


lab.html

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1  <!DOCTYPE html>
2  <html lang="en">
3  <head>
4  <meta charset="UTF-8">
5  <title>Virtual Matrices Lab</title>
6  <link rel="stylesheet" href="lab.css">
7  </head>
8  <body>
9  <header>
10   <h1> Virtual Matrices Lab</h1>
11   <p>Learn Matrices Step by Step with Theory, Examples & Quizzes</p>
12 </header>
13 <nav>
14   <button onclick="showSection('rank')">Rank of Matrix</button>
15   <button onclick="showSection('system')">System of Linear Equations</button>
16   <button onclick="showSection('eigen')">Eigenvalues & Eigenvectors</button>
17   <button onclick="showSection('kirchhoff')">Kirchhoff Laws</button>
18 </nav>
19 <!-- ===== RANK ===== -->
20 <section id="rank" class="content active">
21   <h2>Rank of a Matrix</h2>
22   <p><b>Theory:</b> The rank of a matrix is the maximum number of linearly
23     independent rows or columns in the matrix. It represents the amount of independent
24     information present in the matrix.
25
26     To find the rank, the matrix is usually converted into row echelon form or reduced
27     row echelon form using elementary row operations. The number of non-zero rows
28     obtained after reduction gives the rank of the matrix.
29
30     For a matrix of order
31     
$$\begin{pmatrix} m \\ \times \\ n \end{pmatrix}$$

32      $m \times n$ , the rank is always less than or equal to
33      $\min(m, n)$ .
34     A zero matrix has rank zero. If a square matrix has a non-zero
35     determinant, then its rank is equal to its order.
36
37     The concept of rank is important in solving systems of linear equations. It helps
38     determine whether a system is consistent or inconsistent and whether it has a
39     unique solution or infinitely many solutions.</p>
40   <p><b>Methods to find rank</b>Methods to Find Rank of a Matrix<b></b>
41   <b>1. Row Reduction Method</b>
42
43   Convert the matrix into row echelon form or reduced row echelon form using
44   elementary row operations.

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45
46 Count the number of non-zero rows.
47
48 The number of non-zero rows is the rank.<br>
49
50 <b>2. Using Determinants (for Square Matrices)</b>
51
52 If the determinant of a matrix is non-zero, then the rank equals the order of the
  matrix.
53
54 If the determinant is zero, check the determinants of smaller submatrices.
55
56 The highest order of a non-zero minor gives the rank.</p>
57 <h3>Solved Examples</h3>
58 <div class="example">
59   <b>Example 1:</b> Find rank of A =
60   <span class="matrix">[1 2]<br> [3 6]</span><br>
61   <b>Solution:</b><br>
62   Perform row operation:  $R_2 \rightarrow R_2 - 3R_1$ <br>
63   <span class="matrix">[1 2]<br>[0 0]</span><br>
64   Non-zero rows = 1  $\rightarrow$  Rank(A) = 1
65 </div>
66
67 <div class="example">
68   <b>Example 2:</b> Find rank of A =
69   <span class="matrix">[1 2 3]<br>[2 4 6]<br>[1 1 1]</span><br>
70    $R_2 \rightarrow R_2 - 2R_1 \rightarrow$  <br><span class="matrix">[1 2 3]<br>[0 0 0]<br>[1 1 1]</span>
<br>
71    $R_3 \rightarrow R_3 - R_1 \rightarrow$  <br> <span class="matrix">[1 2 3]<br> [0 0 0] <br>[0 -1 -2]
</span><br>
72   Non-zero rows = 2  $\rightarrow$  Rank(A) = 2
73 </div>
74
75 <div class="example">
76   <b>Example 3:</b> Find rank using determinant A =<br> <span class="matrix">[1
77 2]<br> [3 4]</span><br>
78    $\det(A) = 1 \cdot 4 - 2 \cdot 3 = -2 \neq 0 \rightarrow$  Rank(A) = 2
79 </div>
80
81 <div class="example">
82   <b>Example 4:</b> A =<br><span class="matrix">[1 2]<br>[2 4]<br>[3 6]</span>
<br>
83    $R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - 3R_1 \rightarrow$  <br>
84   <span class="matrix">[1 2]<br>[0 0]<br>[0 0]</span><br>
85   Rank(A) = 1
86 </div>
87 <button class="quiz-btn" onclick="startQuiz('rank')">Start Rank Quiz</button>
88 <p id="rankStatus" class="status"></p>
89 </section>
90
91 <!-- ===== SYSTEM ===== -->
92 <section id="system" class="content">
93   <h2>System of Linear Equations</h2>

