

# VAN (086761) - Exercise 1

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January 28, 2024

## Basic Probability

### Question 1

Consider a random vector  $x$  with a Gaussian distribution :

$$x \sim \mathcal{N}(\mu_x, \Sigma_x)$$

(a) An explicit expression for  $p(x)$

$$p(x) = \det(2\pi\Sigma)^{-\frac{1}{2}} e^{\left(-\frac{1}{2}\|x-\mu\|_{\Sigma}^2\right)}$$

where  $\|\cdot\|_{\Sigma}$  is the Mahalanobis norm, defined as:  $\|x\|_{\Sigma} = x^T \Sigma^{-1} x$

(b) Consider a linear transformation  $y = Ax + b$ , assuming  $A$  is invertible. Show that  $y$  is normally distributed and compute its mean and covariance.

#### Gaussian invariant under Linear Transformations

Using that when  $A$  is invertible we have the relation:

$$f_Y(y) = \frac{f_X(A^{-1}(y-b))}{|\det A|}$$

so,

$$\begin{aligned}
f_Y(y) &= \frac{f_x(A^{-1}(y-b))}{|\det A|} \\
&= \frac{\exp\left(-\frac{1}{2}(A^{-1}(y-b) - \mu_x)^T \Sigma_x^{-1} (A^{-1}(y-b) - \mu_x)\right)}{\sqrt{\det(2\pi\Sigma_x)} |\det A|} \\
&= \frac{\exp\left(-\frac{1}{2}(y - (A\mu_x + b))^T A^{-T} \Sigma_x^{-1} A^{-1} (y - (A\mu_x + b))\right)}{\sqrt{\det(2\pi\Sigma_x) \det(A^T A)}} \\
&\stackrel{1}{=} \frac{\exp\left(-\frac{1}{2}(y - (A\mu_x + b))^T (A\Sigma_x A^T)^{-1} (y - (A\mu_x + b))\right)}{\sqrt{\det(2\pi A\Sigma_x A^T)}}
\end{aligned}$$

1 - Because A is invertible we have that  $A^T \Sigma A$  is invertible too so we can close it with the same parentheses.

We got that Y's density function is of the with the form above (q1a) and therefore is a gaussian with mean  $A\mu_x + b$  and covarivace  $A^T \Sigma A$ .

We can show that these are the right moments with another way:

### Mean

Using the linearity of the mean in 1D we get that: ( $a^i$  is A's row)

$$\mathbb{E}(y_i) \stackrel{1}{=} \mathbb{E}(\langle a^i, x \rangle + b^i) \stackrel{2}{=} \langle a^i, \mathbb{E}(x) \rangle + b^i$$

1 - Definition of y.

2 - Mean's linearity in every coordinate of x

Therefore, we get for y:

$$\mathbb{E}(y) = A\mathbb{E}(x) + b$$

### Covariance

We'll show that:

$$Cov(y) = A^T Cov(x) A$$

for every  $i, j$  we have:

$$\begin{aligned}
Cov(y) &= \mathbb{E}\left[(y - \mathbb{E}(y))(y - \mathbb{E}(y))^T\right] = \mathbb{E}\left[yy^T - \mathbb{E}(y)y^T + \mathbb{E}(y)\mathbb{E}(y)^T - y\mathbb{E}(y)^T\right] \\
&= \mathbb{E}(yy^T) - \mathbb{E}(y)\mathbb{E}(y)^T + \mathbb{E}(y)\mathbb{E}(y)^T - \mathbb{E}(y)\mathbb{E}(y)^T = \mathbb{E}(yy^T) - \mathbb{E}(y)\mathbb{E}(y)^T
\end{aligned}$$

The second equation is just opening the parentheses and then using that mean is linear in 1D and mean of a multivariate Random vector is just the mean on its coordinates and a mean on  $\mathbb{E}(y)$  is simply  $\mathbb{E}(y)$  therefore for example  $\mathbb{E}(y\mathbb{E}(y)^T) = \mathbb{E}(y)\mathbb{E}(y^T)$ .

