DEPARTMENT OF MATHEMATICAL SCIENCES
Intermediate Calculus (MTS-INC 211)
MSE2.

March 22, 2024   Tutorial

Question 1

(a) Evaluate the following integrals:

    i. $\displaystyle\int \frac{5}{(x+1)(x+4)}dx.$

    ii. $\displaystyle\int \frac{3x^2+12x-20}{x^4-8x^2+16}dx.$

    iii. $\displaystyle\int_0^1 \frac{x^2+x+1}{(x+1)^2(x+2)}dx.$

    iv. $\displaystyle\int_2^3 \frac{x(3-5x)}{(3x-1)(x-1)^2}dx.$

(b) Make a substitution to express the integrand as a rational fraction and then evaluate the integrals:

    i. $\displaystyle\int \frac{dx}{x\sqrt{x-1}}.$

    ii. $\displaystyle\int \frac{x^3}{\sqrt[3]{x^2+1}}dx.$

    iii. $\displaystyle\int \frac{1}{\sqrt{x}-\sqrt[3]{x}}dx.$ (**Hint**: substitute $u=\sqrt[6]{x}$)

Question 2

(a) Use the (A) Trapezoidal Rule and (B) Simpson's Rule to approximate the integral with the specified value of n.

    i. $\displaystyle\int_{-1}^{2} e^{x+\cos x}dx, \qquad n=6.$

    ii. $\displaystyle\int_{1}^{3} e^{1/x}dx, \qquad n=8.$

    iii. $\displaystyle\int_{0}^{1} \sqrt{x+x^3}\,dx, \qquad n=10.$

(b)   i. Use Trapezoidal Rule with $n=6$ to approximate $\displaystyle\int_0^{1.5} \sin^2 x\,dx.$

    ii. Estimate the error in the approximation of part (i) above.

    iii. How large should n be so that the Trapezoidal Rule approximation of $\displaystyle\int_0^{1.5} \sin^2 x\,dx$ is within 0.0001?

Question 3

(a) For each of the following problems explain why the integrals are improper and decide if the integral is convergent or divergent. If it converges, find which value it converges to:

  i. $\displaystyle\int_0^\infty \frac{1}{\sqrt[4]{1+x}}dx.$

  ii. $\displaystyle\int_{-2}^2 \frac{1}{x^2}dx.$

  iii. $\displaystyle\int_{-\infty}^0 2^x dx.$

  iv. $\displaystyle\int_{-\infty}^\infty \left(x^3 - 3x^2\right)dx.$

  v. $\displaystyle\int_{-\infty}^\infty \cos \pi t \ dt.$

  vi. $\displaystyle\int_0^1 \frac{\ln x}{\sqrt{x}}dx.$

  vii. $\displaystyle\int_0^\infty \frac{e^x}{e^{2x} + 3}dx.$

  viii. $\displaystyle\int_0^5 \frac{x}{x - 2}dx.$

  ix. $\displaystyle\int_0^\infty \frac{1}{\sqrt{x}}dx.$

  x. $\displaystyle\int_0^\infty \frac{1}{\sqrt{x}(1 + x)}dx.$

  xi. $\displaystyle\int_2^\infty \frac{1}{x\sqrt{x^2 - 4}}dx.$

(b) Use comparison theorem to decide if the following integrals are convergent or divergent:

  i. $\displaystyle\int_0^\infty \frac{x}{x^3 + 1}dx.$

  ii. $\displaystyle\int_1^\infty \frac{2 + \cos x}{\sqrt{x^4 + x^2}}dx.$

  iii. $\displaystyle\int_0^\pi \frac{\sin^2 x}{\sqrt{x}}dx.$

  iv. $\displaystyle\int_0^1 \frac{\sec^2 x}{x\sqrt{x}}dx.$