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Mid-Term Exam

MCA-621

(Q.4)

$$\text{Max } Z = 7x_1^2 + 6x_1 + 5x_2^2$$

Subject to $2x_1 + x_2 \leq 5$, $x_1, x_2 \geq 0$

Using lagrange variable λ_1

$2x_1 + x_2 + \lambda_1 = 5 = C_1$ (let)

(a) $\dfrac{\partial z}{\partial x_1} + \lambda_1 \dfrac{\partial c_1}{\partial x_i} = 0$

i.e $14x_1 + 6 + \lambda_1 (2) = 0$ — (2)

$6x_2 + \lambda_1 (1) = 0$ — (3)

(b) $\lambda_i g_i = 0 \Rightarrow \lambda(2x_1 + x_2 - 5) = 0$

(c) $g_i = 0 \Rightarrow 2x_1 + x_2 - 5 \leq 0$ — (5)

(d) $\lambda_i \geq 0 \Rightarrow \lambda_i \geq 0$ — (6)

from (4) either $\boxed{\lambda_i = 0}$ or $2x_1 + x_2 - 5 = 0$

$\boxed{\text{case-1}}$ when $\lambda_1 = 0$	case-II
from-(2) $14x_1 + 6 = 0$	when $2x_1 + x_2 - 5 = 0$
$x_1 = -6/14$	No need to consider
from (3) $6x_2 = 0 \Rightarrow x_2 = 0$	

so hence the solution is $\left[-6/14, 0\right]$

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(Q.1) Simplex Method

$$\text{Max } Z = 7x_1 + x_2 + 2x_3$$

subject to $x_1 + x_2 - 2x_3 \le 10,$

$$4x_1 + x_2 + x_3 \le 20$$

$$x_1, x_2, x_3 \geq 0$$

$$\text{Max } Z = 7x_1 + x_2 + 2x_3 + 0S_1 + 0S_2$$

subject to,

$$x_1 + x_2 - 2x_3 + S_1 + 0S_2 = 10$$

$$4x_1 + x_2 + x_3 + 0S_1 + S_2 = 20$$

		C_j	7	1	2	0	0	
B	C_B	x_B	x_1	x_2	x_3	S_1	S_2	Min Ratio
S_1	0	10	1	1	-2	1	0	$10/1 = 10$
S_2	0	20	$\boxed{4}$	1	1	0	1	$20/4 = 5$
~~Z_j~~		$Z = 0$	0	0	0	0	0	
~~$Z_j - C_j$~~		$Z_j - C_j$	-7	-1	-2	0	0	

Negative minimum $Z_j - C_j$ is -7 and its column index is 1. So entering variable is x_1.

Min ratio is 5 and its row index is 2. So leaving variable is S_2

Entering = x_1 , Departing = S_2
key element = 4

R_2 (new) = R_2 (old) ÷ 4
R_1 (new) = R_1 (old) − R_2 (new)

Iteration . 2 $\quad C_j$ 7 1 2 0 0

B	C_B	X_B	x_1	x_2	x_3	S_1	S_2	Min Ratio
S_1	0	5	0	0.75	−2.25	1	−0.25	—
x_1	7	5	1	0.25	$\boxed{0.25}$	0	0.25	5/0.25=20
		$Z_j - C_j$	0	0.75	$\boxed{-0.25}$	0	1.75	

Negative minimum $Z_j - C_j$ is −0.25 and its column index is 3. So entering variable is x_3.
and pivot element is $\boxed{0.25}$

R_2 (new) = R_2 (old) ÷ 0.25

R_1 (new) = R_1 (old) + 2.25 R_2 (new)

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		c_j	7	1	2	0	0	
B	c_B	x_B	x_1	x_2	x_3	S_1	S_2	Min Ratio
S_1	0	30	9	3	0	1	2	
x_3	2	20	4	1	1	0	1	
		z_j	8	2	2	0	2	
		$z_j - c_j$	1	1	0	0	2	

Since all $z_j - c_j \geq 0$

hence optimal solution with value of variables as: $x_1 = 0$, $x_2 = 0$, $x_3 = 20$

$$\text{Max } z = 7 \times 0 + 0 + 2 \times 20$$
$$= \boxed{40}$$

(Q.5)

	I	II	III	IV
A	200	150	170	220
B	160	120	150	140
C	190	195	190	200
D	180	175	160	190

maximum monthly sale for representative are 220 lpa.