Now, using mean's linearity as proved before we get:

$$\begin{aligned} & \mathbb{E}\left((Ax+b)(Ax+b)^T\right) - \mathbb{E}(Ax+b)\mathbb{E}(Ax+b)^T = \\ & \mathbb{E}\left(Axx^TA^T + Axb^T + bx^TA^T + bb^T\right) - (A\mathbb{E}(x)+b)(A\mathbb{E}(x)+b)^T = \\ & A\mathbb{E}(xx^T)A^T + A\mathbb{E}(x)b^T + b\mathbb{E}(x^T)A^T + bb^T + \\ & -A\mathbb{E}(x)\mathbb{E}(x)^TA^T - A\mathbb{E}(x)b^T - b\mathbb{E}(x)^TA^T - bb^T = \\ & A\mathbb{E}(xx^T)A^T - A\mathbb{E}(x)\mathbb{E}(x)^TA^T = \\ & A\left(\mathbb{E}(xx^T) - \mathbb{E}(x)\mathbb{E}(x)^T\right)A^T = \\ & ACov(x)A^T \end{aligned}$$

(c)

**We don't have here a complete solution but rather some thought.**

**Option 1 - make H "invertible"**

Every matrix has a finite number of eigen values so we take  $\lambda$  to be one that **isn't**  $H$ 's eigen value. thus we have that  $H - \lambda I$  is invertible. Since, if it wasn't invertible, it means there's a vector  $x \neq 0$  so that  $(H - \lambda I)x = 0 \rightarrow Hx = \lambda x$  means that  $x$  is an eigen vector and  $\lambda$  is an eigen value, contradiction.

so we have that:

$$y = Hx + \lambda Ix - \lambda Ix + b \rightarrow y = (H - \lambda I)x + \lambda Ix + b$$

but from the previous question we know that  $(H - \lambda I)x \sim \mathcal{N}\left((H - \lambda I)\mu_x, (H - \lambda I)\Sigma_x(H - \lambda I)^T\right)$  and  $\lambda Ix + b \sim \mathcal{N}(\lambda\mu_x + b, \lambda^2\Sigma_x)$ . So, we just need to show that the sum of two normal distributed random variable is normal distributed with sum over the mean and the covariance: means :

$$\begin{aligned} \mu_y &= (H - \lambda I)\mu_x + \lambda\mu_x = H\mu_x \\ \Sigma_y &= (H - \lambda I)\Sigma_x(H - \lambda I)^T + \lambda^2\Sigma_x \\ &= H\Sigma_xH^T - \lambda H\Sigma_x - \lambda(H\Sigma_x)^T + \lambda^2\Sigma_x \end{aligned}$$

**Option 2 - Marginalization**

$$p(y) = \int_x p(y|x) \cdot p(x)$$

And we have from the previous question:

## Question 2 |

Let  $p(x) \sim \mathcal{N}(x_0, \Sigma_{x_0})$  be a prior distribution over  $x \in \mathbb{R}^n$  with known mean  $x_0$  and covariance  $\Sigma_{x_0}$ . Consider a measurement  $z = Hx + v$  where  $H$  is known the measurement model and  $v$  is a gaussian noise with zero mean and known covariance  $R$ .

(a) A posteriori probability function

Option 1:

Using Bayes we have:

$$p(x | z) = p(z | x) \cdot p(x) \frac{1}{p(z)}$$

and by the total probability theorem we have

$$p(z) = \int_x p(z | x) p(x)$$

therefore:

$$(\star) : p(x | z) = \frac{p(z | x) \cdot p(x)}{\int_{x'} p(z | x') p(x')}$$

But we have  $p(x)$  and  $p(z | x)$ :

$$p(x) = \det(2\pi\Sigma)^{-\frac{1}{2}} e^{(-\frac{1}{2}\|x-\mu\|_{\Sigma}^2)}$$

and  $z | x \sim \mathcal{N}(Hx, R)$ :

$$p(z | x) = \det(2\pi R)^{-\frac{1}{2}} e^{(-\frac{1}{2}\|z-Hx\|_R^2)}$$

So we just need to plug it in equation  $(\star)$ .

