

Image Size: The number B bits required

to store Digital (gray-scale) image is $B = M \cdot N \cdot k$.

Sampling Digitize coordinate value

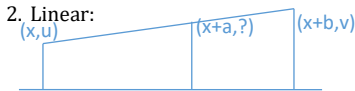
Quantization Digitize amplitude value

Resampling & Interpolation:

First step: align the grid.

Second step: Set the values to new grid.

1. Nearest Neighbor: closest in original



$$y = [(b-a)u + av]/b$$

Adjacency useful for establishing object

Boundary and defining image component

$$N_4(p) = \{(x+1, y), (x-1, y), (x, y+1), (x, y-1)\}$$

$$N_8(p) = \{(x+1, y-1), (x-1, y+1), (x+1, y+1), (x-1, y-1)\}$$

$$N_8(p) = N_4(p) \cup N_4(p)$$

m-adjacency

$$q \in N_m(p) \vee q \in N_8(p) \wedge N_4(p) \cap N_8(p) \neq \emptyset$$

Path $\forall i \in [1, n], p_i$ is adjacent to p_{i-1}

Intensity Transformation $s = T(r)$

Image Negatives $s = (L-1) - r$

Log Transform $s = c \log(1+r)$

Power Law $s = cr^\gamma$

flexible than log transformation.

$\gamma > 1$, lighter; $\gamma < 1$, darker

Piecewise-Linear Find a, b

$$ar_1 + b = s_1, ar_2 + b = s_2$$

pros: customize

cons: not automatic (n photo » n function)

Image Histogram $h(r_k) = n_k$

Normalized $p(r_k) = n_k/n$

mapping $s_k = ((L-1)/MN) \sum_{j=0}^k n_j$

Linear Function $r = T(x)$

Linear if $T(ax_1 + bx_2) = aT(x_1) + bT(x_2)$

w1 w2 w3	Correlation
w4 w5 w6	$s'_5 = \sum_{i=1}^9 w_i s_i$
w7 w8 w9	Convolution
	$s'_5 = \sum_{i=1}^9 w_i s_{9-i}$

Smooth(low-pass) vs Sharp(high-pass)

0.1 0.1 0.1	1 2 1
0.1 0.1 0.1	2 4 2
0.1 0.1 0.1	1 2 1

Average	Time	Gaussian Effect
Average	Fast	Box effect
Median	Slow	remove salt, pepper

High-Boost = (A-1)Original + Highpass

Gradient of image

$$\frac{\partial f}{\partial x} = f(x+1, y) - f(x, y)$$

$$\frac{\partial f}{\partial y} = f(x, y) + f(x, y+1)$$

$$\text{For } z_5, |\nabla \cdot f| = \sqrt{(z_6 - z_5)^2 + (z_5 - z_8)^2}$$

z1 z2 z3	-1 -1 -1
z4 z5 z6	0 0 0
z7 z8 z9	1 1 1

Prewitt

-1 -2 -1	-1 0 1
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Laplacian:

0 1 0
1 4 1
0 1 0

Gradi

$$\nabla^2 f = f(x, y-1) + f(x+1, y) + f(x-1, y) + f(x, y+1) - 4f(x, y)$$

$$f(x, y) = mn / \sum_{(s,t) \in W} g(s, t)^{-1}$$

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Complex number $e^{j\theta} = \cos \theta + j \sin \theta$

Fourier Transform

$$FT[f(t)] = F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j2\pi\mu t} dt$$

$$FT^{-1}[F(\mu)] = f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j2\pi\mu t} d\mu$$

$$f(t) \star h(t) = \int_{-\infty}^{\infty} f(t) h(t - \tau) d\tau$$

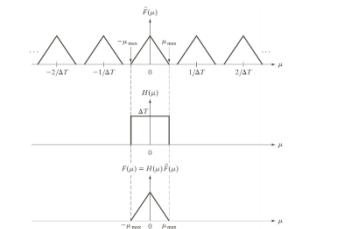
$$\text{Sampling } f(t) s_{\Delta T}(t) = \sum_{n=-\infty}^{\infty} f(t) \delta(t - n\Delta T)$$

Nyquist rate

Sufficient separation guaranteed if:

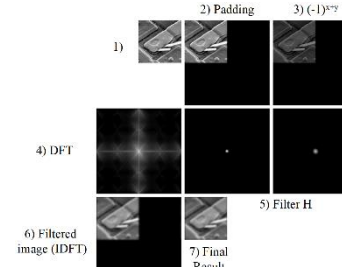
$$\frac{1}{\Delta T} > 2\mu_{max} \text{ ("=" is Nyquist rate)}$$

Aliasing Transform corrupted by frequencies from adjacent periods be done before the sampling



Frequency Domain Filtering

$$g(x, y) = IDFT[H(u, v)F(u, v)]$$



Ideal Lowpass Filter

$$H(u, v) = \begin{cases} 1, & \text{if } D(u, v) \leq D_0 \\ 0, & \text{if } D(u, v) > D_0 \end{cases}$$

$$D(u, v) = \sqrt{(u - P/2)^2 + (v - Q/2)^2}$$

R is small will cause ringing and blurring

Butterworth Lowpass Filter

$$H(u, v) = \frac{1}{1 + [D(u, v)/D_0]^{2n}}$$

Gaussian Lowpass Filter

$$H(u, v) = e^{-D^2(u, v)/2\sigma^2}$$

LowPass(u,v) = 1 - HighPass(u,v)

