ManyBabies 5: The Hunter and Ames Model of Infant Looking Preference

Supplementary Materials: Data Simulation and Power Analysis

ManyBabies Analysis Team

Contents

1 Data Simulation

```
my_sim_data <- function(
    n_subj      = 1280,   # number of subjects
    n_simple  =  12,   # number of complex stimuli
    n_complex =  12,   # number of complex stimuli
    n_small_fam = 8,   #small familiarization time
    n_medium_fam = 8,  #medium familiarization time
    n_high_fam = 8,    #high familiarization time
    n_lab = 40,

    beta_0  =  0, # intercept; i.e., the grand mean
    beta_c  =  0.3, # main effect for complexity
    beta_f = 0.3, # main effect for familiarization time
    beta_a = 0.3, # main effect for age

    beta_ca = 0.3,
    beta_af = 0.3,
    beta_cf = 0.3,

    beta_cfa = 0.3, #main effect for interaction between complexity and familiarization.

    subject_0   = 0.2, # by-subject random intercept sd

    subject_c   =  0.2, # by-subject slope complexity sd
    subject_f = 0.2, # by-subject slope familiarization sd
    subject_a = 0.2, # by-subject slope age sd

    subject_ca = 0.2,# by-subject slope for interaction between age and complexity sd
    subject_af = 0.2, # by-subject slope for interaction between age and familiarization sd
    subject_cf = 0.2, # by-subject slope complexity*familiarization sd

    subject_cfa = 0.2, # by-subject slope for interaction between age, complexity and familiarization sd

    subj_rho     =  .2, # correlations between by-subject random effects

    lab_0 = 0.2, # by-lab random intercept sd

    lab_c = 0.2, # by-lab slope complexity sd
    lab_f = 0.2, # by-lab slope familiarization sd
    lab_a = 0.2, # by-lab slope age sd

    lab_ca = 0.2, # by-lab slope for interaction between age and complexity sd
    lab_af = 0.2, # by-lab slope for interaction between age and familiarization sd
    lab_cf = 0.2, # by-lab random slope complexity*familiarization sd

    lab_cfa = 0.2, # by-lab slope for interaction between age, complexity and familiarization sd

    lab_rho = 0.2, # correlations between by-lab random effects

    item_0 =  0.2, # by-item random intercept sd

    item_c = 0.2, # by-item slope complexity sd
    item_f = 0.2, # by-item slope familiarization sd
    item_a = 0.2, # by-item slope age sd

    item_ca = 0.2, # by-item slope for interaction between age and complexity sd
    item_af = 0.2, # by-item slope for interaction between age and familiarization sd
    item_cf = 0.2, # by-item random slope complexity*familiarization sd

    item_cfa = 0.2, # by-item slope for interaction between age, complexity and familiarization sd
```

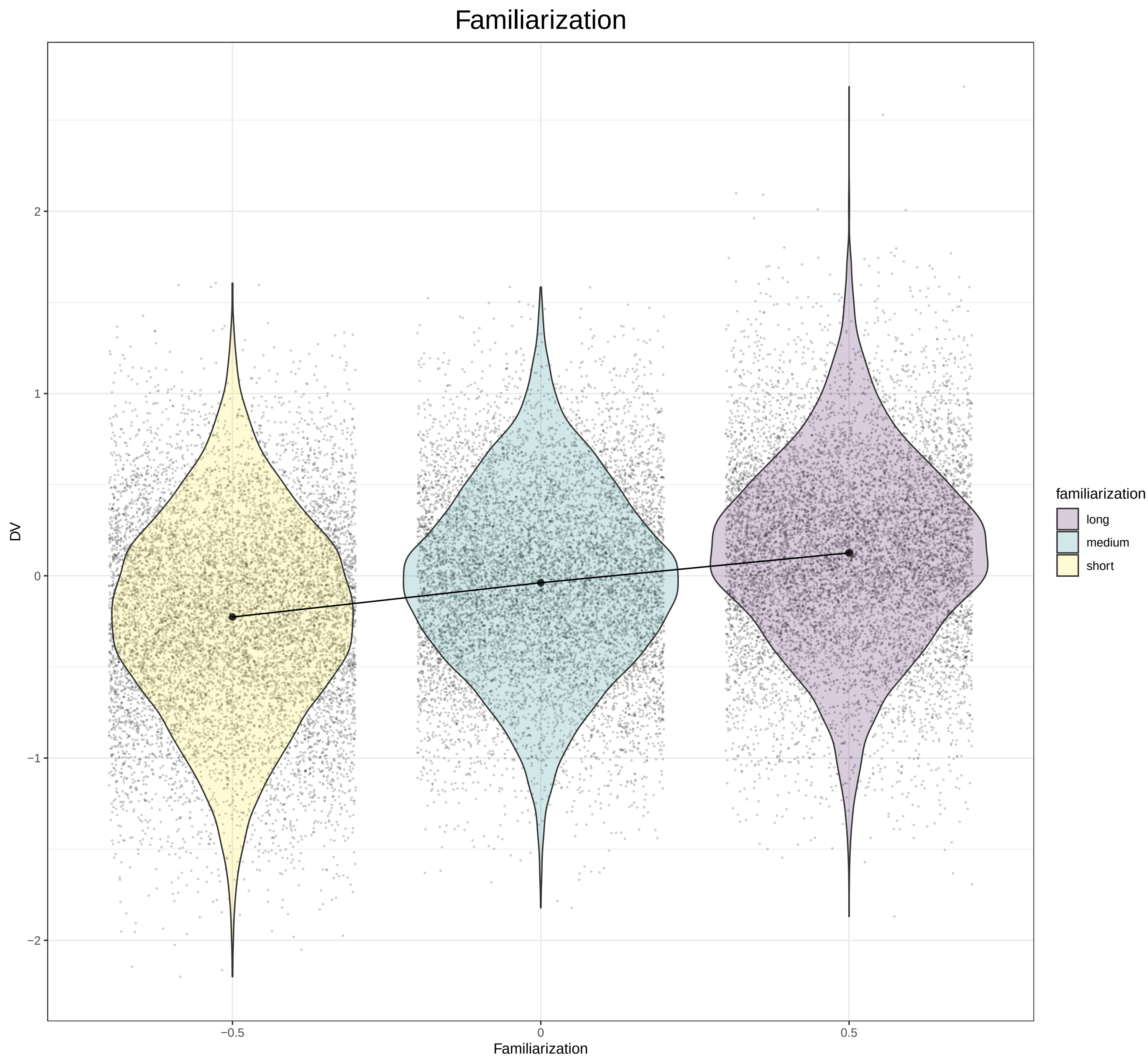
```
  item_rho = 0.2, # correlations between by-item random effects


  sigma = 0.3 # residual (error) sd
  ) { # residual (standard deviation)

# simulate a sample of items
items <- data.frame(
  category = rep(c("simple", "complex"), c(n_simple, n_complex)),
  X_c = rep(c(-0.5, 0.5), c(n_simple, n_complex)),
  familiarization = rep(c("short", "medium", "long"), (n_simple + n_complex)/3),
  X_f = rep(c(-0.5, 0, 0.5), (n_simple + n_complex)/3),
  faux::rnorm_multi(
  n = n_simple + n_complex, mu = 0, sd = c(item_0,
                               item_c,
                               item_f,
                               item_a,
                               item_ca,
                               item_af,
                               item_cf,
                               item_cfa), r = item_rho,
  varnames = c("I_0", "I_c","I_f","I_a",
               "I_ca","I_af", "I_cf",
               "I_cfa"))
  ) %>%
  mutate(item_id = faux::make_id(nrow(.), "I"))

# simulate a sample of subjects
subjects <-
  faux::rnorm_multi(
  n = n_subj, mu = 0, sd = c(subject_0,
                             subject_c,
                             subject_f,
                             subject_a,
                             subject_ca,
                             subject_af,
                             subject_cf,
                             subject_cfa), r = subj_rho,
  varnames = c("S_0", "S_c","S_f","S_a",
               "S_ca","S_af", "S_cf",
               "S_cfa")
) %>%
  mutate(subj_id = faux::make_id(nrow(.), "S")) %>%
  mutate(X_a = runif(n_subj, min = -0.5, max = 0.5))
#add subject age measure, sample from distribution from -0.5 to 0.5. #subjects$subj_id <- 1:n_subj

labs <- faux::rnorm_multi(
  n = n_lab, mu = 0, sd = c(lab_0, lab_c, lab_f, lab_a,
                            lab_ca, lab_af, lab_cf,
                            lab_cfa), r = lab_rho,
  varnames = c("L_0", "L_c","L_f","L_a",
               "L_ca","L_af", "L_cf",
               "L_cfa")
) %>%
  mutate(lab_id = faux::make_id(nrow(.), "L"))

#create lab and subj nesting structure
#Number of subjects must be a multiple of number of labs
lab_multiplier = n_subj/n_lab
lab_subj_dict <- data.frame(
  subj_id = subjects$subj_id,
```

```
    lab_id = rep(labs$lab_id,lab_multiplier)
  )

  # cross subject and item IDs
  temp <- crossing(subjects, items)  %>%
    left_join(lab_subj_dict, by = "subj_id") %>%
    left_join(labs, by = "lab_id") %>%
    group_by(subj_id, item_id) %>%
    mutate(item_id = sample(item_id)) %>%
    ungroup() %>%
    mutate(trial_num = rep(seq(n_simple + n_complex), n_subj))

  temp <- temp %>%
    mutate(
      B_0  = beta_0 + S_0 + L_0 + I_0,

      B_c  = beta_c + S_c + L_c + I_c,
      B_f  = beta_f + S_f + L_f + I_f,
      B_a = beta_a + S_a + L_a + I_a,

      B_ca = beta_ca + S_ca + L_ca + I_ca,
      B_af = beta_af + S_af + L_af + I_af,
      B_cf = beta_cf + S_cf + L_cf + I_cf,

      B_cfa = beta_cfa + S_cfa + L_cfa + I_cfa,

      e_si = rnorm(nrow(temp), mean = 0, sd = sigma),

      DV = B_0 +
        (B_a * X_a) + (B_c * X_c) + (B_f * X_f) +
        (B_cf * X_c * X_f) + (B_af * X_a * X_f) + (B_ca * X_c * X_a) +
        (B_cfa * X_c * X_f * X_a) + e_si
    )
}

dat_sim <- my_sim_data()
```

2 Visualisation of Simulated Data

2.1 Familiarization:

```
dat_sim_plot_familiarization <- dat_sim %>%
    group_by(X_f) %>%
    dplyr::summarise(med_DV = median(DV))

plot_familiarization <- dat_sim %>%
    mutate(X_f = as.factor(X_f)) %>%
    ggplot() + geom_point(aes(y = DV, x = X_f), position = "jitter",
    alpha = 0.2, size = 0.2) + geom_violin(aes(y = DV, x = X_f,
    fill = familiarization), alpha = 0.2) + geom_line(aes(y = med_DV,
    x = as.factor(X_f), group = 1), data = dat_sim_plot_familiarization) +
    geom_point(aes(y = med_DV, x = as.factor(X_f)), alpha = 0.8,
        size = 2, data = dat_sim_plot_familiarization) + scale_fill_manual(values = viridis(n = 3)) +
    ggtitle("Familiarization") + xlab("Familiarization") + theme_bw()

plot_familiarization <- plot_familiarization + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
plot_familiarization
```



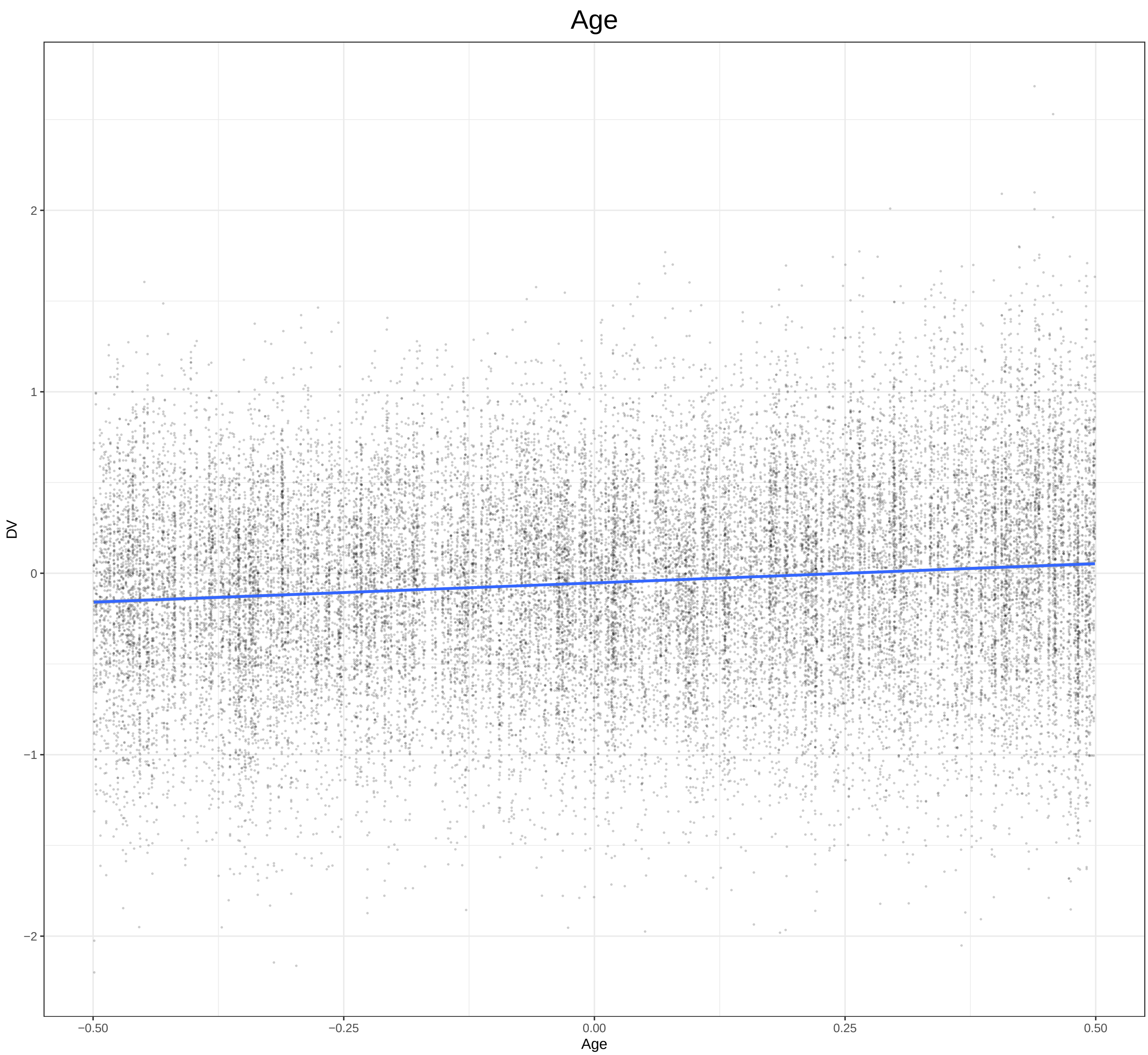
2.2 Complexity:

```
dat_sim_plot_complexity <- dat_sim %>%
    group_by(X_c) %>%
    dplyr::summarise(med_DV = median(DV))

plot_complexity <- dat_sim %>%
    mutate(X_c = as.factor(X_c)) %>%
    ggplot() + geom_point(aes(y = DV, x = X_c), position = "jitter",
    alpha = 0.2, size = 0.2) + geom_violin(aes(y = DV, x = X_c,
    fill = category), alpha = 0.2) + geom_line(aes(y = med_DV,
    x = as.factor(X_c), group = 1), data = dat_sim_plot_complexity) +
    geom_point(aes(y = med_DV, x = as.factor(X_c)), alpha = 0.8,
        size = 2, data = dat_sim_plot_complexity) + scale_fill_manual(values = viridis(n = 2)) +
    ggtitle("Stimulus Complexity") + xlab("Stimulus Complexity") +
    theme_bw()
```

```
plot_complexity <- plot_complexity + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
plot_complexity
```

2.3 Age:

```
plot_age <- dat_sim %>%
    ggplot() + geom_point(aes(y = DV, x = X_a), position = "jitter",
    alpha = 0.2, size = 0.2) + geom_smooth(method = "lm", se = TRUE,
    formula = y ~ x, aes(y = DV, x = X_a)) + ggtitle("Age") +
    xlab("Age") + theme_bw()

plot_age <- plot_age + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
plot_age
```

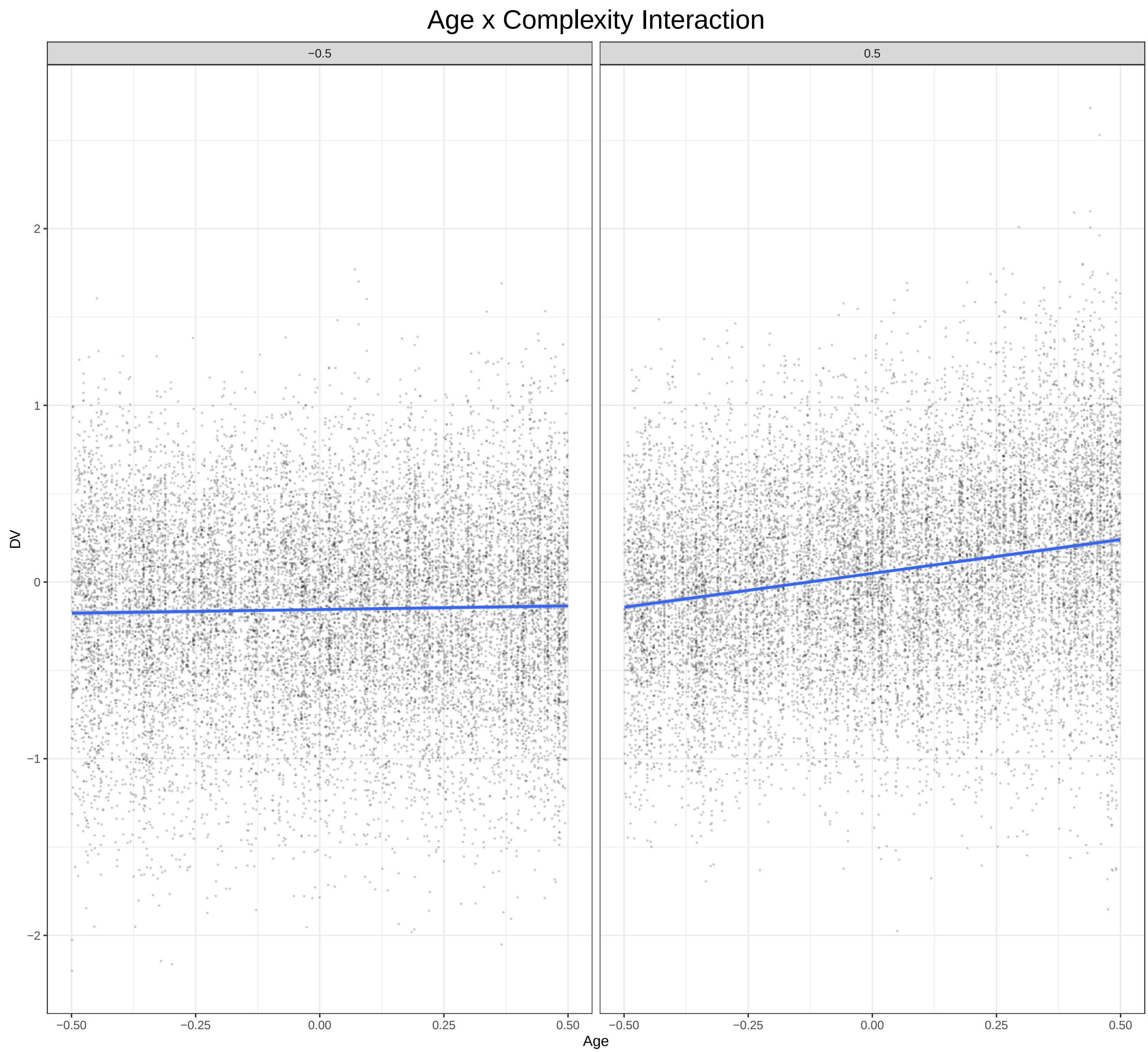


2.4 Age*Familiarization:

```
plot_age_familiarization <- dat_sim %>%
    ggplot() + geom_point(aes(y = DV, x = X_a), position = "jitter",
    alpha = 0.2, size = 0.2) + geom_smooth(method = "lm", formula = y ~
    x, se = TRUE, aes(y = DV, x = X_a)) + facet_wrap(~X_f) +
    ggtitle("Age x Familiarization Interaction") + xlab("Age") +
    theme_bw()
plot_age_familiarization <- plot_age_familiarization + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
plot_age_familiarization
```

2.5 Age*Complexity:

```
plot_age_complexity <- dat_sim %>%
    ggplot() + geom_point(aes(y = DV, x = X_a), position = "jitter",
    alpha = 0.2, size = 0.2) + geom_smooth(method = "lm", formula = y ~
    x, se = TRUE, aes(y = DV, x = X_a)) + facet_wrap(~X_c) +
    ggtitle("Age x Complexity Interaction") + xlab("Age") + theme_bw()

plot_age_complexity <- plot_age_complexity + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
plot_age_complexity
```



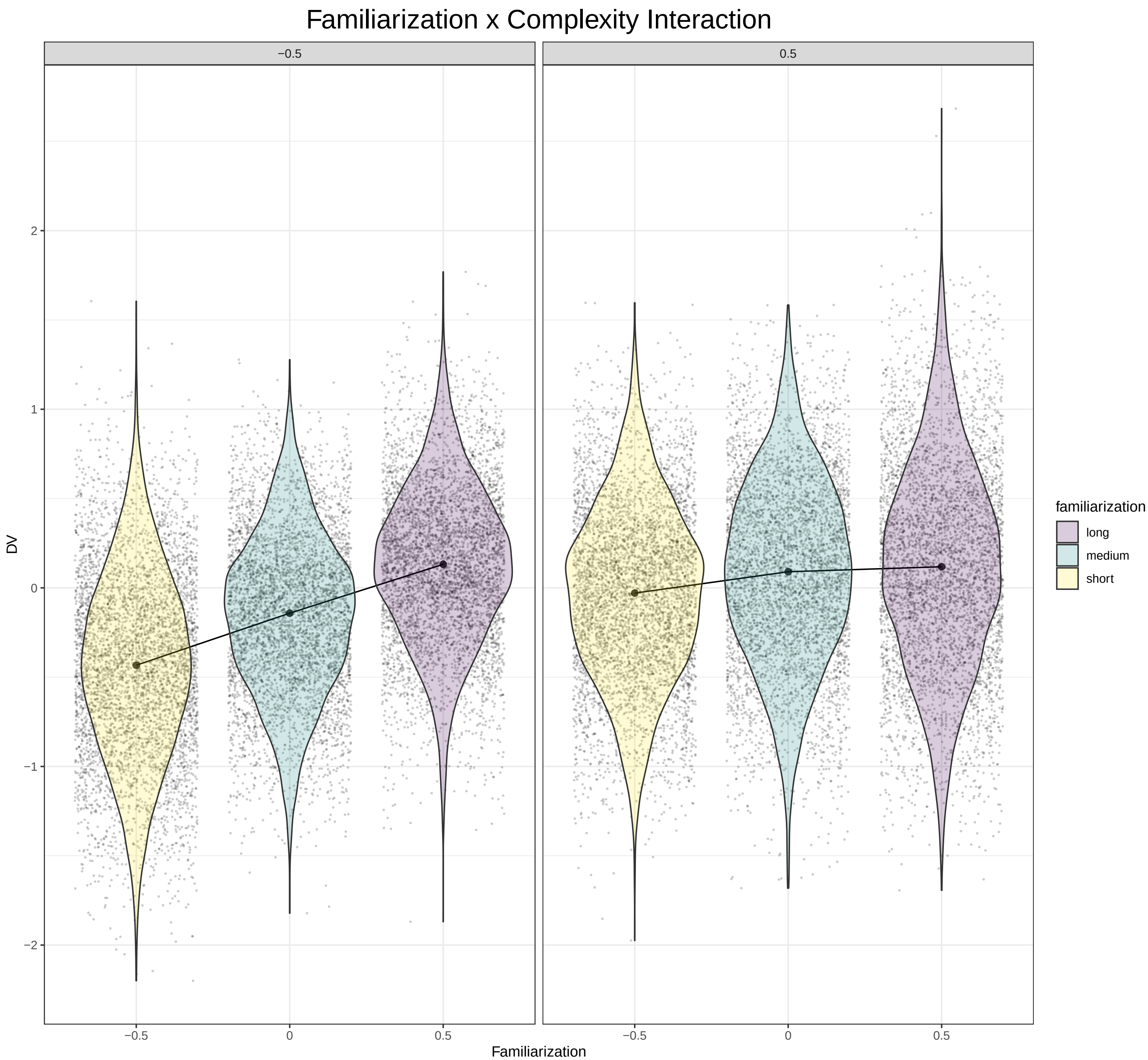
2.6 Familiarization*Complexity:

```
dat_f_c_interaction <- dat_sim %>%
    mutate(X_c = as.factor(X_c)) %>%
    mutate(X_f = as.factor(X_f)) %>%
    group_by(X_f, X_c) %>%
    dplyr::summarise(med_DV = median(DV))
```

