Derivative of length of the path w.r.t θ_1 or θ_2

RSR | RSL

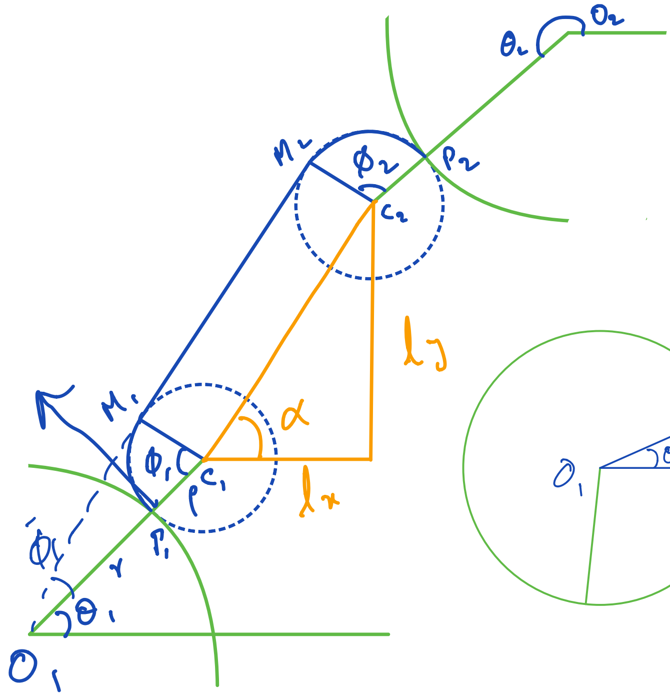
$\mu_{start} = +1$ if start is true

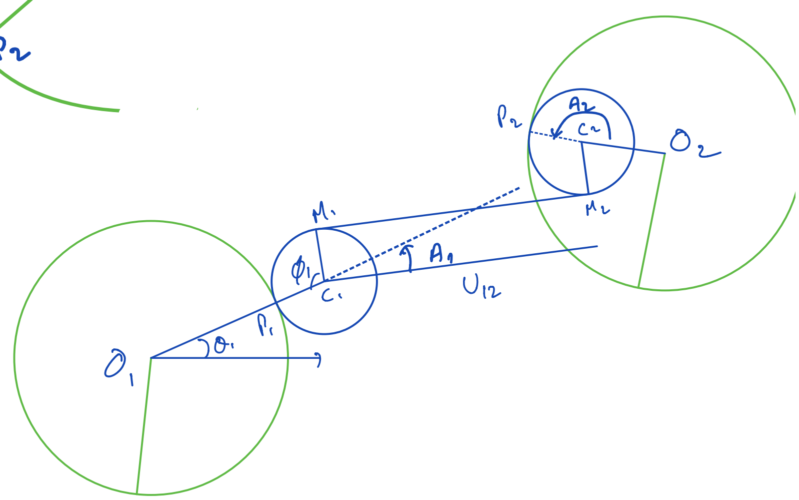
-1 if start is false

$\sin(C_2M_2 \to C_2P_2)$

$= \sin(-2\pi + \phi_2)$

$= \sin\phi_2$





$$\theta_i + \frac{\pi}{2} - \phi_1 = \alpha$$

$$\alpha - \phi_2 = \theta_f + \frac{\pi}{2}$$

RSR

$$\theta_1 + \frac{\pi}{2} - \phi_1 - \phi_2 = \theta_2 + \frac{\pi}{2}$$

$$\phi_1 + \phi_2 = \theta_1 - \theta_2$$

$$\frac{\partial(\phi_1 + \phi_2)}{\partial\theta_1} = 1, \quad \frac{\partial(\phi_1+\phi_2)}{\partial\theta_2} = -1$$

for $i=1,2$

$$\phi_i(\theta_1,\theta_2) = \cos^{-1}(C_iM_i \cdot C_iP_i) \bmod 2\pi$$

$$\boxed{\frac{\partial \phi_i}{\partial \theta_j} = -\frac{1}{\rho^2\sqrt{1-(C_iM_i\cdot C_iP_i)^2}}\left(C_iM_i \cdot \frac{\partial C_iP_i}{\partial \theta_j} + \frac{\partial C_iM_i}{\partial \theta_j}\cdot C_iP_i\right)}$$

$$\sqrt{1-(C_iM_i\cdot C_iP_i)^2} = \sin\phi_i$$

$$\boxed{\begin{array}{l}\text{if } u\cdot v = 0 \text{ or } v\cdot w = 0 \\ u\cdot w = -\|u\|\,\|w\|\left[\sin(\angle uv)\,\sin(\angle vw)\right]\end{array}}$$

