

**Assignment 3:**

# **Projective Geometry**

Computer Vision  
National Taiwan University

Fall 2018

# Part 1: Estimating Homography



# Recap of Homography

- Matrix form:

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} \sim \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

- Equations:

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

# Recap of Homography

$$\begin{bmatrix} v_x \\ v_y \\ 1 \end{bmatrix} \sim \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \\ 1 \end{bmatrix}$$

- Degree of freedom

- There are 9 numbers in  $H$ . Are there 9 DoF?
- No. Note that we can multiply all  $h_{ij}$  by nonzero  $k$  without changing the equations:

$$v_x = \frac{kh_{11}u_x + kh_{12}u_y + kh_{13}}{kh_{31}u_x + kh_{32}u_y + kh_{33}}$$

$$v_y = \frac{kh_{21}u_x + kh_{22}u_y + kh_{23}}{kh_{31}u_x + kh_{32}u_y + kh_{33}}$$



$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

# Enforcing 8 DoF

- **Solution 1:** set  $h_{33} = 1$

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + 1}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + 1}$$

- **Solution 2:** impose unit vector constraint

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

Subject to

$$h_{11}^2 + \dots + h_{33}^2 = 1$$

# Solution 1

- Set  $h_{33} = 1$

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + 1}$$
$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + 1}$$

- Multiply by denominator

$$(h_{31}u_x + h_{32}u_y + 1)v_x = h_{11}u_x + h_{12}u_y + h_{13}$$

$$(h_{31}u_x + h_{32}u_y + 1)v_y = h_{21}u_x + h_{22}u_y + h_{23}$$

- Rearrange

$$h_{11}u_x + h_{12}u_y + h_{13} - h_{31}u_xv_x - h_{32}u_yv_x = v_x$$

$$h_{21}u_x + h_{22}u_y + h_{23} - h_{31}u_xv_y - h_{32}u_yv_y = v_y$$

# Solution 1 (cont.)

- Solve linear system

$$\begin{array}{l}
 \text{Point 1} \\
 \text{Point 2} \\
 \text{Point 3} \\
 \text{Point 4} \\
 \text{Additional points}
 \end{array}
 \begin{array}{c}
 \begin{bmatrix}
 u_{x,1} & u_{y,1} & 1 & 0 & 0 & 0 & -u_{x,1}v_{x,1} & -u_{y,1}v_{x,1} \\
 0 & 0 & 0 & u_{x,1} & u_{y,1} & 1 & -u_{x,1}v_{y,1} & -u_{y,1}v_{y,1} \\
 u_{x,2} & u_{y,2} & 1 & 0 & 0 & 0 & -u_{x,2}v_{x,2} & -u_{y,2}v_{x,2} \\
 0 & 0 & 0 & u_{x,2} & u_{y,2} & 1 & -u_{x,2}v_{y,2} & -u_{y,2}v_{y,2} \\
 u_{x,3} & u_{y,3} & 1 & 0 & 0 & 0 & -u_{x,3}v_{x,3} & -u_{y,3}v_{x,3} \\
 0 & 0 & 0 & u_{x,3} & u_{y,3} & 1 & -u_{x,3}v_{y,3} & -u_{y,3}v_{y,3} \\
 u_{x,4} & u_{y,4} & 1 & 0 & 0 & 0 & -u_{x,4}v_{x,4} & -u_{y,4}v_{x,4} \\
 0 & 0 & 0 & u_{x,4} & u_{y,4} & 1 & -u_{x,4}v_{y,4} & -u_{y,4}v_{y,4}
 \end{bmatrix} \\
 \vdots
 \end{array}
 \begin{array}{c}
 2N \times 8 \\
 \\
 \\
 \\
 \\
 \\
 \\
 \\
 \end{array}
 \begin{array}{c}
 \begin{bmatrix}
 h_{11} \\
 h_{12} \\
 h_{13} \\
 h_{21} \\
 h_{22} \\
 h_{23} \\
 h_{31} \\
 h_{32}
 \end{bmatrix} \\
 = \\
 \begin{bmatrix}
 v_{x,1} \\
 v_{y,1} \\
 v_{x,2} \\
 v_{y,2} \\
 v_{x,3} \\
 v_{y,3} \\
 v_{x,4} \\
 v_{y,4}
 \end{bmatrix} \\
 \vdots
 \end{array}
 \begin{array}{c}
 8 \times 1 \\
 2N \times 1
 \end{array}$$

