8.請嘗試使用一些簡單的財務知識，來驗證本章的計算存續期間的範例程式產生的答案是否合理。假定債券支付的債息為 0，則其存續期間應為多少？ 請問當債息提高（或下降），存續期間應提高還是下降？ 並將推論的結果輸入範例程式中，驗證推論的結果是否和程式的輸出相符合。

MD of a zero-coupon bond is it's final maturity (n).
Proof: because no cash flows before maturity, the MD is

$$MD = \frac{\sum_{i=1}^{n} iC_i(1+y)^{-i}}{\sum_{i=1}^{n} C_i(1+y)^{-i}} = \frac{nC_n(1+y)^{-n}}{C_n(1+y)^{-n}} = n$$

- Define price volatility by

$$-\frac{\partial P / P}{\partial y} \longrightarrow \text{It is also so-call modified duration!}$$

$$\frac{\partial P}{P}(percent\ \ price\ change) \approx -D \times \partial y$$

Price volatility increases as the coupon rate decreases.

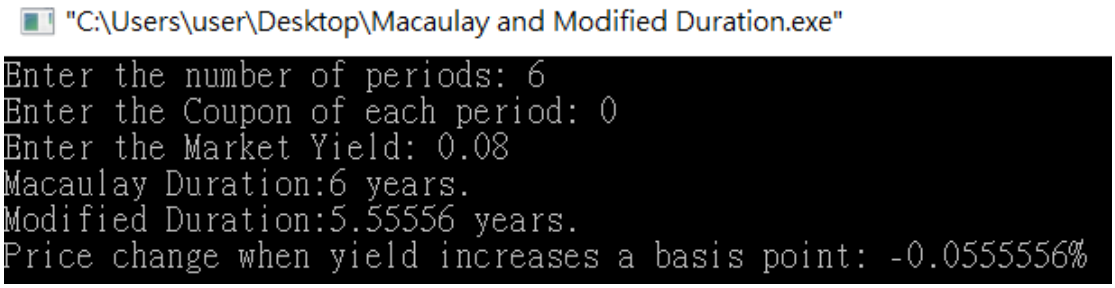
Bonds selling at a deep discount are more volatile than those selling near or above par.

Zero-coupon bonds are the most volatile.

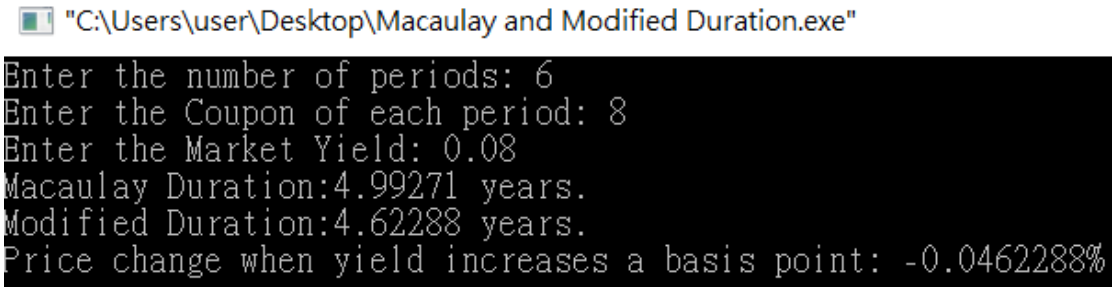
Coupon 越低的話，Face value 影響 bond price 的占比越來越重，那整體的 Price volatility 會增加
Macaulay duration = Modified duration * (1 + y) and Price volatility (Modified duration) increases as the coupon rate decreases.
Thus, if the yield doesn't change, Modified duration decreases as the coupon rate increases.

債券支付的債息為 0，則其存續期間應為 n 期
當債息提高，存續期間應下降

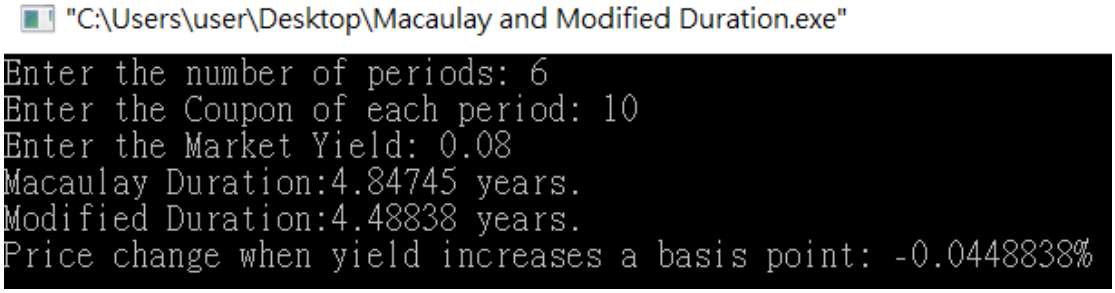
Coupon = 0




Coupon = 8




Coupon = 10




當債息提高，存續期間下降

| Coupon | Macaulay Duration | Modified Duration |
|---|---|---|
| 0 | 6 | 5.5556 |
| 8 | 4.99271 | 4.62288 |
| 10 | 4.84745 | 4.48832 |