lab.html

```html
<!DOCTYPE html>
<html lang="en">
<head>
<meta charset="UTF-8">
<title>Virtual Matrices Lab</title>
<link rel="stylesheet" href="lab.css">
</head>
<body>
<header>
  <h1>🧮 Virtual Matrices Lab</h1>
  <p>Learn Matrices Step by Step with Theory, Examples & Quizzes</p>
</header>
<nav>
  <button onclick="showSection('rank')">Rank of Matrix</button>
  <button onclick="showSection('system')">System of Linear Equations</button>
  <button onclick="showSection('eigen')">Eigenvalues & Eigenvectors</button>
  <button onclick="showSection('kirchhoff')">Kirchhoff Laws</button>
</nav>
<!-- ================= RANK ================= -->
<section id="rank" class="content active">
  <h2>Rank of a Matrix</h2>
  <p><b>Theory:</b> The rank of a matrix is the maximum number of linearly independent rows or columns in the matrix. It represents the amount of independent information present in the matrix.

To find the rank, the matrix is usually converted into row echelon form or reduced row echelon form using elementary row operations. The number of non-zero rows obtained after reduction gives the rank of the matrix.

For a matrix of order
m
×
n
m×n, the rank is always less than or equal to
min

(
m
,
n
)
min(m,n). A zero matrix has rank zero. If a square matrix has a non-zero determinant, then its rank is equal to its order.

The concept of rank is important in solving systems of linear equations. It helps determine whether a system is consistent or inconsistent and whether it has a unique solution or infinitely many solutions.</p>
<p><b>Methods to find rank</b>Methods to Find Rank of a Matrix<br>
<b>1. Row Reduction Method</b>

Convert the matrix into row echelon form or reduced row echelon form using elementary row operations.
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Count the number of non-zero rows.

The number of non-zero rows is the rank.<br>

<b>2. Using Determinants (for Square Matrices)</b>

If the determinant of a matrix is non-zero, then the rank equals the order of the matrix.

If the determinant is zero, check the determinants of smaller submatrices.

The highest order of a non-zero minor gives the rank.</p>
  <h3>Solved Examples</h3>
  <div class="example">
    <b>Example 1:</b> Find rank of A =
    <span class="matrix">[1 2]<br> [3 6]</span><br>
    <b>Solution:</b><br>
    Perform row operation: R2 → R2 − 3R1<br>
    <span class="matrix">[1 2]<br>[0 0]</span><br>
    Non-zero rows = 1 → Rank(A) = 1
  </div>

  <div class="example">
    <b>Example 2:</b> Find rank of A =
    <span class="matrix">[1 2 3]<br>[2 4 6]<br>[1 1 1]</span><br>
    R2 → R2 − 2R1 → <br><span class="matrix">[1 2 3]<br>[0 0 0]<br>[1 1 1]</span><br>
    R3 → R3 − R1 →<br> <span class="matrix">[1 2 3]<br> [0 0 0] <br>[0 -1 -2]</span><br>
    Non-zero rows = 2 → Rank(A) = 2
  </div>

  <div class="example">
    <b>Example 3:</b> Find rank using determinant A =<br> <span class="matrix">[1 2]<br> [3 4]</span><br>
    det(A) = 1*4 − 2*3 = -2 ≠ 0 → Rank(A) = 2
  </div>

  <div class="example">
    <b>Example 4:</b> A =<br><span class="matrix">[1 2]<br>[2 4]<br>[3 6]</span><br>
    R2 → R2 − 2R1, R3 → R3 − 3R1 → <br>
    <span class="matrix">[1 2]<br>[0 0]<br>[0 0]</span><br>
    Rank(A) = 1
  </div>

  <button class="quiz-btn" onclick="startQuiz('rank')">Start Rank Quiz</button>
  <p id="rankStatus" class="status"></p>
</section>

<!-- ================= SYSTEM ================= -->
<section id="system" class="content">
  <h2>System of Linear Equations</h2>
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  <p><b>Theory:</b> A system of linear equations is a set of two or more linear equations involving the same variables. Each equation represents a straight line (or plane in higher dimensions), and the solution of the system is the set of values of variables that satisfy all equations simultaneously.