$\angle uv$ is the angle between $\bar{u}$ & $\bar{v}$ in ccw from $\bar{u}$ to $\bar{v}$

$$\|C_iM_i\| = \rho \qquad C_iP_i = O_iP_i - O_iC_i$$

$$= O_i + r[\cos\theta_i, \sin\theta_i] - O_i - (r+\rho)[\cos\theta_i, \sin\theta_i]$$

$$\|C_iP_i\| = \rho \qquad = -\rho[\cos\theta_i, \sin\theta_i]$$

$$\left\| \frac{\partial C_iP_i}{\partial \theta_j} \right\| = \rho\,\delta_{ij} \quad \left[\begin{array}{ll} \delta_{ij} = 1 & \text{if } i = j \\ = 0 & \text{if } i \neq j \end{array} \right]$$

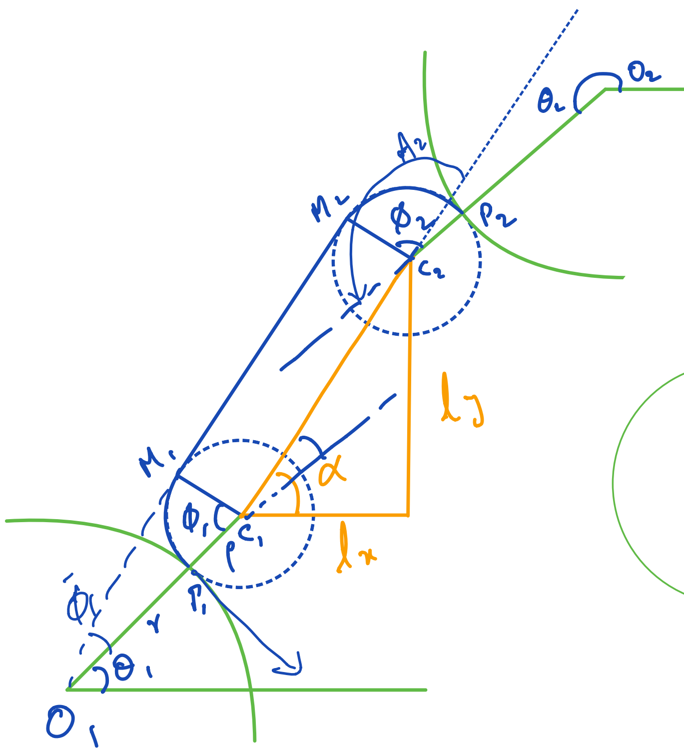
$$C_iM_i \cdot \frac{\partial C_iP_i}{\partial \theta_j} = -\tilde{\rho}\,\delta_{ij}\, \sin(\angle C_iM_i \to C_iP_i)\; \underbrace{\sin\left(\angle C_iP_i \to \frac{\partial C_iP_i}{\partial \theta_i}\right)}_{+1 \text{ always}}$$

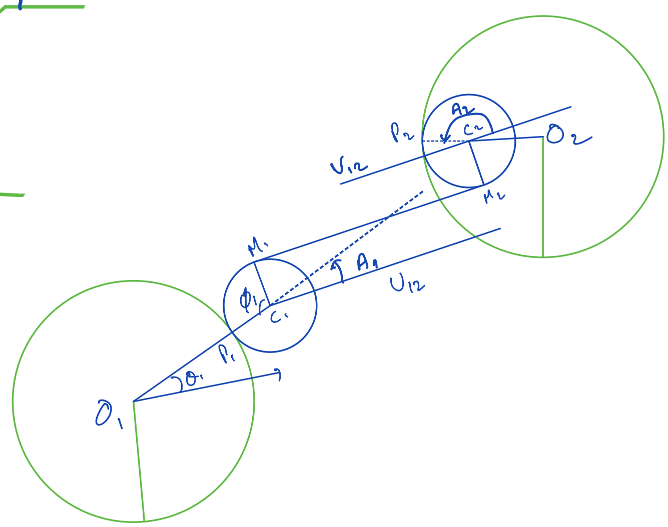
$$\boxed{C_iM_i \cdot \frac{\partial C_iP_i}{\partial \theta_j} = -\rho^2 \delta_{ij}\, \mu_{i=1}\, \mu_{C_i=R} \sin\phi_i}$$

