

Exercises Sheet #7

Introduction to Financial Engineering 2022

Note: The choice of software is up to the students. We recommend that you work in Python, but you can also choose to in R or Matlab. We do not provide solutions for all exercises in all languages.

These exercises deal with portfolio optimisation, especially the efficient frontier. It's useful to have the reading materials for Lecture 7 and/or the slides at hand. It's recommended to program the second exercise such that an arbitrary number of assets can be handled.

1. (returns, portfolios, diversification) Collect 10 years of weekly data for McDonald's, Coca Cola and Microsoft for the period January 1, 2011 to January 1, 2021.
 - (a) Calculate the weekly returns for each of the stocks.
 - (b) Calculate the annualised mean and covariance matrix for the returns of the stocks.
 - (c) Consider the portfolios weights $(0, 0, 1)$, $(0, 0.1, 0.9)$, $(0.1, 0.1, 0.8)$, \dots , $(0.9, 0.1, 0)$, $(1, 0, 0)$. For each of the portfolios¹, calculate the annualised mean and annualised standard deviation. Plot mean against standard deviation for all these portfolios in one single plot.
 - (d) Which of the portfolios considered in (c) has the maximal mean? Could you have answered this question without doing any calculations?
 - (e) Which of the portfolios considered in (c) has the lowest standard deviation? Could you have answered this question without doing the calculations done in (c)?
 - (f) Which of the portfolios considered in (c) has the highest ratio of mean to standard deviation? Why is this portfolio interesting?

¹Recall that a portfolio P with weights w has mean $\mu_P = w'\mu$ and variance $\sigma_P^2 = w'\Sigma w$

2. (portfolios, diversification, efficient frontier) Assume that

- Asset 1 has an annual expected return of 10% and the standard deviation is 10%
- Asset 2 has an annual expected return of 20% and the standard deviation is 20%

(a) Make a function that finds the Efficient Frontier (combinations of $(\mu; \sigma)$) when short selling is allowed and with no riskless lending/borrowing for each of the following correlation coefficient between the returns of assets 1 and 2

- $\rho = 0$
- $\rho = 0.5$
- $\rho = -0.5$

Use different colours for different values of the correlation coefficient. Comment on your findings. What happens to the efficient frontier when $\rho \rightarrow \pm 1$? NOTE: It's a good idea to program this function for an arbitrary number of assets.

3. (portfolios, diversification, efficient frontier) Return to the data in exercise 1. Add the efficient frontier to the plot made in question 1. You have most likely programmed a function that illustrates the efficient frontier, when you did question 2. A sanity check here is that all portfolios from 1 should lie within the efficient frontier (why?)