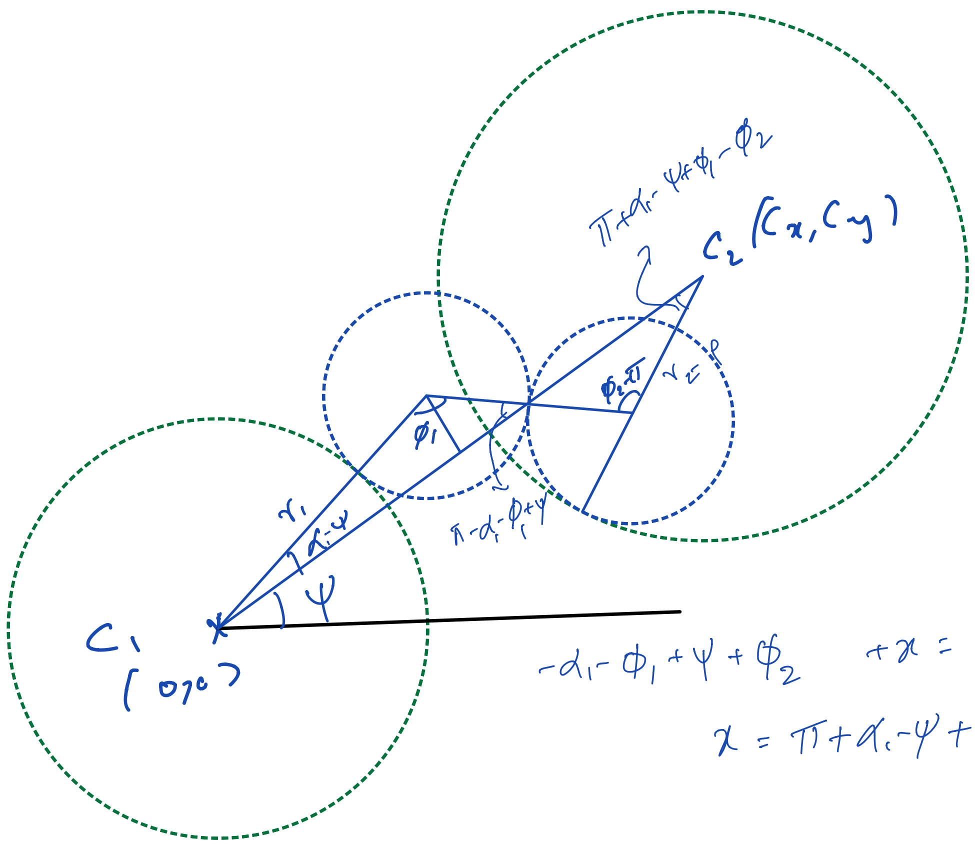
Computation of local minimum LR



$$-\alpha_1-\phi_1+\psi+\phi_2 \quad +x=\pi$$

$$x=\pi+\alpha_1-\psi+\phi_1-\phi_2$$

$$(r_1+\rho)\sin(\alpha_1-\psi)=\rho\sin(\alpha_1+\phi_1-\psi)$$

$$=-(r_2-\rho)\sin\underbrace{(\alpha_1-\psi+\phi_1-\phi_2)}_{\alpha_2-\psi}$$

$$(r_1+\rho)\cos(\alpha_1-\psi)-2\rho\cos(\alpha_1+\phi_1-\psi)-(r_2-\rho)\cos(\alpha_1-\psi+\phi_1-\phi_2)$$

$$=d$$

$$\eta:=\cos(\alpha_1-\psi+\phi_1)$$

$$\xi:=\cos(\alpha_2-\psi) \qquad \alpha_1+\phi_1-\phi_2=\alpha_2$$

$$\rho^2(1-\eta^2) = (r_2-\rho)^2(1-\xi^2)$$
$$= (r_2-\rho)^2 - (r_2-\rho)^2\xi^2$$