$$\frac{\partial C_iM_i}{\partial \theta_j} \cdot C_iP_i = -\left\| \frac{\partial C_iM_i}{\partial \theta_j} \right\| \overbrace{\|C_iP_i\|}^{=\rho} \sin\left(\angle \frac{\partial C_iM_i}{\partial \theta_j} \to C_iM_i\right)$$

$$\underbrace{\sin(\angle C_iM_i \to C_iP_i)}_{\mu_{i=1}\, \mu_{C_i=R} \sin\phi_i}$$

$$\boxed{\frac{\partial \phi_i}{\partial \theta_j} = \mu_{i=1}\mu_{C_2=R}\left[\delta_{ij} + \frac{1}{\rho}\left\|\frac{\partial C_iM_i}{\partial \theta_j}\right\| \sin\left(\angle \frac{\partial C_iM_i}{\partial \theta_j} \to C_iM_i\right)\right]}$$





$$\sin(\angle U_{12} - O_1C_1)$$
$$= -\sin\left(\frac{\pi}{2} - \phi_1\right)$$
$$= -\cos \phi_1$$

$$\angle U_{12} \to O_2C_2 = \angle U_{12} \to U_{21} + \angle U_{21} \to O_2C_2 = \pi + \frac{\pi}{2} - \phi_2$$

$$\sin(\angle U_{12} \to O_2C_2) = -\cos \phi_2$$

$$\angle U_{12} \to O_1C_1$$
$$= \frac{\pi}{2} - \pi + \phi_1$$
$$= -\frac{\pi}{2} + \phi_1$$
$$\sin(\angle U_{12} \to O_1C_1) = -\cos \phi_1$$

$$\angle U_{12} \to O_2C_2 = \phi_2 - \frac{\pi}{2}$$

$$\sin(\angle U_{12} \to O_2C_2) = -\cos \phi_2$$

$$O_1O_2 = O_1C_1 + C_1M_1 + M_1M_2 + M_2C_2 + C_2O_2$$

$$M_1M_2 = O_2C_2 + C_2M_2 - O_1C_1 - C_1M_1 + O_1O_2$$

For RSR (LSL)

$$C_2M_2 = C_1M_1$$

$$M_1M_2 = O_2C_2 - O_1C_1$$

RSL (LSR)

or $C_2M_2 = -C_1M_1$

$$M_1M_2 = O_2C_2 - O_1C_1 - 2C_1M_1$$

$$\frac{\partial O_iC_i}{\partial \theta_j} = 0 \quad \text{if } i \neq j$$

$$\frac{\partial M_1M_2}{\partial \theta_j} = -\mu_{j=1} \frac{\partial O_jC_j}{\partial \theta_j}$$

$$L_s^2 = M_1M_2 \cdot M_1M_2$$

$$L_s \frac{\partial L_s}{\partial \theta_j} = M_1M_2 \cdot \frac{\partial M_1M_2}{\partial \theta_j}$$

$$\frac{\partial L_s}{\partial \theta_j} = U_{12} \cdot \frac{\partial M_1M_2}{\partial \theta_j}$$

$$\left(U_{12} = \frac{\overline{M_1M_2}}{L_s} \text{, unit vector} \right)$$

$$\frac{\partial L_s}{\partial \theta_j} = -\mu_{j=1}\, U_{12} \cdot \frac{\partial O_jC_j}{\partial \theta_j}$$

$$= \mu_{j=1} \left\| \frac{\partial O_jC_j}{\partial \theta_j} \right\| \sin\left(\angle U_{12} \to O_jC_j\right)$$