(b) An expresion for the MAP and  $p(x | z)$  mean and covariance

The MAP is :

$$\operatorname{argmax}_x p(x | z)$$

We notice that

$$\begin{aligned} p(x | z) &= p(z | x) \cdot p(x) \frac{1}{p(z)} = \eta \cdot p(z | x) \cdot p(x) \\ &= \eta \cdot \det(2\pi\Sigma_z)^{-\frac{1}{2}} e^{(-\frac{1}{2}\|z-\mu_z\|_{\Sigma_z}^2)} \cdot \det(2\pi\Sigma_x)^{-\frac{1}{2}} e^{(-\frac{1}{2}\|x-\mu_x\|_{\Sigma_x}^2)} \\ &= \eta \cdot \det(4\pi^2\Sigma_z\Sigma_x)^{-\frac{1}{2}} e^{-\frac{1}{2}(\|z-\mu_z\|_{\Sigma_z}^2 + \|x-\mu_x\|_{\Sigma_x}^2)} \end{aligned}$$

It means the  $p(x | z)$  is of the form  $\eta e^{-\frac{1}{2}f}$  where  $f$  is a quadratic equation so it's a Normal distribution. Now we'll find

its Mean and Covariance.

Before we do that we mention that the maximum value of a normal distribution is obtained in the mean. so the **MAP is actually the mean.**

In addition for a normal distribution

$$p(x) = \det(2\pi\Sigma)^{-\frac{1}{2}} e^{(-\frac{1}{2}\|x-\mu\|_{\Sigma}^2)}$$

With denoting  $f = -\frac{1}{2}\|x - \mu\|_{\Sigma}^2$  we have that:

$$f'(\mu) = 0$$

$$f'' = \Sigma^{-1}$$

So, we back to our density function  $p(x | z)$  we have the deriving the power  $\|z - \mu_z\|_{\Sigma_z}^2 + \|x - \mu_x\|_{\Sigma_x}^2$  once and setting it to zero will give us the mean and twice will give us the  $\Sigma^{-1}$ .

First we write:

$$\begin{aligned} \|z - \mu_z\|_{\Sigma_z}^2 + \|x - \mu_x\|_{\Sigma_x}^2 &= \|z - Hx\|_R^2 + \|x - \mu_x\|_{\Sigma_x}^2 \\ &= \|R^{-1/2}(Hx - z)\|_2^2 + \|\Sigma_x^{-1/2}(x - \mu_x)\|_2^2 \end{aligned}$$

So, we drive and set to zero:

$$\begin{aligned} \|R^{-1/2}Hx - R^{-1/2}z\|_2^2 + \|\Sigma_x^{-1/2}x - \Sigma_x^{-1/2}\mu_x\|_2^2 &= 0 \\ 2\left(R^{-1/2}H\right)^T \left(R^{-1/2}Hx - R^{-1/2}z\right) + 2\Sigma_x^{-T/2} \left(\Sigma_x^{-1/2}x - \Sigma_x^{-1/2}\mu_x\right) &= 0 \\ H^T R^{-1}(Hx - z) + \Sigma_x^{-1}(x - \mu_x) &= 0 \\ \Sigma_x^{-1}x + H^T R^{-1}Hx - H^T R^{-1}z - \Sigma_x^{-1}\mu_x &= 0 \\ \Sigma_x^{-1}x + H^T R^{-1}Hx &= H^T R^{-1}z + \Sigma_x^{-1}\mu_x \\ x &= (H^T R^{-1}H + \Sigma_x^{-1})^{-1} (H^T R^{-1}z + \Sigma_x^{-1}\mu_x) \end{aligned}$$

so

$$x = (H^T R^{-1}H + \Sigma_x^{-1})^{-1} (H^T R^{-1}z + \Sigma_x^{-1}\mu_x)$$

And deriving it again we get:

$$(\Sigma_x^{-1}x + H^T R^{-1}Hx - H^T R^{-1}z - \Sigma_x^{-1}\mu_x)' = \Sigma_x^{-1} + H^T R^{-1}H$$