# Solution 1 (cont.)

- What might be wrong with solution 1?
- If  $h_{33}$  is actually 0, we can not get the right answer



# Solution 2

- A more general solution by confining  $h_{11}^2 + \dots + h_{33}^2 = 1$

$$v_x = \frac{h_{11}u_x + h_{12}u_y + h_{13}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

$$v_y = \frac{h_{21}u_x + h_{22}u_y + h_{23}}{h_{31}u_x + h_{32}u_y + h_{33}}$$

- Multiply by denominator

$$(h_{31}u_x + h_{32}u_y + h_{33})v_x = h_{11}u_x + h_{12}u_y + h_{13}$$

$$(h_{31}u_x + h_{32}u_y + h_{33})v_y = h_{21}u_x + h_{22}u_y + h_{23}$$

- Rearrange

$$h_{11}u_x + h_{12}u_y + h_{13} - h_{31}u_xv_x - h_{32}u_yv_x - h_{33}v_x = 0$$

$$h_{21}u_x + h_{22}u_y + h_{23} - h_{31}u_xv_y - h_{32}u_yv_y - h_{33}v_y = 0$$

# Solution 2

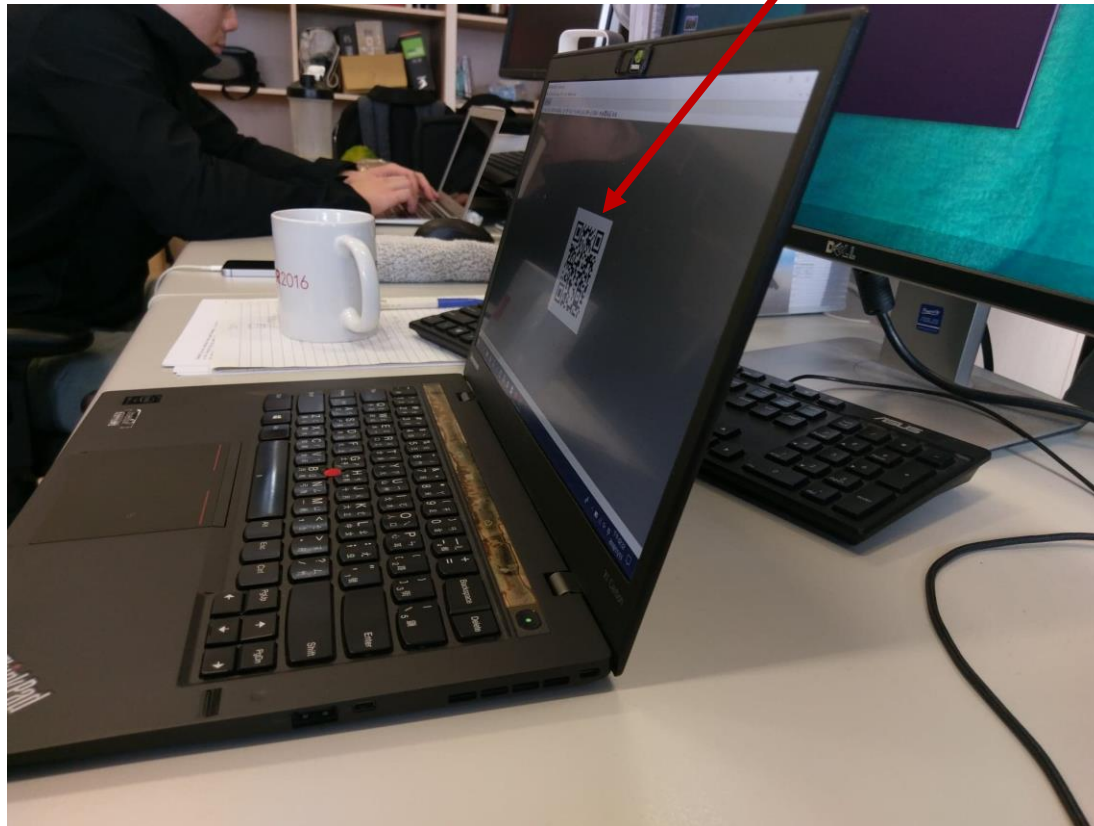
- Similarly, we have a linear system like this:

$$\begin{matrix} 2N \times 9 & 9 \times 1 & & 2N \times 1 \\ \mathbf{A} & \mathbf{h} & = & \mathbf{b} \end{matrix}$$

- Here,  $\mathbf{b}$  is all zero, so above equation is a [homogeneous system](#)
- Solve:
  - $Ah = 0$
  - $A^T Ah = A^T 0 = 0$
  - SVD of  $A^T A = U\Sigma V^T$
  - Let  $h$  be the column of  $U$  (unit eigenvector) associated with the smallest eigenvalue in  $\Sigma$

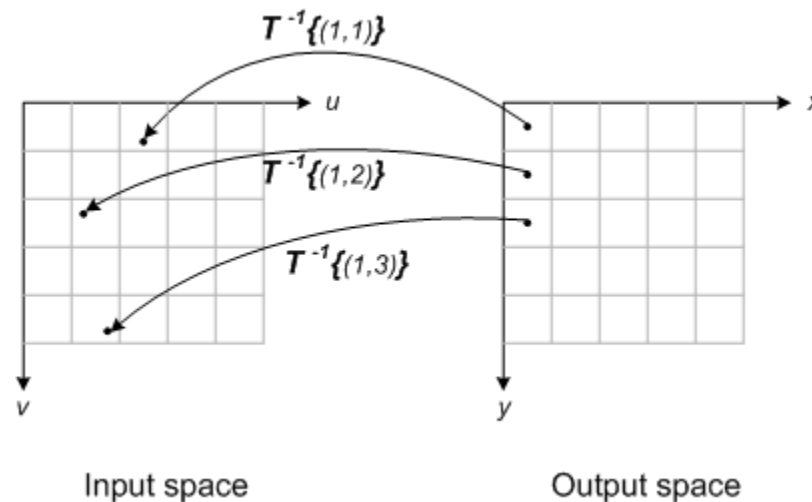
# Part 2: Unwarp the Screen

Make the QR code frontal parallel



# Backward Warping

- Why?
  - Prevent holes in output space
- Pixel value at sub-pixel location like (30.21, 22.74)?
  - Bilinear interpolation
  - Nearest neighbor