$$\underbrace{\sin\left(\angle O_jC_j \to \frac{\partial O_jC_j}{\partial \theta_j}\right)}_{+1}$$

$$= \mu_{j=1} (\gamma + \rho) \sin\left(\angle U_{12} \to O_jC_j\right)$$

for RSR

$$\boxed{\frac{\partial L_s}{\partial \theta_j} = -\mu_{j=1} (\gamma + \rho) \cos \phi_j}$$

$$\boxed{\frac{\partial \phi_i}{\partial \theta_j} = \mu_{i=1}\,\mu_{C_2=R}\left[\delta_{ij} + \frac{1}{\rho}\left\|\frac{\partial C_iM_i}{\partial \theta_j}\right\| \sin\left(\angle\, \frac{\partial C_iM_i}{\partial \theta_j} \to C_iM_i\right)\right]}$$

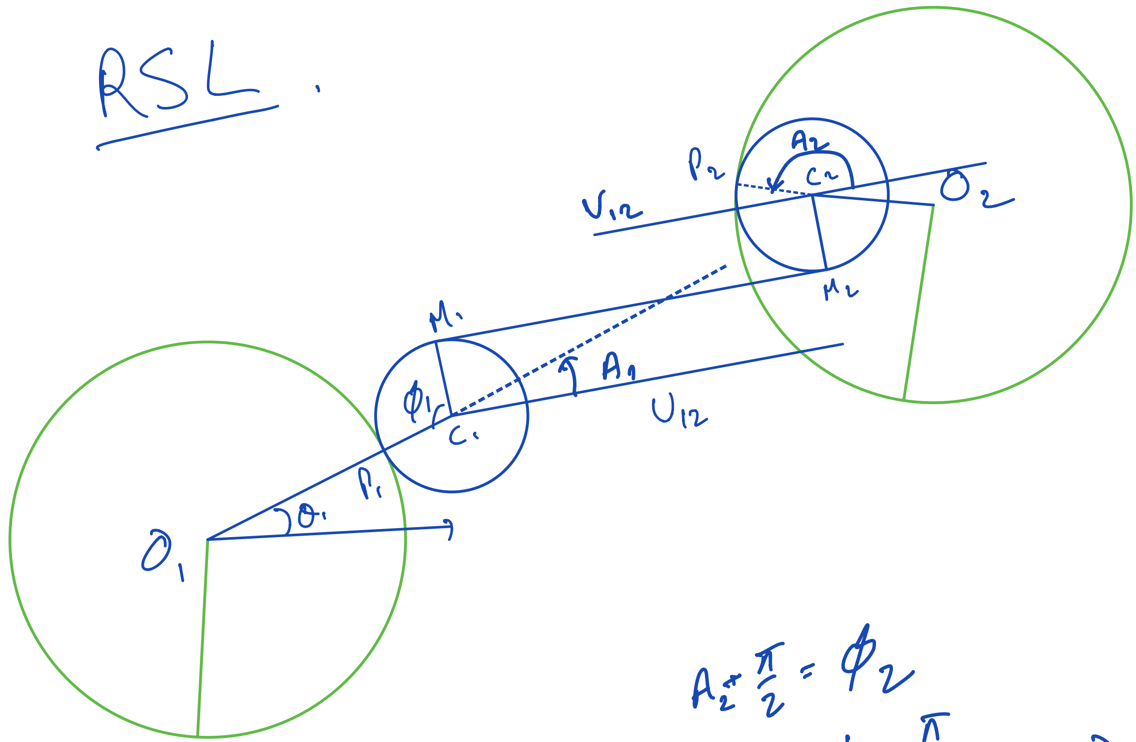
$$\frac{\partial \phi_1}{\partial \theta_j} = \delta_{1j} + \frac{1}{\rho}\left\|\frac{\partial C_1M_1}{\partial \theta_j}\right\| \sin\left(\angle\, \frac{\partial C_1M_1}{\partial \theta_j} \to C_1M_1\right)$$

$$\frac{\partial \phi_2}{\partial \theta_j} = -\delta_{2j} - \frac{1}{\rho}\left\|\frac{\partial C_1M_1}{\partial \theta_j}\right\| \sin\left(\angle\, \frac{\partial C_1M_1}{\partial \theta_j} \to C_1M_1\right)$$

$$\frac{\partial(\phi_1 + \phi_2)}{\partial \theta_j} = (\delta_{1j} - \delta_{2j}) = \mu_{j=1}$$

$$\boxed{\frac{\partial L_{RSR}}{\partial \theta_j} = \mu_{j=1}\left[\rho - (r+\rho)\cos \phi_j\right]}$$