So the covariance is:

$$\Sigma_x^{-1} + H^T R^{-1}H$$

Finally we have:

$$\mathcal{N}\left(\mu = \left(H^T R^{-1} H + \Sigma_x^{-1}\right)^{-1} \left(H^T R^{-1} z + \Sigma_x^{-1} \mu_x\right), \Sigma = \Sigma_x^{-1} + H^T R^{-1} H\right)$$

# Hands-on Exercises

All code can be found in the APPENDIX part at the end of the HW

## Question 1

### (a) Rotations

We just need to multiply the 3 matrices:

```
def euler_to_rot_mat(yaw, pitch, roll):
    # Rz = yaw | Ry = pitch | Rx = roll
    Rx = np.array([[1., 0., 0.],
                   [0., np.cos(roll), np.sin(roll)],
                   [0., -np.sin(roll), np.cos(roll)]])

    Ry = np.array([[np.cos(pitch), 0., -np.sin(pitch)],
                   [0., 1., 0.],
                   [np.sin(pitch), 0., np.cos(pitch)]])

    Rz = np.array([[np.cos(yaw), np.sin(yaw), 0.],
                   [-np.sin(yaw), np.cos(yaw), 0.],
                   [0., 0., 1.]])

    return Rz @ Ry @ Rx
```

### (b) Rotation matrix

The rotation matrix from Body to global for  $\psi = \pi/7, \theta = \pi/5, \phi = \pi/4$  is:

$$R = \begin{bmatrix} 0.7288913 & 0.6812907 & -0.0676648 \\ -0.35101932 & 0.45676743 & 0.81741497 \\ 0.58778525 & -0.5720614 & 0.5720614 \end{bmatrix}$$

Our code print:

```
Question 1b:
Rotation Matrix for yaw 25.714285714285715 pitch 36.0 roll 45.0

[[ 0.72889913  0.68126907 -0.0676648 ]
 [-0.35101932  0.45674743  0.81741497]
 [ 0.58778525 -0.5720614   0.5720614 ]]
```

### (c) Rotation Matrix to Euler angles

Code can be seen in function

#### Theory

Multiplying all the 3 matrices we get: (Note that these matrix performing a rotation clock-wise)

$$\begin{aligned}
 R_z(\psi) R_y(\theta) R_x(\phi) &= \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{bmatrix} \\
 &= \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & \sin(\theta) \sin(\phi) & -\sin(\theta) \cos(\phi) \\ 0 & \cos(\phi) & \sin(\phi) \\ \sin(\theta) & -\sin(\phi) \cos(\theta) & \cos(\phi) \cos(\theta) \end{bmatrix} \\
 &= \begin{bmatrix} \cos(\psi) \cos(\theta) & \cos(\psi) \sin(\theta) \sin(\phi) + \sin(\psi) \cos(\phi) & -\cos(\psi) \sin(\theta) \cos(\phi) + \sin(\psi) \sin(\phi) \\ -\sin(\psi) \cos(\theta) & -\sin(\psi) \sin(\theta) \sin(\phi) + \cos(\psi) \cos(\phi) & \sin(\psi) \sin(\theta) \cos(\phi) + \cos(\psi) \sin(\phi) \\ \sin(\theta) & -\sin(\phi) \cos(\theta) & \cos(\phi) \cos(\theta) \end{bmatrix}
 \end{aligned}$$

And that equals to the given  $R_{3 \times 3}$  matrix.

From this we can see that:

$$\theta = \sin^{-1}(R_1^3)$$

So we have 2 options:

We have 2 options.  $\theta_1 = \sin^{-1}(R_1^3)$  and  $\theta_2 = -\pi - \theta_1$ . We use  $-\pi$  because it's clock-wise.