$$\boxed{\pm\sqrt{-\rho^2(1-\eta^2)+(r_2-\rho)^2} = (r_2-\rho)\xi}$$

$$(r_1+\rho)^2 = (r_2-\rho)^2(1-\xi^2) + \left(d+2\rho\eta+(r_2-\rho)\xi\right)^2$$

$$(r_1+\rho)^2 = (r_2-\rho)^2 + d^2 + 4\rho^2\eta^2 + 4\rho d\eta$$
$$+ 4\rho(r_2-\rho)\eta\xi + 2d(r_2-\rho)\xi$$

$$(r_1+\rho)^2 - (r_2-\rho)^2 - d^2 - 4\rho^2\eta^2 - 4\rho d\eta$$
$$= \pm(4\rho\eta+2d)\sqrt{r_2^2 - 2r_2\rho + \rho^2\eta^2}$$

$$\gamma := r_1^2 - r_2^2 + 2\rho(r_1+r_2) - d^2$$
$$(\gamma - 4\rho^2\eta^2 - 4\rho d\eta)^2 = (16\rho^2\eta^2 + 4d^2 + 16\rho d\eta)$$
$$\times(r_2^2 - 2r_2\rho + \rho^2\eta^2)$$

$$\gamma^2 + \cancel{16\rho^4\eta^4} + 16\rho^2 d^2\eta^2 - 8\gamma\rho^2\eta^2 - 8\gamma\rho d\eta + 32\rho^3 d\eta^3$$

$$= 16\rho^2 r_2^2\eta^2 - 32\rho^3 r_2\eta^2 + \cancel{16\rho^4\eta^4} + 4d^2 r_2^2$$

$$- 8\rho r_2 d^2 + 4\rho^2 d^2\eta^2 + 16\rho r_2^2 d\eta - 32\rho^2 r_2 d\eta$$

$$+ 16\rho^3 d\eta^3$$

$$16\rho^3 d\eta^3 + \eta^2\left(12\rho^2 d^2 - 8\gamma\rho^2 - 16\rho^2 r_2^2 + 32\rho^3 r_2\right)$$

$$+\eta\left(-8\gamma\rho d - 16\rho r_2^2 d + 32\rho^2 r_2 d\right) + \gamma^2$$

$$-4d^2 r_2^2 + 8\rho r_2 d^2 = 0$$

$$C_2 = 12\rho^2 d^2 - 16\rho^2 r_2^2 + 32\rho^3 r_2 - 8\rho^2\left(r_1^2 - r_2^2 + 2r_1\rho + 2r_2\rho - d^2\right)$$

$$= 20\rho^2 d^2 - 8\rho^2 r_2^2 + 16\rho^3 r_2 - 16\rho^3 r_1 - 8\rho^2 r_1^2$$

$$C_3 = 32\rho^2 r_2 d - 16\rho r_2 d - 8\rho d\left(r_1^2 - r_2^2 + 2\rho r_1 + 2\rho r_2 - d^2\right)$$

$$= 16\rho^2 r_2 d - 16\rho r_2 d - 8\rho r_1^2 d + 8\rho r_2^2 d - 16\rho^2 r_1 d$$

$$+ 8\rho d^3$$

solve for η

$$(r_1+\rho)\sin(\alpha_1-\psi) = \rho\sin(\alpha_1+\phi_1-\psi)$$

$$= \rho\sqrt{1-\eta^2}$$

$$\alpha_1 = \sin^{-1}\left(\pm\frac{\rho\sqrt{1-\eta^2}}{r_1+\rho}\right) + \psi$$

$$\rho\sin(\alpha_1+\phi_1-\psi) = -(r_2-\rho)\sin\underbrace{(\alpha_1-\psi+\phi_1-\phi_2)}_{\alpha_2-\psi}$$

$$\left(-\frac{\rho}{r_2-\rho}\right)\sqrt{1-\eta^2} = \sin(\alpha_2-\psi)$$

$$\alpha_2 = \psi + \pi + \sin^{-1}\left(\frac{\rho\sqrt{1-\eta^2}}{r_2-\rho}\right)$$