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94  <p><b>Theory:</b> A system of linear equations is a set of two or more linear
    equations involving the same variables. Each equation represents a straight line
    (or plane in higher dimensions), and the solution of the system is the set of
    values of variables that satisfy all equations simultaneously.
95
96  A system can be represented in matrix form as:
97
98   $AX=B$ 
99  <br>
100 where
101
102  $A$ 
103  $A$  is the coefficient matrix,<br>
104
105  $X$ 
106  $X$  is the variable matrix, and<br>
107
108  $B$ 
109  $B$  is the constant matrix.<br>
110
111 The consistency of a system is determined using the Rouché–Capelli theorem:<br>
112
113 1.If
114
115
116 <b>rank(A)=rank([A|B])</b>, the system is consistent
117 <br>
118 2.If <b>
119 rank
120 (
121  $A$ 
122 )
123  $\neq$ 
124 rank
125 (
126 [
127  $A$ 
128 |
129  $B$ 
130 ]
131 )
132 ,</b> the system is inconsistent.<br>
133
134 If the system is consistent:
135 <br>
136 If <b>rank = number of variables</b>  $\rightarrow$  unique solution<br>
137
138 If <b>rank < number of variables</b>  $\rightarrow$  infinitely many solutions</p>
139
140 <h3>Solved Examples</h3>
141 <div class="example">
142   <b>Example 1 (Unique Solution):</b><br>
143    $x + y = 5$ <br>
144    $x - y = 1$ <br>
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145     Add equations:  $2x=6 \rightarrow x=3$ <br>
146     Substitute:  $3 + y=5 \rightarrow y=2$ <br>
147     Solution:  $x=3, y=2$ 
148 </div>
149
150 <div class="example">
151     <b>Example 2 (No Solution):</b><br>
152      $x + y = 2$ <br>
153      $2x + 2y = 5$ <br>
154     Multiply first eq by 2:  $2x + 2y = 4$ <br>
155     Contradiction  $\rightarrow$  No solution
156 </div>
157
158 <div class="example">
159     <b>Example 3 (Infinite Solutions):</b><br>
160      $x + y = 4$ <br>
161      $2x + 2y = 8$ <br>
162     Second eq is multiple of first  $\rightarrow$  infinitely many solutions
163 </div>
164
165 <div class="example">
166     <b>Example 4 (Three variables):</b><br>
167      $x+y+z=6$ <br>
168      $x+2y+3z=14$ <br>
169      $2x+y+z=7$ <br>
170     Augmented matrix:<br>
171      $[1 \ 1 \ 1 \ | \ 6; \ 1 \ 2 \ 3 \ | \ 14; \ 2 \ 1 \ 1 \ | \ 7]$ <br>
172      $R2 \rightarrow R2 - R1, \ R3 \rightarrow R3 - 2R1 \rightarrow [1 \ 1 \ 1 \ | \ 6; \ 0 \ 1 \ 2 \ | \ 8; \ 0 \ -1 \ -1 \ | \ -5]$ <br>
173      $R3 \rightarrow R3 + R2 \rightarrow [1 \ 1 \ 1 \ | \ 6; \ 0 \ 1 \ 2 \ | \ 8; \ 0 \ 0 \ 1 \ | \ 3]$ <br>
174     Back substitution:  $z=3, y=2, x=1$ 
175 </div>
176
177 <button class="quiz-btn" onclick="startQuiz('system')">Start System
Quiz</button>
178 <p id="systemStatus" class="status"></p>
179 </section>
180
181 <!-- ===== EIGEN ===== -->
182 <section id="eigen" class="content">
183     <h2>Eigenvalues & Eigenvectors</h2>
184     <p><b>Theory:</b> For a square matrix
185     A
186     A, a non-zero vector
187     v
188     v is called an eigenvector of
189     A
190     A if multiplication by
191     A
192     A only scales the vector, not changes its direction. The scalar factor
193      $\lambda$ 
194      $\lambda$  is called the eigenvalue. <br>
195     Mathematically:<br>
196      $Av=\lambda v$ <br>
197     where:

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198
199 A
200 A = square matrix<br>
201
202 v
203 v = eigenvector (
204 v
205 ≠
206 0)<br>
207 λ = eigenvalue
208 <h3>Solved Examples</h3>
209 <div class="example">
210   <b>Example 1:</b> A=[2 0;0 3]<br>
211   Characteristic eq:  $\det(A-\lambda I)=0 \rightarrow (2-\lambda)(3-\lambda)=0 \rightarrow \lambda_1=2, \lambda_2=3$ <br>
212   Eigenvectors:  $\lambda=2 \rightarrow v=[1 \ 0], \lambda=3 \rightarrow v=[0 \ 1]$ 
213 </div>
214
215 <div class="example">
216   <b>Example 2:</b> A=[1 1;1 1]<br>
217    $\det(A-\lambda I)=\lambda(\lambda-2)=0 \rightarrow \lambda_1=0, \lambda_2=2$ <br>
218   Eigenvectors:  $\lambda=0 \rightarrow v=[1 \ -1], \lambda=2 \rightarrow v=[1 \ 1]$ 
219 </div>
220
221 <div class="example">
222   <b>Example 3:</b> Diagonal A=[3 0 0;0 4 0;0 0 5]<br>
223    $\lambda=3,4,5 \rightarrow$  Eigenvectors along axes:  $v_1=[1 \ 0 \ 0], v_2=[0 \ 1 \ 0], v_3=[0 \ 0 \ 1]$ 
224 </div>
225
226 <div class="example">
227   <b>Example 4:</b> A=[0 1; -2 -3]<br>
228    $\det(A-\lambda I)=\lambda^2+3\lambda+2=0 \rightarrow \lambda=-1, -2$ <br>
229   Eigenvectors:  $\lambda=-1 \rightarrow v=[1 \ -1], \lambda=-2 \rightarrow v=[1 \ -2]$ 
230 </div>
231
232 <button class="quiz-btn" onclick="startQuiz('eigen')">Start Eigen Quiz</button>
233 <p id="eigenStatus" class="status"></p>
234 </section>
235
236 <!-- ===== QUIZ ===== -->
237 <section id="quiz" class="content">
238   <h2 id="quizTitle"></h2>
239   <form id="quizForm"></form>
240   <button onclick="submitQuiz()">Submit Quiz</button>
241   <div id="result"></div>
242 </section>
243 <section>
244   <section id="kirchhoff" class="content">
245     <div class="card">
246       <h2>Application of Matrices in Kirchhoff's Laws</h2>
247
248       <p>
249         Kirchhoff's laws are used to analyze electrical circuits.
250         Matrix methods help solve multiple loop or node equations
251         efficiently using linear algebra.

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252     </p>
253
254     <h3>Theory</h3>
255     <p>
256         Kirchhoff's Voltage Law (KVL) states that the algebraic sum of
257         voltages around any closed loop is zero.
258         Kirchhoff's Current Law (KCL) states that the algebraic sum of
259         currents at a junction is zero.
260     </p>
261
262     <p>
263         When a circuit has multiple loops, each loop gives one linear
264         equation. These equations can be written in matrix form:
265     </p>
266
267     <p class="matrix">
268          $A \cdot X = B$ 
269     </p>
270
271     <ul>
272         <li>A → coefficient matrix (resistances)</li>
273         <li>X → current matrix (unknown currents)</li>
274         <li>B → voltage matrix (sources)</li>
275     </ul>
276
277     <p>
278         Solving this matrix equation using row reduction gives the
279         values of currents in the circuit.
280     </p>
281 </div>
282
283 <div class="card">
284     <h3>Solved Examples</h3>
285
286     <ol>
287         <li>
288             <b>Single Loop:</b><br>
289              $R = 5\Omega$ ,  $V = 10V$ <br>
290              $5I = 10 \rightarrow I = 2A$ 
291         </li>
292
293         <li>
294             <b>Two Loop Circuit:</b><br>
295              $3I_1 + I_2 = 10$ <br>
296              $I_1 + 2I_2 = 5$ <br>
297             Matrix form:
298             <div class="matrix">
299                  $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$ <br>
300             </div>
301             Solution:  $I_1 = 3A$ ,  $I_2 = 1A$ 
302         </li>
303     </ol>
304
305     <li>
```

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306     <b>Using KCL:</b><br>
307      $I_1 + I_2 - I_3 = 0 \rightarrow$  matrix equation
308 </li>
309
310 <li>
311     <b>Three-loop circuit:</b><br>
312     Solved using augmented matrix and row operations
313 </li>
314 </ol>
315 </div>
316
317 <button class="quiz-btn" onclick="startQuiz('kirchhoff')">
318     Start Kirchhoff Quiz
319 </button>
320
321 <p id="kirchhoffStatus" class="status"></p>
322 </section>
323
324 </section>
325
326
327 <script src="lab.js"></script>
328 </body>
329 </html>
330
```