RSL .



$$\frac{\pi}{2} - A_1 = \pi - \phi_1$$

$$A_1 = \phi_1 - \frac{\pi}{2}$$

$$\sin A_1 = -\cos\phi_1$$

$$A_2 + \frac{\pi}{2} = \phi_2$$

$$A_2 = \phi_2 - \frac{\pi}{2}$$

$$\sin A_2 = \sin\left(\phi_2 - \frac{\pi}{2}\right)$$

$$= -\cos\phi_2$$

$$M_1M_2 = O_2C_2 - O_1C_1 - 2C_1M_1$$

$$\frac{\partial M_1M_2}{\partial \theta_j} = -\mu_{j=1} \frac{\partial O_jC_j}{\partial \theta_j} - 2\frac{\partial C_1M_1}{\partial \theta_j}$$

$$\frac{\partial L_s}{\partial \theta_j} = U_{12} \cdot \frac{\partial M_1M_2}{\partial \theta_j}$$

$$U_{12} \cdot \frac{\partial O_j C_j}{\partial \theta_j} = -\overbrace{\left\| \frac{\partial O_j C_j}{\partial \theta_j} \right\|}^{(r + \mu_{c_j = R}\, \rho)} \overbrace{\sin\left(U_{12} \to O_j C_j\right)}^{-\cos(\phi_j)} \underbrace{\sin\left(O_j C_j \to \frac{\partial O_j C_j}{\partial \theta_j}\right)}_{+1}$$

$$U_{12} \cdot \frac{\partial O_j C_j}{\partial \theta_j} = (r + \mu\, c_j = R \;\; \rho) \cos \phi_j$$

$$U_{12} \cdot \frac{\partial C_1 M_1}{\partial \theta_j} = -\left\| \frac{\partial C_1 M_1}{\partial \theta_j} \right\| \overbrace{\sin\left(U_{12} \to C_1 M_1\right)}^{+1} \sin\left(C_1 M_1 \to \frac{\partial C_1 M_1}{\partial \theta_j}\right)$$

$$= -\left\| \frac{\partial C_1 M_1}{\partial \theta_j} \right\| \sin\left(C_1 M_1 \to \frac{\partial C_1 M_1}{\partial \theta_j}\right)$$

$$\frac{\partial L_S}{\partial \theta_j} = - \sum_{j=1} U_{12} \cdot \frac{\partial O_j C_j}{\partial \theta_j} - 2 U_{12} \cdot \frac{\partial C_1 M_1}{\partial \theta_j}$$

$$= - \sum_{j=1} \left(r + \sum_{C_j = R} \rho \right) \cos \phi_j$$

$$+ 2 \left\| \frac{\partial C_1 M_1}{\partial \theta_j} \right\| \sin\left(C_1 M_1 \to \frac{\partial C_1 M_1}{\partial \theta_j} \right)$$

$$\frac{\partial (\phi_1 + \phi_2)}{\partial \theta_j} = \overbrace{\left(\delta_{1j} + \delta_{2j} \right)}^{=1} +$$

$$\frac{2}{\rho} \left\| \frac{\partial C_1 M_1}{\partial \theta_j} \right\| \sin\left(\angle \frac{\partial C_1 M_1}{\partial \theta_j} \to C_1 M_1 \right)$$

$$\boxed{\frac{\partial L_{RSL}}{\partial \theta_j} = \rho - \mu_{j=1}\left(r + \mu_{c_j = R}\,\rho\right)\cos\phi_j}$$

The first derivatives w.r.t θ_1 or θ_2 vanish when the straight line segment is collinear with center of the arc_1 or arc_2, respectively.