$$\cos^{-1}\eta = \alpha_1-\psi+\phi_1$$

$$\phi_1 = \cos^{-1}\eta + \psi - \alpha_1$$

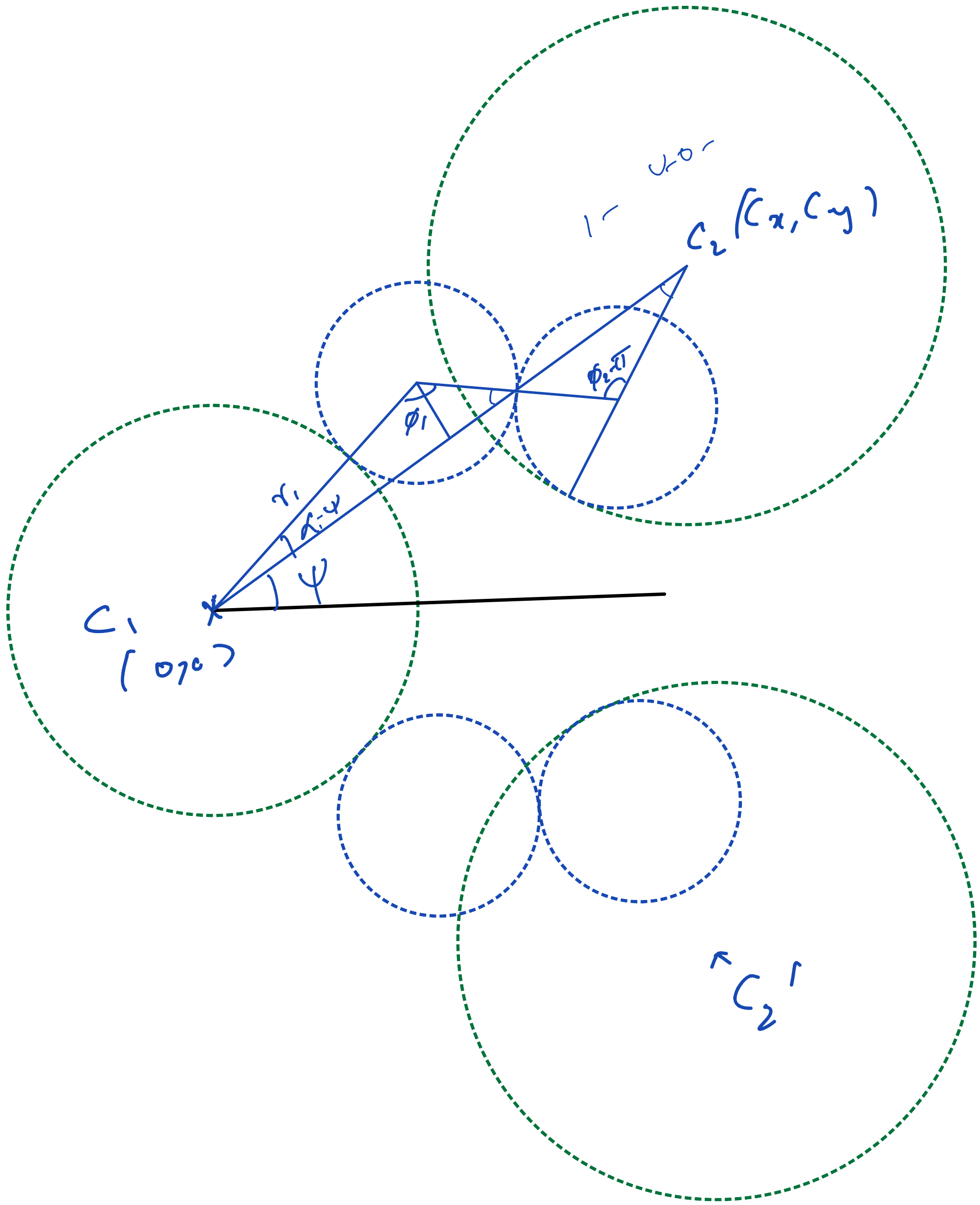
$$\alpha_1 + \phi_1 - \phi_2 = \alpha_2$$