A system can be represented in matrix form as:

AX=B
<br>
where

A
A is the coefficient matrix,<br>

X
X is the variable matrix, and<br>

B
B is the constant matrix.<br>

The consistency of a system is determined using the Rouché–Capelli theorem:<br>

1.If


<b>rank(A)=rank([A|B])</b>, the system is consistent
<br>
2.If <b>
rank
(
A
)
≠
rank
(
[
A
|
B
]
)
,</b> the system is inconsistent.<br>

If the system is consistent:
<br>
If <b>rank = number of variables</b> → unique solution<br>

If <b>rank < number of variables</b> → infinitely many solutions</p>

  <h3>Solved Examples</h3>
  <div class="example">
    <b>Example 1 (Unique Solution):</b><br>
    x + y = 5<br>
    x − y = 1<br>
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    Add equations: 2x=6 → x=3<br>
    Substitute: 3 + y=5 → y=2<br>
    Solution: x=3, y=2
  </div>

  <div class="example">
    <b>Example 2 (No Solution):</b><br>
    x + y = 2<br>
    2x + 2y = 5<br>
    Multiply first eq by 2: 2x + 2y = 4<br>
    Contradiction → No solution
  </div>

  <div class="example">
    <b>Example 3 (Infinite Solutions):</b><br>
    x + y = 4<br>
    2x + 2y = 8<br>
    Second eq is multiple of first → infinitely many solutions
  </div>

  <div class="example">
    <b>Example 4 (Three variables):</b><br>
    x+y+z=6<br>
    x+2y+3z=14<br>
    2x+y+z=7<br>
    Augmented matrix:<br>
    [1 1 1 |6; 1 2 3 |14; 2 1 1 |7]<br>
    R2→R2−R1, R3→R3−2R1 → [1 1 1 |6; 0 1 2 |8; 0 −1 −1 |−5]<br>
    R3→R3+R2 → [1 1 1 |6; 0 1 2 |8; 0 0 1 |3]<br>
    Back substitution: z=3, y=2, x=1
  </div>

  <button class="quiz-btn" onclick="startQuiz('system')">Start System Quiz</button>
  <p id="systemStatus" class="status"></p>
</section>

<!-- ================= EIGEN ================= -->
<section id="eigen" class="content">
  <h2>Eigenvalues & Eigenvectors</h2>
  <p><b>Theory:</b> For a square matrix
A
A, a non-zero vector
v
v is called an eigenvector of
A
A if multiplication by
A
A only scales the vector, not changes its direction. The scalar factor
λ
λ is called the eigenvalue. <br>
Mathematically:<br>
Av=λv<br>
where:
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```
A
A = square matrix<br>

v
v = eigenvector (
v
≠
0)<br>
λ = eigenvalue
  <h3>Solved Examples</h3>
  <div class="example">
    <b>Example 1:</b> A=[2 0;0 3]<br>
    Characteristic eq: det(A−λI)=0 → (2−λ)(3−λ)=0 → λ1=2, λ2=3<br>
    Eigenvectors: λ=2 → v=[1 0], λ=3 → v=[0 1]
  </div>

  <div class="example">
    <b>Example 2:</b> A=[1 1;1 1]<br>
    det(A−λI)=λ(λ−2)=0 → λ1=0, λ2=2<br>
    Eigenvectors: λ=0 → v=[1 −1], λ=2 → v=[1 1]
  </div>

  <div class="example">
    <b>Example 3:</b> Diagonal A=[3 0 0;0 4 0;0 0 5]<br>
    λ=3,4,5 → Eigenvectors along axes: v1=[1 0 0], v2=[0 1 0], v3=[0 0 1]
  </div>

  <div class="example">
    <b>Example 4:</b> A=[0 1; −2 −3]<br>
    det(A−λI)=λ²+3λ+2=0 → λ=−1,−2<br>
    Eigenvectors: λ=−1 → v=[1 −1], λ=−2 → v=[1 −2]
  </div>

  <button class="quiz-btn" onclick="startQuiz('eigen')">Start Eigen Quiz</button>
  <p id="eigenStatus" class="status"></p>
</section>

<!-- ================= QUIZ ================= -->
<section id="quiz" class="content">
  <h2 id="quizTitle"></h2>
  <form id="quizForm"></form>
  <button onclick="submitQuiz()">Submit Quiz</button>
  <div id="result"></div>
</section>
<section>
  <section id="kirchhoff" class="content">
  <div class="card">
    <h2>Application of Matrices in Kirchhoff’s Laws</h2>

    <p>
      Kirchhoff’s laws are used to analyze electrical circuits.
      Matrix methods help solve multiple loop or node equations
      efficiently using linear algebra.
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    </p>

    <h3>Theory</h3>
    <p>
      Kirchhoff’s Voltage Law (KVL) states that the algebraic sum of
      voltages around any closed loop is zero.
      Kirchhoff’s Current Law (KCL) states that the algebraic sum of
      currents at a junction is zero.
    </p>

    <p>
      When a circuit has multiple loops, each loop gives one linear
      equation. These equations can be written in matrix form:
    </p>

    <p class="matrix">
      A · X = B
    </p>

    <ul>
      <li>A → coefficient matrix (resistances)</li>
      <li>X → current matrix (unknown currents)</li>
      <li>B → voltage matrix (sources)</li>
    </ul>

    <p>
      Solving this matrix equation using row reduction gives the
      values of currents in the circuit.
    </p>
  </div>

  <div class="card">
    <h3>Solved Examples</h3>

    <ol>
      <li>
        <b>Single Loop:</b><br>
        R = 5Ω, V = 10V<br>
        5I = 10 → I = 2A
      </li>

      <li>
        <b>Two Loop Circuit:</b><br>
        3I₁ + I₂ = 10<br>
        I₁ + 2I₂ = 5<br>
        Matrix form:
        <div class="matrix">
          [ 3  1 ] [ I₁ ] = [10]<br>
          [ 1  2 ] [ I₂ ]   [5]
        </div>
        Solution: I₁ = 3A, I₂ = 1A
      </li>

      <li>
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        <b>Using KCL:</b><br>
        I₁ + I₂ − I₃ = 0 → matrix equation
      </li>

      <li>
        <b>Three-loop circuit:</b><br>
        Solved using augmented matrix and row operations
      </li>
    </ol>
  </div>

  <button class="quiz-btn" onclick="startQuiz('kirchhoff')">
    Start Kirchhoff Quiz
  </button>

  <p id="kirchhoffStatus" class="status"></p>
</section>

</section>


<script src="lab.js"></script>
</body>
</html>

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