UNIVERSITY OF MALAWI

The Polytechnic

END OF YEAR EXAMINATIONS

BSc. in
TECHNICAL EDUCATION

Year 2

CALCULUS & LINEAR ALGEBRA
MAT 220

December, 2008 **Time Allowed: 3 Hours**

1. This paper has **SIX** questions. Please check.
2. All questions carry equal marks. Answer any **FIVE** questions.
3. Show all your work clearly.
4. Answer each question on a separate sheet of paper

(Do not turn over the paper until you are told to do so)

QUESTION ONE

a) Define carefully what it means for the series $\sum_{n=1}^{\infty} a_n$ to converge to S. (3)

b) Determine whether the series is convergent or divergent. If it is convergent, find its sum:

(i) $\sum_{k=1}^{\infty} \frac{1}{2^k}$. (3)

(ii) $\sum_{k=0}^{\infty} \frac{1}{7}\left(\frac{3}{2}\right)^k$. (3)

(iii) $\sum_{k=2}^{\infty} 3\left(-\frac{1}{5}\right)^k$. (3)

c) Find the values of x for which the series $\sum_{n=0}^{\infty} x^n$ converges. (4)

d) If the n^{th} partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $S_n = 1 - \frac{1}{2^n}$, find $\sum_{n=1}^{\infty} a_n$. (4)

QUESTION TWO

a) Explain what it means for the two functions f_1 and f_2 to be linearly dependent on the interval (a,b). (3)

b) Consider two functions f_1 and f_2 defined on $\Re$ as $f_1(x) = e^x$, $f_2(x) = e^{-x}$.

(i) Evaluate the *Wronskian*, $W(f_1, f_2)$ of f_1 and f_2. (5)

(ii) Hence or otherwise, determine if the two functions, f_1 and f_2 are linearly independent on $\Re$. (2)

c) When is a first order ordinary differential equation said to be separable? (2)

d) Determine whether the equation $xy' = y^2 + \sqrt{y}$ is separable. (2)

e) Use the method of undetermined coefficients to find the general solution to the second order differential equation.

$$\frac{d^2 y}{dx^2} - 9\frac{dy}{dx} + 20y = x.$$ (6)

QUESTION THREE

a) Find x, y, z, w if $\begin{bmatrix} 2 & 4 \\ 3 & z \end{bmatrix} + \begin{bmatrix} x & y \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ w & 0 \end{bmatrix}$. (4)

b) Evaluate $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 3 & 1 & 0 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 3 & 0 & 1 \end{pmatrix}$. (4)

c) Given that $A = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$, find

(i) A^2. (2)

(ii) A^3. (2)

(iii) A^{-1}. (4)

(iv) Suggest a likely matrix for A^{10}. (4)

QUESTION FOUR

a) To solve a Cauchy-Euler differential equation $a_2 x^2 \frac{d^2 y}{dx^2} + a_1 x \frac{dy}{dx} + a_0 y = g(x)$ where $a_2, a_1, a_0 \in \Re$ we use the substitution $x = e^u$, where u is a function of x. Show that

(i) $x \frac{dy}{dx} = \frac{dy}{du}$. (4)

(ii) $x^2 \frac{d^2 y}{dx^2} = \frac{d^2 y}{du^2} - \frac{dy}{du}$. (4)

b) Hence solve the following Cauchy-Euler equation

$$x^2 y'' - 2xy' + 2y = x^3.$$ (12)

QUESTION FIVE

a) Let $\underset{\sim}{a} = 2\underset{\sim}{i} - 3\underset{\sim}{j} + \underset{\sim}{k}$, $\underset{\sim}{b} = \underset{\sim}{i} + \underset{\sim}{j} + \underset{\sim}{k}$ and $\underset{\sim}{c} = 2\underset{\sim}{i} - 3\underset{\sim}{k}$. Find

(i) $\underset{\sim}{a} + \underset{\sim}{b} + \underset{\sim}{c}$. (3)

(ii) $2\underset{\sim}{b} - 3\underset{\sim}{c}$. (3)

b) State whether each expression is meaningful. If not, explain why. If so, state whether it is a vector or a scalar.

(i) $\underset{\sim}{a} \bullet (\underset{\sim}{b} \times \underset{\sim}{c})$. (2)

(ii) $\underset{\sim}{a} \times (\underset{\sim}{b} \bullet \underset{\sim}{c})$. (2)

(iii) $\underset{\sim}{a} \times (\underset{\sim}{b} \times \underset{\sim}{c})$. (2)

(iv) $(\underset{\sim}{a} \bullet \underset{\sim}{b}) \times (\underset{\sim}{c} \bullet \underset{\sim}{d})$. (2)

(v) $(\underset{\sim}{a} \times \underset{\sim}{b}) \bullet (\underset{\sim}{c} \times \underset{\sim}{d})$. (2)

c) Show that $\underset{\sim}{0} \times \underset{\sim}{a} = \underset{\sim}{o} = \underset{\sim}{a} \times \underset{\sim}{0}$ for any vector $\underset{\sim}{a}$ in $\Re^3$. (4)