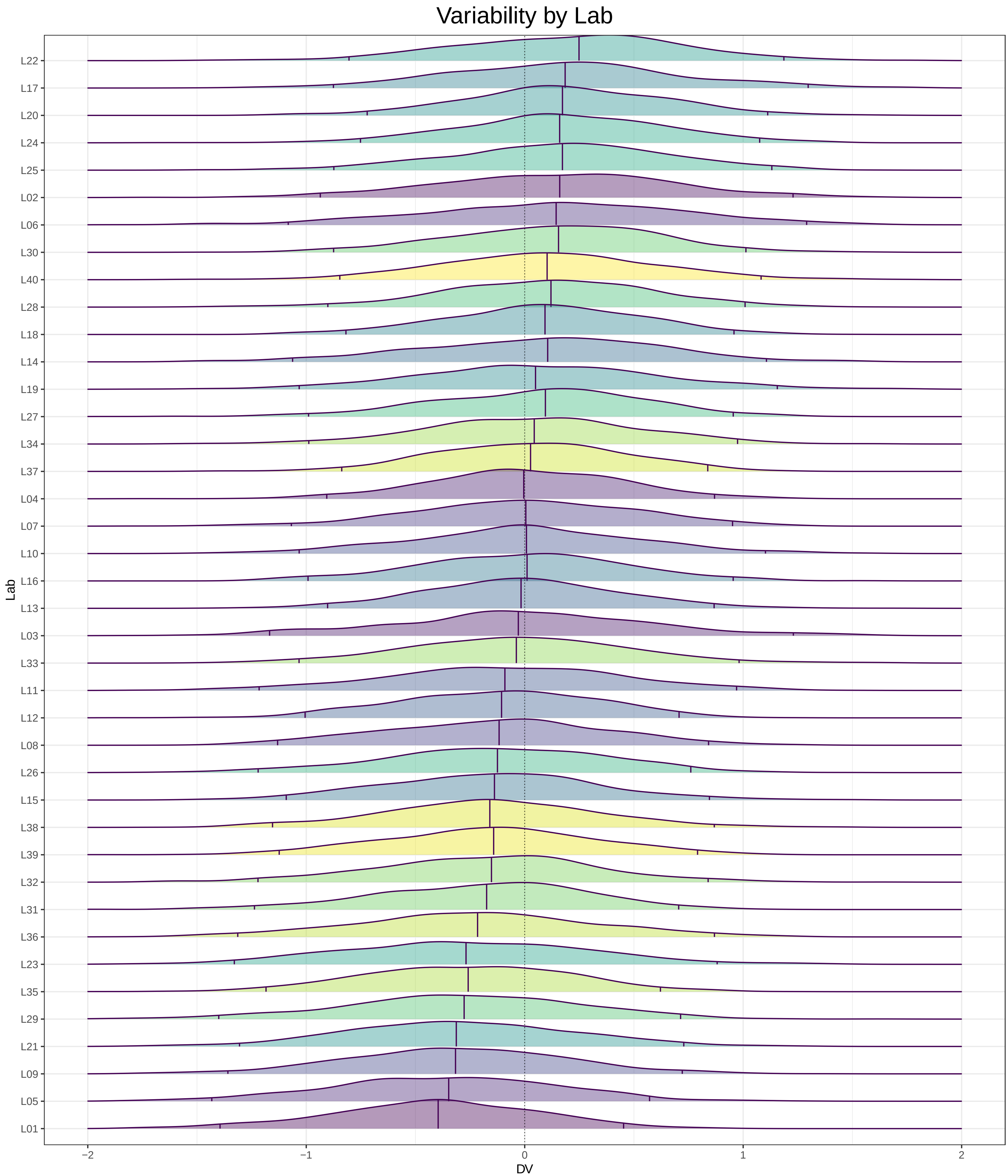
```
## `summarise()` has grouped output by 'X_f'. You can override using the `.groups`
## argument.
```

```
plot_familiarization_complexity <- dat_sim %>%
    mutate(X_c = as.factor(X_c)) %>%
    mutate(X_f = as.factor(X_f)) %>%
    ggplot() + geom_point(aes(y = DV, x = X_f), position = "jitter",
```

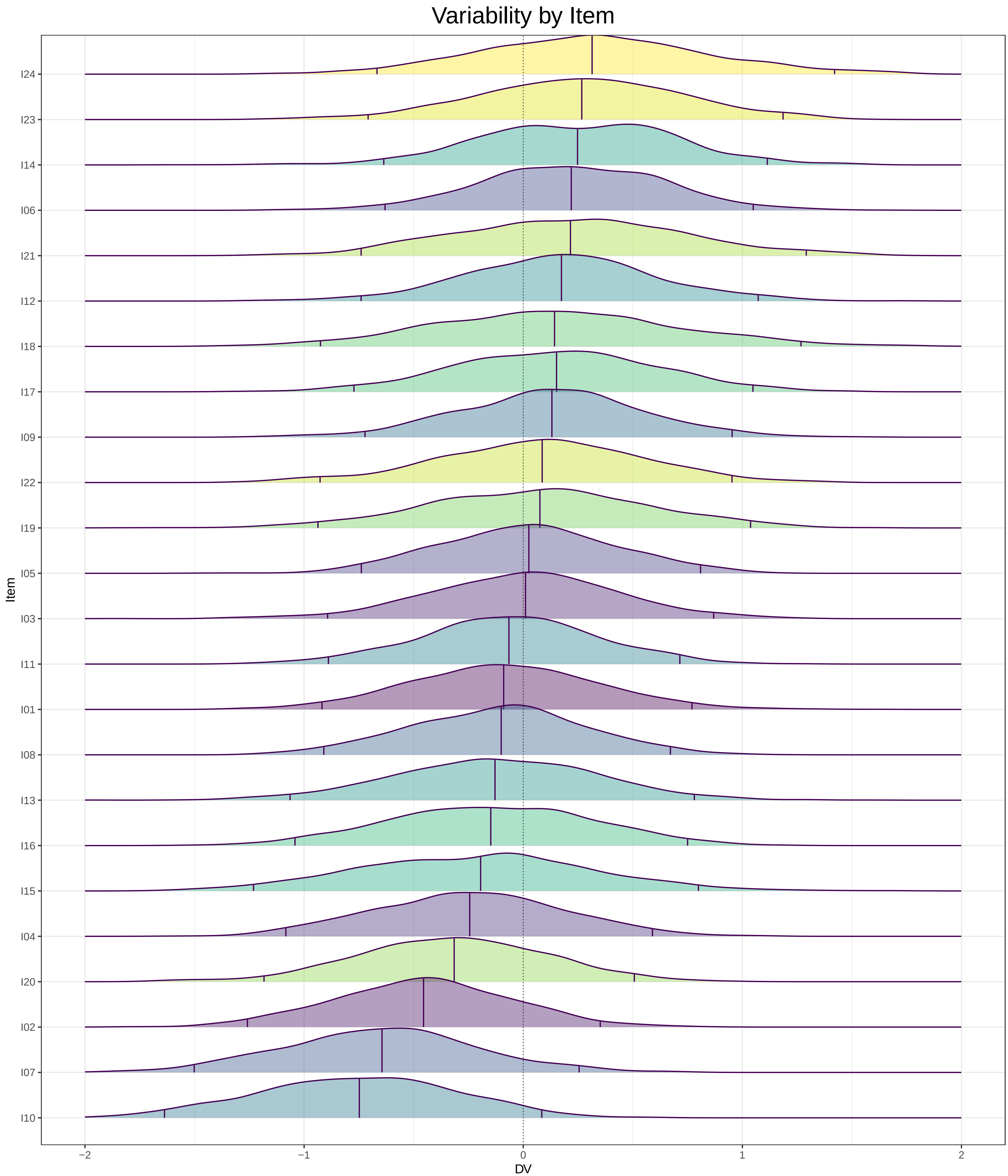
```
    alpha = 0.2, size = 0.2) + geom_point(aes(y = med_DV, x = as.factor(X_f)),
    alpha = 0.8, size = 2, data = dat_f_c_interaction) + geom_line(aes(y = med_DV,
    x = as.factor(X_f), group = 1), data = dat_f_c_interaction) +
    geom_violin(aes(y = DV, x = X_f, fill = familiarization),
        alpha = 0.2) + scale_fill_manual(values = viridis(n = 3)) +
    facet_wrap(~X_c) + ggtitle("Familiarization x Complexity Interaction") +
    xlab("Familiarization") + theme_bw()
plot_familiarization_complexity <- plot_familiarization_complexity +
    theme(plot.title = element_text(hjust = 0.5, size = 20))
plot_familiarization_complexity
```



2.7 Variability by Lab



2.8 Variability by Item



3 Power Calculation with Full Data and Varying Intercepts and Varying Slopes

3.1 Effect Size = 0.5

```
# Number of simulations:
reps <- 350

# Simulation function:
run_sims <- function(filename_full, ef) {

  dat_sim <- my_sim_data(beta_c  =  ef,
                         beta_f = ef,
                         beta_a = ef,

                         beta_ca = ef,
                         beta_af = ef,
                         beta_cf = ef,

                         beta_cfa = ef)

  mod_sim <- lmer(DV ~ 1 + X_a * X_c * X_f +
                    (1 + X_c * X_f | subj_id) +
                    (1 | lab_id) +
                    (1 | item_id),
                  data=dat_sim)

  sim_results <- broom.mixed::tidy(mod_sim)

  # append the results to a file
  append <- file.exists(filename_full)
  write_csv(sim_results, filename_full, append = append)

  # return the tidy table
  sim_results
}

filename_full_0.5 = 'run_sims_full_0.5.csv'
start_time <- Sys.time()
sims <- purrr::map_df(1:reps, ~run_sims(filename_full = filename_full_0.5, ef = 0.5))
end_time <- Sys.time()
end_time - start_time
```

3.2 Effect Size = 0.4

```
filename_full_0.4 = "run_sims_full_0.4.csv"
start_time <- Sys.time()
sims <- purrr::map_df(1:reps, ~run_sims(filename_full = filename_full_0.4,
    ef = 0.4))
end_time <- Sys.time()
end_time - start_time
```

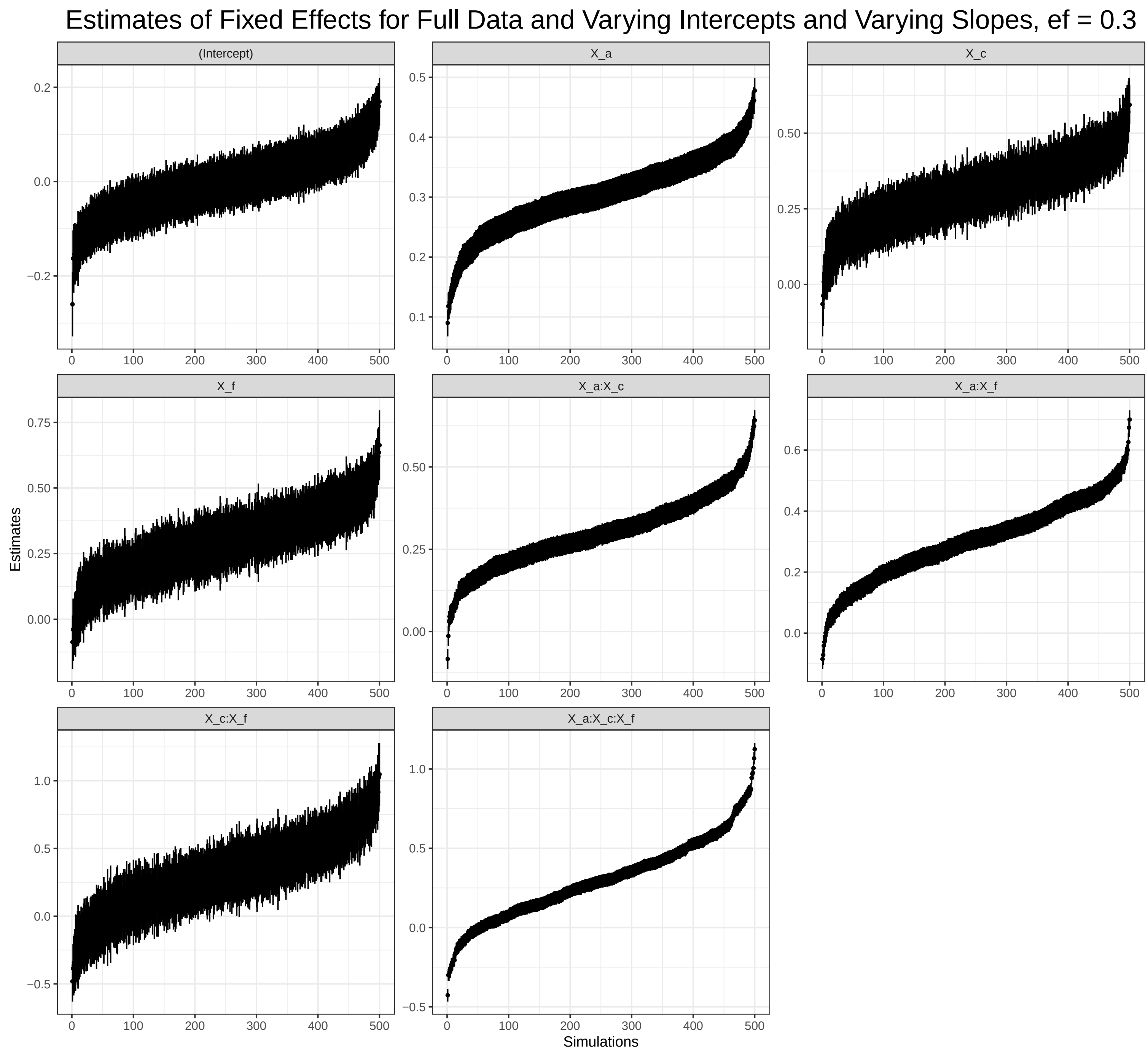
3.3 Effect Size = 0.3

```
filename_full_0.3 = "run_sims_full_0.3.csv"
start_time <- Sys.time()
sims <- purrr::map_df(1:reps, ~run_sims(filename_full = filename_full_0.3,
```

```r
    ef = 0.3))
end_time <- Sys.time()
end_time - start_time
```

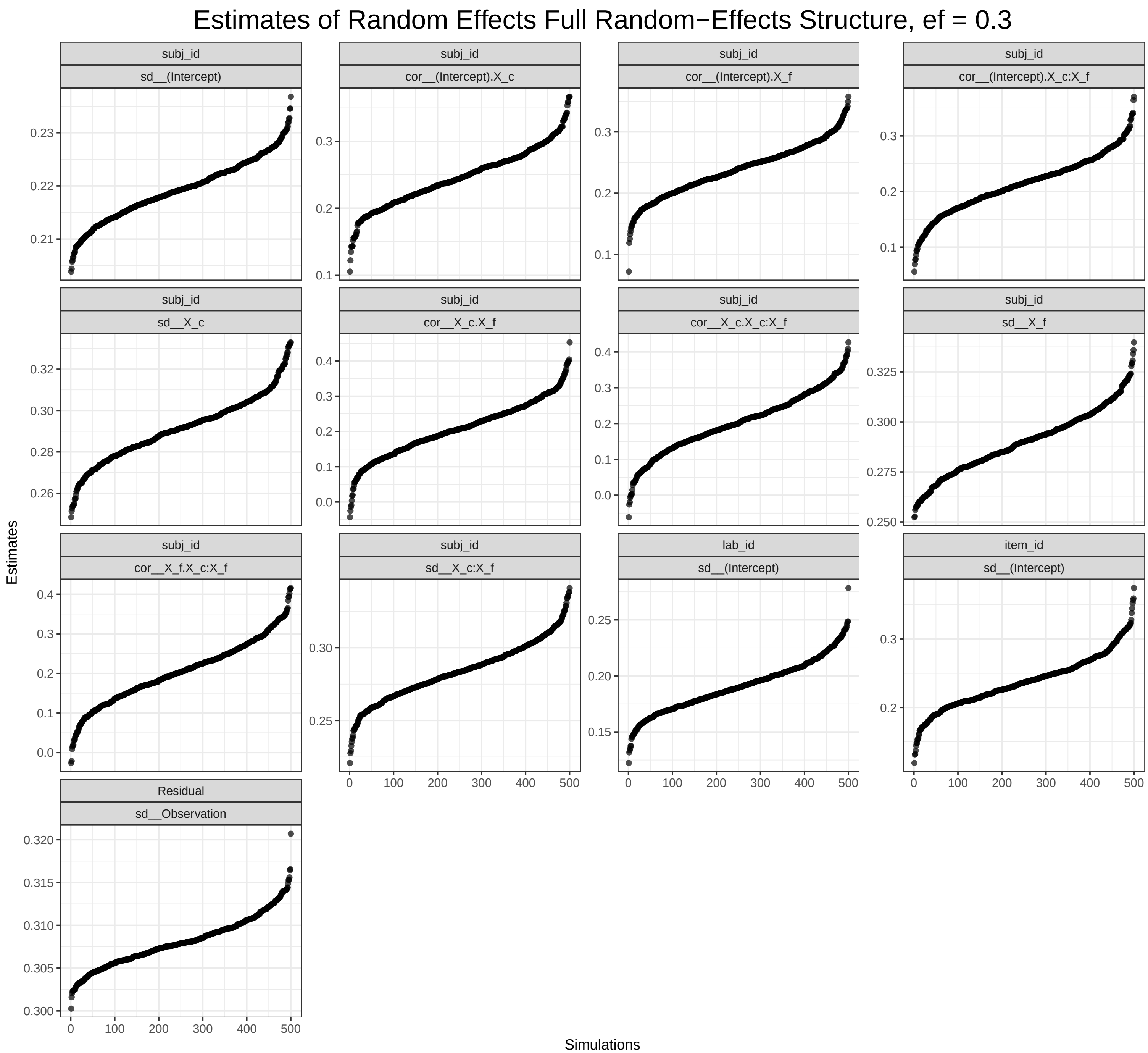
3.3.1 Visualise Estimates for Fixed Effects:

```r
sims_full_0.3 <- read_csv(filename_full_0.3, col_types = cols(group = col_factor(ordered = TRUE),
    term = col_factor(ordered = TRUE)))

fixed_full_plot <- sims_full_0.3 %>%
    filter(effect == "fixed") %>%
    ungroup() %>%
    arrange(term, estimate) %>%
    mutate(row = rep(seq(1:reps), 8)) %>%
    ggplot(aes(x = row, y = estimate, ymin = estimate - std.error,
        ymax = estimate + std.error)) + facet_wrap(~term, scales = "free") +
    geom_pointrange(fatten = 1/2) + ylab("Estimates") + xlab("Simulations") +
    ggtitle("Estimates of Fixed Effects for Full Data and Varying Intercepts and Varying Slopes, ef = 0.3") +
    theme_bw()

fixed_full_plot <- fixed_full_plot + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
fixed_full_plot
```



3.3.2 Visualise Estimates for Random Effects:

```
ran_full_plot <- sims_full_0.3 %>%
    filter(effect == "ran_pars") %>%
    ungroup() %>%
    arrange(group, term, estimate) %>%
    mutate(row = rep(seq(1:reps), 13)) %>%
    ggplot(aes(x = row, y = estimate)) + geom_point(alpha = 0.7) +
    facet_wrap(~group + term, scales = "free_y") + theme_bw() +
    ylab("Estimates") + xlab("Simulations") + ggtitle("Estimates of Random Effects Full Random-Effects Structure,
    theme_bw()
ran_full_plot <- ran_full_plot + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
ran_full_plot
```



3.4 Effect Size = 0.2

```
filename_full_0.2 = "run_sims_full_0.2.csv"
start_time <- Sys.time()
sims <- purrr::map_df(1:reps, ~run_sims(filename_full = filename_full_0.2,
    ef = 0.2))
end_time <- Sys.time()
end_time - start_time
```

3.5 Effect Size = 0.1

```
filename_full_0.1 = "run_sims_full_0.1.csv"
start_time <- Sys.time()
sims <- purrr::map_df(1:reps, ~run_sims(filename_full = filename_full_0.1,
```

```
    ef = 0.1))
end_time <- Sys.time()
end_time - start_time
```

4 Power Calculation with Full Data and Varying Intercepts

4.1 Effect Size = 0.5

```
# Simulation function:
run_sims <- function(filename_full, ef) {

  dat_sim <- my_sim_data(beta_c  =  ef,
                         beta_f = ef,
                         beta_a = ef,

                         beta_ca = ef,
                         beta_af = ef,
                         beta_cf = ef,

                         beta_cfa = ef)

  mod_sim <- lmer(DV ~ 1 + X_a * X_c * X_f +
                    (1 | subj_id) +
                    (1 | lab_id) +
                    (1 | item_id),
                  data=dat_sim)

  sim_results <- broom.mixed::tidy(mod_sim)

  # append the results to a file
  append <- file.exists(filename_full)
  write_csv(sim_results, filename_full, append = append)

  # return the tidy table
  sim_results
}

filename_full_int_0.5 = 'run_sims_full_int_0.5.csv'
start_time <- Sys.time()
sims <- purrr::map_df(1:reps, ~run_sims(filename_full = filename_full_int_0.5, ef = 0.5))
end_time <- Sys.time()
end_time - start_time
```

4.2 Effect Size = 0.4

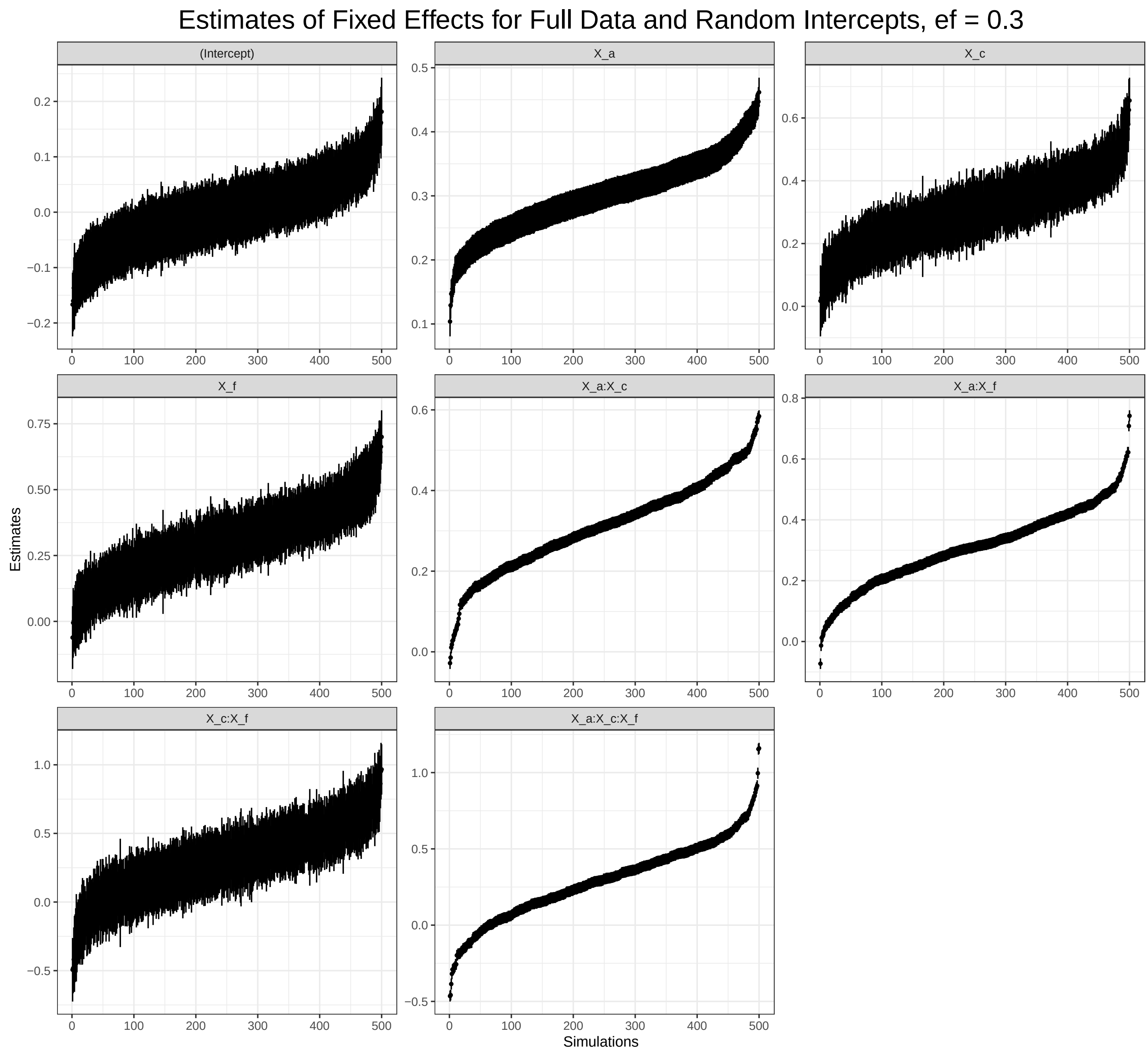
```
filename_full_int_0.4 = "run_sims_full_int_0.4.csv"
start_time <- Sys.time()
sims <- purrr::map_df(1:reps, ~run_sims(filename_full = filename_full_int_0.4,
    ef = 0.4))
end_time <- Sys.time()
end_time - start_time
```

4.3 Effect Size = 0.3

```
filename_full_int_0.3 = "run_sims_full_int_0.3.csv"
start_time <- Sys.time()
sims <- purrr::map_df(1:reps, ~run_sims(filename_full = filename_full_int_0.3,
    ef = 0.3))
end_time <- Sys.time()
end_time - start_time
```