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	I	II	III	IV
A	20	70	50	0
B	60	100	70	80
C	30	25	30	20
D	40	45	60	30

doing Row - Reduction we get,

	I	II	III	IV
A	20	70	50	0
B	0	40	10	20
C	10	5	10	0
D	10	15	30	0

doing column reduction,

	I	II	III	IV
A	20	65	40	0
B	0	35	0	20
C	10	0	0	0
D	10	10	20	0

	I	II	III	IV
A	10	55	30	$\boxed{0}$
B	$\boxed{0}$	35	10	30
C	10	$\boxed{0}$	10	10
D	~~0~~	0	10	0

	I	II	III	IV
A	$\boxed{0}$	45	30	0
B	0	25	$\boxed{0}$	20
C	20	$\boxed{0}$	10	80
D	0	0	20	$\boxed{0}$

	I	II	III	IV
A	0	45	30	$\boxed{0}$
B	$\boxed{0}$	35	40	10
C	20	0	$\boxed{0}$	20
D	10	$\boxed{0}$	10	0

So the optimum assignment is

A → IV

B → I

C → III

D → II

total maximum sales increas per month

$= 220 + 160 + 190 + \cancel{180}\ 175$

$= \boxed{745}$

(5) (b) :-

	I	II	III	IV
A	200	150	170	220
B	160	120	—	140
C	190	195	190	200
D	180	175	160	190

We will take sale representative as ∞ to sales territory III.

	I	II	III	IV
A	200	180	170	220
B	160	120	∞	140
C	190	195	190	200
D	180	175	160	190

Row-Reduction

	I	II	III	IV
A	50	0	20	70
B	40	0	∞	20
C	0	5	0	10
D	20	15	0	30

column Reduction

	I	II	III	IV
A	50	0	20	60
B	40	0	∞	10
C	0	5	0	0
D	20	15	0	20

	I	II	III	IV
A	40	$\boxed{0}$	20	50
B	30	0	∞	$\boxed{0}$
C	$\boxed{0}$	15	10	0
D	10	15	$\boxed{0}$	10

A → II
B → IV
C → I
D → III

so, total sales = 180 + 140 + 190 + 160

= $\boxed{640}$

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Big-M Method

(Q.2)

$$\text{Max } z = 4x_1 + 2x_2$$

subject to $3x_1 + x_2 \geq 27$

$x_1 + x_2 \geq 21$

$x_1, x_2 \geq 0$

$$\text{Max } Z = 4x_1 + 2x_2 + 0S_1 + 0S_2 - mA_1 - mA_2$$

Subject to $3x_1 + 1x_2 - 1S_1 + 0S_2 + 1A_1 + 0A_2 = 27$

$1x_1 + 1x_2 + 0S_1 - 1S_2 + 0A_1 + 1A_2 = 21$

Table-1

	C_j	4	2	0	0	$-m$	$-m$	
C_B	x_B	x_1	x_2	S_1	S_2	A_1	A_2	Ratio
$-m$	A_1	3	1	-1	0	1	0	27
$-m$	A_2	1	1	0	-1	0	1	21
	$Z_j - C_j$	$-4m-4$	$-2m-2$	m	m	0	0	$-48m$

pivot element is 3 and x_1 variable enters and A_1 leaves

Table - 2

	C_j	4	2	0	0	$-m$	$-m$	
C_B	x_B	x_1	x_2	S_1	S_2	A_1	A_2	Ratio
.	x_1	1	$1/3$	$-1/3$	0	$1/3$	0	9
	A_2	0	$2/3$	$1/3$	-1	$-1/3$	1	12
	$Z_j - C_j$	0	$-\frac{2}{3}m - \frac{2}{3}$	$-\frac{1}{3}m - \frac{4}{3}$	m	$\frac{4}{3}m + \frac{4}{3}$	0	$-12m/+36$

$x_2 \rightarrow$ enter

$A_2 \rightarrow$ leaves

pivot element $2/3$

Table-3

		C_j	4	2	0	0	$-M$	$-M$	
C_B	x_B		x_1	x_2	S_1	S_2	A_1	A_2	Ratio
4	x_1		1	0	$-1/2$	$1/2$	$1/2$	$-1/2$	3
2	x_2		0	1	$1/2$	$-3/2$	$-1/2$	$3/2$	18
	$Z_j - C_j$		0	0	-1	-1	$M+1$	$M+1$	48

$S_1 \rightarrow$ enters

$x_2 \rightarrow$ leaves

pivot $\rightarrow$ $1/2$

Table-4

		C_j	4	2	0	0	$-M$	$-M$	
C_B	x_B		x_1	x_2	S_1	S_2	A_1	A_2	Ratio
4	x_1		1	1	0	-1	0	1	21
0	S_1		0	2	1	-3	-1	3	36
	$Z_j - C_j$		0	2	0	-4	M	$M+4$	84

The problem has infinite solution variable S_2 must enter but no variable can leave..