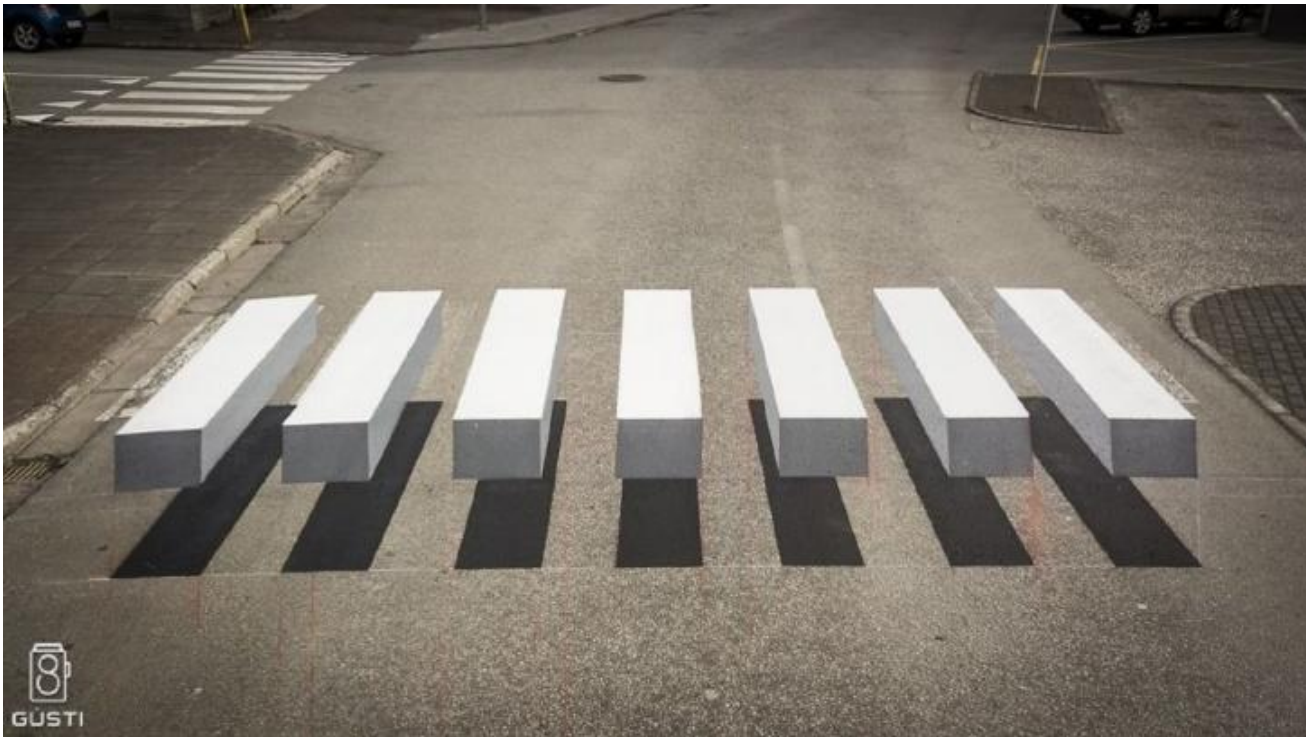
# Part 3: Unwarp the 3D Illusion

- 3D illusion art



# Part 3: Unwarp the 3D Illusion

- Input:





# Part 3: Unwarp the 3D Illusion

- Ground-truth top view:



**Can you unwarp the input image to match the ground-truth top view?**

# Assignment Description

- Part 1
  - Implement solution 1 or 2 for estimating homography.
  - Map 5 images of different people to the target surfaces (given in main.py). You can use whatever images you like. Include these images in your submission.
  - Include the function `solve_homography(u, v)` in your report.
- Part 2
  - Choose the unwarp region yourself.
  - The output image should contain the detectable QR code.
  - Include `the QR code` and the `decoded link` in your report.
- Part 3
  - Unwarp the image to the `top view`.
  - Can you get the parallel bars from the top view?
  - If not, why? Discuss in your report.



# Bonus (Optional)

- Simple AR
  - Given a [short video](#) (~6 sec) and [a template](#)
  - Paste an image (it's up to you) on the surface to stick to the marker
  - Include your algorithm in your report



# Submission

- Code: main.py (Python 3.5+)
- Input images for part 1
- Output images
  - part1.png, part2.png, part3.png
- A PDF report, containing
  - Your student ID, name
  - Your answers to each part
  - (Optional) algorithm to the simple AR
- (Optional) Input image and output video of the bonus part
- Compress all above files in a zip file named StudentID.zip
  - e.g. R07654321.zip
- Submit to CEIBA
- Deadline: 12/4 11:00 pm