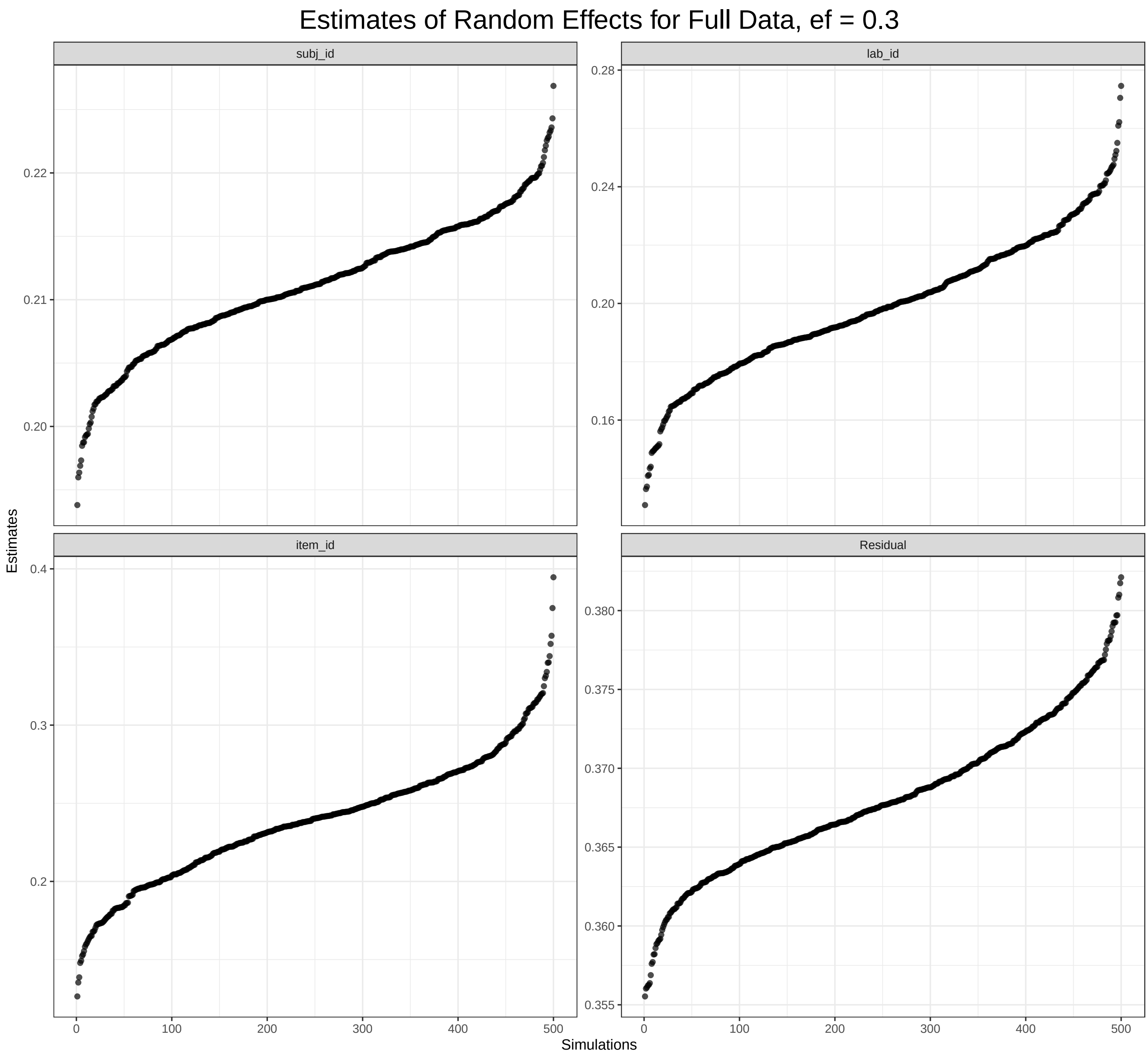
4.3.1 Visualise Estimates for Fixed Effects:

```
sims_full_int_0.3 <- read_csv(filename_full_int_0.3, col_types = cols(group = col_factor(ordered = TRUE),
    term = col_factor(ordered = TRUE)))

fixed_full_int_plot <- sims_full_int_0.3 %>%
    filter(effect == "fixed") %>%
    ungroup() %>%
    arrange(term, estimate) %>%
    mutate(row = rep(seq(1:reps), 8)) %>%
    ggplot(aes(x = row, y = estimate, ymin = estimate - std.error,
        ymax = estimate + std.error)) + facet_wrap(~term, scales = "free") +
    geom_pointrange(fatten = 1/2) + ylab("Estimates") + xlab("Simulations") +
    ggtitle("Estimates of Fixed Effects for Full Data and Random Intercepts, ef = 0.3") +
    theme_bw()

fixed_full_int_plot <- fixed_full_int_plot + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
fixed_full_int_plot
```



4.3.2 Visualise Estimates for Random Effects:

```
ran_full_int_plot <- sims_full_int_0.3 %>%
    filter(effect == "ran_pars") %>%
    ungroup() %>%
    arrange(group, estimate) %>%
    mutate(row = rep(seq(1:reps), 4)) %>%
    ggplot(aes(x = row, y = estimate)) + geom_point(alpha = 0.7) +
    facet_wrap(~group, scales = "free_y") + theme_bw() + ylab("Estimates") +
    xlab("Simulations") + ggtitle("Estimates of Random Effects for Full Data, ef = 0.3") +
    theme_bw()
ran_full_int_plot <- ran_full_int_plot + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
ran_full_int_plot
```



4.4 Effect Size = 0.2

```
filename_full_int_0.2 = "run_sims_full_int_0.2.csv"
start_time <- Sys.time()
sims <- purrr::map_df(1:reps, ~run_sims(filename_full = filename_full_int_0.2,
    ef = 0.2))
end_time <- Sys.time()
end_time - start_time
```

4.5 Effect Size = 0.1

```
filename_full_int_0.1 = "run_sims_full_int_0.1.csv"
start_time <- Sys.time()
sims <- purrr::map_df(1:reps, ~run_sims(filename_full = filename_full_int_0.1,
```

```
    ef = 0.1))
end_time <- Sys.time()
end_time - start_time
```

5 Power Calculation with 20 pct. Missing Data and Varying Intercepts and Varying Slopes

5.1 Effect Size = 0.5

```
run_sims_missing <- function(filename_missing, ef) {

  dat_sim <- my_sim_data(beta_c  =  ef,
                       beta_f = ef,
                       beta_a = ef,

                       beta_ca = ef,
                       beta_af = ef,
                       beta_cf = ef,

                       beta_cfa = ef)

  missing_samples <- dat_sim %>%
    mutate(nas = rbinom(nrow(dat_sim), 1, 1 - .20)) %>%
    mutate(DV = ifelse(nas == 1, DV, NA)) %>%
    drop_na()

  mod_sim <- lmer(DV ~ 1 + X_a * X_c * X_f +
                    (1 + X_c * X_f | subj_id) +
                    (1 | lab_id) +
                    (1 | item_id),
                  data=missing_samples)

  sim_results <- broom.mixed::tidy(mod_sim)

  # append the results to a file
  append <- file.exists(filename_missing)
  write_csv(sim_results, filename_missing, append = append)

  # return the tidy table
  sim_results
}

filename_20_missing_0.5 = 'run_sims_20_missing_0.5.csv'
start_time <- Sys.time()
sims_missing <- purrr::map_df(1:reps, ~run_sims_missing(filename_missing = filename_20_missing_0.5, ef = 0.5))
end_time <- Sys.time()
end_time - start_time
```

5.2 Effect Size = 0.4

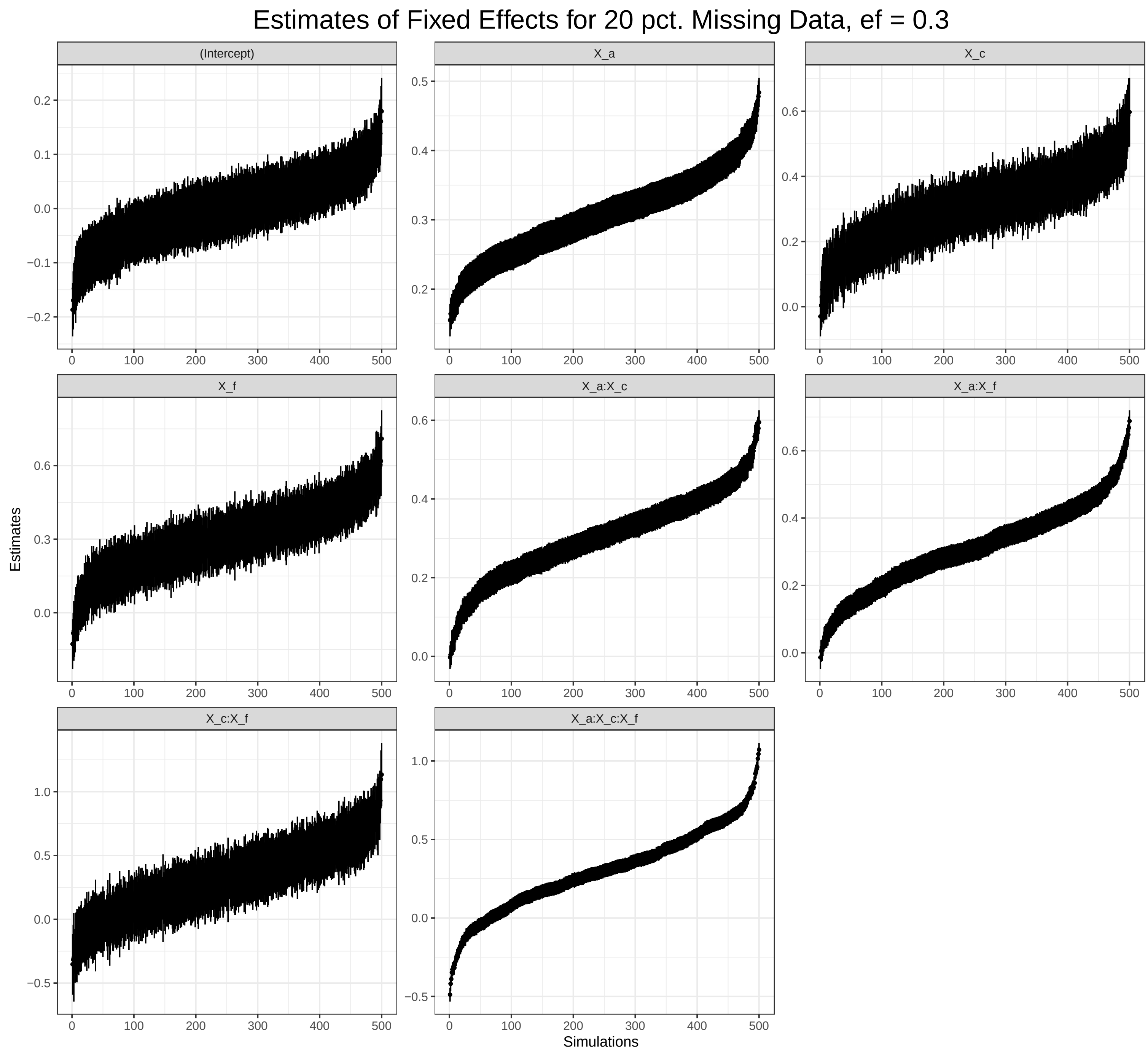
```
filename_20_missing_0.4 = "run_sims_20_missing_0.4.csv"
start_time <- Sys.time()
sims_missing <- purrr::map_df(1:reps, ~run_sims_missing(filename_missing = filename_20_missing_0.4,
    ef = 0.4))
end_time <- Sys.time()
end_time - start_time
```

5.3 Effect Size = 0.3

```
filename_20_missing_0.3 = "run_sims_20_missing_0.3.csv"
start_time <- Sys.time()
sims_missing <- purrr::map_df(1:reps, ~run_sims_missing(filename_missing = filename_20_missing_0.3,
    ef = 0.3))
end_time <- Sys.time()
end_time - start_time
```

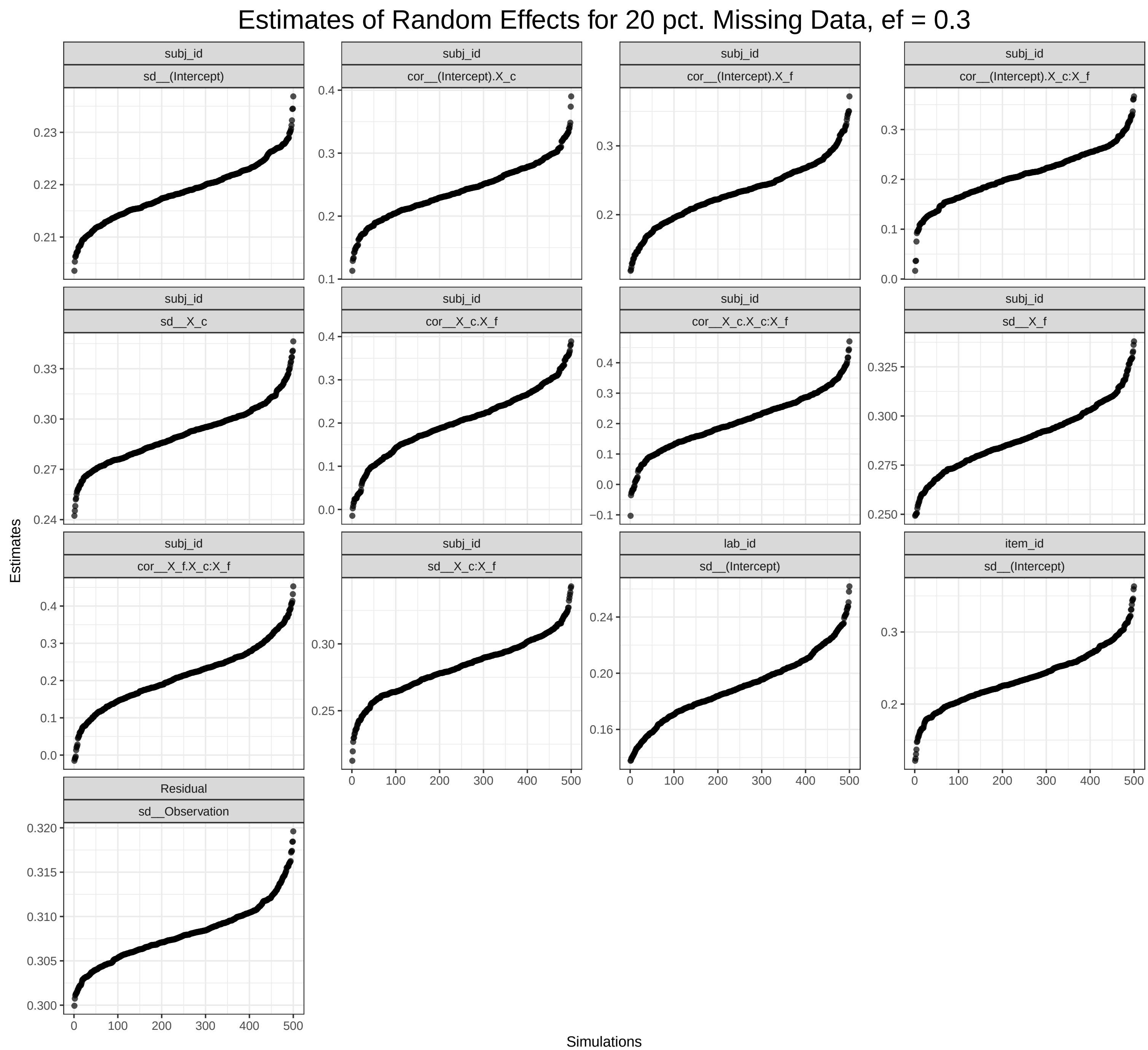
5.3.1 Visualise Estimates for Fixed Effects:

```
# read saved simulation data
sims_20_missing_0.3 <- read_csv(filename_20_missing_0.3, col_types = cols(
  # makes sure plots display in this order
  group = col_factor(ordered = TRUE),
  term = col_factor(ordered = TRUE)
))

fixed_missing_plot <- sims_20_missing_0.3 %>%
  filter(effect == "fixed") %>%
  ungroup() %>%
  arrange(term, estimate) %>%
  mutate(row = rep(seq(1:reps), 8)) %>%
  ggplot(aes(x = row, y = estimate, ymin = estimate-std.error, ymax = estimate+std.error)) +
  facet_wrap(~term, scales = "free") +
  geom_pointrange(fatten = 1/2) +
  ylab("Estimates") +
  xlab("Simulations") +
  ggtitle('Estimates of Fixed Effects for 20 pct. Missing Data, ef = 0.3') +
  theme_bw()

fixed_missing_plot <- fixed_missing_plot + theme(plot.title = element_text(hjust = 0.5, size=20))
fixed_missing_plot
```



5.3.2 Visualise Estimates for Random Effects:

```
ran_missing_plot <- sims_20_missing_0.3 %>%
    filter(effect == "ran_pars") %>%
    ungroup() %>%
    arrange(group, term, estimate) %>%
    mutate(row = rep(seq(1:reps), 13)) %>%
    ggplot(aes(x = row, y = estimate)) + geom_point(alpha = 0.7) +
    facet_wrap(~group + term, scales = "free_y") + theme_bw() +
    ylab("Estimates") + xlab("Simulations") + ggtitle("Estimates of Random Effects for 20 pct. Missing Data, ef =
    theme_bw()
ran_missing_plot <- ran_missing_plot + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
ran_missing_plot
```



5.4 Effect Size = 0.2

```
filename_20_missing_0.2 = "run_sims_20_missing_0.2.csv"
start_time <- Sys.time()
sims_missing <- purrr::map_df(1:reps, ~run_sims_missing(filename_missing = filename_20_missing_0.2,
    ef = 0.2))
end_time <- Sys.time()
end_time - start_time
```

5.5 Effect Size = 0.1

```
filename_20_missing_0.1 = "run_sims_20_missing_0.1.csv"
start_time <- Sys.time()
sims_missing <- purrr::map_df(1:reps, ~run_sims_missing(filename_missing = filename_20_missing_0.1,
```

```
    ef = 0.1))
end_time <- Sys.time()
end_time - start_time
```

6 Power Calculation with 50 pct. Missing Data and Varying Intercepts and Varying Slopes

6.1 Effect Size = 0.5

```
run_sims_missing <- function(filename_missing, ef) {

  dat_sim <- my_sim_data(beta_c  =  ef,
                         beta_f = ef,
                         beta_a = ef,

                         beta_ca = ef,
                         beta_af = ef,
                         beta_cf = ef,

                         beta_cfa = ef)

  missing_samples <- dat_sim %>%
    mutate(nas = rbinom(nrow(dat_sim), 1, 1 - .50)) %>%
    mutate(DV = ifelse(nas == 1, DV, NA)) %>%
    drop_na()

  mod_sim <- lmer(DV ~ 1 + X_a * X_c * X_f +
                    (1 + X_c * X_f | subj_id) +
                    (1 | lab_id) +
                    (1 | item_id),
                  data=missing_samples)

  sim_results <- broom.mixed::tidy(mod_sim)

  # append the results to a file
  append <- file.exists(filename_missing)
  write_csv(sim_results, filename_missing, append = append)

  # return the tidy table
  sim_results
}

filename_50_missing_0.5 = 'run_sims_50_missing_0.5.csv'
start_time <- Sys.time()
sims_missing <- purrr::map_df(1:reps, ~run_sims_missing(filename_missing = filename_50_missing_0.5, ef = 0.5))
end_time <- Sys.time()
end_time - start_time
```

6.2 Effect Size = 0.4

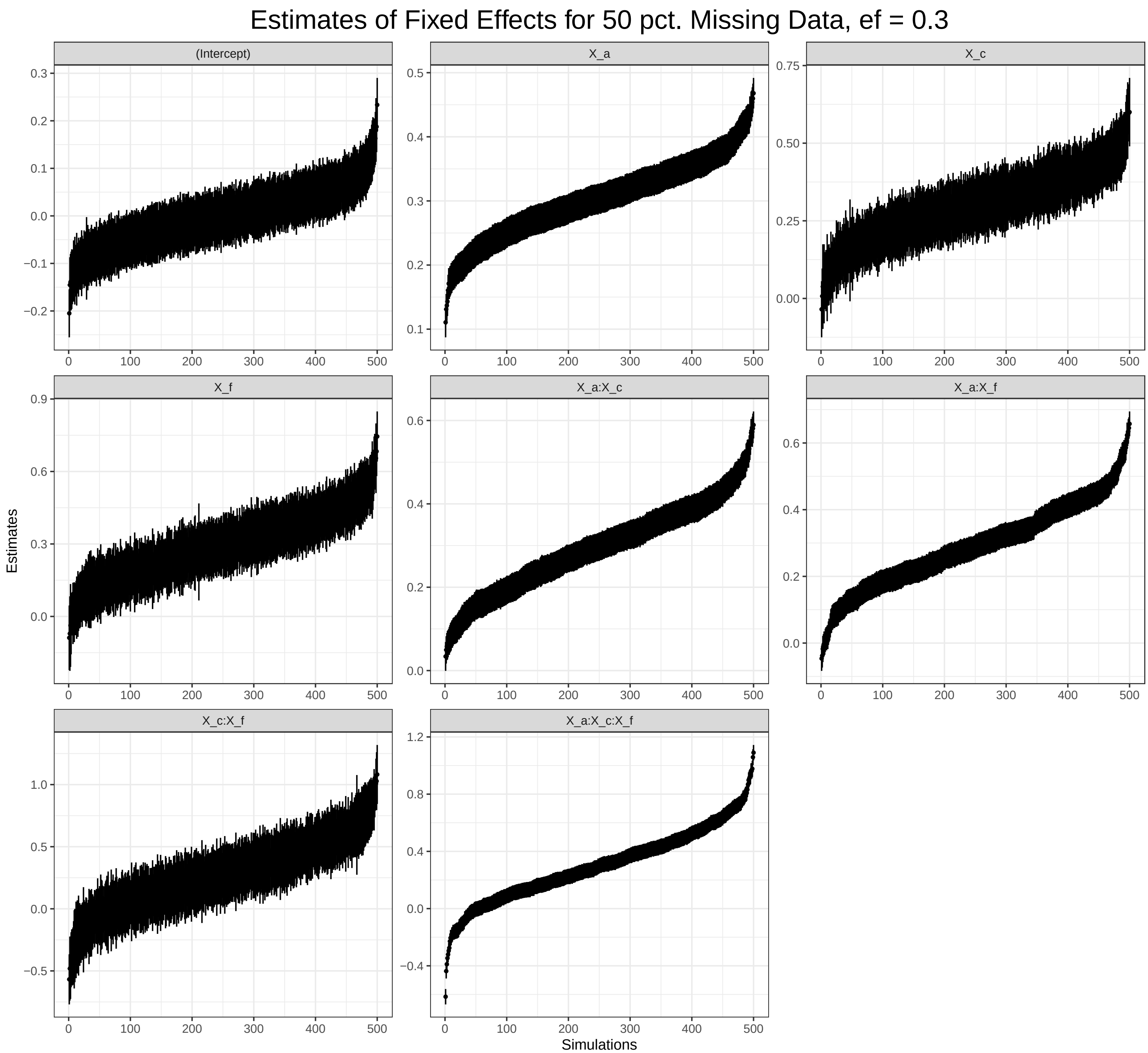
```
filename_50_missing_0.4 = "run_sims_50_missing_0.4.csv"
start_time <- Sys.time()
sims_missing <- purrr::map_df(1:reps, ~run_sims_missing(filename_missing = filename_50_missing_0.4,
    ef = 0.4))
end_time <- Sys.time()
end_time - start_time
```

6.3 Effect Size = 0.3

```
filename_50_missing_0.3 = "run_sims_50_missing_0.3.csv"
start_time <- Sys.time()
sims_missing <- purrr::map_df(1:reps, ~run_sims_missing(filename_missing = filename_50_missing_0.3,
    ef = 0.3))
end_time <- Sys.time()
end_time - start_time
```

