## Introduction to Algorithms

## Homework 2

## Due 10/18/2018 in class

This part of homework is not for programming but helps clarify some ideas and concepts. Hand in your homework on due day **before** the class starts. **Late**homework will not be accepted.

- 1. Write an efficient MAX-HEAPIFY that uses an iterative control construct (a loop) instead of recursion.
- 2. Consider the following version of quicksort, which simulates tail recursion:

```
TAIL-RECURSIVE-QUICKSORT (A, p, r)

1 while p < r

2  // Partition and sort left subarray.

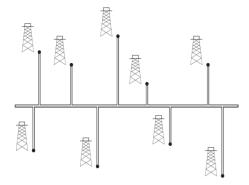
3  q = \text{PARTITION}(A, p, r)

4  TAIL-RECURSIVE-QUICKSORT (A, p, q - 1)

5  p = q + 1
```

- (a) Argue that TAIL-RECURSIVE-QUICKSORT(A, 1, A.length) correctly sorts the array A.
- (b) Describe a scenario in which TAIL-RECURSIVE-QUICKSORT's stack depth is  $\Theta(n)$  on an n-element input array.
- 3. Show how to sort n integers in the range 0 to  $n^3 1$  in O(n) time.

4.



Professor Olay is consulting for an oil company, which is planning a large pipeline running east to west through an oil field of n wells. The company wants to connect a spur pipeline from each well directly to the main pipeline along a shortest route (either north or south), as shown in the above figure. Given the x- and y-coordinates of the wells, how should the professor pick the optimal location of the main pipeline, which would be the one that minimizes the total length of the spurs? Show how to determine the optimal location in linear time.