And now we'll separate into 2 cases,

$\cos(\theta) \neq 0$ :

we can compute the rest with, let's compute  $\phi$ :

$$\begin{aligned}
 R_2^3 &= -\sin(\phi) \cos(\theta) \\
 R_3^3 &= \cos(\phi) \cos(\theta)
 \end{aligned}
 \rightarrow \tan(\phi) = -\frac{R_2^3}{R_3^3} \rightarrow \phi = \arctan\left(\frac{-R_2^3}{R_3^3}\right)$$

But arctan has 2 options. We Notice that because the rotation matrix values are fixed and we have two options for  $\theta$  we get that:

1. if  $-\frac{R_2^3}{\cos \theta} < 0$  that we must have that  $\phi \in [-\pi, 0]$
2. else:  $\phi \in [0, \pi]$

To avoid divisions we can write the condition  $R_2^3 \cdot \cos \theta > 0$

and we notice that  $\cos(\theta) = -\cos(-\pi - \theta)$  so when for one option for  $\theta$  we now what are the angles for  $\theta_2$ .

so :

1. if  $-\frac{R_2^3}{\cos \theta_1} < 0$  that we must have that  $\phi_1 \in [-\pi, 0]$  and  $\phi_2 \in [0, \pi]$



2. else:  $\phi_1 \in [0, \pi]$  and  $\phi_2 \in [-\pi, 0]$

We can do the same for  $\psi$ :

$$\begin{aligned} R_1^1 &= \cos(\psi) \cos(\theta) \\ R_1^2 &= -\sin(\psi) \cos(\theta) \end{aligned} \rightarrow \tan(\psi) = \frac{-R_1^2}{R_1^1} \rightarrow \psi = \arctan\left(\frac{-R_1^2}{R_1^1}\right)$$

1. if  $-\frac{R_1^2}{\cos \theta_1} < 0$  that we must have that  $\psi_1 \in [-\pi, 0]$  and  $\psi_2 \in [0, \pi]$

2. else:  $\psi_1 \in [0, \pi]$  and  $\psi_2 \in [-\pi, 0]$

and

$\cos(\theta) = 0$ :

It means that  $\theta = \frac{\pi}{2}$  OR  $\frac{3\pi}{2}$ .

If  $\theta = \frac{\pi}{2}$ :

$$\begin{aligned} &= \begin{bmatrix} 0 & \cos(\psi) \sin(\phi) + \sin(\psi) \cos(\phi) & -\cos(\psi) \cos(\phi) + \sin(\psi) \sin(\phi) \\ 0 & -\sin(\psi) \sin(\phi) + \cos(\psi) \cos(\phi) & \sin(\psi) \cos(\phi) + \cos(\psi) \sin(\phi) \\ 1 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & \sin(\psi + \phi) & -\cos(\psi + \phi) \\ 0 & \cos(\psi + \phi) & \sin(\psi + \phi) \\ 1 & 0 & 0 \end{bmatrix} \end{aligned}$$

So we have,

$$\begin{aligned} \psi + \phi &= \tan^{-1}\left(\frac{R_2^1}{R_2^2}\right) \\ \psi + \phi &= \tan^{-1}\left(-\frac{R_3^2}{R_3^1}\right) \end{aligned}$$

So:

This linear system has an infinite solutions. So we can choose  $\psi$  to be whatever we want (including 0) and we get that  $\phi_1 = \tan^{-1}\left(\frac{R_2^1}{R_2^2}\right)$  and  $\phi_2 = \pi - \tan^{-1}\left(\frac{R_2^1}{R_2^2}\right)$ .

and else:

$$\begin{aligned} &= \begin{bmatrix} 0 & -\cos(\psi) \sin(\phi) + \sin(\psi) \cos(\phi) & \cos(\psi) \cos(\phi) + \sin(\psi) \sin(\phi) \\ 0 & \sin(\psi) \sin(\phi) + \cos(\psi) \cos(\phi) & -\sin(\psi) \cos(\phi) + \cos(\psi) \sin(\phi) \\ -1 & 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & \sin(\psi - \phi) & \cos(\psi - \phi) \\ 0 & \cos(\psi - \phi) & -\sin(\psi - \phi) \\ -1 & 0 & 0 \end{bmatrix} \end{aligned}$$

Same as above we get:

$$\phi_1 = -\tan^{-1}\left(\frac{R_2^1}{R_2^2}\right) \text{ and } \phi_2 = \pi + \tan^{-1}\left(\frac{R_2^1}{R_2^2}\right)$$

### Implementation

Can be found at the implementation python file in the `rot_mat_to_euler_angles(rot_mat)` function.