Objective of Restoration

$$MSE = \frac{1}{MN} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [f(x, y) - \hat{f}(x, y)]^2$$

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Contraharmonic mean for salt and pepper

but not both

$$\hat{f}(x, y) = \sum_{(s,t) \in W} g(s, t)^{Q+1} / \sum_{(s,t) \in W} g(s, t)^Q$$

Order-Statistic Filters

Median, Max, Min, Mid-pt, α trimmed

$d = mn - 1$, median, $d = 0$, mean

$\{0, 3, 55, 65, 70, \dots, 250\}$ take mean

Adaptive Filters

- change behavior base on statistical

characteristics of image in window

- superior to that no-adaptive filter

Ada. Local noise reduction

$$\hat{f}(x, y) = (1 - \frac{\sigma_n^2}{\sigma_L^2})g(x, y) + \frac{\sigma_n^2}{\sigma_L^2}[m_L]$$

Adaptive Median Filter

Wiener Filtering

Based on aforementioned conditions, the image estimate in the frequency domain is given by

$$\hat{F}(u, v) = \frac{1}{H(u, v) [H(u, v)]^2 + S_g(u, v)/S_f(u, v)} G(u, v) \quad (\text{Eq. 5.8-2})$$

$$H^*(u, v) = \text{complex conjugate of } H(u, v)$$

$$|H(u, v)|^2 = H^*(u, v)H(u, v)$$

$$S_g(u, v) = |N(u, v)|^2 = \text{power spectrum density of the noise}$$

$$S_f(u, v) = |F(u, v)|^2 = \text{power spectrum density of the degraded image}$$

Bilateral Filter

$$BF(I)_v = \frac{1}{N_p} \sum_{q \in N_p} G_s(\|p-q\|) G_r(\|I_p - I_q\|) I_q$$

normalization factor

space weight

range weight

Edge

Detection of short linear edge segments

Aggregation of edgels into extended edges

Image Gradient $\nabla f, \theta = \tan^{-1} f_y/f_x$

discrete: $f'_x[x, y] = f[x+1, y] - f[x, y]$

Sobel

Effect on noise

smooth then take edge

Canny Edge

detection, localization

(reliability)

Linear filtering, addition iid Gauss-noise

response to edge, no noise

reduce the precision of localization

detect edge near true edge

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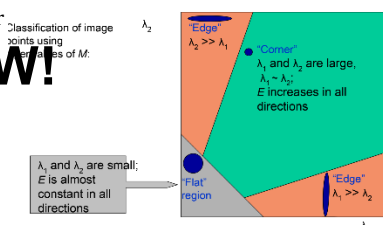
detect edge near true edge

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Measure of corner response:

$$R = \det M - k (\text{trace } M)^2$$

The brightness constancy constraint

$$I(x, y, t-1) \text{ displacement } = (u, v) \quad I(x, y, t) = I(x+u, y+v, t)$$

• Brightness Constancy Equation:

$$I(x, y, t-1) = I(x+u(x, y), y+v(x, y), t)$$

Can be written as: shorthand: $I_x =$

$$I(x, y, t-1) \approx I(x, y, t) + I_x \cdot u(x, y) + I_y \cdot v(x, y)$$

So, $I_x \cdot u + I_y \cdot v + I_t \approx 0$

Lucas-Kanade:

How to get more equations for a pixel?

Spatial coherence constraint: pretend the pixel's neighbors have the same (u,v)

- If we use a 5x5 window, that gives us 25 equations per pixel

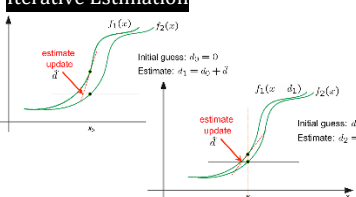
$$0 = I_t(p_1) + \nabla I(p_1) \cdot [u \ v]$$

$$\begin{bmatrix} I_x(p_1) & I_y(p_1) \\ I_x(p_2) & I_y(p_2) \\ \vdots & \vdots \\ I_x(p_{25}) & I_y(p_{25}) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(p_1) \\ I_t(p_2) \\ \vdots \\ I_t(p_{25}) \end{bmatrix}$$

$$A \cdot d = b$$

$$25 \times 2 \quad 2 \times 1 \quad 25 \times 1$$

Iterative Estimation



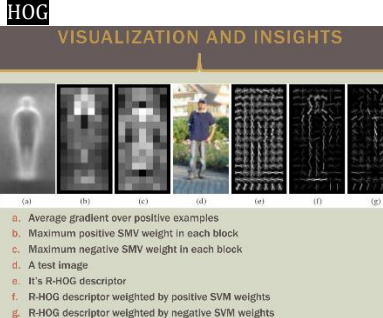
Conditions for solvability

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

When is this solvable?

- $A^T A$ should be invertible
- $A^T A$ should not be too small
- eigenvalues λ_1 and λ_2 of $A^T A$ should not be too small
- $A^T A$ should be well-conditioned
- λ_1/λ_2 should not be too large (λ_1 = larger eigenvalue)

HOG



a. Average gradient over positive examples

b. Maximum positive SVM weight in each block

c. Maximum negative SVM weight in each block

d. A test image

e. It's RHOG descriptor

f. RHOG descriptor weighted by positive SVM weights