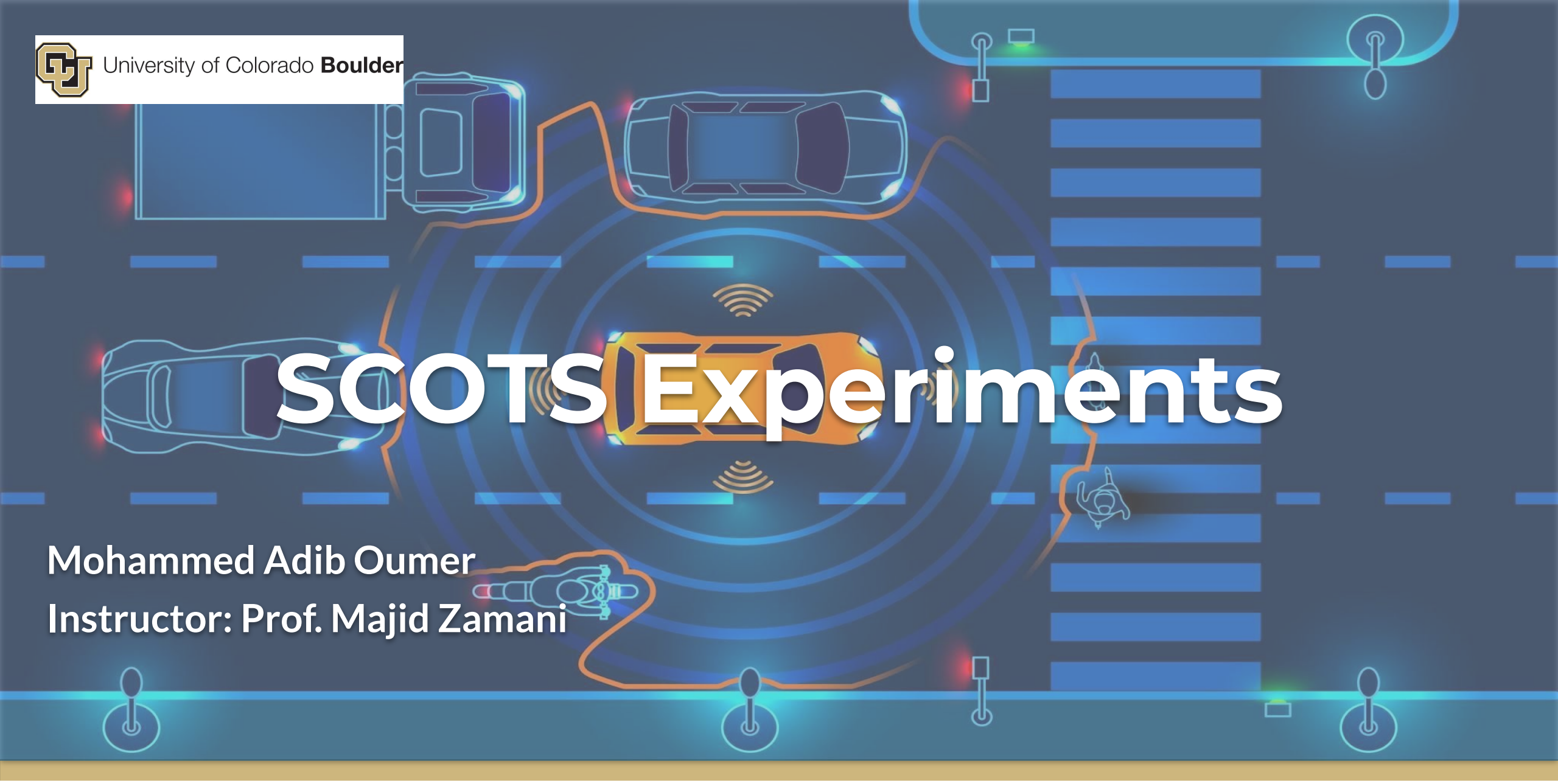
University of Colorado **Boulder**



SCOTS Experiments

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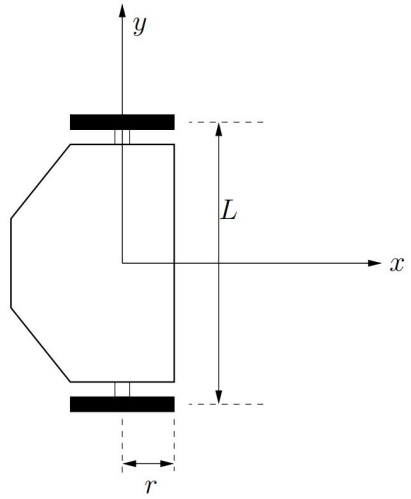
Instructor: Prof. Majid Zamani

Intro

- Systems of interest 1 - Roomba:



Differential Drive Robot

$$\dot{x} = \frac{r}{2}(u_l + u_r)cos\theta$$

$$\dot{y} = \frac{r}{2}(u_l + u_r)sin\theta$$

$$\dot{\theta} = \frac{r}{L}(u_r - u_l)$$

States $\mathbf{x} = [x, y, \theta]^T$

Inputs $\mathbf{u} = [u_l, u_r]^T$

Smooth Differential Drive (w/ integrators)

$$\dot{x} = \frac{r}{2}(\omega_l + \omega_r)cos\theta$$

$$\dot{y} = \frac{r}{2}(\omega_l + \omega_r)sin\theta$$

$$\dot{\theta} = \frac{r}{L}(\omega_r - \omega_l)$$

$$\dot{\omega_l} = u_l$$

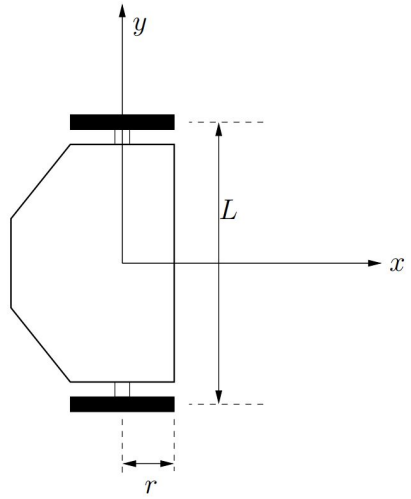
$$\dot{\omega_r} = u_r$$

$$\mathbf{x} = [x, y, \theta, \omega_l, \omega_r]^T$$

$$\mathbf{u} = [u_l, u_r]^T$$

Intro

- Growth Bound - Roomba:



Differential Drive Robot

$$\beta(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} \frac{r}{2} \left| u_l + u_r \right| x_2 \\ \frac{r}{2} \left| u_l + u_r \right| x_2 \\ 0 \end{bmatrix}$$

Smooth Differential Drive (w/ integrators)

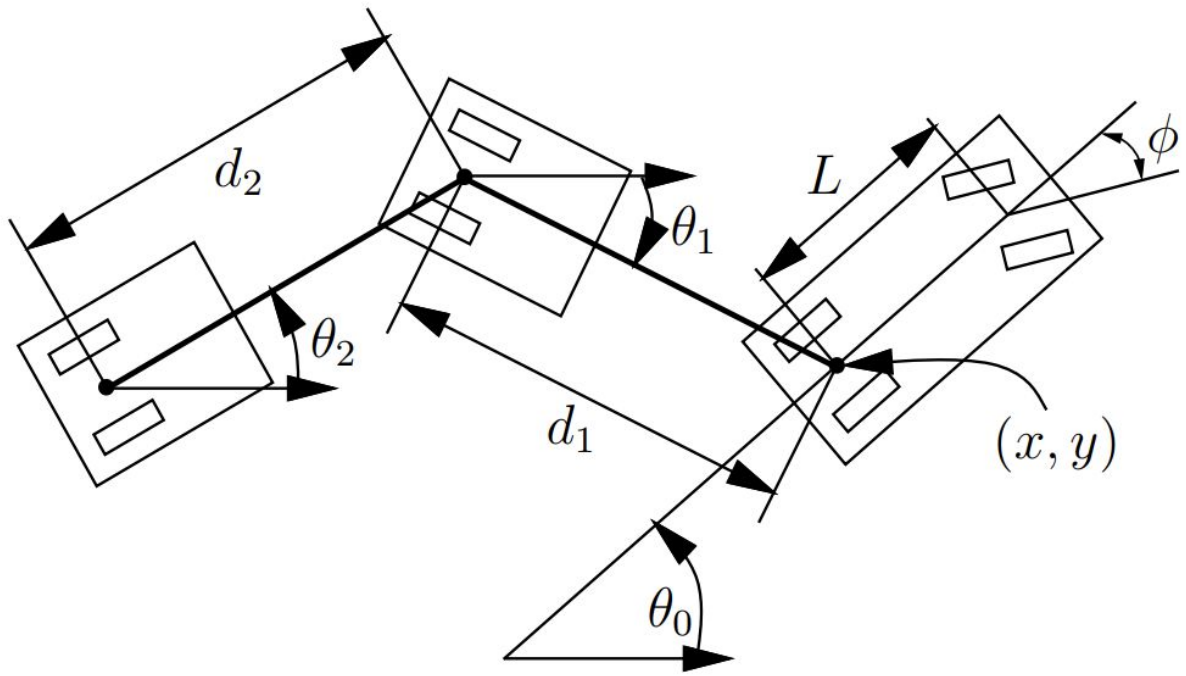
$$\beta(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} x_2 + \frac{r}{2}(x_3 + x_4) \\ x_2 + \frac{r}{2}(x_3 + x_4) \\ \frac{r}{L}(x_3 + x_4) \\ 0 \\ 0 \end{bmatrix}$$