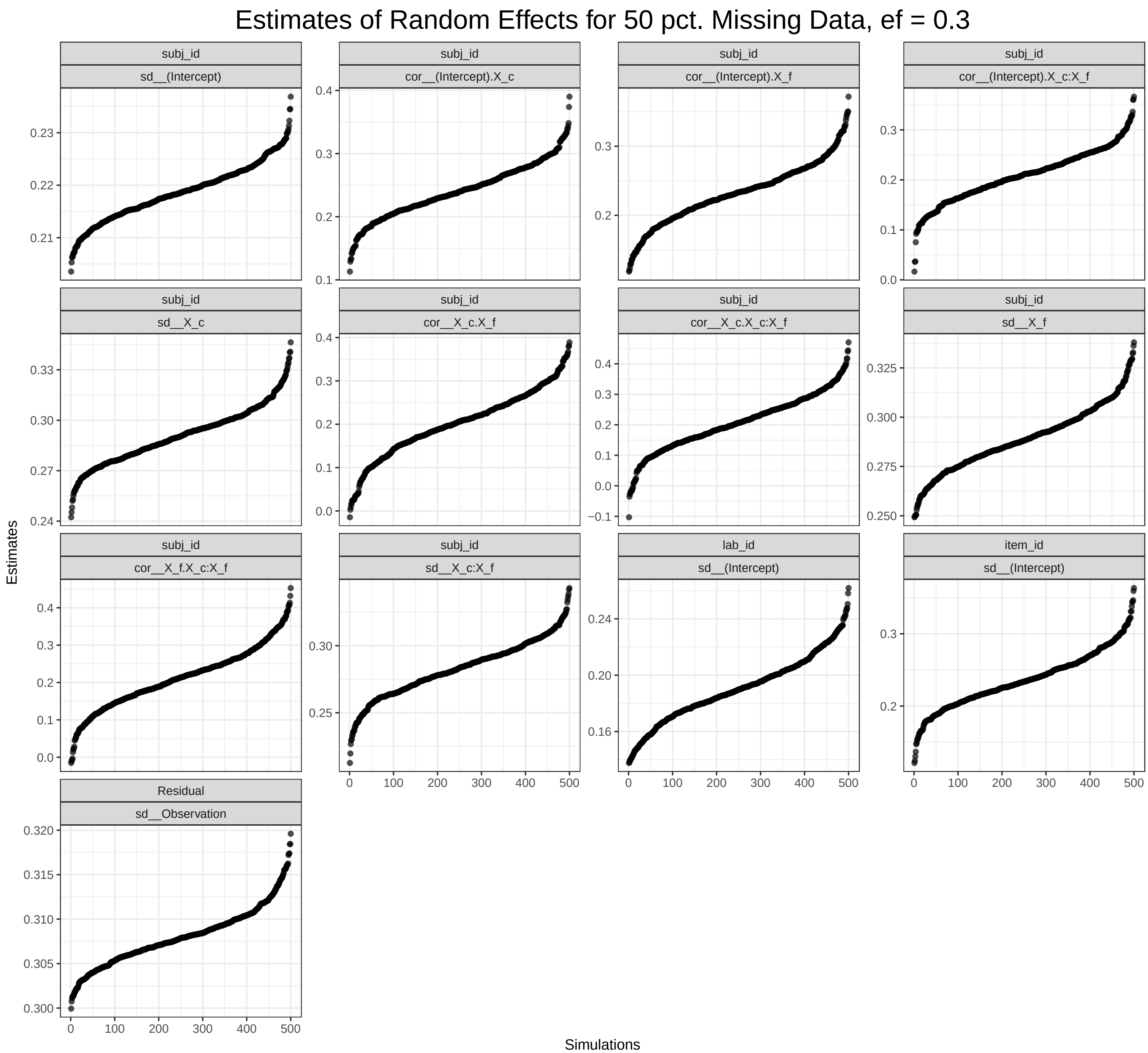
6.3.1 Visualise Estimates for Fixed Effects:

```
# read saved simulation data
sims_50_missing_0.3 <- read_csv(filename_50_missing_0.3, col_types = cols(
  # makes sure plots display in this order
  group = col_factor(ordered = TRUE),
  term = col_factor(ordered = TRUE)
))

fixed_missing_plot <- sims_50_missing_0.3 %>%
  filter(effect == "fixed") %>%
  ungroup() %>%
  arrange(term, estimate) %>%
  mutate(row = rep(seq(1:reps), 8)) %>%
  ggplot(aes(x = row, y = estimate, ymin = estimate-std.error, ymax = estimate+std.error)) +
  facet_wrap(~term, scales = "free") +
  geom_pointrange(fatten = 1/2) +
  ylab("Estimates") +
  xlab("Simulations") +
  ggtitle('Estimates of Fixed Effects for 50 pct. Missing Data, ef = 0.3') +
  theme_bw()

fixed_missing_plot <- fixed_missing_plot + theme(plot.title = element_text(hjust = 0.5, size=20))
fixed_missing_plot
```



6.3.2 Visualise Estimates for Random Effects:

```
ran_missing_plot <- sims_20_missing_0.3 %>%
    filter(effect == "ran_pars") %>%
    ungroup() %>%
    arrange(group, term, estimate) %>%
    mutate(row = rep(seq(1:reps), 13)) %>%
    ggplot(aes(x = row, y = estimate)) + geom_point(alpha = 0.7) +
    facet_wrap(~group + term, scales = "free_y") + theme_bw() +
    ylab("Estimates") + xlab("Simulations") + ggtitle("Estimates of Random Effects for 50 pct. Missing Data, ef =
    theme_bw()
ran_missing_plot <- ran_missing_plot + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
ran_missing_plot
```



6.4 Effect Size = 0.2

```
filename_50_missing_0.2 = "run_sims_50_missing_0.2.csv"
start_time <- Sys.time()
sims_missing <- purrr::map_df(1:reps, ~run_sims_missing(filename_missing = filename_50_missing_0.2,
    ef = 0.2))
end_time <- Sys.time()
end_time - start_time
```

6.5 Effect Size = 0.1

```
filename_50_missing_0.1 = "run_sims_50_missing_0.1.csv"
start_time <- Sys.time()
sims_missing <- purrr::map_df(1:reps, ~run_sims_missing(filename_missing = filename_50_missing_0.1,
```

```
    ef = 0.1))
end_time <- Sys.time()
end_time - start_time
```

7 Overview of Power Simulation Results

7.1 Summary Statistics for Power Calculation with Full Data and Varying Intercepts and Varying Slopes:

Table 1: Power for Simulations with Full Data and Varying Intercepts and Varying Slopes

term	power, ef = 0.1	power, ef = 0.2	power, ef = 0.3	power, ef = 0.4	power, ef = 0.5
(Intercept)	0.050	0.056	0.070	0.056	0.026
X_a	0.844	0.990	1.000	1.000	1.000
X_c	0.170	0.474	0.838	0.966	0.992
X_f	0.118	0.360	0.634	0.864	0.968
X_a:X_c	0.690	0.894	0.986	1.000	1.000
X_a:X_f	0.748	0.864	0.966	0.994	1.000
X_c:X_f	0.072	0.124	0.244	0.350	0.486
X_a:X_c:X_f	0.770	0.808	0.856	0.938	0.956

7.2 Summary Statistics for Power Calculation with Full Data and Varying Intercepts:

Table 2: Power for Simulations with Full Data and Varying Intercepts

term	power, ef = 0.1	power, ef = 0.2	power, ef = 0.3	power, ef = 0.4	power, ef = 0.5
(Intercept)	0.050	0.056	0.050	0.080	0.058
X_a	0.822	0.994	1.000	1.000	1.000
X_c	0.162	0.482	0.810	0.998	0.966
X_f	0.136	0.352	0.644	0.970	0.884
X_a:X_c	0.868	0.958	0.990	1.000	0.998
X_a:X_f	0.848	0.932	0.990	1.000	0.998
X_c:X_f	0.092	0.136	0.220	0.564	0.348
X_a:X_c:X_f	0.798	0.832	0.882	0.964	0.934

7.3 Summary Statistics for Power Calculation with 20 pct. Missing Data and Varying Intercepts and Varying Slopes:

Table 3: Power for Simulations with 20 pct. Missing Data and Varying Intercepts and Slopes

term	power, ef = 0.1	power, ef = 0.2	power, ef = 0.3	power, ef = 0.4	power, ef = 0.5
(Intercept)	0.040	0.054	0.042	0.064	0.052
X_a	0.832	0.996	1.000	1.000	1.000
X_c	0.162	0.476	0.806	0.950	0.998
X_f	0.122	0.348	0.658	0.842	0.972
X_a:X_c	0.686	0.928	0.982	1.000	1.000
X_a:X_f	0.674	0.872	0.974	0.996	1.000
X_c:X_f	0.062	0.132	0.266	0.376	0.498
X_a:X_c:X_f	0.762	0.800	0.862	0.926	0.950

7.4 Summary Statistics for Power Calculation with 50 pct. Missing Data and Varying Intercepts and Varying Slopes:

Table 4: Power for Simulations with 50 pct. Missing Data and Varying Intercepts and Slopes

term	power, ef = 0.1	power, ef = 0.2	power, ef = 0.3	power, ef = 0.4	power, ef = 0.5
(Intercept)	0.036	0.058	0.054	0.044	0.046
X_a	0.822	0.986	1.000	1.000	1.000
X_c	0.168	0.540	0.840	0.970	0.996
X_f	0.120	0.366	0.658	0.836	0.964
X_a:X_c	0.666	0.902	0.992	0.998	1.000
X_a:X_f	0.680	0.826	0.966	0.992	1.000
X_c:X_f	0.066	0.174	0.232	0.372	0.460
X_a:X_c:X_f	0.690	0.754	0.840	0.896	0.942

8 Overview of Bias Results

```
sim_stats_full_bias_int_slope %>%
    kbl(caption = "Bias for Simulations with Full Data and Varying Intercepts and Varying Slopes",
        digits = 3, align = "c") %>%
    kable_styling(full_width = T, latex_options = c("striped",
        "HOLD_position"))
```

Table 5: Bias for Simulations with Full Data and Varying Intercepts and Varying Slopes

term	bias, ef = 0.1	bias, ef = 0.2	bias, ef = 0.3	bias, ef = 0.4	bias, ef = 0.5
(Intercept)	0.000	0.003	0.003	-0.003	0.003
X_a	0.000	-0.001	-0.003	0.002	-0.003
X_c	0.001	-0.003	-0.007	0.000	-0.004
X_f	0.012	0.002	0.002	0.014	-0.021
X_a:X_c	-0.001	0.004	0.005	0.006	0.006
X_a:X_f	-0.007	0.013	-0.007	0.000	-0.002
X_c:X_f	0.024	-0.001	-0.015	0.001	0.011
X_a:X_c:X_f	0.009	0.003	0.008	0.018	-0.018

```
sim_stats_full_bias_int %>%
    kbl(caption = "Bias for Simulations with Full Data and Varying Intercepts",
        digits = 3, align = "c") %>%
    kable_styling(full_width = T, latex_options = c("striped",
        "HOLD_position"))
```

Table 6: Bias for Simulations with Full Data and Varying Intercepts

term	bias, ef = 0.1	bias, ef = 0.2	bias, ef = 0.3	bias, ef = 0.4	bias, ef = 0.5
(Intercept)	0.002	0.006	0.001	0.003	-0.003
X_a	0.001	0.002	-0.002	-0.098	0.094
X_c	0.007	0.000	-0.003	-0.103	0.095
X_f	0.004	0.012	-0.001	-0.094	0.094
X_a:X_c	-0.011	0.008	-0.013	-0.105	0.105
X_a:X_f	-0.003	0.006	-0.011	-0.105	0.093
X_c:X_f	-0.005	0.014	-0.005	-0.136	0.101
X_a:X_c:X_f	0.004	0.027	-0.003	-0.075	0.113

```
sim_stats_20_missing_bias %>%
    kbl(caption = "Bias for Simulations with 20 pct. Missing Data and Varying Intercepts and Slopes",
        digits = 3, align = "c") %>%
```

```
    kable_styling(full_width = T, latex_options = c("striped",
        "HOLD_position"))
```

Table 7: Bias for Simulations with 20 pct. Missing Data and Varying Intercepts and Slopes

term	bias, ef = 0.1	bias, ef = 0.2	bias, ef = 0.3	bias, ef = 0.4	bias, ef = 0.5
(Intercept)	0.001	0.002	0.002	-0.005	0.001
X_a	0.010	0.000	-0.006	0.006	0.001
X_c	0.002	-0.003	-0.011	0.007	0.000
X_f	0.000	0.001	-0.006	0.005	-0.002
X_a:X_c	-0.001	-0.009	-0.005	0.004	-0.014
X_a:X_f	0.014	0.005	-0.007	0.001	0.000
X_c:X_f	0.008	0.002	-0.008	0.011	-0.008
X_a:X_c:X_f	0.009	-0.014	-0.003	0.003	0.020

```
sim_stats_50_missing_bias %>%
    kbl(caption = "Bias for Simulations with 50 pct. Missing Data and Varying Intercepts and Slopes",
        digits = 3, align = "c") %>%
    kable_styling(full_width = T, latex_options = c("striped",
        "HOLD_position"))
```

Table 8: Bias for Simulations with 50 pct. Missing Data and Varying Intercepts and Slopes

term	bias, ef = 0.1	bias, ef = 0.2	bias, ef = 0.3	bias, ef = 0.4	bias, ef = 0.5
(Intercept)	-0.006	-0.003	0.002	0.000	-0.003
X_a	-0.002	0.002	-0.004	0.004	0.000
X_c	0.006	-0.011	0.000	-0.005	0.000
X_f	-0.003	-0.008	0.007	0.016	0.005
X_a:X_c	0.000	-0.002	0.002	0.004	-0.007
X_a:X_f	0.004	0.004	0.010	0.012	0.007
X_c:X_f	0.010	-0.031	0.029	-0.012	0.014
X_a:X_c:X_f	0.000	0.016	0.000	0.020	-0.009

9 Increasing Variance with Trial Number

9.1 Extracting estimates of increasing variance from the adult pilot data

```
# download data from adult pilot study:
adult_data <- read.csv("02_processed_lt_data.csv")

stimulus_info_df <- adult_data %>%
    filter(phase == "pref") %>%
    mutate(familiar_item = case_when(left_stimulus == "familiar" ~
        stimulus_processed_left, right_stimulus == "familiar" ~
        stimulus_processed_right), novel_item = case_when(left_stimulus ==
        "novel" ~ stimulus_processed_left, right_stimulus ==
        "novel" ~ stimulus_processed_right)) %>%
    select(subject, block_number, familiar_item, novel_item) %>%
    distinct(subject, block_number, .keep_all = TRUE)

lt_comparison_df <- adult_data %>%
    filter(phase == "pref") %>%
    group_by(subject, exposure_time, block_number, complexity,
        gaze_location_type) %>%
    summarise(sum_dwell_time = sum(dwell_time)) %>%
    pivot_wider(names_from = gaze_location_type, values_from = sum_dwell_time) %>%
    ungroup() %>%
```

```
    left_join(stimulus_info_df, by = c("subject", "block_number")) %>%
    rename(familiarization_time = exposure_time, stimulus_complexity = complexity,
        trial_number = block_number, familiar_looking_time = familiar,
        novel_looking_time = novel, participant = subject) %>%
    # currently excluding all the empty trials
filter(!is.na(familiar_looking_time) & !is.na(novel_looking_time))
```

```
## `summarise()` has grouped output by 'subject', 'exposure_time', 'block_number',
## 'complexity'. You can override using the `.groups` argument.
```

```
lt_comparison_df <- lt_comparison_df %>%
    mutate(familiarization_time_scaled_centered = log10(familiarization_time) -
        log10(2000), contrast_coded_complexity = ifelse(stimulus_complexity ==
        "complex", 0.5, -0.5))

proportion_df <- adult_data %>%
    filter(phase == "pref") %>%
    group_by(block_number, exposure_time, complexity, gaze_location_type,
        subject) %>%
    summarise(sum_dwell_time = sum(dwell_time, na.rm = TRUE)) %>%
    pivot_wider(names_from = gaze_location_type, values_from = sum_dwell_time) %>%
    mutate(familiar = ifelse(is.na(familiar), 0, familiar), novel = ifelse(is.na(novel),
        0, novel)) %>%
    mutate(novelty_looking_proportion = novel/(familiar + novel))
```

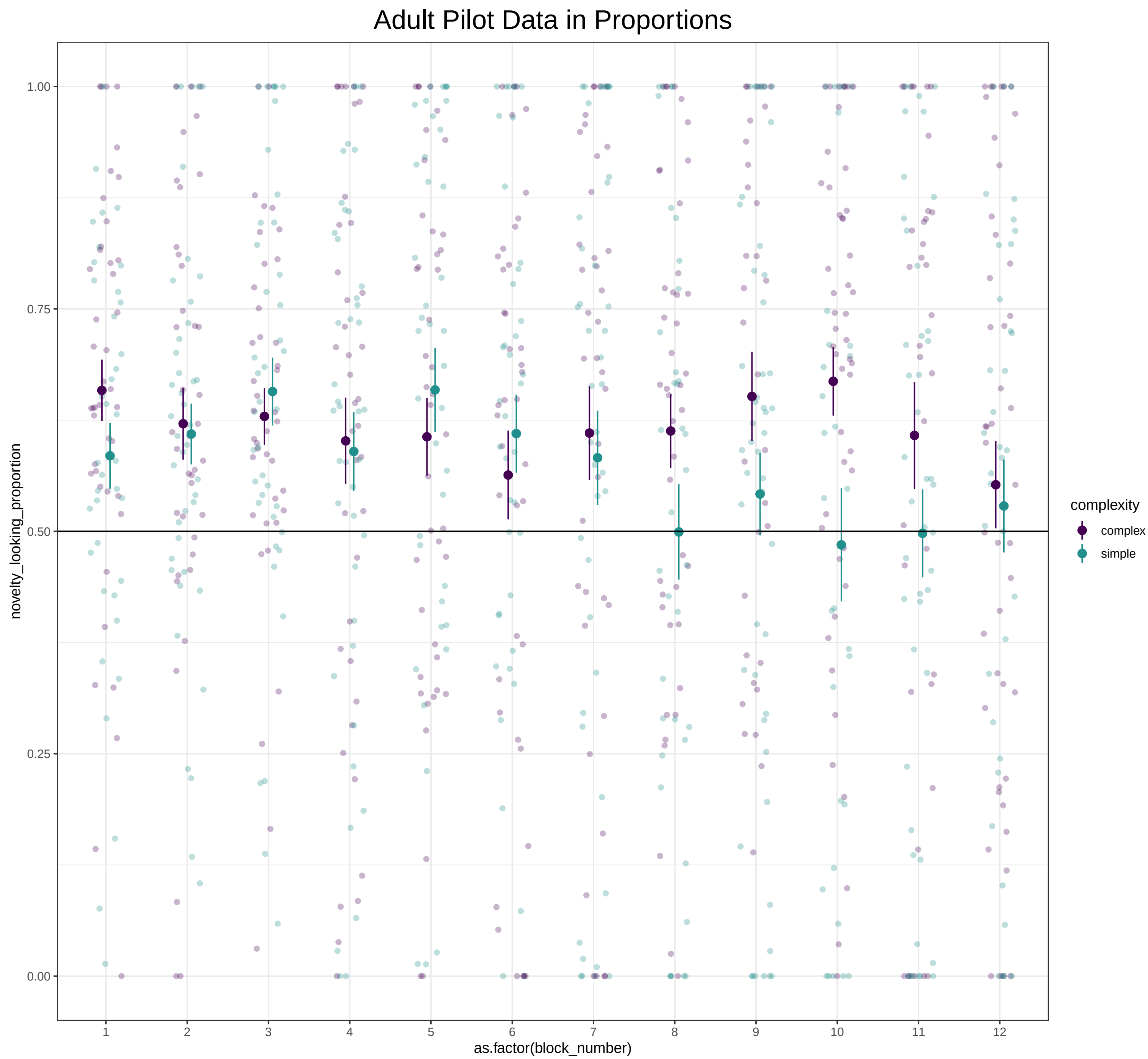
```
## `summarise()` has grouped output by 'block_number', 'exposure_time',
## 'complexity', 'gaze_location_type'. You can override using the `.groups`
## argument.
```

```
dwell_proportion_mean <- mean(proportion_df$novelty_looking_proportion,
    na.rm = T)
dwell_proportion_sd <- sd(proportion_df$novelty_looking_proportion,
    na.rm = T)

adult_data_z <- proportion_df %>%
    mutate(dwell_time_z = (novelty_looking_proportion - dwell_proportion_mean)/dwell_proportion_sd)

data_on_increase_in_sd <- adult_data_z %>%
    group_by(block_number) %>%
    summarise(n = n(), dwell_time_median = median(dwell_time_z,
        na.rm = T), sd_block = sd(dwell_time_z, na.rm = T))

increase_sd_value <- data_on_increase_in_sd$sd_block[12] - data_on_increase_in_sd$sd_block[1]
increase_sd_value
```

```
## [1] 0.3098641
```

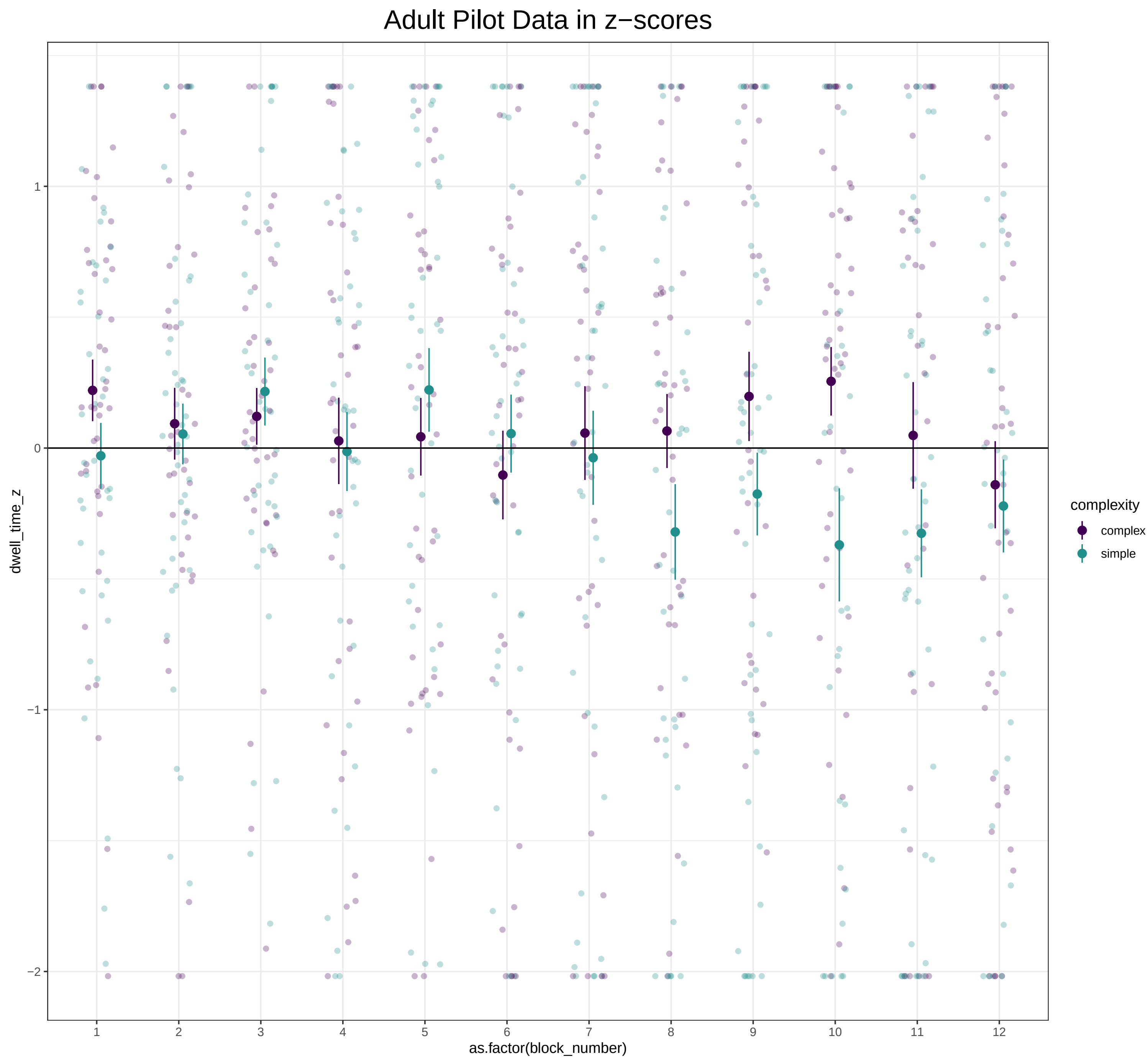
9.2 Visualisation of adult data in proportions

```
adult_proportion_plot <- proportion_df %>%
    ggplot(aes(x = as.factor(block_number), y = novelty_looking_proportion,
        color = complexity)) + geom_jitter(alpha = 0.3, width = 0.2) +
    stat_summary(aes(x = as.factor(block_number), y = novelty_looking_proportion,
        color = complexity), position = position_dodge(width = 0.2)) +
    geom_hline(yintercept = 0.5) + scale_color_manual(values = viridis(n = 3)) +
    ggtitle("Adult Pilot Data in Proportions") + theme_bw()
adult_proportion_plot <- adult_proportion_plot + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
adult_proportion_plot
```



9.3 Visualisation of adult data in z-score

```
adult_data_z_plot <- adult_data_z %>%
    ggplot(aes(x = as.factor(block_number), y = dwell_time_z,
        color = complexity)) + geom_jitter(alpha = 0.3, width = 0.2) +
    stat_summary(aes(x = as.factor(block_number), y = dwell_time_z,
        color = complexity), position = position_dodge(width = 0.2)) +
    geom_hline(yintercept = 0) + scale_color_manual(values = viridis(n = 3)) +
    ggtitle("Adult Pilot Data in z-scores") + theme_bw()

adult_data_z_plot <- adult_data_z_plot + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
adult_data_z_plot
```



9.4 Modification of simulation function to include variance increase

```
my_sim_data_variance <- function(
    increase_sd = increase_sd_value, #Add variance to increase with trial number

    n_subj      = 1280,   # number of subjects
    n_simple  =  12,   # number of complex stimuli
    n_complex =  12,   # number of complex stimuli
    n_small_fam = 8,   #small familiarization time
    n_medium_fam = 8,  #medium familiarization time
    n_high_fam = 8,    #high familiarization time
    n_lab = 40,

    beta_0  =  0, # intercept; i.e., the grand mean
    beta_c  =  0.3, # main effect for complexity
    beta_f = 0.3, # main effect for familiarization time
```