#### (d) Rotation matrix to euler angles

For the matrix:

$$R = \begin{bmatrix} 0.813797681 & 0.440969611 & 0.378522306 \\ 0.46984631 & 0.882564119 & 0.0180283112 \\ -0.342020143 & 0.163175911 & 0.925416578 \end{bmatrix}$$

Is :

$$\begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} = \begin{pmatrix} -10 \\ -20 \\ -30 \end{pmatrix} \text{ or } \begin{pmatrix} 170 \\ -160 \\ 150 \end{pmatrix}$$

Quesion 1d:

Two possible angle values:

```
-10.0000000078920193 -19.999999980143038 -29.99999998990158  
169.99999992107982 -160.00000001985697 150.00000001009843
```

## Question 2

We want to calculate the transformation from Global to Camera, Means we want:  $R_G^C, t_{C \rightarrow G}^C$ . We are given  $R_G^C$  and  $t_{C \rightarrow G}^G$  and we have that:

$$t_{C \rightarrow G}^C = R_G^C t_{C \rightarrow G}^G$$

So, our desired matrix is:

$$\begin{bmatrix} \ell^C \\ 1 \end{bmatrix} = \begin{pmatrix} R_G^C & R_G^C t_{C \rightarrow G}^G \\ 0 & 1 \end{pmatrix} \begin{bmatrix} \ell^G \\ 1 \end{bmatrix}$$

Result:

$$\ell^C = \begin{pmatrix} -555.2033 \\ 351.31482 \\ 450 \end{pmatrix}$$

```
Quesiton 2:  
l_cam is:  
[[-555.20335297]  
 [ 351.31482926]  
 [ 450.         ]]
```

## Question 3

An autonomous ground vehicle (robot) is commanded to move forward by 1 meter each time step. Due to imperfect control system, the robot instead moves forward by 1.01 meter and also rotates by 1 degree.

(a)

In a 2D scenario, the pose of a robot is represented by a 3-element vector:  $(x, y, \vartheta)^T$ , where  $(x, y)$  represents the position coordinates, and  $\vartheta$  represents the orientation angle.

Translation is relative to robot's frame, therefore  $t = (0, 1)^T$  - We asume that "forward" means in the Y'th Global CS direction. Commanded:

$$T_k^{k+1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Actual (assuming angle's error is counter clock-wise):

