BER Optimization for MIMO with Three Active Layers

Group-2 (Project-8)

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Contents

Introduction

- In Multiple Input Multiple Output (MIMO) transmission, Bit Error Rate (BER) introduced and it is minimized using the power allocation method.
- To find this power allocation factors we have used Lagrange multiplier method.

Multiple Input Multiple Output (MIMO)

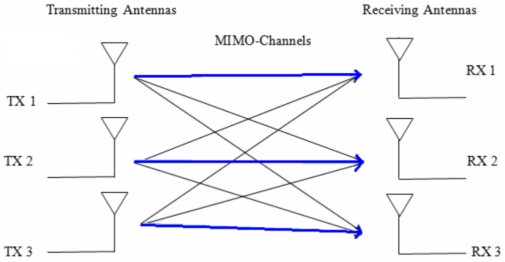

Figure: 3X3 MIMO channel

- It causes inter-channel interference.
- Equation for 3X3 MIMO channel
  - U1 = h11*a1+h12*a2+h13*a3+n1
  - U2 = h21*a1+h22*a2+h23*a3+n2
  - U3 = h31*a1+h32*a2+h33*a3+n3

Singular Value Decomposition (SVD)

- Eliminate interference
- Time-variant system
- Power distribution

Bit Error Rate (BER)

- BER is the percentage of bits that have errors relative to the total number of bits received in transmission.

$$P_b = \frac{2}{\sum_{i=1}^{L} \log_2 M_l} \sum_{i=1}^{L} \left(1 - \frac{1}{\sqrt{M_l}}\right) * erfc\left(\frac{\pi_l \lambda_l}{2\sigma} \sqrt{\frac{3P_s}{L(M_l - 1)}}\right)$$

  - π_l = Power allocation factors
  - P_s = Over all available transmit power
  - P_b = Over all system BER
- Optimal power distribution (Power allocation)

Lagrange Multiplier Method

- Lagrange multiplier method is used to find maxima or minima of multi-variable function.
- The equation to find power allocation parameters and Lagrange multiplier is as follows,

$$J\left(\pi_1 ... \pi_3, \mu\right) = \frac{2}{\sum_{i=1}^{L} \log_2 M_l} \sum_{i=1}^{L} \left(1 - \frac{1}{\sqrt{M_l}}\right) * erfc\left(\frac{\pi_l \lambda_l}{2\sigma} \sqrt{\frac{3P_s}{L(M_l - 1)}}\right) + \mu\left(\sum_{i=1}^{L} \pi_l^2 - L\right)$$

  - μ = Lagrange multiplier,
  - The factors (π_1, π_2, π_3) can be derived using Lagrange multiplier method.

Lagrange Multiplier Method

- $\frac{\partial}{\partial \pi_1} = \frac{2}{\log_2 M_1 + \log_2 M_2 + \log_2 M_3} * \left(\left(1 - \frac{1}{\sqrt{M_1}}\right) * \frac{2}{\sqrt{\pi}}\left(\frac{\lambda_1}{2\sigma}\sqrt{\frac{3P_s}{L(M_1-1)}}\right)\left(-e^{-}\left(\frac{\lambda_1 \pi_1}{2\sigma}\sqrt{\frac{3P_s}{L(M_1-1)}}\right)^2\right)\right) + 2\mu\pi_1$
- $\frac{\partial}{\partial \pi_2} = \frac{2}{\log_2 M_1 + \log_2 M_2 + \log_2 M_3} * \left(\left(1 - \frac{1}{\sqrt{M_2}}\right) * \frac{2}{\sqrt{\pi}}\left(\frac{\lambda_2}{2\sigma}\sqrt{\frac{3P_s}{L(M_2-1)}}\right)\left(-e^{-}\left(\frac{\lambda_2 \pi_2}{2\sigma}\sqrt{\frac{3P_s}{L(M_2-1)}}\right)^2\right)\right) + 2\mu\pi_2$
- $\frac{\partial}{\partial \pi_3} = \frac{2}{\log_2 M_1 + \log_2 M_2 + \log_2 M_3} * \left(\left(1 - \frac{1}{\sqrt{M_3}}\right) * \frac{2}{\sqrt{\pi}}\left(\frac{\lambda_3}{2\sigma}\sqrt{\frac{3P_s}{L(M_3-1)}}\right)\left(-e^{-}\left(\frac{\lambda_3 \pi_3}{2\sigma}\sqrt{\frac{3P_s}{L(M_3-1)}}\right)^2\right)\right) + 2\mu\pi_3$
- $\frac{\partial}{\partial \mu} = \pi_1^2 + \pi_2^2 + \pi_3^2 - 3$

Results

- M1 $= 64$, M2 $= 2$, M3 $= 2$
- $\sigma \approx 0.223607$ for SNR $=$ 10dB

```
SNR = 10dB
System BER without PA =    [     3.0749166591104E-002,     3.0749166591105E-002]

Power Allocation factors computed using Lagrange Multiplier Method are,
             Candidate: 1
             pi1      = [ 1.530142232163761E+000, 1.530146449673016E+000]
             pi2      = [ 5.178380628361379E-001, 5.178386889508021E-001]
             pi3      = [ 6.249012727038828E-001, 6.249014548045145E-001]
             meu      = [ 2.097896627614002E-002, 2.097897914521249E-002]

System BER with PA     =    [ 5.882860441283604E-003, 5.882947717195338E-003]
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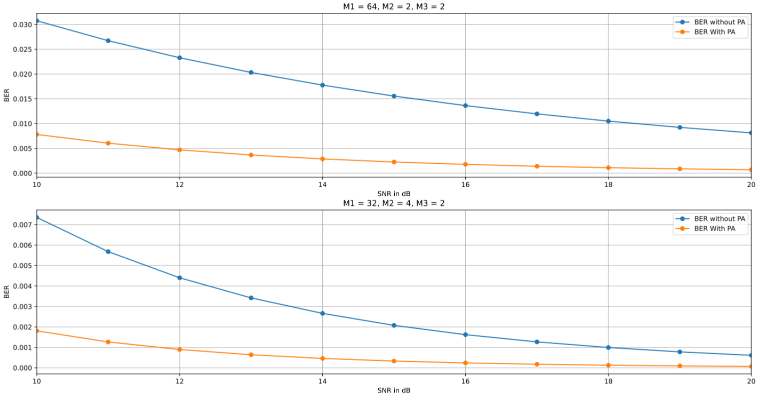
	M1=64, M2=2, M3=2		M1=32, M2=4, M3=2	
SNR in dB	BER Without PA	BER With PA	BER Without PA	BER With PA
10	3.07E-02	5.88E-03	7.35E-03	1.41E-03
11	2.67E-02	4.28E-03	5.68E-03	9.04E-04
12	2.33E-02	3.12E-03	4.40E-03	5.86E-04
13	2.03E-02	2.29E-03	3.42E-03	3.84E-04
14	1.78E-02	1.68E-03	2.66E-03	2.54E-04
15	1.56E-02	1.24E-03	2.08E-03	1.70E-04
16	1.36E-02	9.14E-04	1.62E-03	1.15E-04
17	1.20E-02	6.78E-04	1.27E-03	7.81E-05
18	1.05E-02	5.04E-04	9.93E-04	5.37E-05
19	9.24E-03	3.76E-04	7.78E-04	3.72E-05
20	8.13E-03	2.81E-04	6.11E-04	2.60E-05

Graphical Representation



Conclusion

- In conclusion, using a C-XSC library we have solved the Lagrangian expression, and found the power allocation parameters for MIMO System.
- The BER in MIMO system is improved with the help of optimal power allocation.

References

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