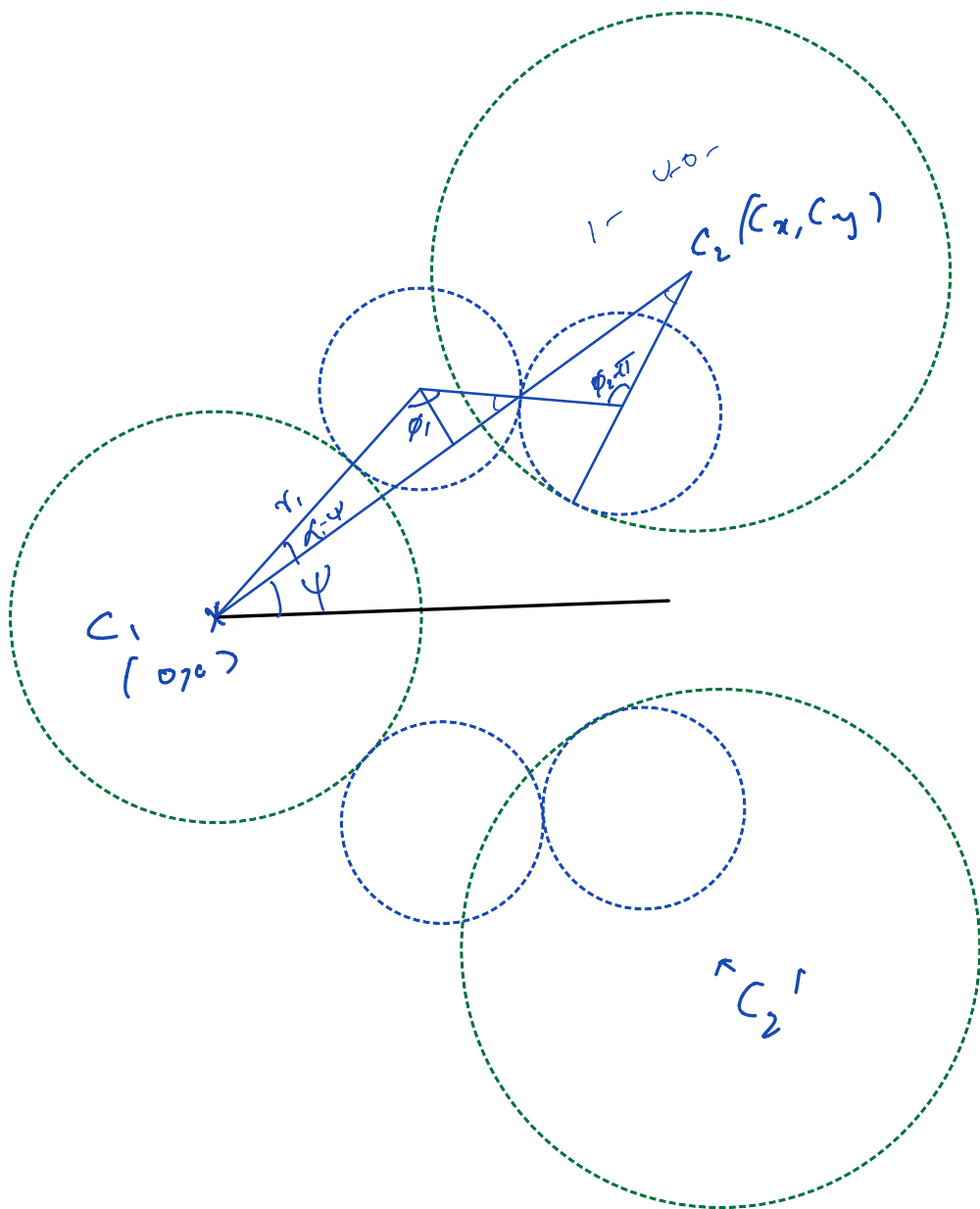
$$\alpha_1 + \phi - \phi_2 = \alpha_2$$

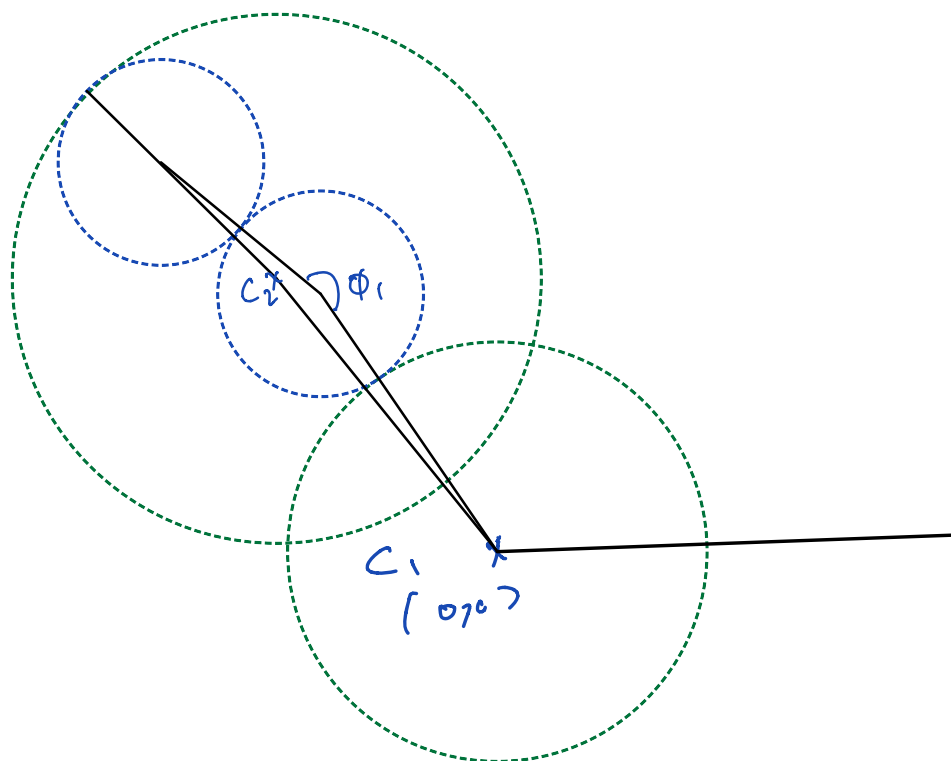
$$\phi_2 = \alpha_1 - \alpha_2 + \phi_1$$

$$\frac{\pm \sqrt{-p^2(1-\gamma^v) + (r_2 - p)^2}}{r_2 - p} = \xi = \cos(\alpha_2 - \psi)$$

$$\alpha_2 = \cos^{-1} \left(\frac{\sqrt{(r_2 - p)^2 - p^2(1-\gamma^v)}}{(r_2 - p)} \right) + \psi$$

$$r_2^2 - 2pr_2 - + p^v \gamma^v$$





$$(r_1 + p) \sin(\alpha_1 - \psi) = p \sin(\alpha_1 + \phi_1 - \psi)$$

$$= - (r_2 - p) \sin \underbrace{(\alpha_1 - \psi + \phi_1 - \phi_2)}_{\alpha_2 - \psi}$$

$$(r_1 + p) \cos(\alpha_1 - \psi) - 2p \cos(\alpha_1 + \phi_1 - \psi) - (r_2 - p) \cos(\alpha_1 - \psi + \phi_1 - \phi_2) = d$$

$$\eta := \cos(\alpha_1 - \psi + \phi_1)$$

$$\xi_p := \cos(\alpha_2 - \psi)$$

$$x = G \theta$$

$$\theta = G^T x \quad \text{or} \quad -G^T x$$