財工 HW8

0616098
黃秉茂

year	swap rate
1	$F_1 = 2.26\%$
2	$F_2 = 2.275\%$
3	$F_3 = 2.285\%$
4	$F_4 = \frac{2.285\% + 2.355\%}{2} = 2.32\%$
5	$F_5 = 2.355\%$
6	$F_6 = \frac{2.355\% + 2.44\%}{2} = 2.3975\%$
7	$F_7 = 2.44\%$

假設swap rate已是continuous

assume zero coupon rate s_i is discrete

$$\sum_{i=1}^{n-1} \frac{F_n \cdot P}{(1+s_i)^i} + \frac{(1+F_n) \cdot P}{(1+s_n)^n} = P$$

$$\sum_{i=1}^{n-1} \frac{F_n}{(1+s_i)^i} + \frac{1+F_n}{(1+s_n)^n} = 1$$

$$\frac{1+F_n}{(1+s_n)^n} = 1 - \sum_{i=1}^{n-1} \frac{F_n}{(1+s_i)^i}$$

$$(1+s_n)^n = \frac{1+F_n}{1 - \sum_{i=1}^{n-1} \frac{F_n}{(1+s_i)^i}}$$

$$s_n = \sqrt[n]{\frac{1+F_n}{1 - \sum_{i=1}^{n-1} \frac{F_n}{(1+s_i)^i}}} - 1$$

$$S_1 = \sqrt[1]{\frac{1+F_1}{1}} - 1$$

$$= \frac{1+2.26\%}{1} - 1$$

$$= 2.26\%$$

$$S_2 = \sqrt{\frac{1+F_2}{1-\frac{F_2}{(1+S_1)^1}}} - 1$$

$$= \sqrt{\frac{1+2.275\%}{1-\frac{2.275\%}{1+2.26\%}}} - 1$$

$$= 2.27517\%$$

$$S_3 = \sqrt[3]{\frac{1+F_3}{1-\frac{F_3}{(1+S_1)^1}-\frac{F_3}{(1+S_2)^2}}} - 1$$

$$= \sqrt[3]{\frac{1+2.285\%}{1-\frac{2.285\%}{1+2.26\%}-\frac{2.285\%}{(1+2.27517\%)^2}}} - 1$$

$$= 2.28534\%$$

$$S_n = \sqrt[n]{\frac{1+F_n}{1-\sum_{i=1}^{n-1}\frac{F_n}{(1+S_i)^i}}} - 1$$

以此類推

$$S_4 = \sqrt[4]{\frac{1+F_4}{1-\sum_{i=1}^{4-1}\frac{F_4}{(1+S_i)^i}}} - 1$$

$$= 2.3215\%$$

$$S_5 = \sqrt[5]{\frac{1+F_5}{1-\sum_{i=1}^{5-1}\frac{F_5}{(1+S_i)^i}}} - 1$$

$$= 2.35791\%$$

$$S_6 = \sqrt[6]{\frac{1+F_6}{1-\sum_{i=1}^{6-1}\frac{F_6}{(1+S_i)^i}}} - 1$$

$$= 2.40261\%$$

$$S_7 = \sqrt[7]{\frac{1+F_7}{1-\sum_{i=1}^{7-1}\frac{F_7}{(1+S_i)^i}}} - 1$$

$$= 2.44771\%$$

assume zero coupon rate s_i is continuous

$$\sum_{i=1}^{n-1} p \cdot F_n \cdot e^{-s_i \times i} + p \cdot (1+F_n) \cdot e^{-s_n \times n} = p$$

$$F_n \sum_{i=1}^{n-1} e^{-s_i \times i} + (1+F_n) \cdot e^{-s_n \times n} = 1$$

$$(1+F_n) e^{-s_n \times n} = 1 - F_n \sum_{i=1}^{n-1} e^{-s_i \times i}$$

$$e^{-s_n \times n} = \frac{1 - F_n \sum_{i=1}^{n-1} e^{-s_i \times i}}{1+F_n}$$

$$-s_n \times n = \ln\left(\frac{1 - F_n \sum_{i=1}^{n-1} e^{-s_i \times i}}{1+F_n} \right)$$

$$s_n = - \frac{\ln\left(\frac{1 - F_n \sum_{i=1}^{n-1} e^{-s_i \times i}}{1+F_n} \right)}{n}$$

$$S_1 = -\frac{\ln\left(\frac{1}{1+F_1}\right)}{1}$$

$$= -\frac{\ln\left(\frac{1}{1+2.26\%}\right)}{1}$$

$$= 2.23484\%$$

$$S_2 = -\frac{\ln\left(\frac{1-F_2\cdot e^{-S_1\times 1}}{1+F_2}\right)}{2}$$

$$= -\frac{\ln\left(\frac{1-2.275\%\times e^{-2.23484\%\times 1}}{1+2.275\%}\right)}{2}$$

$$= 2.24967\%$$

$$S_3 = -\frac{\ln\left[\dfrac{1 - F_3 \cdot \left(e^{-S_1 \times 1} + e^{-S_2 \times 2}\right)}{1 + F_3}\right]}{3}$$

$$= -\frac{\ln\left[\dfrac{1 - 2.285\% \times \left(e^{-2.23484\% \times 1} + e^{-2.24967\% \times 2}\right)}{1 + 2.285\%}\right]}{3}$$

$$= 2.25962\%$$

$$S_n = -\frac{\ln\left(\dfrac{1 - F_n \sum_{i=1}^{n-1} e^{-S_i x_i}}{1 + F_n}\right)}{n}$$

以此類推

$$s_4 = -\frac{\ln\left(\dfrac{1 - F_4 \sum_{i=1}^{4-1} e^{-s_i x_i}}{1 + F_4}\right)}{4}$$

$$= 2.29496\%$$

$$s_5 = -\frac{\ln\left(\dfrac{1 - F_5 \sum_{i=1}^{5-1} e^{-s_i x_i}}{1 + F_5}\right)}{5}$$

$$= 2.33054\%$$

$$s_6 = -\frac{\ln\left(\dfrac{1 - F_6 \sum_{i=1}^{6-1} e^{-s_i x_i}}{1 + F_6}\right)}{6}$$

$$= 2.3742\%$$

$$s_7 = -\frac{\ln\left(\dfrac{1 - F_7 \sum_{i=1}^{7-1} e^{-s_i x_i}}{1 + F_7}\right)}{7}$$

$$= 2.41824\%$$