We want to add an angle constraint loss c on the controls c in order to enforce that the angle between the first and second control points is approximately θ. Meaning:

$c_1.x + \cos\theta \cdot r \approx c_2.x$

$c_1.y + \sin\theta \cdot r \approx c_2.y$

For some $r > 0$.

Let's choose MSE as the loss function to impose this:

$c(c,r) = |c_1.x + cos\theta \cdot r - c_2.x|^2 + |c_1.y + sin\theta \cdot r - c_2.y|^2$

Write the control vector c:

$c = [c_1.x, \quad c_2.x, \dots, c_N.x, c_1.y, c_2.y, \dots, C_N.y] \in \mathbb{R}^{2N}$

Let's rewrite our objective c as $\tilde{c}$ s.t. we add the variable r (that the LS solver finds) in the end:

$\tilde{c} = [c_1.x, \quad c_2.x, \dots, c_N.x, c_1.y, c_2.y, \dots, C_N.y, r] \in \mathbb{R}^{2N+1}$

And now we wish to solve for $\tilde{c}$.

We can easily write our original objective functions (MSE, smoothing) with respect to $\tilde{c}$ by just adding rows or columns of zeros (to zero out the effect of r on the MSE, smoothing calculations).

Let's write our new constraint c in matrix form with respect to $\tilde{c}$:

$c(c) = |c_1.x + cos\theta \cdot r - c_2.x|^2 + |c_1.y + sin\theta \cdot r - c_2.y|^2$

$$\hat{B} = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & cos\theta \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & \dots & 0 & sin\theta \end{bmatrix}$$

$\hat{B} \in \mathbb{R}^{2\times 2N+1}$

$$c(c,r) = c(\tilde{c}) = \left\|\hat{B}\tilde{c}\right\|_2^2$$

We add this loss to our general loss, and solve (not shown)

$\ell(\tilde{c}) = (1-\lambda_2)[(1-\lambda)MSE(B\tilde{c}, y) + \lambda \cdot smoothing(B\tilde{c})] + \lambda_2 c(\tilde{c})$

After finding $\tilde{c}$, we throw away r at the end to keep c.