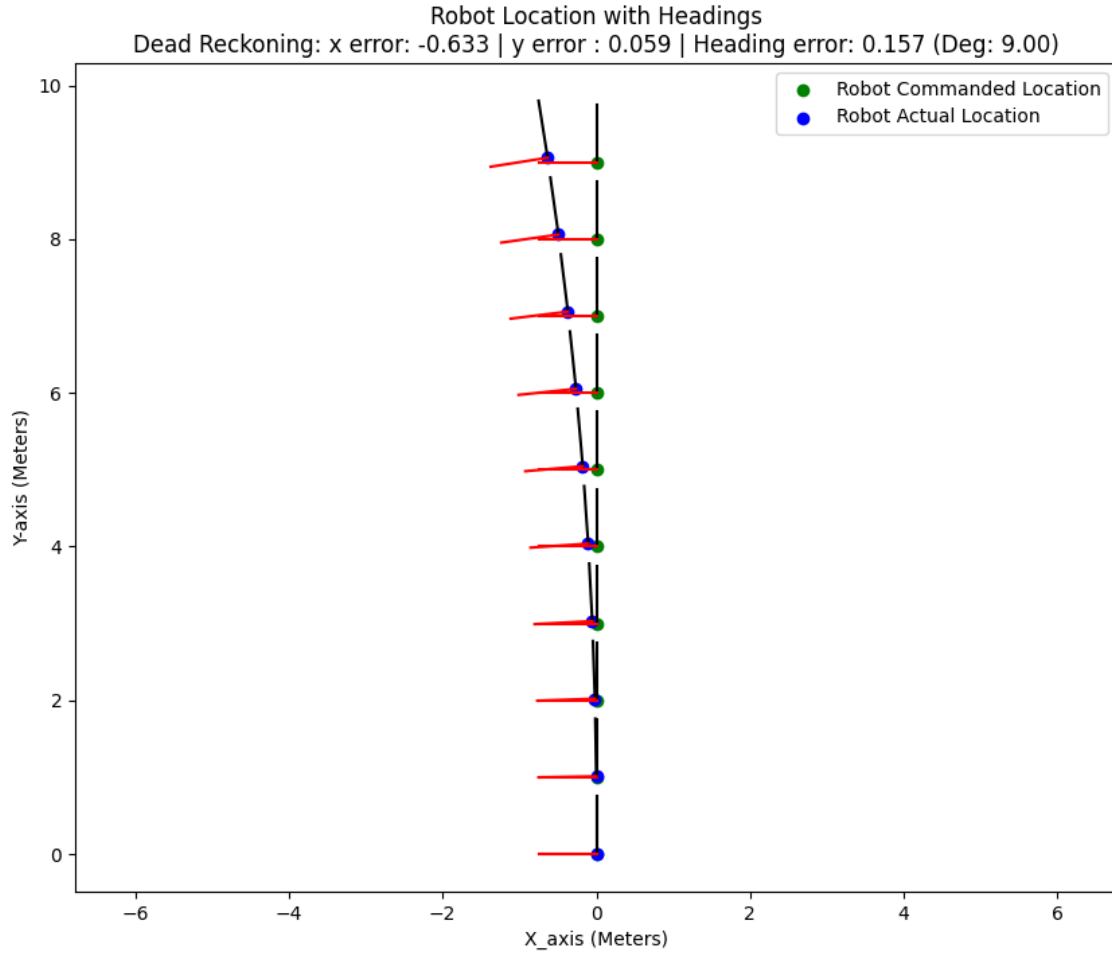
$$T_k^{k+1} = \begin{bmatrix} \cos(1) & -\sin(1) & 0 \\ \sin(1) & \cos(1) & 1.01 \\ 0 & 0 & 1 \end{bmatrix}$$

(b)

Evolution of robot pose for 10 steps using pose composition. In terms of x-y position and orientation angle for consecutive times  $k$ ,  $k + 1$ :

Notice that the heading is actually the image of  $e_2$  under the transformation so it'll be  $R_1$ (means the first column).

We thought that “Forward” means in the Y direction.



## Appendix

### Code

#### question 1

```
def euler_to_rot_mat(yaw, pitch, roll):  
    # Rz = yaw | Ry = pitch | Rx = roll  
    Rx = np.array([[1., 0., 0.], [0., np.cos(roll), np.sin(roll)], [0., -np.sin(roll), np.co
```

```

Ry = np.array([[np.cos(pitch), 0., -np.sin(pitch)], [0., 1., 0.], [np.sin(pitch), 0., np.cos(pitch)]])
Rz = np.array([[np.cos(yaw), np.sin(yaw), 0.], [-np.sin(yaw), np.cos(yaw), 0.], [0., 0., 1.]])
return Rz @ Ry @ Rx

def rot_mat_to_euler_angles(rot_mat):
    # psi = yaw | theta = pitch | phi = roll
    yaw1, yaw2, pitch1, pitch2, roll1, roll2 = 0, 0, 0, 0, 0, 0
    if rot_mat[2, 0] != 1 or rot_mat[2, 0] != -1:
        pitch1 = np.arcsin(rot_mat[2, 0])
        pitch2 = -np.pi - pitch1
        atan_roll = np.arctan(-rot_mat[2, 1] / rot_mat[2, 2])
        atan_yaw = np.arctan(-rot_mat[1, 0] / rot_mat[0, 0])
        if rot_mat[2, 1] * np.cos(pitch1) > 0:
            roll1 = atan_roll if -np.pi < atan_roll < 0 else atan_roll - np.pi
            roll2 = np.pi + roll1
        else:
            roll1 = atan_roll if 0 < atan_roll < np.pi else atan_roll - np.pi
            roll2 = np.pi + roll1
        if rot_mat[1, 0] * np.cos(pitch1) > 0:
            yaw1 = atan_yaw if -np.pi < atan_yaw < 0 else atan_yaw - np.pi
            yaw2 = np.pi + yaw1
        else:
            yaw1 = atan_yaw if 0 < atan_yaw < np.pi else atan_yaw - np.pi
            yaw2 = np.pi + yaw1
    elif rot_mat[2, 0] == 1:
        pitch1 = np.pi / 2
        pitch2 = np.pi / 2
        roll1 = np.arctan(rot_mat[0, 1] / rot_mat[1, 1])
        roll2 = roll1 - np.pi
        yaw1 = 0
        yaw2 = 0
    elif rot_mat[2, 0] == -1:
        pitch1 = 3 * np.pi / 2
        pitch2 = 3 * np.pi / 2
        roll1 = -np.arctan(rot_mat[0, 1] / rot_mat[1, 1])
        roll2 = roll1 - np.pi
        yaw1 = 0
        yaw2 = 0
    return convert_to_range(yaw1), convert_to_range(yaw2), \
        convert_to_range(pitch1), \
        convert_to_range(roll1), convert_to_range(roll2)

def convert_to_range(angle):
    if angle < -np.pi:
        return 2 * np.pi + angle
    elif angle > np.pi:
        return angle - 2 * np.pi
    else:
        return angle