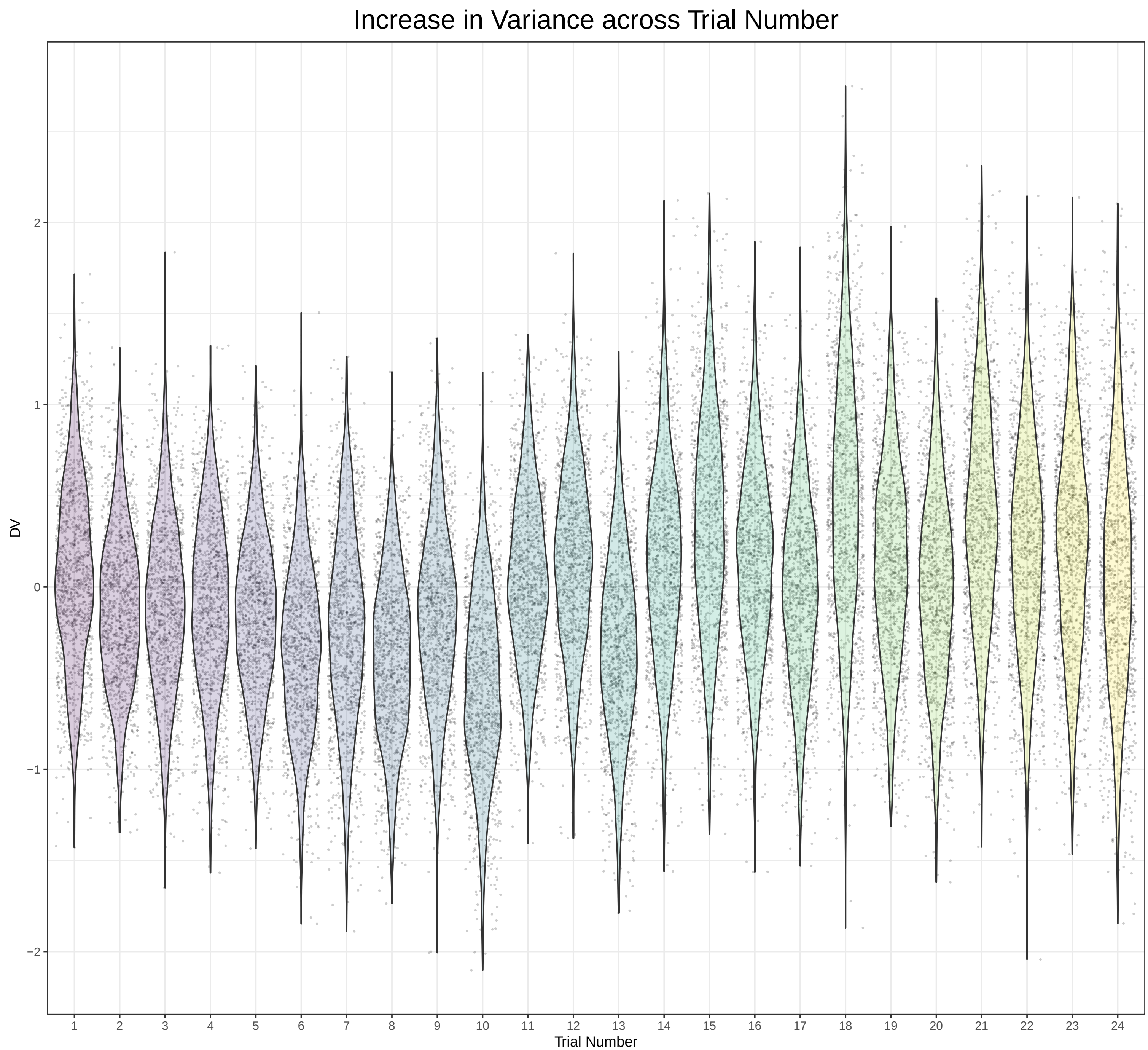
```
    beta_a = 0.3, # main effect for age

    beta_ca = 0.3,
    beta_af = 0.3,
    beta_cf = 0.3,

    beta_cfa = 0.3, #main effect for interaction between complexity and familiarization.

    subject_0   = 0.2, # by-subject random intercept sd

    subject_c   =  0.2, # by-subject slope complexity sd
    subject_f = 0.2, # by-subject slope familiarization sd
    subject_a = 0.2, # by-subject slope age sd

    subject_ca = 0.2,# by-subject slope for interaction between age and complexity sd
    subject_af = 0.2, # by-subject slope for interaction between age and familiarization sd
    subject_cf = 0.2, # by-subject slope complexity*familiarization sd

    subject_cfa = 0.2, # by-subject slope for interaction between age, complexity and familiarization sd

    subj_rho     =  .2, # correlations between by-subject random effects

    lab_0 = 0.2, # by-lab random intercept sd

    lab_c = 0.2, # by-lab slope complexity sd
    lab_f = 0.2, # by-lab slope familiarization sd
    lab_a = 0.2, # by-lab slope age sd

    lab_ca = 0.2, # by-lab slope for interaction between age and complexity sd
    lab_af = 0.2, # by-lab slope for interaction between age and familiarization sd
    lab_cf = 0.2, # by-lab random slope complexity*familiarization sd

    lab_cfa = 0.2, # by-lab slope for interaction between age, complexity and familiarization sd

    lab_rho = 0.2, # correlations between by-lab random effects

    item_0 =  0.2, # by-item random intercept sd

    item_c = 0.2, # by-item slope complexity sd
    item_f = 0.2, # by-item slope familiarization sd
    item_a = 0.2, # by-item slope age sd

    item_ca = 0.2, # by-item slope for interaction between age and complexity sd
    item_af = 0.2, # by-item slope for interaction between age and familiarization sd
    item_cf = 0.2, # by-item random slope complexity*familiarization sd

    item_cfa = 0.2, # by-item slope for interaction between age, complexity and familiarization sd

    item_rho = 0.2, # correlations between by-item random effects


    sigma = 0.3 # residual (error) sd
    ) { # residual (standard deviation)

  # simulate a sample of items
  items <- data.frame(
    item_id = seq_len(n_simple + n_complex),
    category = rep(c("simple", "complex"), c(n_simple, n_complex)),
    X_c = rep(c(-0.5, 0.5), c(n_simple, n_complex)),
    familiarization = rep(c("short", "medium", "long"), (n_simple + n_complex)/3),
    X_f = rep(c(-0.5, 0, 0.5), (n_simple + n_complex)/3),
    faux::rnorm_multi(
```

```
  n = n_simple + n_complex, mu = 0, sd = c(item_0,
                              item_c,
                              item_f,
                              item_a,
                              item_ca,
                              item_af,
                              item_cf,
                              item_cfa), r = item_rho,
  varnames = c("I_0", "I_c","I_f","I_a",
               "I_ca","I_af", "I_cf",
               "I_cfa"))
  ) %>%
  mutate(item_id = faux::make_id(nrow(.), "I"))

# simulate a sample of subjects
subjects <-
  faux::rnorm_multi(
  n = n_subj, mu = 0, sd = c(subject_0,
                              subject_c,
                              subject_f,
                              subject_a,
                              subject_ca,
                              subject_af,
                              subject_cf,
                              subject_cfa), r = subj_rho,
  varnames = c("S_0", "S_c","S_f","S_a",
               "S_ca","S_af", "S_cf",
               "S_cfa")
) %>%
  mutate(subj_id = faux::make_id(nrow(.), "S")) %>%
  mutate(X_a = runif(n_subj, min = -0.5, max = 0.5))
#add subject age measure, sample from distribution from -0.5 to 0.5. #subjects$subj_id <- 1:n_subj

labs <- faux::rnorm_multi(
  n = n_lab, mu = 0, sd = c(lab_0, lab_c, lab_f, lab_a,
                             lab_ca, lab_af, lab_cf,
                             lab_cfa), r = lab_rho,
  varnames = c("L_0", "L_c","L_f","L_a",
               "L_ca","L_af", "L_cf",
               "L_cfa")
) %>%
  mutate(lab_id = faux::make_id(nrow(.), "L"))

#create lab and subj nesting structure
#Number of subjects must be a multiple of number of labs
lab_multiplier = n_subj/n_lab
lab_subj_dict <- data.frame(
  subj_id = subjects$subj_id,
  lab_id = rep(labs$lab_id,lab_multiplier)
)

# cross subject and item IDs
temp <- crossing(subjects, items)  %>%
  left_join(lab_subj_dict, by = "subj_id") %>%
  left_join(labs, by = "lab_id") %>%
  group_by(subj_id, item_id) %>% mutate(item_id = sample(item_id)) %>%
  ungroup() %>%
  mutate(trial_num = rep(seq(n_simple + n_complex), n_subj))

temp <- temp %>%
  mutate(trial_i_var = ifelse(trial_num > 0, rnorm(nrow(temp), 0, trial_num*(increase_sd / 24)), 0))
```

```r
  temp %>%
    mutate(
      B_0  = beta_0 + S_0 + L_0 + I_0,

      B_c  = beta_c + S_c + L_c + I_c,
      B_f  = beta_f + S_f + L_f + I_f,
      B_a = beta_a + S_a + L_a + I_a,

      B_ca = beta_ca + S_ca + L_ca + I_ca,
      B_af = beta_af + S_af + L_af + I_af,
      B_cf = beta_cf + S_cf + L_cf + I_cf,

      B_cfa = beta_cfa + S_cfa + L_cfa + I_cfa,

      e_si = rnorm(nrow(temp), mean = 0, sd = sigma) + trial_i_var,

      DV = B_0 +
        (B_a * X_a) + (B_c * X_c) + (B_f * X_f) +
        (B_cf * X_c * X_f) + (B_af * X_a * X_f) + (B_ca * X_c * X_a) +
        (B_cfa * X_c * X_f * X_a) + e_si
    )
}

dat_sim <- my_sim_data_variance()
```

9.5 Visualise the variance increase

```r
# The increase in variance is over 24 trials in this
# ManyBabies5 case:
dat_sim <- my_sim_data_variance(increase_sd = increase_sd_value)

plot_trial_num <- dat_sim %>%
    mutate(trial_num = as.factor(trial_num)) %>%
    ggplot() + geom_point(aes(y = DV, x = trial_num), position = "jitter",
    alpha = 0.2, size = 0.2) + geom_violin(aes(y = DV, x = trial_num,
    fill = trial_num), alpha = 0.2, show.legend = FALSE) + scale_fill_manual(values = viridis(n = 24)) +
    ggtitle("Increase in Variance across Trial Number") + xlab("Trial Number") +
    theme_bw()

plot_trial_num <- plot_trial_num + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
plot_trial_num
```



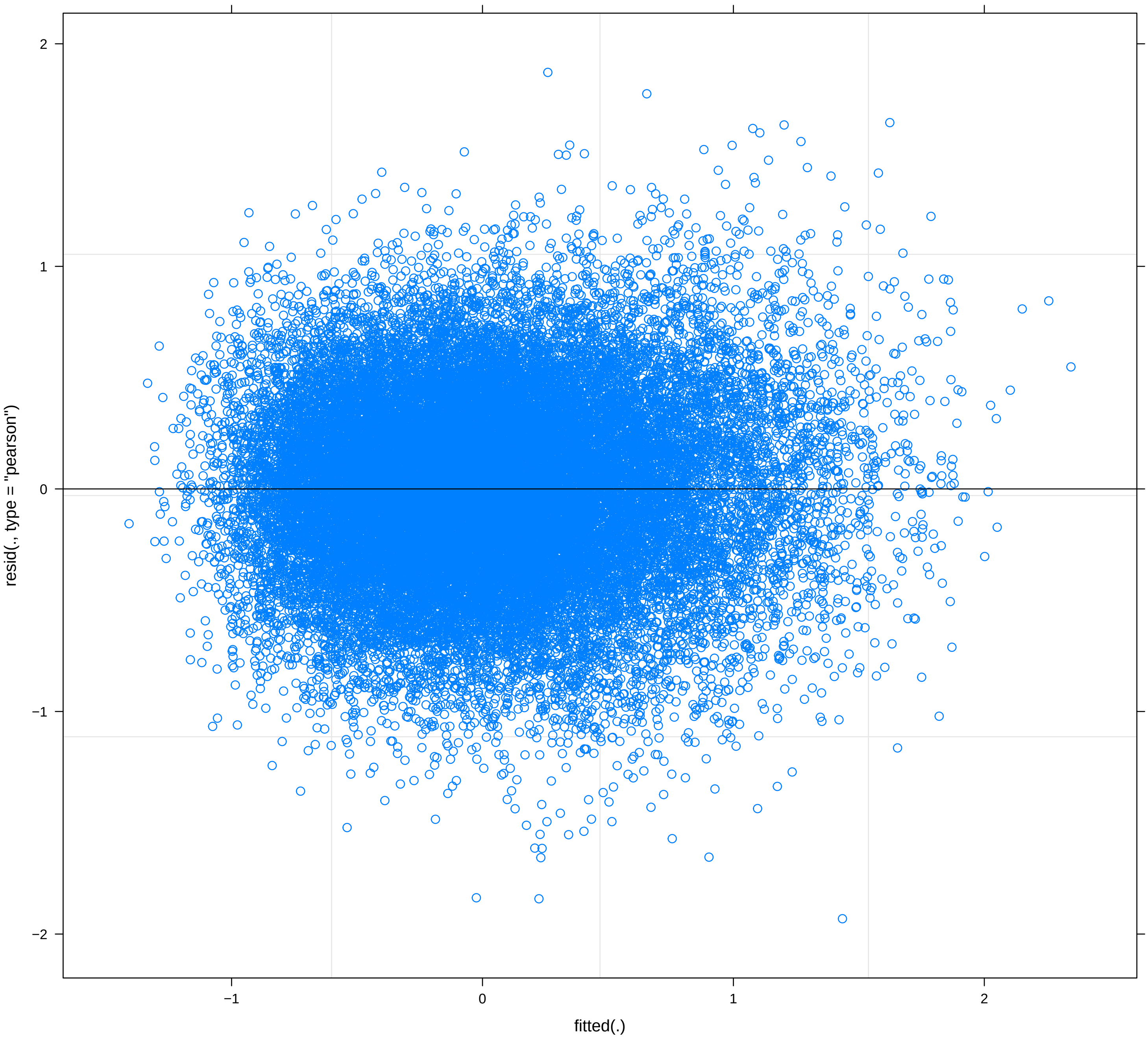
9.6 Model building and bias assessment

```
dat_sim <- my_sim_data_variance(increase_sd = increase_sd_value)

mod_sim <- lmer(DV ~ 1 + X_a * X_c * X_f + (1 | subj_id) + (1 |
    lab_id) + (1 | item_id), data = dat_sim)

summary(mod_sim)
```

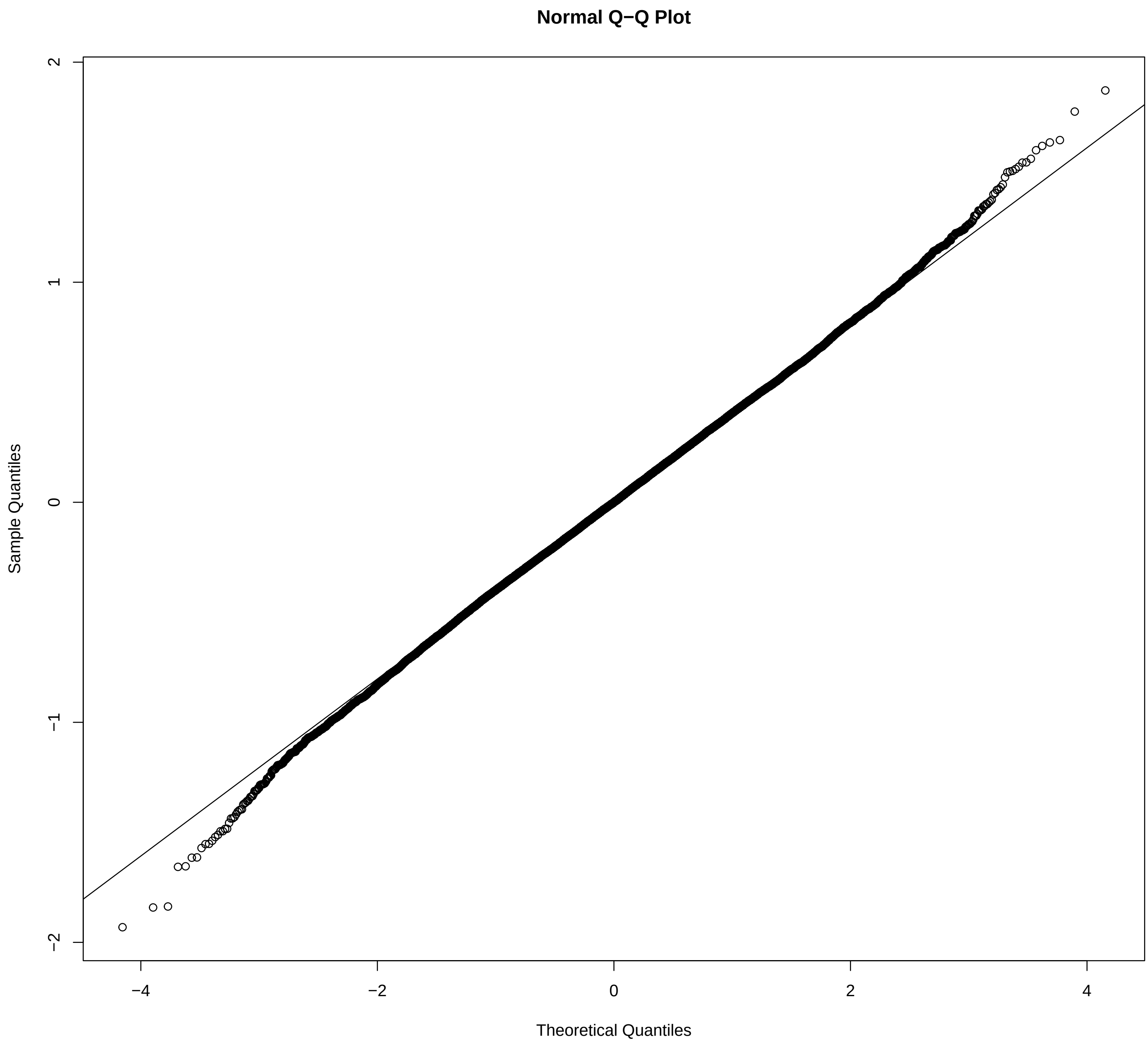
```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: DV ~ 1 + X_a * X_c * X_f + (1 | subj_id) + (1 | lab_id) + (1 |
##     item_id)
##    Data: dat_sim
##
```

```
## REML criterion at convergence: 35957.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.6460 -0.6486 -0.0005  0.6571  4.5024
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  subj_id  (Intercept) 0.04078  0.2019
##  lab_id   (Intercept) 0.02656  0.1630
##  item_id  (Intercept) 0.06894  0.2626
##  Residual             0.17278  0.4157
## Number of obs: 30720, groups:  subj_id, 1280; lab_id, 40; item_id, 24
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 2.737e-02  5.978e-02 2.993e+01   0.458  0.65040
## X_a         3.720e-01  2.133e-02 1.243e+03  17.437  < 2e-16 ***
## X_c         5.855e-01  1.073e-01 2.000e+01   5.457 2.43e-05 ***
## X_f         4.687e-01  1.314e-01 2.000e+01   3.567  0.00193 **
## X_a:X_c     1.976e-01  1.627e-02 2.941e+04  12.144  < 2e-16 ***
## X_a:X_f     3.844e-01  1.992e-02 2.941e+04  19.294  < 2e-16 ***
## X_c:X_f     6.772e-01  2.628e-01 2.000e+01   2.577  0.01801 *
## X_a:X_c:X_f 7.447e-01  3.985e-02 2.941e+04  18.690  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##             (Intr) X_a    X_c    X_f    X_:X_c X_:X_f X_c:X_
## X_a         -0.002
## X_c          0.000  0.000
## X_f          0.000  0.000  0.000
## X_a:X_c      0.000  0.000 -0.001  0.000
## X_a:X_f      0.000  0.000  0.000 -0.001  0.000
## X_c:X_f      0.000  0.000  0.000  0.000  0.000  0.000
## X_a:X_c:X_f  0.000  0.000  0.000  0.000  0.000  0.000 -0.001
```

```
plot(mod_sim)
```




```
qqnorm(resid(mod_sim))
qqline(resid(mod_sim))
```




```
check_normality(mod_sim)
```

```
## OK: residuals appear as normally distributed (p = 0.250).
```

```
check_outliers(mod_sim)
```

```
## OK: No outliers detected.
## - Based on the following method and threshold: cook (0.92).
## - For variable: (Whole model)
```

```
check_collinearity(mod_sim)
```

```
## # Check for Multicollinearity
##
## Low Correlation
##
```

```
##         Term  VIF  VIF 95% CI Increased SE Tolerance Tolerance 95% CI
##          X_a 1.00 [1.00,    ]         1.00      1.00     [    , 1.00]
##          X_c 1.00 [1.00, Inf]         1.00      1.00     [0.00, 1.00]
##          X_f 1.00 [1.00, Inf]         1.00      1.00     [0.00, 1.00]
##      X_a:X_c 1.00 [1.00, Inf]         1.00      1.00     [0.00, 1.00]
##      X_a:X_f 1.00 [1.00, Inf]         1.00      1.00     [0.00, 1.00]
##      X_c:X_f 1.00 [1.00, Inf]         1.00      1.00     [0.00, 1.00]
##  X_a:X_c:X_f 1.00 [1.00, Inf]         1.00      1.00     [0.00, 1.00]
```

```
check_heteroscedasticity(mod_sim)
```

```
## Warning: Heteroscedasticity (non-constant error variance) detected (p = 0.014).
```

9.7 Test how many of the models violate homoskedasticity

```
reps <- 200

run_sims_heteroskedasticity <- function(filename_heteroskedasticity) {

    dat_sim <- my_sim_data_variance(increase_sd = increase_sd_value)

    mod_sim <- lmer(DV ~ 1 + X_a * X_c * X_f + (1 | subj_id) +
        (1 | lab_id) + (1 | item_id), data = dat_sim)

    heteroskedasticity_results <- as_tibble(check_heteroscedasticity(mod_sim)[1])

    # append the results to a file
    append <- file.exists(filename_heteroskedasticity)
    write_csv(heteroskedasticity_results, filename_heteroskedasticity,
        append = append)

    # return the tidy table
    heteroskedasticity_results
}

filename_heteroskedasticity = "sims/filename_heteroskedasticity.csv"
start_time <- Sys.time()
heteroskedasticity_results_data <- purrr::map_df(1:reps, ~run_sims_heteroskedasticity(filename_heteroskedasticity
end_time <- Sys.time()
end_time - start_time
```

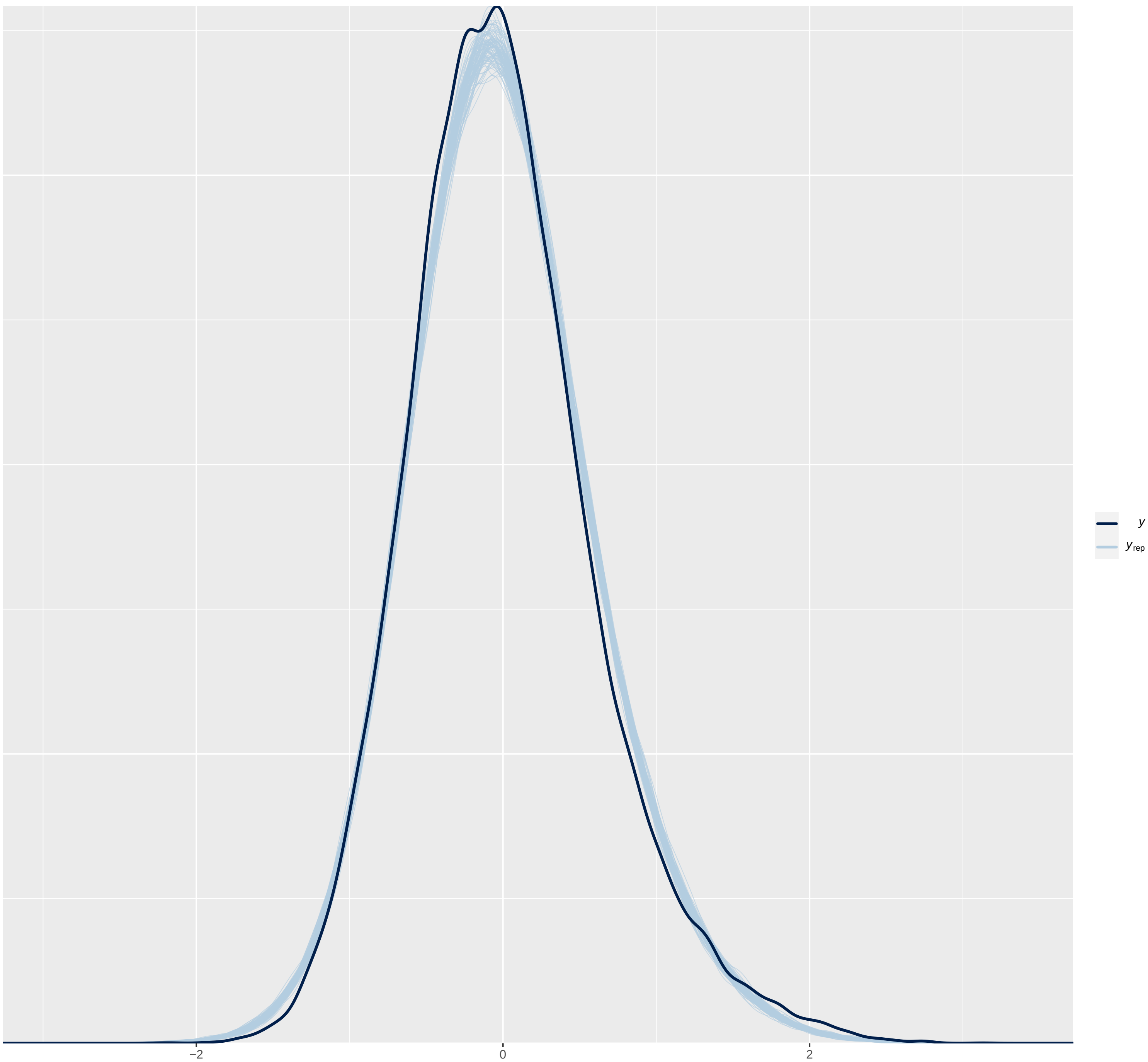
9.8 Overview of how many of the models exhibit homoskedasticity

```
heteroskedasticity_results <- read.csv(filename_heteroskedasticity)

homoskedastic_models <- heteroskedasticity_results %>%
    filter(value > 0.05)

print(paste0(nrow(homoskedastic_models), "/", nrow(heteroskedasticity_results),
    " of the models exhibit homoskedasticity"))
```

```
## [1] "156/200 of the models exhibit homoskedasticity"
```

