$$\ell = -\theta_1 \sum_{i=0}^{N-1} v_i + \theta_2 \sum_{i=1}^{N-2} (v_{i+1} - 2v_i + v_{i-1})^2 + \theta_2 (v_1 - 2v_0 + v_{init})^2 + \theta_2 (v_0 - v_{init} - a_{init})^2$$

$$\forall i \in [2, N-3]: \frac{\partial \ell}{\partial v_i} =$$
$$-\theta_1 + 2\theta_2 (v_{i+2} - 2v_{i+1} + v_i) - 4\theta_2 (v_{i+1} - 2v_i + v_{i-1}) + 2\theta_2 (v_i - 2v_{i-1} + v_{i-2}) =$$
$$-\theta_1 + v_{i-2}(2\theta_2) + v_{i-1}\big((-4-4)\theta_2\big) + v_i\big((2+8+2)\theta_2\big) + v_{i+1}\big((-4-4)\theta_2\big) + v_{i+2}\big((2)\theta_2\big) =$$
$$-\theta_1 + 2\theta_2 v_{i-2} - 8\theta_2 v_{i-1} + 12\theta_2 v_i - 8\theta_2 v_{i+1} + 2\theta_2 v_{i+2}$$

$$i = N-2: \frac{\partial \ell}{\partial v_i} = -\theta_1 - 4\theta_2 (v_{i+1} - 2v_i + v_{i-1}) + 2\theta_2 (v_i - 2v_{i-1} + v_{i-2}) =$$
$$-\theta_1 + 2\theta_2 v_{i-2} - 8\theta_2 v_{i-1} + 10\theta_2 v_i - 4\theta_2 v_{i+1}$$

$$i = N-1: \frac{\partial \ell}{\partial v_i} = -\theta_1 + 2\theta_2 (v_i - 2v_{i-1} + v_{i-2}) = -\theta_1 + 2\theta_2 v_{i-2} - 4\theta_2 v_{i-1} + 2\theta_2 v_i$$

$$i = 1: \frac{\partial \ell}{\partial v_1} = -\theta_1 + 2\theta_2 (v_3 - 2v_2 + v_1) - 4\theta_2 (v_2 - 2v_1 + v_0) + 2\theta_2 (v_1 - 2v_0 + v_{init}) =$$
$$-\theta_1 - 2\theta_2 v_3 - 8\theta_2 v_2 + 12\theta_2 v_1 - 8\theta_2 v_0 + 2\theta_2 v_{init}$$

$$i = 0: \frac{\partial \ell}{\partial v_0} = -\theta_1 + 2\theta_2 (v_2 - 2v_1 + v_0) - 4\theta_2 (v_1 - 2v_0 + v_{init}) + 2\theta_2 (v_0 - v_{init} - a_{init}) =$$
$$-\theta_1 + 2\theta_2 v_2 - 8\theta_2 v_1 + 12\theta_2 v_0 - 6\theta_2 v_{init} - 2\theta_2 a_{init}$$