BER optimization for MIMO with three active layers

Group 2 (Project 8)

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Multiple Input Multiple Output:

In Multiple Input Multiple Output (MIMO), N_t is number of transmitter antennas and the N_r is number of receiving antennas, where $N_t = N_r$. In MIMO transmission system, interference occurs between the channels due to multiple transmitters, which in turn produces bit error rate (BER). The BER is error in the transmission relative to the number of bits received on receiver side.

The expression for calculating the overall system BER is as follows:

$$P_b = \frac{2}{\sum_{i=1}^{L} \log_2 M_l} \sum_{i=1}^{L} \left(1 - \frac{1}{\sqrt{M_l}}\right) * erfc\left(\frac{\pi_l \lambda_l}{2\sigma} \sqrt{\frac{3P_s}{L(M_l - 1)}}\right)$$

where,

P_s = Over all available transmit power, P_b = Over all system BER, σ = Noise power variance

This BER can be reduced by using the singular value decomposition (SVD) technique in MIMO system. In SVD the power distribution is same for all the layers during the transmission. And the overall system bit error rate is less than the normal MIMO system.

To further reduce the BER and to make the MIMO system more error free, the power allocation method can be used. In the power allocation method, higher number layer is given more power availability so the overall BER of the system can reduce further.

Lagrange multiplier method:

The Lagrange multiplier method is used to find the maxima and minima of a multi-variable function. The following Lagrangian expression provides the power allocation factors in respect with the number of layers.

Lagrangian expression:

$$J(\pi_1 ... \pi_3, \mu) = \frac{2}{\sum_{i=1}^{L} \log_2 M_l} \sum_{i=1}^{L} \left(1 - \frac{1}{\sqrt{M_l}}\right) * erfc\left(\frac{\pi_l \lambda_l}{2\sigma} \sqrt{\frac{3P_s}{L(M_l - 1)}}\right) + \mu \left(\sum_{i=1}^{L} \pi_l^2 - L\right)$$

Where,

μ = Lagrange multiplier, π_l = Power allocation factor, L = Number of layers

Taking the partial derivative of the above Lagrangian expression provides the following non -linear equations.

Partial derivative expressions:

$$\frac{\partial}{\partial \pi_1} = \frac{2}{\log_2 M_1 + \log_2 M_2 + \log_2 M_3} * \left(\left(1 - \frac{1}{\sqrt{M_1}}\right) * \frac{2}{\sqrt{\pi}} \left(\frac{\lambda_1}{2\sigma}\sqrt{\frac{3P_s}{L(M_1-1)}}\right) \left(-e^{-}\left(\frac{\lambda_1\pi_1}{2\sigma}\sqrt{\frac{3P_s}{L(M_1-1)}}\right)^2\right)\right) + 2\mu\pi_1$$

$$\frac{\partial}{\partial \pi_2} = \frac{2}{\log_2 M_1 + \log_2 M_2 + \log_2 M_3} * \left(\left(1 - \frac{1}{\sqrt{M_2}}\right) * \frac{2}{\sqrt{\pi}} \left(\frac{\lambda_2}{2\sigma}\sqrt{\frac{3P_s}{L(M_2-1)}}\right) \left(-e^{-}\left(\frac{\lambda_2\pi_2}{2\sigma}\sqrt{\frac{3P_s}{L(M_2-1)}}\right)^2\right)\right) + 2\mu\pi_2$$

$$\frac{\partial}{\partial \pi_3} = \frac{2}{\log_2 M_1 + \log_2 M_2 + \log_2 M_3} * \left(\left(1 - \frac{1}{\sqrt{M_3}}\right) * \frac{2}{\sqrt{\pi}} \left(\frac{\lambda_3}{2\sigma}\sqrt{\frac{3P_s}{L(M_3-1)}}\right) \left(-e^{-}\left(\frac{\lambda_3\pi_3}{2\sigma}\sqrt{\frac{3P_s}{L(M_3-1)}}\right)^2\right)\right) + 2\mu\pi_3$$

$$\frac{\partial}{\partial \mu} = \pi_1^2 + \pi_2^2 + \pi_3^2 - 3$$

These non-linear equations are not solvable analytically. However, with the help of C-XSC tool the values of the power allocation factor for each layer is calculated as shown in figure.

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Power Allocation factors computed using Lagrange Multiplier Method are,
             Candidate: 1
             pi1      = [ 1.530142232163761E+000, 1.530146449673016E+000]
             pi2      = [ 5.178380628361379E-001, 5.178386889508021E-001]
             pi3      = [ 6.249012727038828E-001, 6.249014548045145E-001]
             meu      = [ 2.097896627614002E-002, 2.097897914521249E-002]
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Figure 1: Power-allocation factors

	M1=64, M2=2, M3=2		M1=32, M2=4, M3=2	
SNR in dB	BER Without PA	BER With PA	BER Without PA	BER With PA
10	3.07E-02	5.88E-03	7.35E-03	1.41E-03
11	2.67E-02	4.28E-03	5.68E-03	9.04E-04
12	2.33E-02	3.12E-03	4.40E-03	5.86E-04
13	2.03E-02	2.29E-03	3.42E-03	3.84E-04
14	1.78E-02	1.68E-03	2.66E-03	2.54E-04
15	1.56E-02	1.24E-03	2.08E-03	1.70E-04
16	1.36E-02	9.14E-04	1.62E-03	1.15E-04
17	1.20E-02	6.78E-04	1.27E-03	7.81E-05
18	1.05E-02	5.04E-04	9.93E-04	5.37E-05
19	9.24E-03	3.76E-04	7.78E-04	3.72E-05
20	8.13E-03	2.81E-04	6.11E-04	2.60E-05

Table:1 SNR VS BER

The above table represents the comparison between the system BER without and with power allocation, for three layers M1 = 64, M2 = 2, M3 = 2 and M1 = 32, M2 = 4, M3 = 2 respectively with SNR variation from 10 to 20 dB.