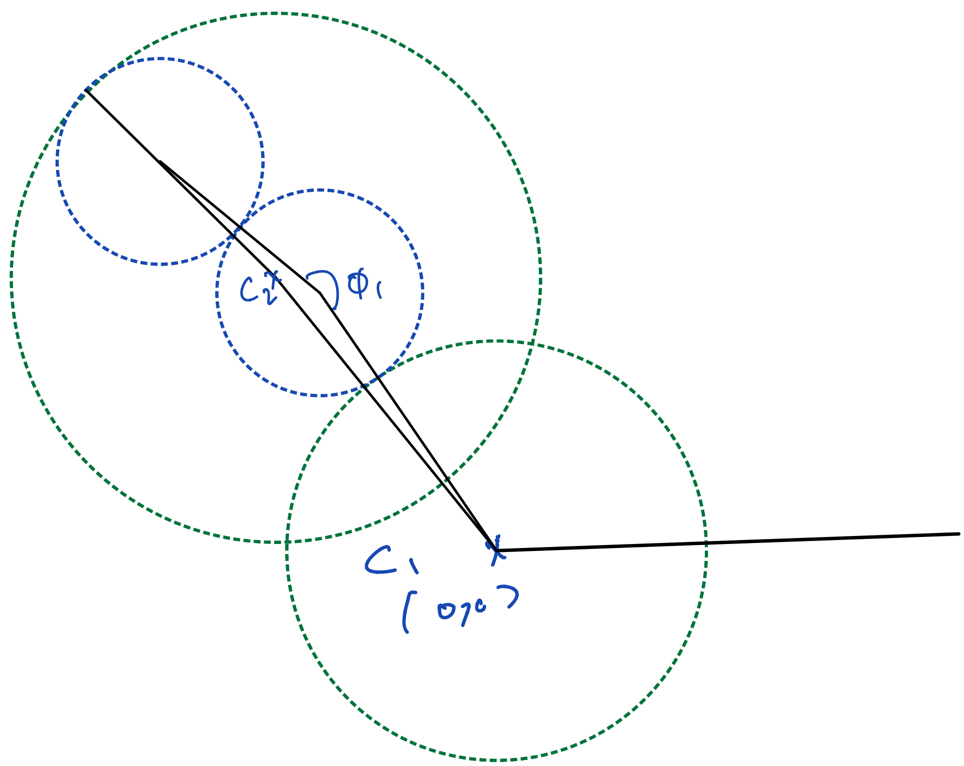
$$\phi_2 = \alpha_1 - \alpha_2 + \phi_1$$

$$\frac{\pm\sqrt{-\rho^2(1-\eta^2) + (r_2-\rho)^2}}{r_2-\rho} = \xi = \cos(\alpha_2 - \psi)$$

$$\alpha_2 = \cos^{-1}\left(\frac{\sqrt{(r_2-\rho)^2 - \rho^2(1-\eta^2)}}{(r_2-\rho)}\right) + \psi$$

$$r_2^2 \quad - 2\rho r_2 - \quad + \rho^2\eta^2$$


C_2 (C_x, C_y)
φ_1
φ_2-π
r_1
ψ
C_1
(0,0)
C_2'



$$(r_1+\rho)\sin(\alpha_1-\psi) = \rho\sin(\alpha_1+\phi_1-\psi)$$

$$= -(r_2-\rho)\sin\underbrace{(\alpha_1-\psi+\phi_1-\phi_2)}_{\alpha_2-\psi}$$

$$(r_1+\rho)\cos(\alpha_1-\psi) - 2\rho\cos(\alpha_1+\phi_1-\psi) - (r_2-\rho)\cos(\alpha_1-\psi+\phi_1-\phi_2)$$

$$= d$$

$$\eta := \cos(\alpha_1-\psi+\phi_1)$$

$$\xi := \cos(\alpha_2-\psi)$$

$$x = \cos\theta$$

$$\theta = \cos^{-1} x \quad \text{or} \quad -\cos^{-1} x$$