Department of Mathematical Sciences

Bachelor of Education (Business Studies)

Year 2

Business Statistics I

(MTS-BST-211)

Binomial probability distribution

By the end of this lesson you must be able to

- Explain why a random variable is binomial or not from a given context
- Identify the parameters of a binomial random variable given a context
- Calculate probabilities of events for a binomial random variable using its probability distribution function
- Calculate probabilities using the cumulative distribution of a binomial random variable by using either a binomial distribution table or a calculator,
- Calculate probabilities using the cumulative distribution of a binomial random variable involving the complement rule and/or the difference rule
- Calculate the mean for binomial distributions
- Calculate the variance and standard deviation for binomial distributions
- Solve real-world problems involving binomial probability distributions

Introduction

- You can model many complex business problems by using probability distributions.
- These distributions provide answers to questions such as: “What is the likelihood that oil prices will rise during the coming year?”, “What is the probability of a stock market crash next month?”, or “How likely is it that a corporation’s earnings will fall below expectations this year?”
- Many types of probability problems have only two types of outcomes or can be reduced to two outcomes. For example, customers may like a product or not.
- Such probability problems that have only two types of outcomes can be modelled using a Binomial probability distribution.

A binomial experiment

- A **binomial experiment** is a statistical experiment that has the following properties:
    1. The experiment consists of n repeated trials.
    2. Each trial can result in just two possible outcomes. We call one of these outcomes a success and the other, a failure.
    3. The probability of success, denoted by P, is the same on every trial.
    4. The trials are independent; that is, the outcome on one trial does not affect the outcome on other trials.
- The outcomes of a binomial experiment and the corresponding probabilities of these outcomes are called a binomial distribution.
- The random variable in a binomial experiment counts the number of successes in n repeated trials.

Probability mass function

Let X be a random variable that a follows a binomial distribution. Its probability mass function is given by

$$P(X=x)=\binom{n}{x}p^x q^{n-x}, \qquad x=0,1,2,\ldots,n.$$

The following notation is helpful, when we talk about binomial probability.

- x: The number of successes that result from the binomial experiment.
- n: The number of trials in the binomial experiment.
- p: The probability of success on an individual trial.
- q: The probability of failure on an individual trial. (This is equal to $1 - p$).
- $n!$: The factorial of n (also known as n factorial).
- $\binom{n}{x}$ or ${}_nC_x$: The number of combinations of n things, taken x at a time.

Example 1

A survey found that 30% of the teenage consumers receive their spending money from part time jobs. If 5 teenagers are selected at random, calculate the probability that

(a) exactly 2 of them will have part time jobs.
(b) at least 3 of them will have part time jobs.
(c) at most 2 of them will have part time jobs.

Solution

(a) Let X be the number of teenagers who will have part time jobs. We are required to find

$$P(X=2) = {}_5C_2 \times 0.3^2(1-0.3)^{5-2} = 10 \times 0.3^2 \times 0.7^3 = 0.3087$$

Example 1…cont'd

(b) We are required to find

$$P(X \geq 3) = P(X = 3) + P(X = 4) + P(X = 5)$$

$$= \binom{5}{3} \times 0.3^3 \times 0.7^2 + \binom{5}{4} \times 0.3^4 \times 0.7^1 + \binom{5}{5} \times 0.3^5 \times 0.7^0$$

$$= 0.16308$$

Example 1…cont'd

(c) We are required to find

$$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$$

$$= \binom{5}{0} \times 0.3^0 \times 0.7^5 + \binom{2}{1} \times 0.3^1 \times 0.7^1 + \binom{5}{2} \times 0.3^2 \times 0.7^3 = 0.83692$$

Note that this is just the cumulative probability, $F(2) = P(X \le 2)$

We have an alternative way to find $P(X \le 2)$:

Since we have calculated $P(X \ge 3)$ in (b) then

$$P(X \le 2) = 1 - P(X \ge 3) = 1 - 0.1608 = 0.83692$$

The Mean and Variance for a Binomial Random variable

If X follows a binomial probability distribution then;

- The mean(expected value) is

$$\boxed{E(X)=np}$$

- The variance is

$$\boxed{Var(X)=np(1-p)}$$

- The stabdard deviation is

$$\boxed{\sigma=\sqrt{np(1-p)}}$$

Example 2

Twenty-three percent of automobiles are not covered by insurance in Blantyre city. On a particular weekend, 35 automobiles are involved in traffic accidents.

(a) What is the expected number of these automobiles that are not covered by insurance?

(b) What are the variance and standard deviation?

Solution

(a) Expected number:

$$E[X] = np = 35 \times 0.23 = 8.05 \approx 8$$

(b) Variance and Standard deviation:

$$* \; V[X] = npq = 35 \times 0.23 \times 0.77 = 6.1985$$

$$* \; \sigma = \sqrt{npq} = \sqrt{6.1985} \approx 2.49$$

Activity

1. A university found that 20% of its students withdraw without completing the introductory statistics course. Assume that 20 students registered for the course.
   (a) Compute the probability that two or fewer will withdraw.
   (b) Compute the probability that exactly four will withdraw.
   (c) Compute the probability that more than three will withdraw.
   (d) Compute the expected number of withdrawals.

2. According to a survey conducted by TD Consultants, one out of four investors have exchange-traded funds in their portfolios. Consider a sample of 20 investors.
   (a) If you found that exactly 12 of the investors have exchange-traded funds in their portfolios, would you doubt the accuracy of the survey results?
   (b) Compute the expected number of investors who have exchange-traded funds in their portfolios.

Conclusion

- A **binomial experiment** is a statistical experiment that has the following properties:
  1. The experiment consists of *n* repeated trials.
  2. Each trial can result in just two possible outcomes. We call one of these outcomes a success and the other, a failure.
  3. The probability of success, denoted by *P*, is the same on every trial.
  4. The trials are independent; that is, the outcome on one trial does not affect the outcome on other trials.
- The mean and the variance for a binomial random variable are given by
  - $E[X] = np$
  - $V[X] = npq$

References

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