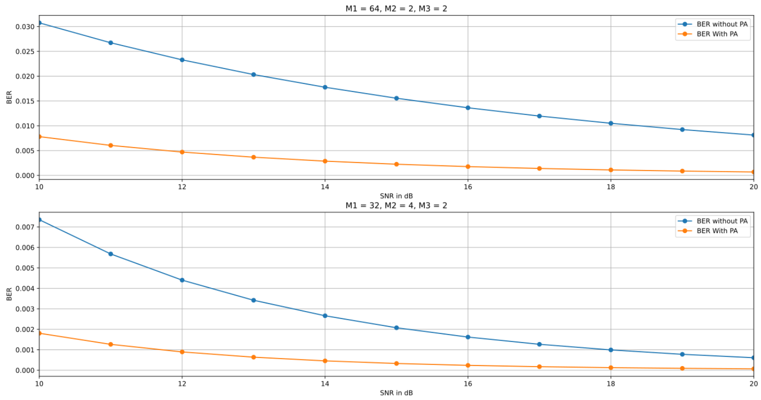

Figure 2: SNR vs BER (a): for M1 = 64,M2 = 2,M3 = 2, (b): for M1 = 32,M2 = 4,M3 = 2

It can be observed from the graph that the optimized overall system BER is less than the overall system BER. It can also be observed that, as the SNR increases, the overall system BER is decreasing.

Unit Design of the Implementation

- Static Design:
  - Figure 3 elaborates, static class diagram to understand the mapping and dependencies between multiple classes. It also state the attributes and operations of the particular classes.
- Dynamic Design:
  - Sequence diagram in Figure 4, helps to understand the dynamic behavior of the code along with its timing interfaces.

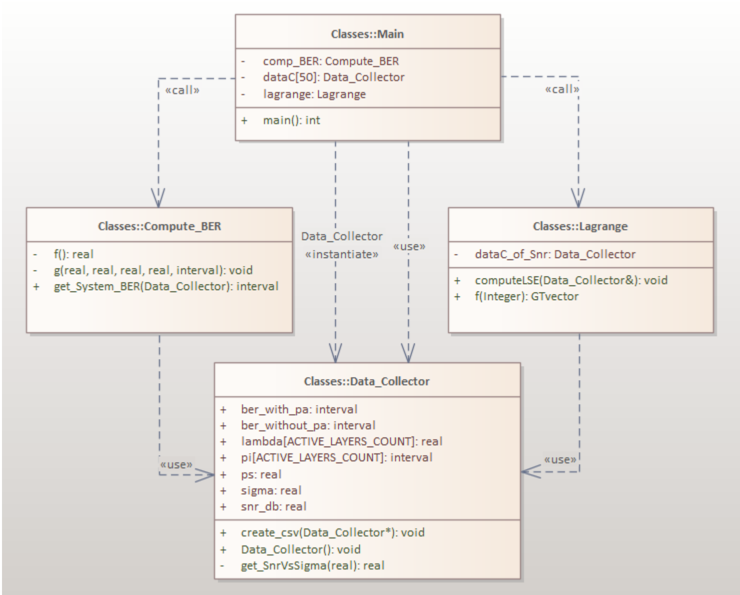

Figure 3: Static Design

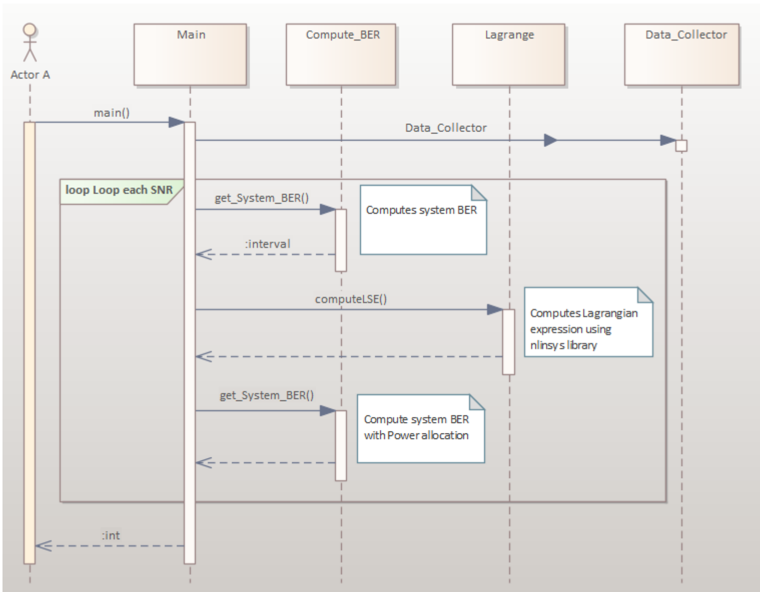

Figure 4: Dynamic Design

References

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