8.請嘗試使用一些簡單的財務知識,來驗證本章的計算存續期間的範例程式產生的答案是否合理。假定債券支付的債息為 0,則其存續期間應為多少? 請問當債息提高(或下降),存續期間應提高還是下降? 並將推論的結果輸入範例程式中,驗證推論的結果是否和程式的輸出相符合。

MD of a zero-coupon bond is it's final maturity (n).

Proof: because no cash flows before maturity, the MD is

$$MD = \frac{\sum_{i=1}^{n} iC_i (1+y)^{-i}}{\sum_{i=1}^{n} C_i (1+y)^{-i}} = \frac{nC_n (1+y)^{-n}}{C_n (1+y)^{-n}} = n$$

Define price volatility by

$$-\frac{\partial P/P}{\partial y} \longrightarrow \text{It is also so-call modified duration!}$$

$$\frac{\partial P}{P}(percent \ price \ change) \approx -D \times \partial y$$

Price volatility increases as the coupon rate decreases.

Bonds selling at a deep discount are more volatile than those selling near or above par.

Zero-coupon bonds are the most volatile.

Coupon 越低的話,Face value 影響 bond price 的占比越來越重,那整體的 Price volatility 會增加

Macaulay duration = Modified duration *(1 + y) and Price volatility (Modified duration) increases as the coupon rate decreases.

Thus, if the yield doesn't change, Modified duration decreases as the coupon rate increases.

債券支付的債息為 0,則其存續期間應為 n 期當債息提高,存續期間應下降

Coupon = 0

"C:\Users\user\Desktop\Macaulay and Modified Duration.exe"

```
Enter the number of periods: 6
Enter the Coupon of each period: 0
Enter the Market Yield: 0.08
Macaulay Duration:6 years.
Modified Duration:5.55556 years.
Price change when yield increases a basis point: -0.0555556%
```

Coupon = 8

"C:\Users\user\Desktop\Macaulay and Modified Duration.exe"

```
Enter the number of periods: 6
Enter the Coupon of each period: 8
Enter the Market Yield: 0.08
Macaulay Duration:4.99271 years.
Modified Duration:4.62288 years.
Price change when yield increases a basis point: -0.0462288%
```

Coupon = 10

"C:\Users\user\Desktop\Macaulay and Modified Duration.exe"

```
Enter the number of periods: 6
Enter the Coupon of each period: 10
Enter the Market Yield: 0.08
Macaulay Duration:4.84745 years.
Modified Duration:4.48838 years.
Price change when yield increases a basis point: -0.0448838%
```

當債息提高,存續期間下降

Coupon	Macaulay Duration	Modified Duration
0	6	5.5556
8	4.99271	4.62288
10	4.84745	4.48832