9.9 Bayesian robust location-scale regression model

```r
model_formula <- bf(DV ~ 1 + X_a * X_c * X_f +
                      (1 | subj_id) + (1 | lab_id) + (1 | item_id),
                    sigma ~ 1 + trial_num + (1 | subj_id) + (1 | lab_id) + (1 | item_id))

#get_prior(model_formula, data = dat_sim, family = student)

priors1 <- c(prior(normal(0, 0.5), class = Intercept),
             prior(normal(0, 0.5), class = b),
             prior(normal(0, 0.5), class = b, dpar = sigma),
             prior(normal(0.25, 0.3), class = sd),
             prior(gamma(2, 0.1), class = nu)
             )

heteroskedasticity_fit <-
  brm(data = dat_sim,
           family = student,
           model_formula,
           prior = priors1,
           sample_prior = "yes",
           iter = 4000,
           warmup = 500,
           #backend = "cmdstanr",
           #threads = threading(2),
           file = "heteroskedasticity_fit",
           cores = 64,
           chains = 2,
           save_pars = save_pars(all = TRUE))

pp_check(heteroskedasticity_fit, ndraws = 100)
```



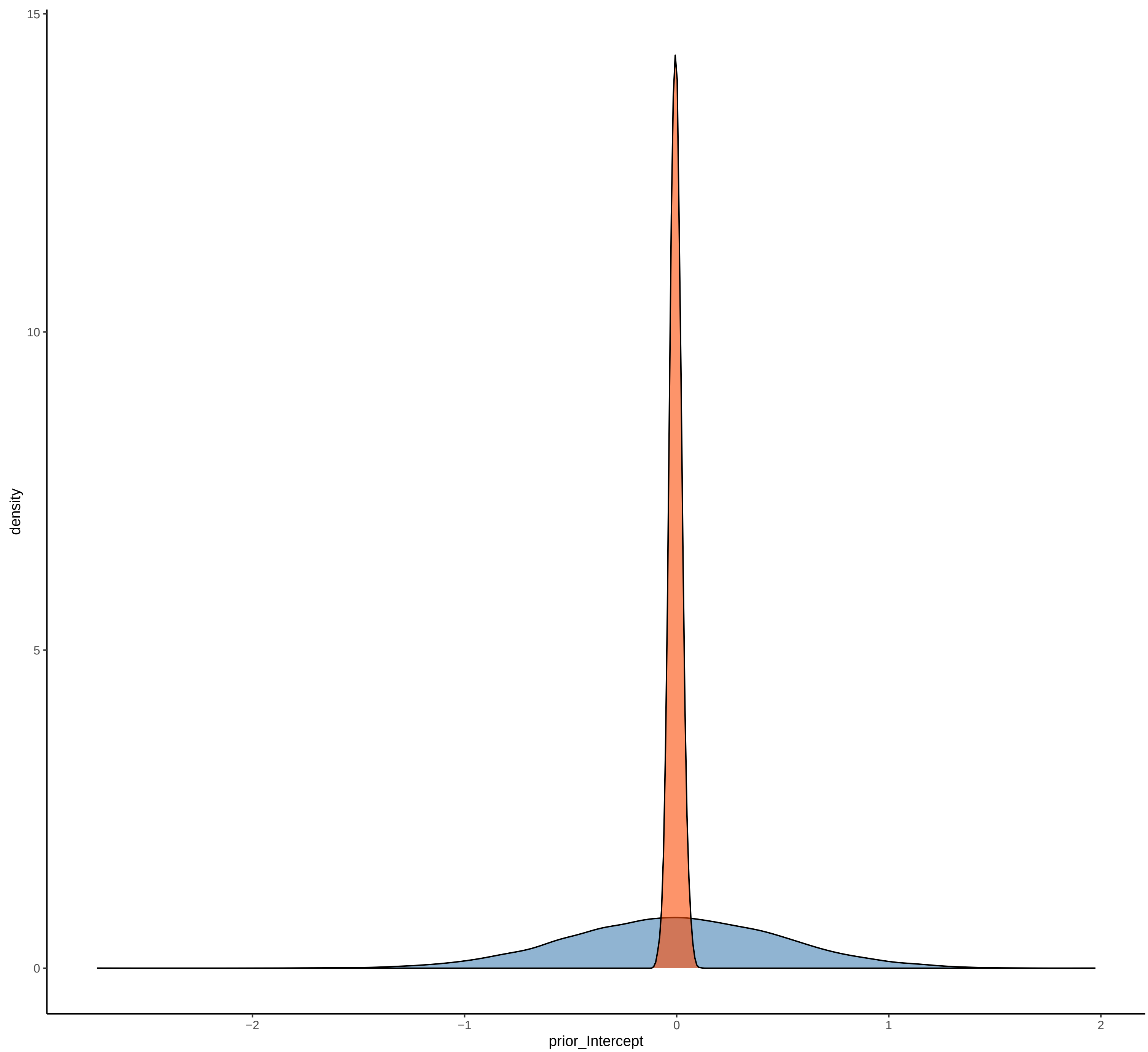

```
heteroskedasticity_fit
```

```
##  Family: student
##   Links: mu = identity; sigma = log; nu = identity
## Formula: DV ~ 1 + X_a * X_c * X_f + (1 | subj_id) + (1 | lab_id) + (1 | item_id)
##          sigma ~ 1 + trial_num + (1 | subj_id) + (1 | lab_id) + (1 | item_id)
##    Data: dat_sim (Number of observations: 30720)
##   Draws: 2 chains, each with iter = 4000; warmup = 500; thin = 1;
##          total post-warmup draws = 7000
##
## Group-Level Effects:
## ~item_id (Number of levels: 24)
##                     Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)           0.00      0.00     0.00     0.01 1.00     3967     4291
## sd(sigma_Intercept)     0.01      0.01     0.00     0.02 1.00     3111     3651
##
## ~lab_id (Number of levels: 40)
```

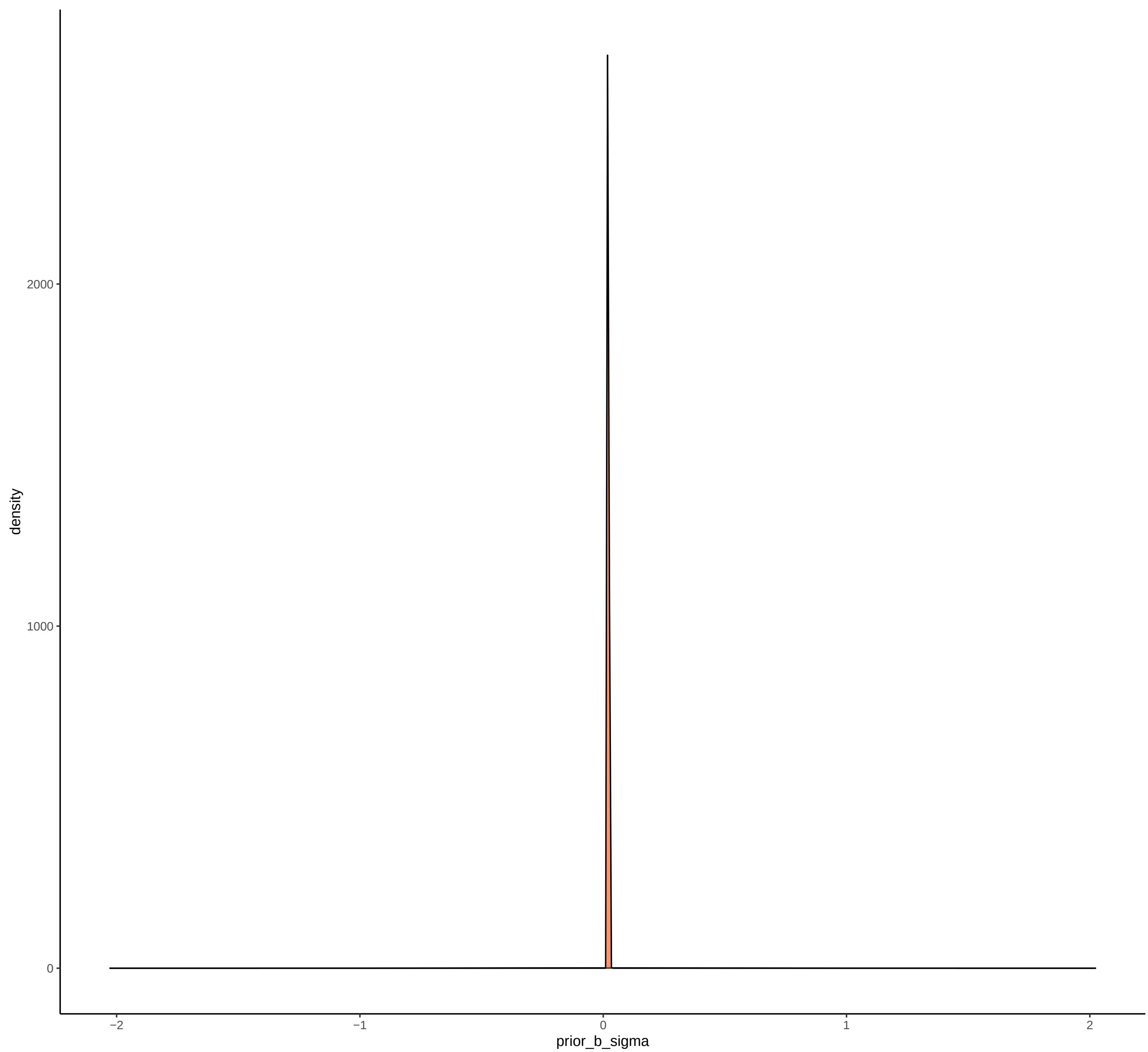
```
##                      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)            0.17      0.02     0.13     0.22 1.00     1881     3066
## sd(sigma_Intercept)      0.04      0.01     0.03     0.06 1.00     3176     5234
##
## ~subj_id (Number of levels: 1280)
##                      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## sd(Intercept)            0.21      0.01     0.20     0.22 1.00     2481     3755
## sd(sigma_Intercept)      0.05      0.01     0.03     0.07 1.00     1260     1282
##
## Population-Level Effects:
##                  Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## Intercept           -0.01      0.03    -0.06     0.05 1.00     1051     1816
## sigma_Intercept     -1.07      0.01    -1.09    -1.04 1.00     6208     5953
## X_a                  0.26      0.02     0.22     0.31 1.00     2180     3628
## X_c                  0.30      0.01     0.29     0.32 1.00    11577     5326
## X_f                  0.39      0.01     0.38     0.40 1.00    14033     5762
## X_a:X_c              0.45      0.02     0.42     0.49 1.00    17307     5375
## X_a:X_f              0.67      0.02     0.62     0.71 1.00    14751     5860
## X_c:X_f              0.73      0.01     0.70     0.75 1.00    16307     5246
## X_a:X_c:X_f          0.36      0.04     0.27     0.44 1.00    15476     5133
## sigma_trial_num      0.02      0.00     0.02     0.02 1.00     7882     5137
##
## Family Specific Parameters:
##    Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
## nu    81.74     19.02    52.07   125.80 1.00    11984     5331
##
## Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
## and Tail_ESS are effective sample size measures, and Rhat is the potential
## scale reduction factor on split chains (at convergence, Rhat = 1).
```

9.9.1 Prior-Posterior Update Checks

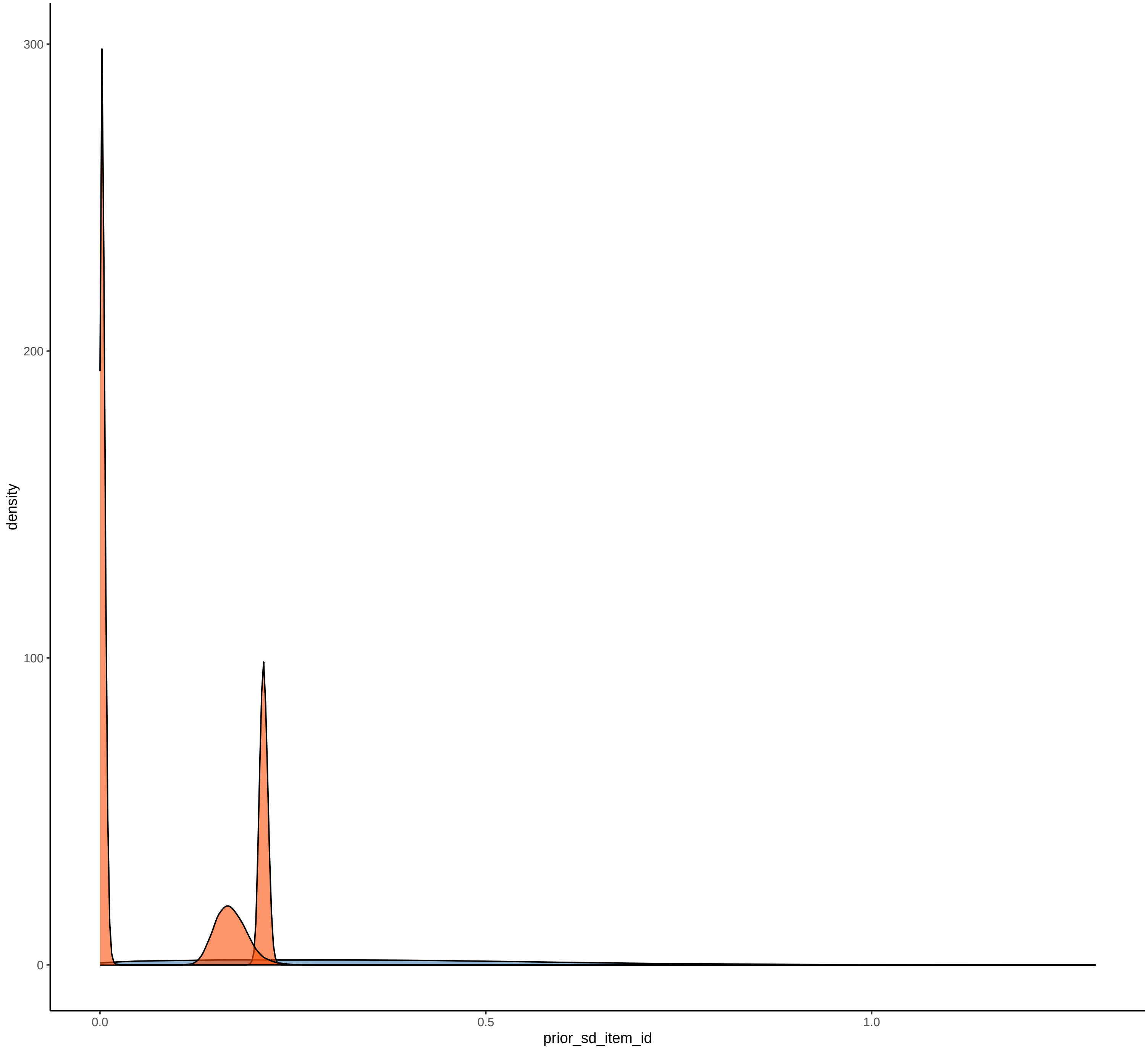
```
# Sample the parameters of interest:
Posterior_m1 <- as_draws_df(heteroskedasticity_fit)

# Plot the prior-posterior update plot for the intercept:
ggplot(Posterior_m1) + geom_density(aes(prior_Intercept), fill = "steelblue",
    color = "black", alpha = 0.6) + geom_density(aes(b_Intercept),
    fill = "#FC4E07", color = "black", alpha = 0.6) + theme_classic()
```



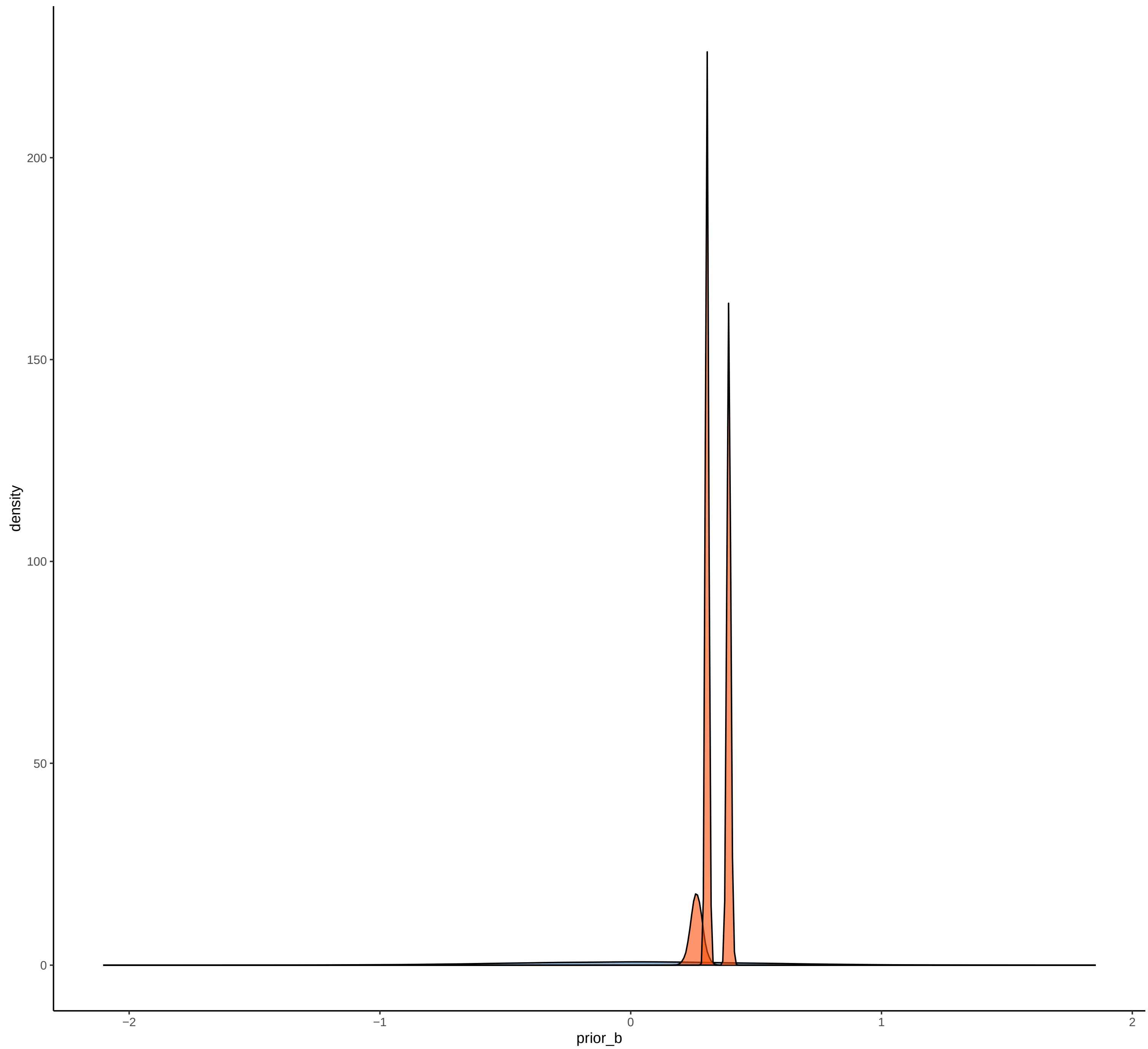

```
ggplot(Posterior_m1) + geom_density(aes(prior_b_sigma), fill = "steelblue",
    color = "black", alpha = 0.6) + geom_density(aes(b_sigma_trial_num),
    fill = "#FC4E07", color = "black", alpha = 0.6) + theme_classic()
```



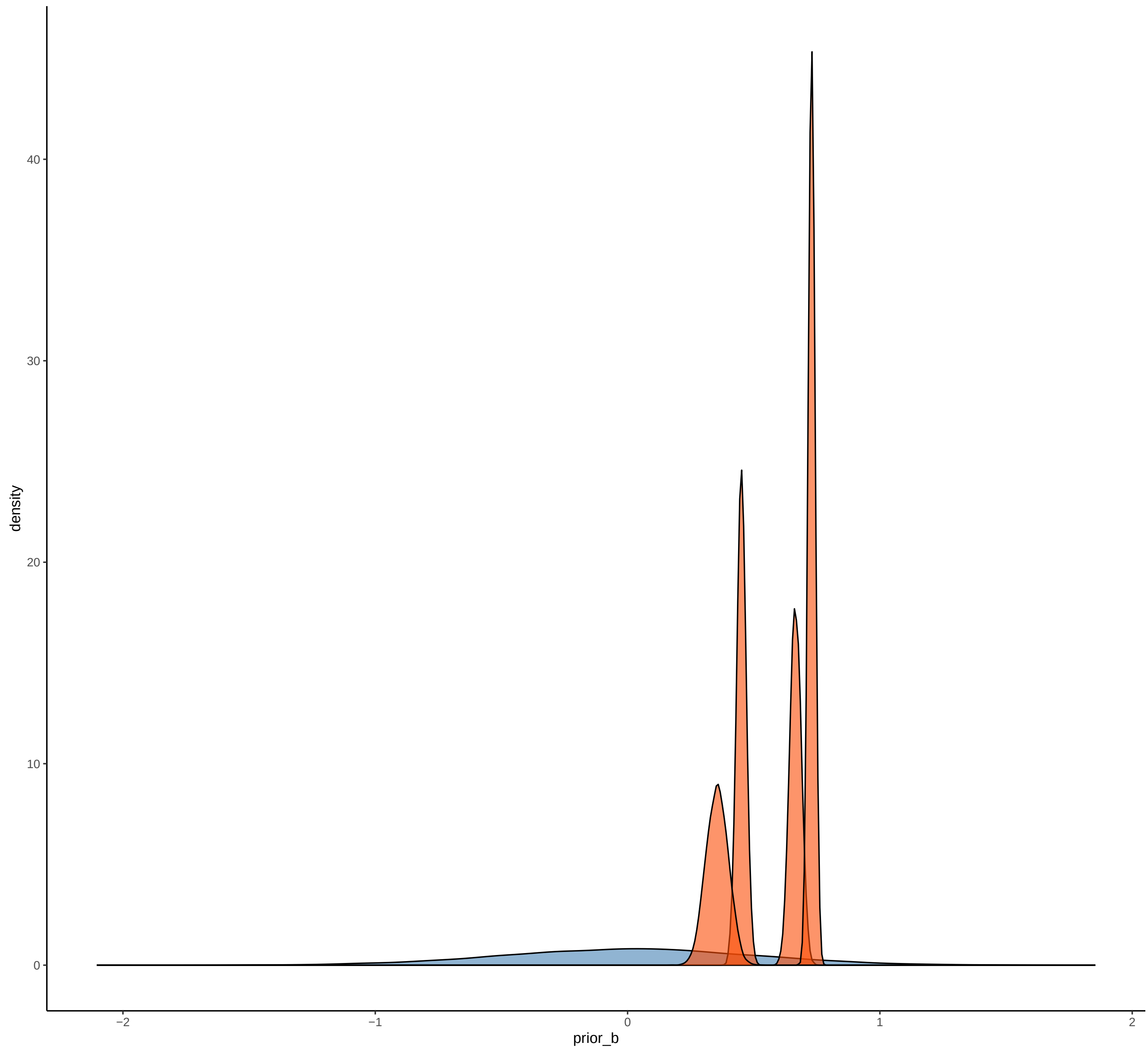

```
ggplot(Posterior_m1) + geom_density(aes(prior_sd_item_id), fill = "steelblue",
    color = "black", alpha = 0.6) + geom_density(aes(sd_subj_id__Intercept),
    fill = "#FC4E07", color = "black", alpha = 0.6) + geom_density(aes(sd_lab_id__Intercept),
    fill = "#FC4E07", color = "black", alpha = 0.6) + geom_density(aes(sd_item_id__Intercept),
    fill = "#FC4E07", color = "black", alpha = 0.6) + theme_classic()
```