```

## question 2

```
def q2():
    R_global2cam = np.array([[0.5363, -0.8440, [0.8440, 0.5363, [0, 0, 1]]])
    t_cam2global_globalCS = np.array([-451.2459, 257.0322, 400]).reshape(3, 1)
    t_cam2global_camCS = R_global2cam @ t_cam2global_globalCS
    l_global = np.array([450, 400, 50]).reshape(3, 1)
    l_cam = R_global2cam @ l_global + t_cam2global_camCS      # l_cam = R_global2cam @ l_global - t_cam2global
# l_cam = R_global2cam @ (l_global - t_cam2global)
    print("Quesiton 2:")      print("l_cam is: ")      print(l_cam)
    print()      print("Check with inverse on l_cam:")      print("Test inv: ", np.allclose(np.linalg.pinv(R_global2cam) @ l_cam, l_global + t_cam2global_camCS))
    return
```

## question 3b

```
def q3b():
    commanded_T = np.array([[1, 0, 0.], [0, 1, 1], [0, 0, 1]])
    actual_angle = 1 * DEG_TO_RAD
    actual_T = np.array([[np.cos(actual_angle), -np.sin(actual_angle), 0.], [np.sin(actual_angle), np.cos(actual_angle), 0.], [0, 0, 1]])

    commanded_T_lst = create_composed_T_arr(commanded_T)
    actual_T_lst = create_composed_T_arr(actual_T)
    commanded_loc = get_locations_from_T(commanded_T_lst)
    actual_loc = get_locations_from_T(actual_T_lst)
    commanded_headings = get_headings(commanded_T_lst)
    actual_headings = get_headings(actual_T_lst)
    loc_diff = actual_loc[-1] - commanded_loc[-1]
    heading_err = np.arccos(np.dot(actual_headings[-1, :], commanded_headings[-1, :]))
    x_err = loc_diff[0]
    y_err = loc_diff[1]
    err_title = "Dead Reckoning: x error: {:.3f} | y error : {:.3f} | Heading error: {:.3f}"
    print(err_title.format(x_err, y_err, heading_err * RAD_TO_DEG))
    draw_robot_pose_actual_and_commanded(commanded_loc, commanded_headings, actual_loc)

def create_composed_T_arr(T, arr_len=10):
    arr = [None] * arr_len
    arr[0] = np.eye(3)
    for i in range(1, arr_len):
        arr[i] = T @ arr[i - 1]
    return arr

def get_locations_from_T(Ts_lst):
    return np.array([T[0:2, 2] for T in Ts_lst])
```

```
def get_headings(Ts_lst):  
    return np.array([T[0:2, 1] for T in Ts_lst])
```