QUESTION SIX

a) Given that $m(1,2) + n(0,2) = (2,4)$, find the values of m and n (4)

b) Find $3\underset{\sim}{a} + 2\underset{\sim}{b}$ if $\underset{\sim}{a} = (2,1,4)$ and $\underset{\sim}{b} = (1,1,0)$ (3)

c) L_1 is a line through $(1,1,1)$ and $(3,3,0)$ and L_2 is the line through $(2,1,-2)$ and $(3,4,5)$. Do L_1 and L_2 meet? If so where? (6)

d) ABC is a triangle. H, K are mid points of AB and AC respectively. Show using vectors that $HK = \frac{1}{2}BC$ and $HK \parallel BC$ (7)

END OF QUESTION PAPER.

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END OF SEMESTER ONE EXAMINATIONS

BSc. in

Technical Education

Year 2

CALCULUS II

CAL-210

June, 2012 | Time Allowed: 3 Hours

Instructions

1. This paper has **FOUR** questions. Please check.
2. All questions carry equal marks.
3. Answer all the questions.
4. Show all your work clearly.
5. Start each question on a fresh page.

(Do not turn over until you are told to do so)

QUESTION ONE

a) Let $I_n = \int x^n e^{2x} dx.$

i) Integrating by parts, show that $I_n = \frac{1}{2}x^n e^{2x} - \frac{n}{2} I_{n-1}.$ (6 marks)

ii) Hence evaluate I_2. (4 marks)

b) Decompose $\frac{-x+1}{x^2+3x+2}$ into a sum of partial fractions. (4 marks)

c) Show that if $g(x) = \sec x + \tan x$, then $\frac{g'(x)}{g(x)} = \sec x$. Hence evaluate $\int \sec x\, dx$.

(4 marks)

d) Evaluate the following integrals:

i) $\int x^3 e^{x^4} dx.$ (3 marks)

ii) $\int \ln x\, dx.$ (4 marks)

QUESTION TWO

a) If $5e^x - 4e^{-x} \equiv A \sinh x + B \cosh x$, find A and B. (4 marks)

b) Show that $\cosh^2 x - \sinh^2 x = 1.$ (3 marks)

c) A thermometer is taken from an inside room and placed outside where the air temperature is 5°F. After 1 minute the thermometer reads 55°F and after 5 minutes the reading is 30°F. What is the initial temperature of the room? (6 marks)

d) The half-life of a certain radioactive substance is 1659 years.

i) What percentage of the original substance will remain after 12 years?

(5 marks)

ii) In how many years will only 20% of the substance remain? (4 marks)

e) Evaluatesinh$(0.72)$to 4 significant figures,. (3 marks)

QUESTION THREE

a) To convert any rational function of $\sin x$ and $\cos x$ into an ordinary rational function of t, we use the substitution $t = \tan\left(\frac{x}{2}\right)$. Sketch a right angled triangle or use trigonometric identities to show:

i) $dx = \frac{2}{1+t^2}\,dt.$ (3 marks)

ii) $\sin x = \frac{2t}{1+t^2}.$ (3 marks)

iii) $\cos x = \frac{1-t^2}{1+t^2}.$ (3 marks)

Hence evaluate $\int \frac{dx}{3-5\sin x}$. (3 marks)

b) Show that the improper integral $\int_1^\infty \frac{dx}{x^P}$ converges for $P > 1$ and diverges for $P \leq 1$. (7 marks)

c) Use (i) Simpson's Rule (ii) Trapezoidal Rule with $n = 10$ to approximate the integral $\int_1^2 \ln x\, dx$. (6 marks)

QUESTION FOUR

a) Define carefully when we say the series $\sum_{n=1}^{\infty} a_n$ converges to sum S. (2 marks)

b) Let $a_n = \frac{1}{n}$.

i) Determine whether $\{a_n\}$ is convergent. (3 marks)

ii) Determine whether $\sum_{n=1}^{\infty} \frac{1}{n}$ is convergent. (4 marks)

c) What is wrong with the following calculation?

$$0 = 0 + 0 + 0 + 0 + \cdots$$
$$= (1-1) + (1-1) + (1-1) + (1-1) + \cdots$$
$$= 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 + \cdots$$
$$= 1 + (-1+1) + (-1+1) + (-1+1) + \cdots$$
$$= 1 + 0 + 0 + 0 + 0 + \cdots$$
$$= 1$$

(2 marks)

d) Show that the volume of the sphere of radius r is $V = \frac{4}{3}\pi r^3$. (6 marks)

e) The curve $y = \sqrt{4 - x^2}$, $-1 \leq x \leq 1$, is an arc of the circle $x^2 + y^2 = 4$. Find the area of the surface obtained by rotating this arc about the $x - axis$. (8 marks)

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