Homework 1
Binomial Tree

I. Binomial Model Derivation

(20%) In the binomial model, suppose that the initial stock price is S_0, and the life of the option is T. S_0 can either move up from S_0 to a new level, $S_0 u$, where $u > 1$, or down to a new level, $S_0 d$, where $0 < d < 1$. Suppose the payoff from option is f_u in the up state, and is f_d in the down state. Denote the risk-free rate by *r*.

Please construct a riskless portfolio in a one-step tree and show **<u>in detail</u>**

that $f = e^{-rT}[pf_u + (1-p)f_d]$ where $p = \dfrac{e^{rT} - d}{u - d}$

II. Binomial Trees in Practice

Consider a non-dividend-paying stock with current stock price S_0 =\$50, volatility σ=0.3, strike price K=\$52, time to maturity T=2 years, interest rate r=5%.

Please use binomial model to price European put options. You may refer to the materials in Section 21.1 of the textbook. Consider the following three alternative settings of time steps: Δt =1 month (12*T steps); 1 week (52*T steps); and 1 day (252*T steps).

(a) (10%) First compute the up step size *u*, the down step size *d*, and the probability of up move *p* under these three settings.

(b) (40%) Use binomial model to compute the put option prices under these three settings. Report your results and compare them with that of the Black-Scholes formula. Briefly explain your findings.

(c) (20%) Modify your program in (b) to compute the American put option values. Report your result.

(d) (10%) Change the number of time steps from 1 to 2 to 3 all the way to 252 to calculate European put option prices. Plot your results as well as the Black-Scholes closed form solution. Briefly explain your findings.

Bonus: (20%) For 6, 12, and 52 time steps, compute the terminal stock prices as well as their corresponding probabilities. Plot the terminal stock price distribution. Briefly explain your findings.

<u>Matlab function and syntax:</u>

1. zeros(): to create an matrix of all zeros. e.g. S = zeros(m,n)
2. sqrt(): square root
3. exp(): exponential function
4. max(): max function
5. for loop

    e.g.

```
for j=1:1:10
        statement
end
```

*You have to submit the results of your homework in a word (or pdf) file as well as **programs by e3**. Your computer program is part of this assignment. You can use either C++ or Matlab for programming.