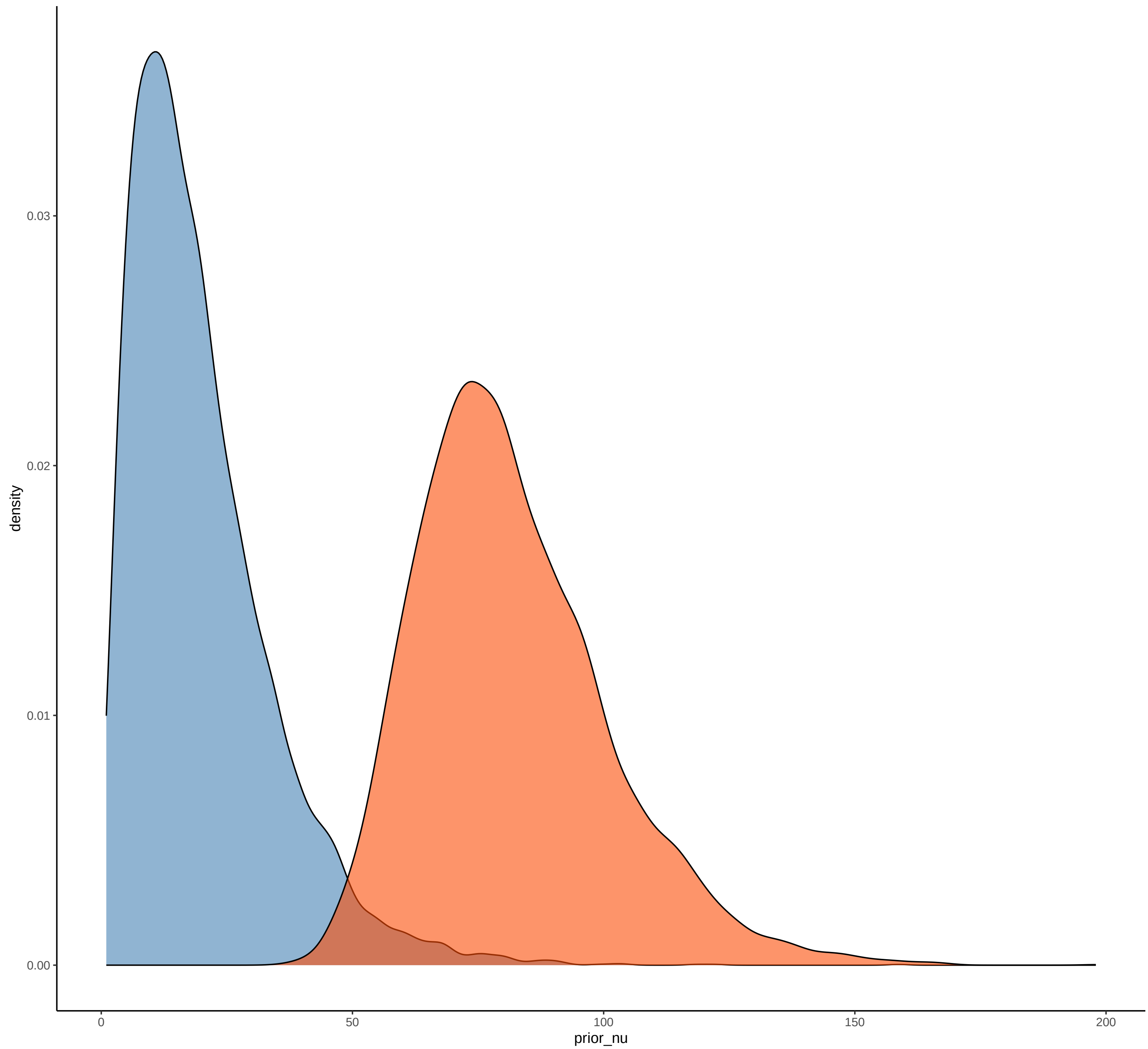

```
ggplot(Posterior_m1) + geom_density(aes(prior_b), fill = "steelblue",
    color = "black", alpha = 0.6) + geom_density(aes(b_X_a),
    fill = "#FC4E07", color = "black", alpha = 0.6) + geom_density(aes(b_X_c),
    fill = "#FC4E07", color = "black", alpha = 0.6) + geom_density(aes(b_X_f),
    fill = "#FC4E07", color = "black", alpha = 0.6) + theme_classic()
```




```r
ggplot(Posterior_m1) + geom_density(aes(prior_b), fill = "steelblue",
    color = "black", alpha = 0.6) + geom_density(aes(`b_X_a:X_c`),
    fill = "#FC4E07", color = "black", alpha = 0.6) + geom_density(aes(`b_X_a:X_f`),
    fill = "#FC4E07", color = "black", alpha = 0.6) + geom_density(aes(`b_X_c:X_f`),
    fill = "#FC4E07", color = "black", alpha = 0.6) + geom_density(aes(`b_X_a:X_c:X_f`),
    fill = "#FC4E07", color = "black", alpha = 0.6) + theme_classic()
```




```
ggplot(Posterior_m1) + geom_density(aes(prior_nu), fill = "steelblue",
    color = "black", alpha = 0.6) + geom_density(aes(nu), fill = "#FC4E07",
    color = "black", alpha = 0.6) + theme_classic()
```

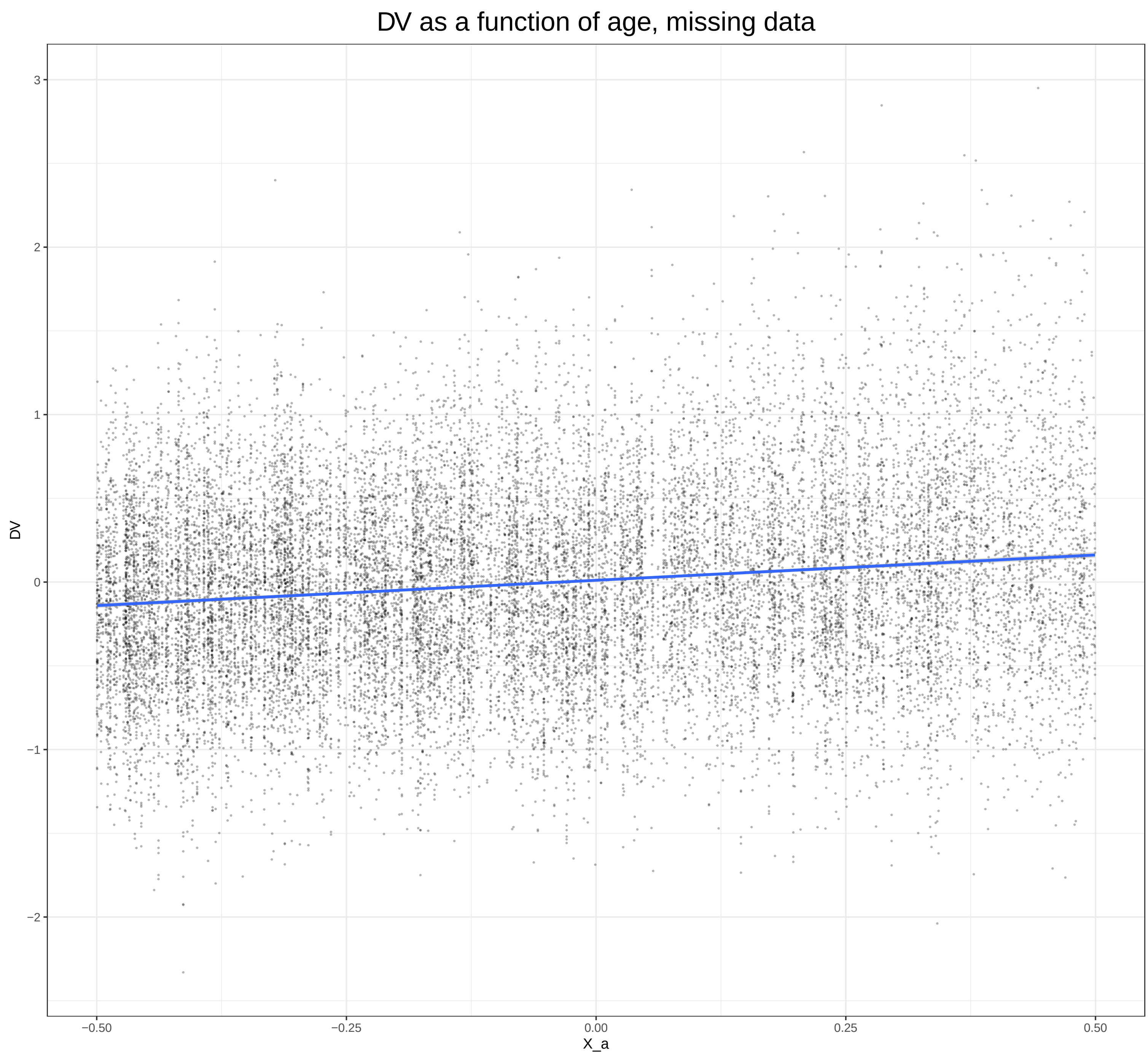


10 Data missing not completely at random

10.1 Simulation of missing data according to increasing infant age

```
# back to assumptions of equal variance across trials:
dat_sim <- my_sim_data()

# Proportion of missing data increases with age:
missing_samples_age <- dat_sim %>%
    mutate(nas = rbinom(n(), 1, 0.95 - ifelse(X_a > -0.5, (X_a +
        0.5) * 0.5, 0))) %>%
    mutate(DV = ifelse(nas == 1, DV, NA))

missing_age_plot <- missing_samples_age %>%
    drop_na() %>%
```

```r
    ggplot() + ggtitle("DV as a function of age, missing data") +
    geom_point(aes(x = X_a, y = DV), alpha = 0.3, size = 0.2,
        position = "jitter") + geom_smooth(aes(x = X_a, y = DV),
    method = "lm", se = TRUE, formula = y ~ x) + theme_bw()

missing_age_plot <- missing_age_plot + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
missing_age_plot
```



10.1.1 Simulation of models

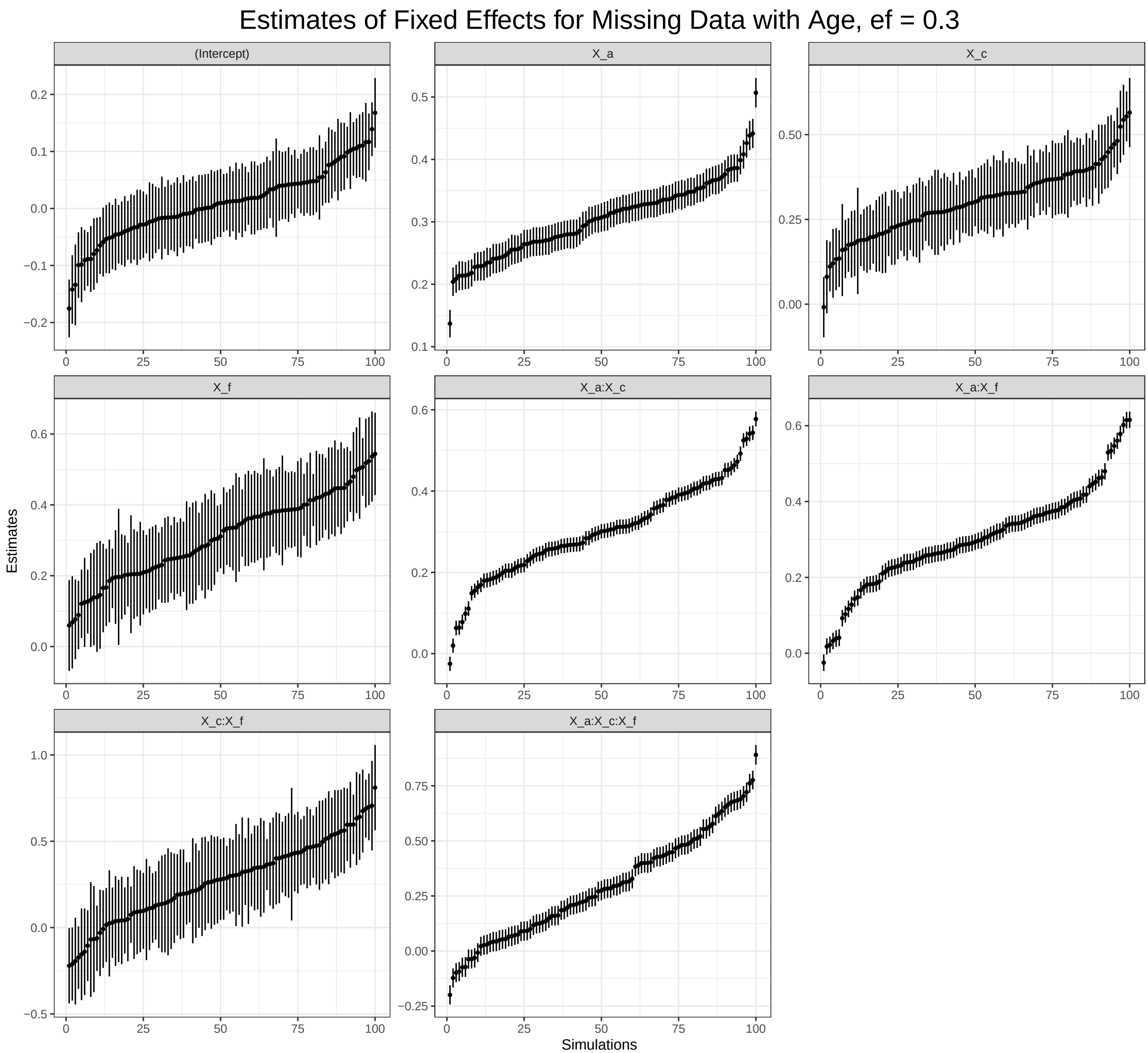
```r
# Number of simulations:
reps <- 100

# Simulation function:
```

```
run_sims <- function(filename_full, ef) {

  dat_sim <- my_sim_data(beta_c  =  ef,
                         beta_f = ef,
                         beta_a = ef,

                         beta_ca = ef,
                         beta_af = ef,
                         beta_cf = ef,

                         beta_cfa = ef)

  missing_samples_age <- dat_sim %>%
    mutate(nas = rbinom(n(), 1, 0.95 - ifelse(X_a > -0.5, (X_a+0.5)*0.50, 0))) %>%
    mutate(DV = ifelse(nas == 1, DV, NA))

  mod_sim <- lmer(DV ~ 1 + X_a * X_c * X_f +
                    (1 | subj_id) +
                    (1 | lab_id) +
                    (1 | item_id),
                  data=missing_samples_age)

  sim_results <- broom.mixed::tidy(mod_sim)

  # append the results to a file
  append <- file.exists(filename_full)
  write_csv(sim_results, filename_full, append = append)

  # return the tidy table
  sim_results
}

filename_full_0.3_missing_age = 'sims/run_sims_0.3_age_missing.csv'
start_time <- Sys.time()
sims <- purrr::map_df(1:reps, ~run_sims(filename_full = filename_full_0.3_missing_age, ef = 0.3))
end_time <- Sys.time()
end_time - start_time
```

10.1.2 Visualise Estimates for Fixed Effects:

```
# read saved simulation data
sims_50_missing_age_0.3 <- read_csv(filename_full_0.3_missing_age, col_types = cols(
  # makes sure plots display in this order
  group = col_factor(ordered = TRUE),
  term = col_factor(ordered = TRUE)
))

reps <- 100

fixed_missing_age_plot <- sims_50_missing_age_0.3 %>%
  filter(effect == "fixed") %>%
  ungroup() %>%
  arrange(term, estimate) %>%
  mutate(row = rep(seq(1:reps), 8)) %>%
  ggplot(aes(x = row, y = estimate, ymin = estimate-std.error, ymax = estimate+std.error)) +
  facet_wrap(~term, scales = "free") +
  geom_pointrange(fatten = 1/2) +
  ylab("Estimates") +
  xlab("Simulations") +
  ggtitle('Estimates of Fixed Effects for Missing Data with Age, ef = 0.3') +
```

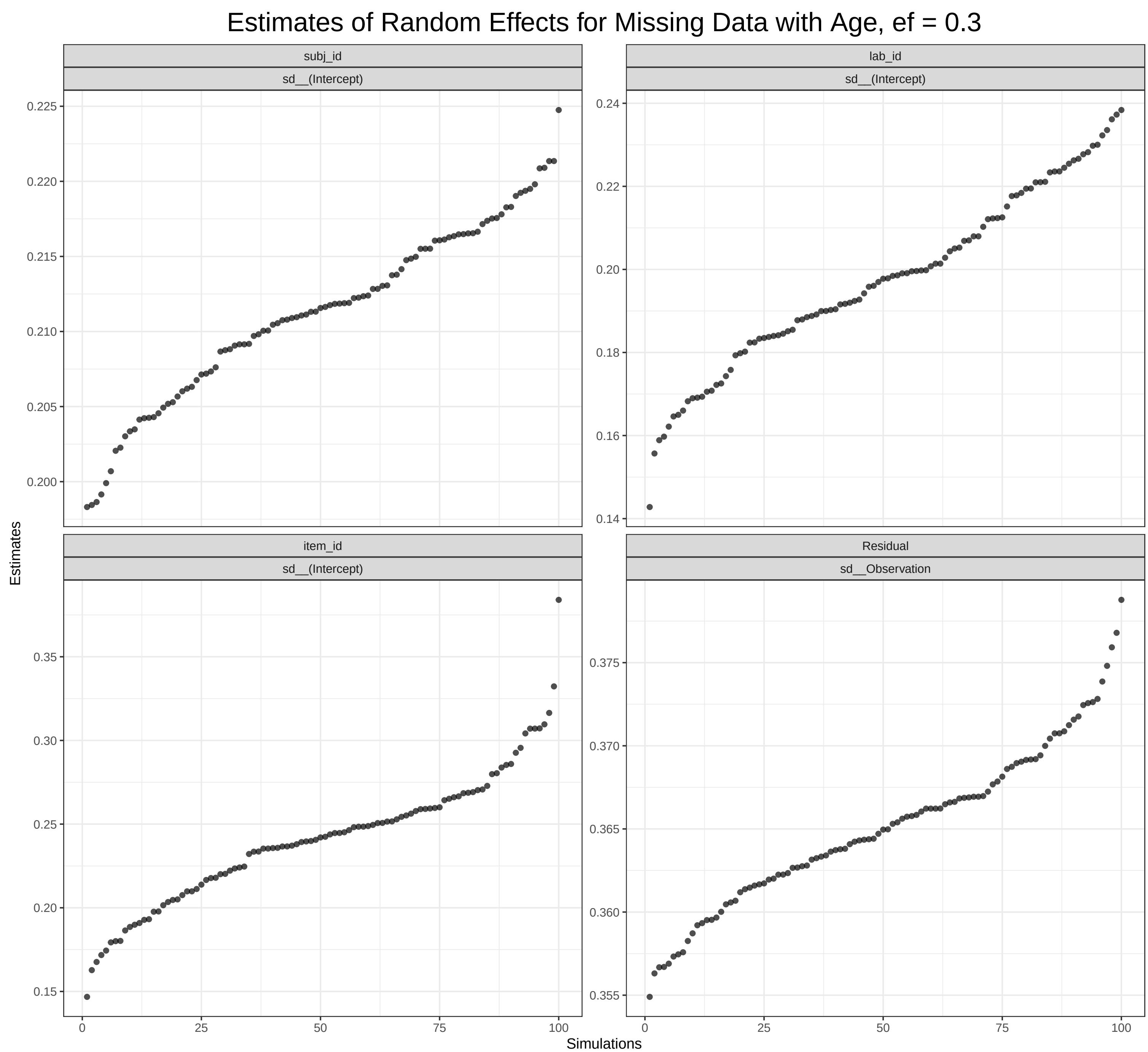
```
  theme_bw()

fixed_missing_age_plot <- fixed_missing_age_plot + theme(plot.title = element_text(hjust = 0.5, size=20))
fixed_missing_age_plot
```



10.1.3 Visualise Estimates for Random Effects:

```
ran_missing_age_plot <- sims_50_missing_age_0.3 %>%
    filter(effect == "ran_pars") %>%
    ungroup() %>%
    arrange(group, term, estimate) %>%
    mutate(row = rep(seq(1:reps), 4)) %>%
    ggplot(aes(x = row, y = estimate)) + geom_point(alpha = 0.7) +
    facet_wrap(~group + term, scales = "free_y") + theme_bw() +
    ylab("Estimates") + xlab("Simulations") + ggtitle("Estimates of Random Effects for Missing Data with Age, ef
```

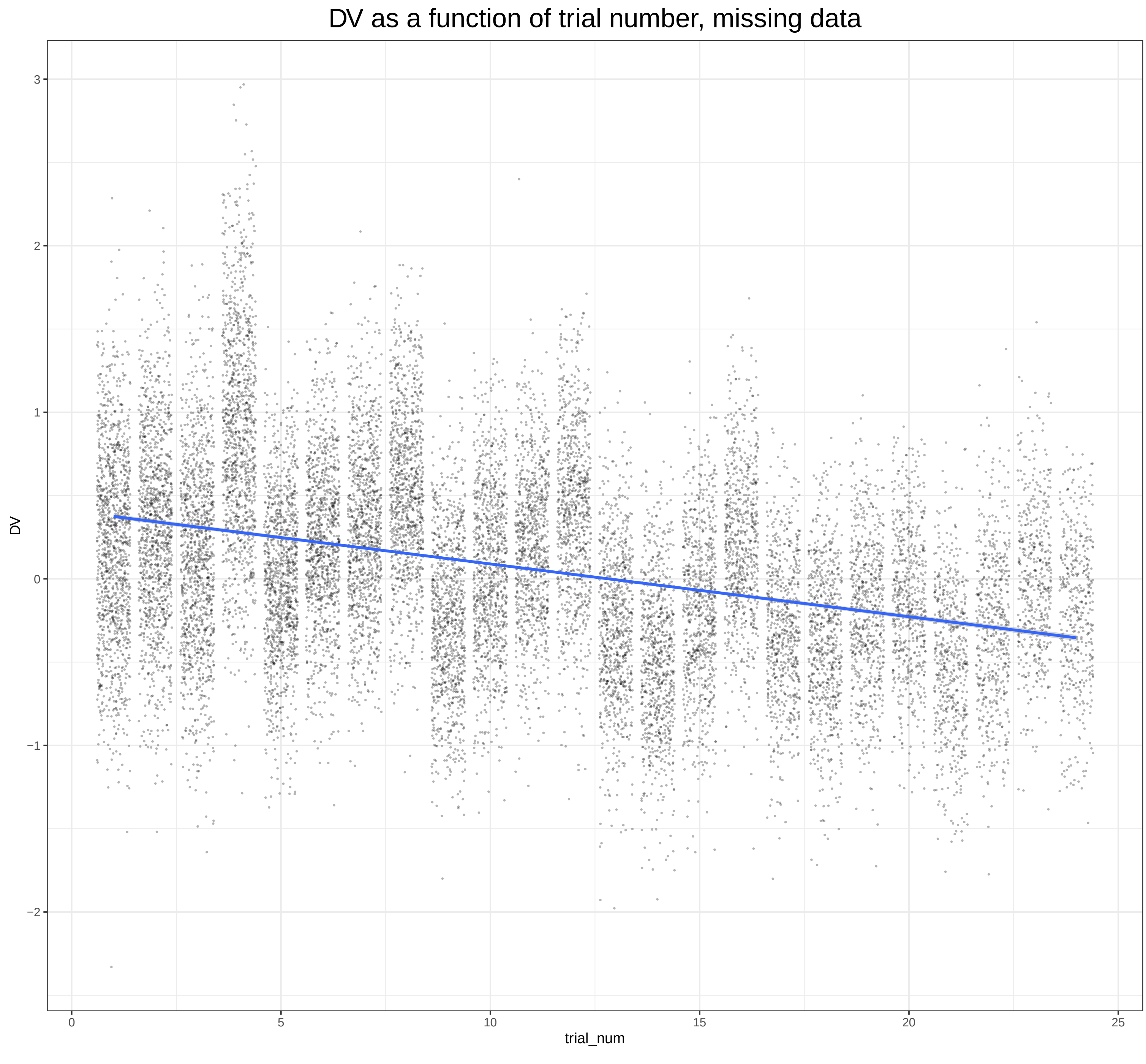
```
    theme_bw()
ran_missing_age_plot <- ran_missing_age_plot + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
ran_missing_age_plot
```



10.2 Simulation of missing data according to increasing trial number

```
# Proportion of missing data increases with trial number:
missing_samples_trial <- dat_sim %>%
    mutate(nas = rbinom(n(), 1, 0.95 - ifelse(trial_num > 0,
        trial_num * 0.025, 0))) %>%
    mutate(DV = ifelse(nas == 1, DV, NA))

missing_trialnum_plot <- missing_samples_trial %>%
    drop_na() %>%
```

```
    ggplot() + geom_point(aes(x = trial_num, y = DV), alpha = 0.3,
    size = 0.2, position = "jitter") + geom_smooth(aes(x = trial_num,
    y = DV), method = "lm", se = TRUE, formula = y ~ x) + ggtitle("DV as a function of trial number, missing data
    theme_bw()

missing_trialnum_plot <- missing_trialnum_plot + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
missing_trialnum_plot
```



10.2.1 Simulation of models

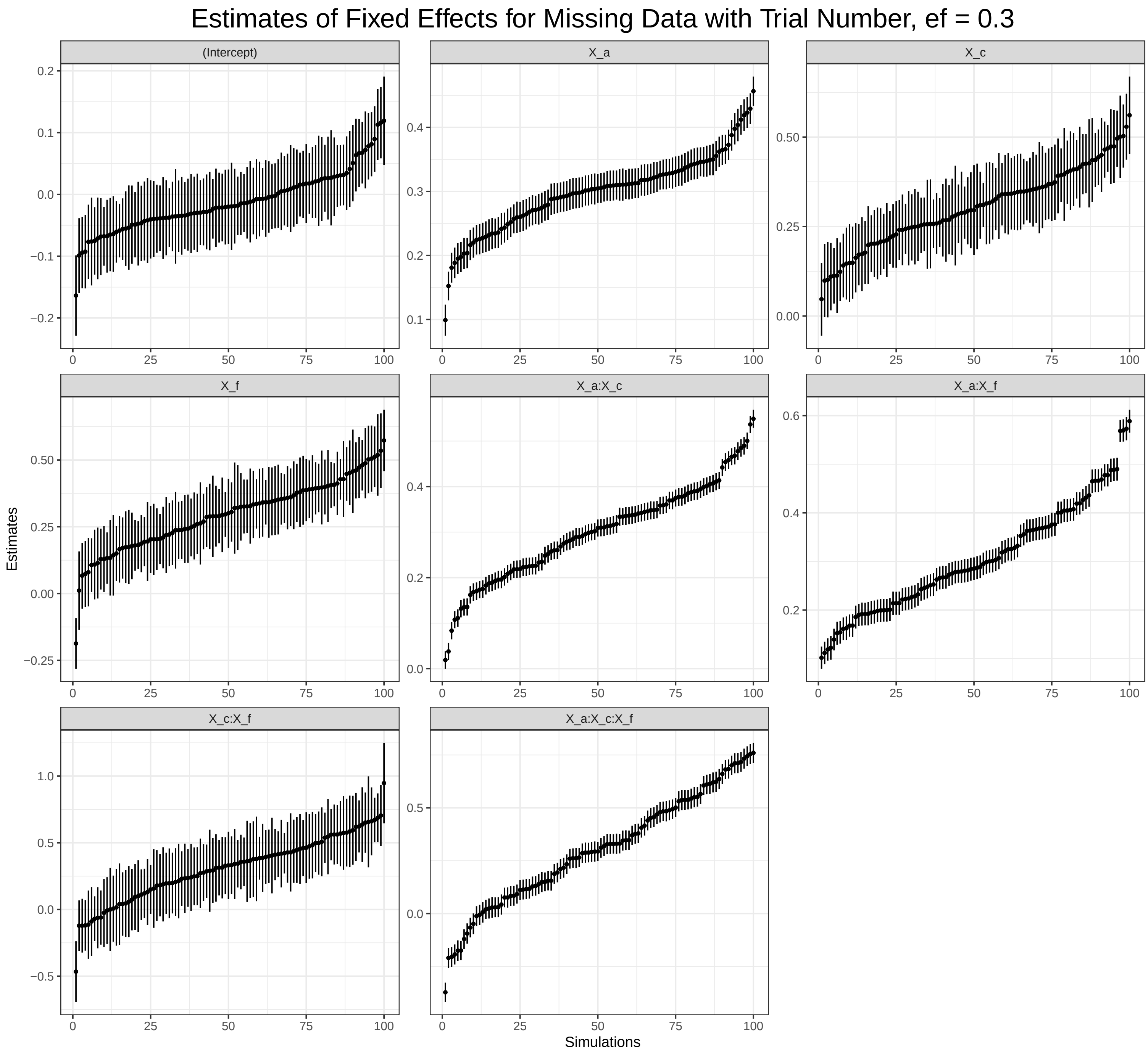
```
# Number of simulations:
reps <- 100

# Simulation function:
```

```
run_sims <- function(filename_full, ef) {

  dat_sim <- my_sim_data(beta_c  =  ef,
                         beta_f = ef,
                         beta_a = ef,

                         beta_ca = ef,
                         beta_af = ef,
                         beta_cf = ef,

                         beta_cfa = ef)

  missing_samples_trial <- dat_sim %>%
    mutate(nas = rbinom(n(), 1, 0.95 - ifelse(trial_num > 0, trial_num*0.025, 0))) %>%
    mutate(DV = ifelse(nas == 1, DV, NA))

  mod_sim <- lmer(DV ~ 1 + X_a * X_c * X_f +
                    (1 | subj_id) +
                    (1 | lab_id) +
                    (1 | item_id),
                  data=missing_samples_trial)

  sim_results <- broom.mixed::tidy(mod_sim)

  # append the results to a file
  append <- file.exists(filename_full)
  write_csv(sim_results, filename_full, append = append)

  # return the tidy table
  sim_results
}

filename_full_0.3_missing_trial = 'sims/run_sims_0.3_trial_missing.csv'
start_time <- Sys.time()
sims <- purrr::map_df(1:reps, ~run_sims(filename_full = filename_full_0.3_missing_trial, ef = 0.3))
end_time <- Sys.time()
end_time - start_time
```

10.2.2 Visualise Estimates for Fixed Effects:

```
# read saved simulation data
sims_50_missing_trial_0.3 <- read_csv(filename_full_0.3_missing_trial, col_types = cols(
  # makes sure plots display in this order
  group = col_factor(ordered = TRUE),
  term = col_factor(ordered = TRUE)
))

fixed_missing_trial_plot <- sims_50_missing_trial_0.3 %>%
  filter(effect == "fixed") %>%
  ungroup() %>%
  arrange(term, estimate) %>%
  mutate(row = rep(seq(1:reps), 8)) %>%
  ggplot(aes(x = row, y = estimate, ymin = estimate-std.error, ymax = estimate+std.error)) +
  facet_wrap(~term, scales = "free") +
  geom_pointrange(fatten = 1/2) +
  ylab("Estimates") +
  xlab("Simulations") +
  ggtitle('Estimates of Fixed Effects for Missing Data with Trial Number, ef = 0.3') +
  theme_bw()
```

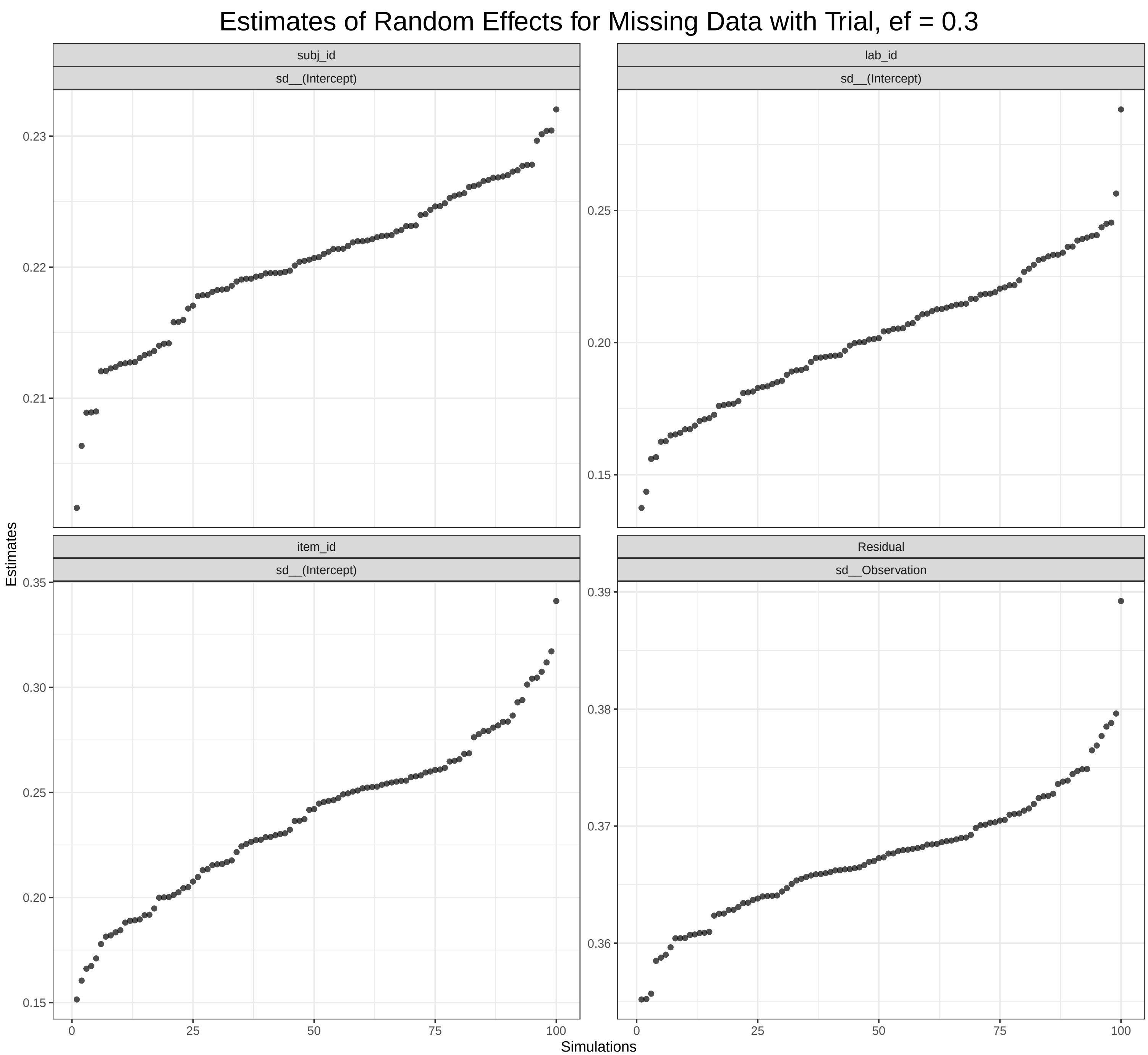
```
fixed_missing_trial_plot <- fixed_missing_trial_plot + theme(plot.title = element_text(hjust = 0.5, size=20))
fixed_missing_trial_plot
```



10.2.3 Visualise Estimates for Random Effects:

```
ran_missing_trial_plot <- sims_50_missing_trial_0.3 %>%
    filter(effect == "ran_pars") %>%
    ungroup() %>%
    arrange(group, term, estimate) %>%
    mutate(row = rep(seq(1:reps), 4)) %>%
    ggplot(aes(x = row, y = estimate)) + geom_point(alpha = 0.7) +
    facet_wrap(~group + term, scales = "free_y") + theme_bw() +
    ylab("Estimates") + xlab("Simulations") + ggtitle("Estimates of Random Effects for Missing Data with Trial, e
    theme_bw()
ran_missing_trial_plot <- ran_missing_trial_plot + theme(plot.title = element_text(hjust = 0.5,
```

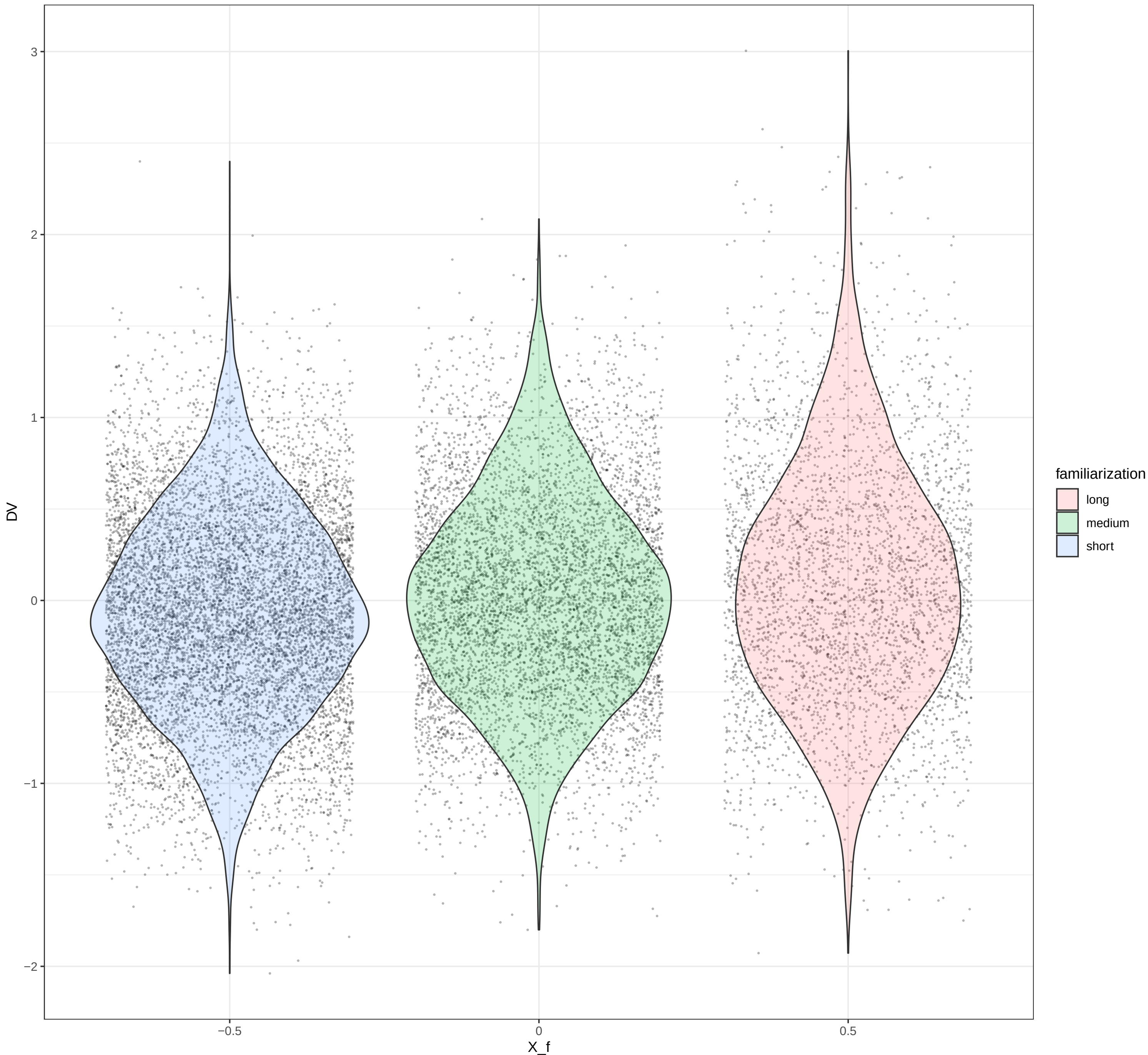
```
    size = 20))
ran_missing_trial_plot
```



10.3 Simulation of missing data according to increasing familiarisation

```
# Proportion of missing data increases with
# familiarisation:
missing_samples_fam <- dat_sim %>%
    mutate(nas = rbinom(n(), 1, 0.95 - ifelse(X_f > -0.5, (X_f +
        0.5) * 0.6, 0))) %>%
    mutate(DV = ifelse(nas == 1, DV, NA))

missing_samples_fam %>%
    mutate(X_f = as.factor(X_f)) %>%
    drop_na() %>%
```

```
ggplot() + geom_point(aes(x = X_f, y = DV), alpha = 0.3,
size = 0.2, position = "jitter") + geom_violin(aes(y = DV,
x = X_f, fill = familiarization), alpha = 0.2) + theme_bw()
```



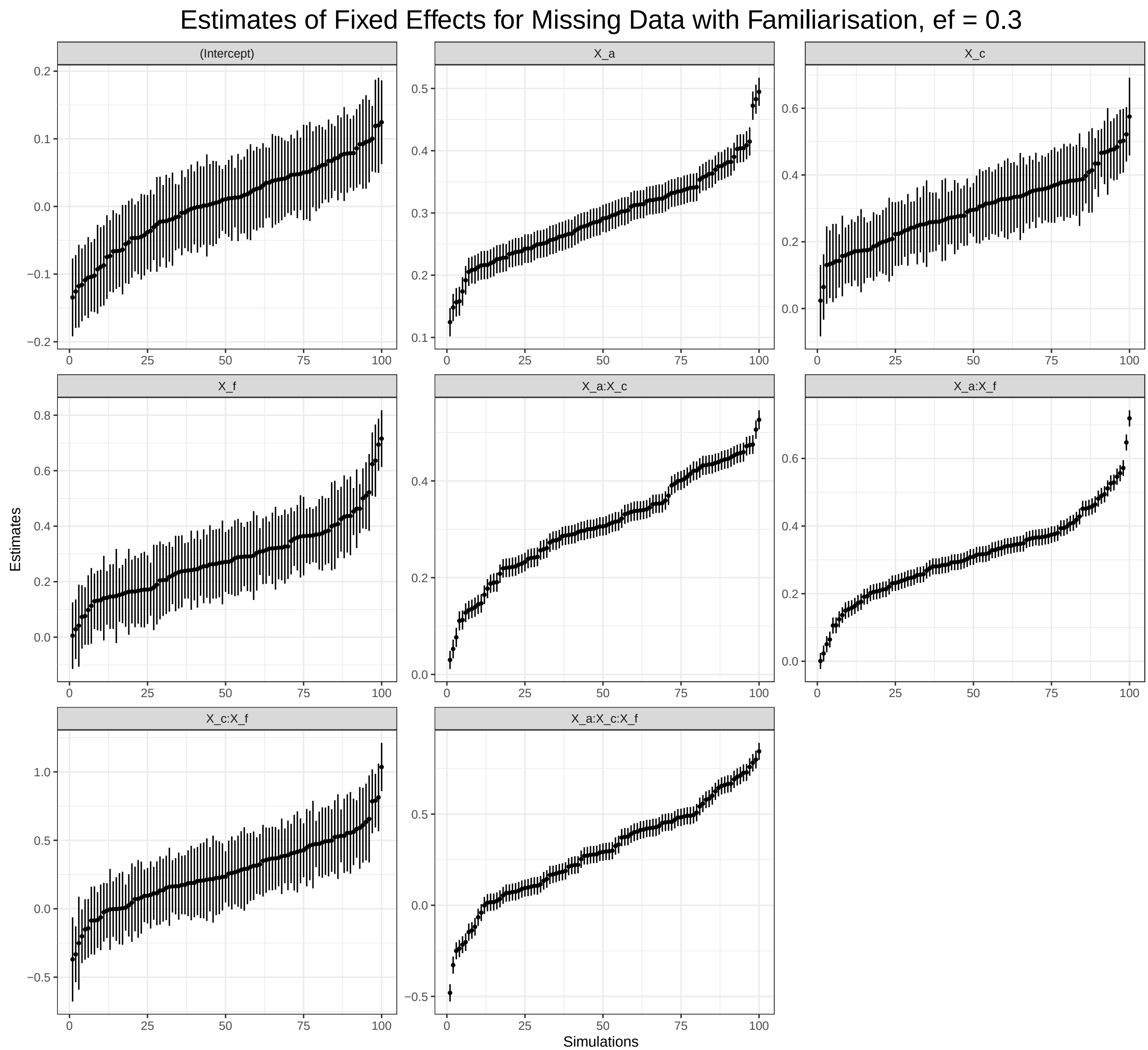
10.3.1 Simulation of models

```
# Number of simulations:
reps <- 100

# Simulation function:
run_sims <- function(filename_full, ef) {

  dat_sim <- my_sim_data(beta_c  =  ef,
                         beta_f = ef,
                         beta_a = ef,
```

```
                        beta_ca = ef,
                        beta_af = ef,
                        beta_cf = ef,

                        beta_cfa = ef)

  missing_samples_fam <- dat_sim %>%
    mutate(nas = rbinom(n(), 1, 0.95 - ifelse(X_f > -0.5, (X_f+0.5)*0.6, 0))) %>%
    mutate(DV = ifelse(nas == 1, DV, NA))

  mod_sim <- lmer(DV ~ 1 + X_a * X_c * X_f +
                    (1 | subj_id) +
                    (1 | lab_id) +
                    (1 | item_id),
                  data=missing_samples_fam)

  sim_results <- broom.mixed::tidy(mod_sim)

  # append the results to a file
  append <- file.exists(filename_full)
  write_csv(sim_results, filename_full, append = append)

  # return the tidy table
  sim_results
}

filename_full_0.3_missing_fam = 'sims/run_sims_0.3_fam_missing.csv'
start_time <- Sys.time()
sims <- purrr::map_df(1:reps, ~run_sims(filename_full = filename_full_0.3_missing_fam, ef = 0.3))
end_time <- Sys.time()
end_time - start_time
```

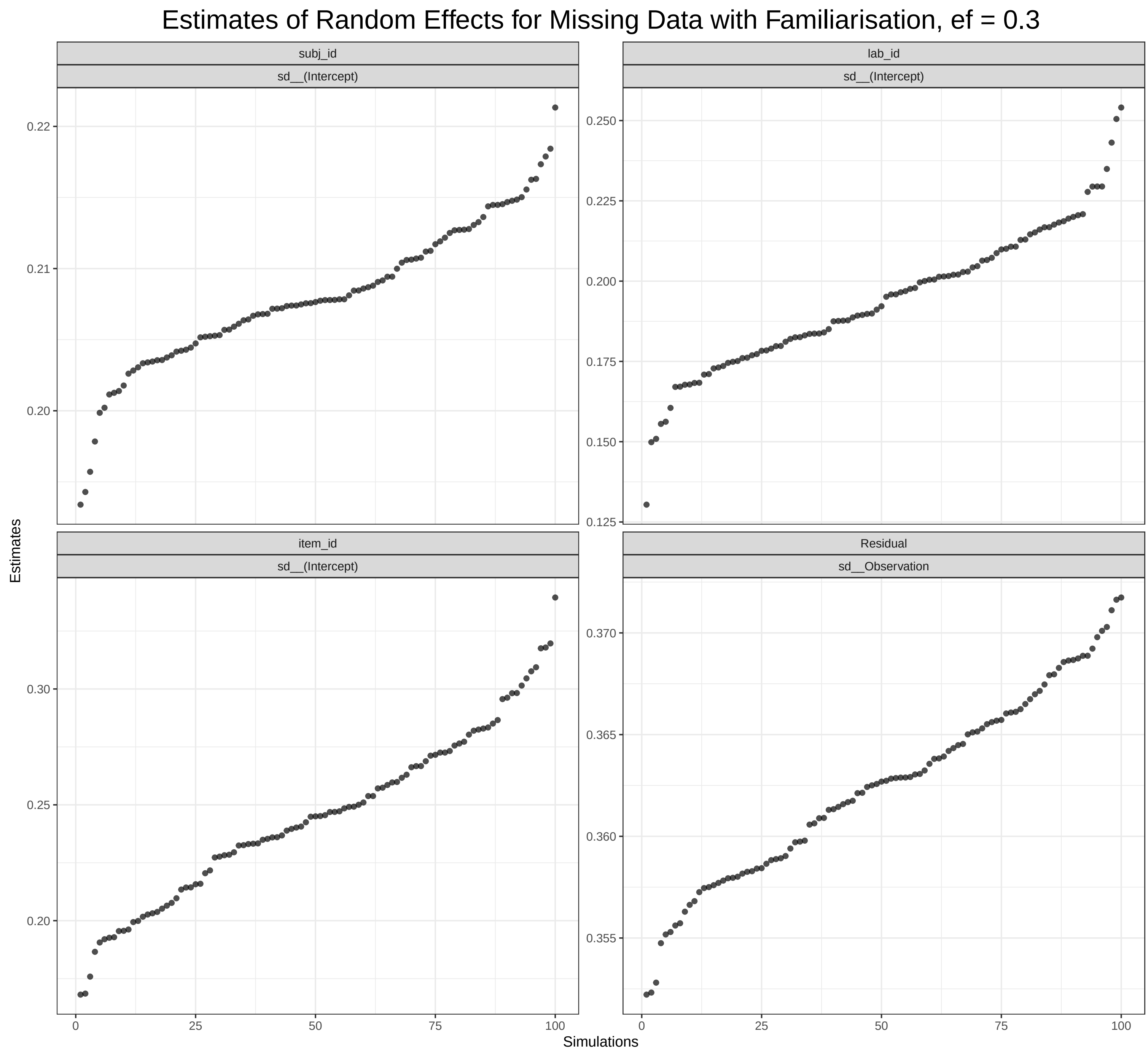
10.3.2 Visualise Estimates for Fixed Effects:

```
# read saved simulation data
sims_50_missing_fam_0.3 <- read_csv(filename_full_0.3_missing_fam, col_types = cols(
  # makes sure plots display in this order
  group = col_factor(ordered = TRUE),
  term = col_factor(ordered = TRUE)
))

fixed_missing_fam_plot <- sims_50_missing_fam_0.3 %>%
  filter(effect == "fixed") %>%
  ungroup() %>%
  arrange(term, estimate) %>%
  mutate(row = rep(seq(1:reps), 8)) %>%
  ggplot(aes(x = row, y = estimate, ymin = estimate-std.error, ymax = estimate+std.error)) +
  facet_wrap(~term, scales = "free") +
  geom_pointrange(fatten = 1/2) +
  ylab("Estimates") +
  xlab("Simulations") +
  ggtitle('Estimates of Fixed Effects for Missing Data with Familiarisation, ef = 0.3') +
  theme_bw()

fixed_missing_fam_plot <- fixed_missing_fam_plot + theme(plot.title = element_text(hjust = 0.5, size=20))
fixed_missing_fam_plot
```



Visualise Estimates for Random Effects:

```
ran_missing_fam_plot <- sims_50_missing_fam_0.3 %>%
    filter(effect == "ran_pars") %>%
    ungroup() %>%
    arrange(group, term, estimate) %>%
    mutate(row = rep(seq(1:reps), 4)) %>%
    ggplot(aes(x = row, y = estimate)) + geom_point(alpha = 0.7) +
    facet_wrap(~group + term, scales = "free_y") + theme_bw() +
    ylab("Estimates") + xlab("Simulations") + ggtitle("Estimates of Random Effects for Missing Data with Familiar
    theme_bw()
ran_missing_fam_plot <- ran_missing_fam_plot + theme(plot.title = element_text(hjust = 0.5,
    size = 20))
ran_missing_fam_plot
```



11 Summary Statistics for Power Calculation with Data missing not completely at random

Table 9: Power for Simulations with Non-Random Missing Data and Varying Intercepts

term	missing with age	missing with trial	missing with fam
(Intercept)	0.07	0.01	0.06
X__a	1.00	1.00	1.00
X__c	0.79	0.79	0.78
X__f	0.66	0.62	0.58
X__a:X__c	0.98	0.99	0.99
X__a:X__f	0.94	1.00	0.98
X__c:X__f	0.22	0.24	0.12
X__a:X__c:X__f	0.81	0.85	0.85

Table 10: Bias for Simulations with Non-Random Missing Data and Varying Intercepts

term	age bias, ef = 0.3	trial bias, ef = 0.3	fam bias, ef = 0.3
(Intercept)	-0.010	0.020	-0.011
X__a	-0.008	-0.005	0.008
X__c	-0.003	-0.001	0.004
X__f	-0.019	-0.003	0.029
X__a:X__c	-0.001	-0.009	-0.007
X__a:X__f	0.005	0.014	-0.012
X__c:X__f	0.019	-0.031	0.055
X__a:X__c:X__f	0.022	-0.002	0.006

12 Logistic Regression Models

```
m_missing_age <- glmer(nas ~ X_f + X_a + X_c + trial_num + (1 |
    subj_id) + (1 | lab_id) + (1 | item_id), data = missing_samples_age,
    family = binomial)

summary(m_missing_age)

m_missing_fam <- glmer(nas ~ X_f + X_a + X_c + trial_num + (1 |
    subj_id) + (1 | lab_id) + (1 | item_id), data = missing_samples_fam,
    family = binomial)

summary(m_missing_fam)

m_missing_trial <- glmer(nas ~ X_f + X_a + X_c + trial_num +
    (1 | subj_id) + (1 | lab_id) + (1 | item_id), data = missing_samples_trial,
    family = binomial)

summary(m_missing_trial)
```