Intro

- System of interest 2 - Vehicle with one trailer:

$$\dot{x} = ucos\theta_0$$
$$\dot{y} = usin\theta_0$$
$$\dot{\theta_0} = \frac{u}{L}tan\phi$$
$$\dot{\theta_1} = \frac{u}{d_1}sin(\theta_0 - \theta_1)$$

$$\mathbf{x} = [x, y, \theta_0, \theta_1]^T$$
$$\mathbf{u} = [u, \phi]^T$$



Intro

- System of interest 2 - Vehicle with one trailer:

$$\dot{x} = ucos\theta_0$$
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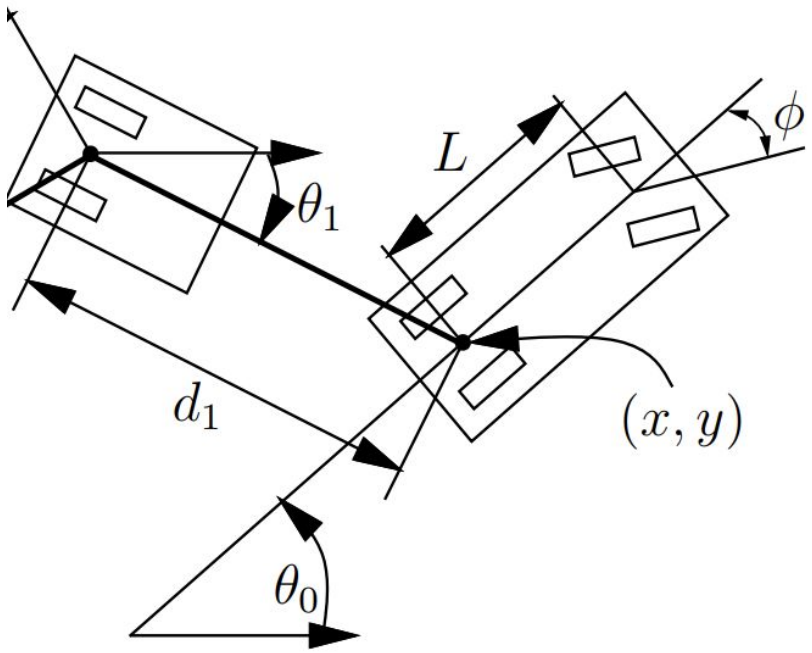
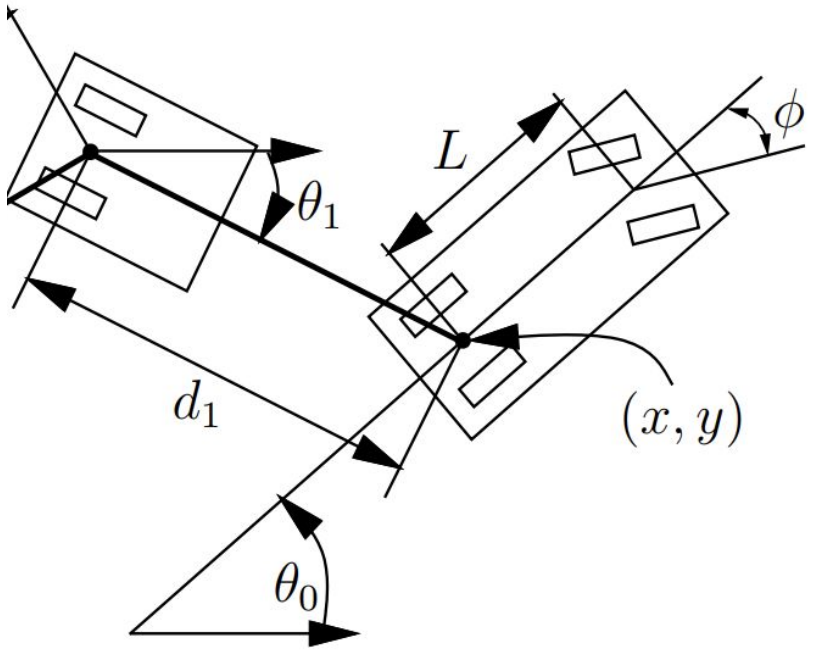
$$\mathbf{x} = [x, y, \theta_0, \theta_1]^T$$
$$\mathbf{u} = [u, \phi]^T$$



Intro

- Growth Bound - Vehicle with one trailer:

$$\beta(\mathbf{x}, \mathbf{u}) = \begin{bmatrix} |u|\, x_2 \\ |u|\, x_2 \\ 0 \\ \frac{|u|}{d_1}(x_2 + x_3) \end{bmatrix}$$



Properties Tested

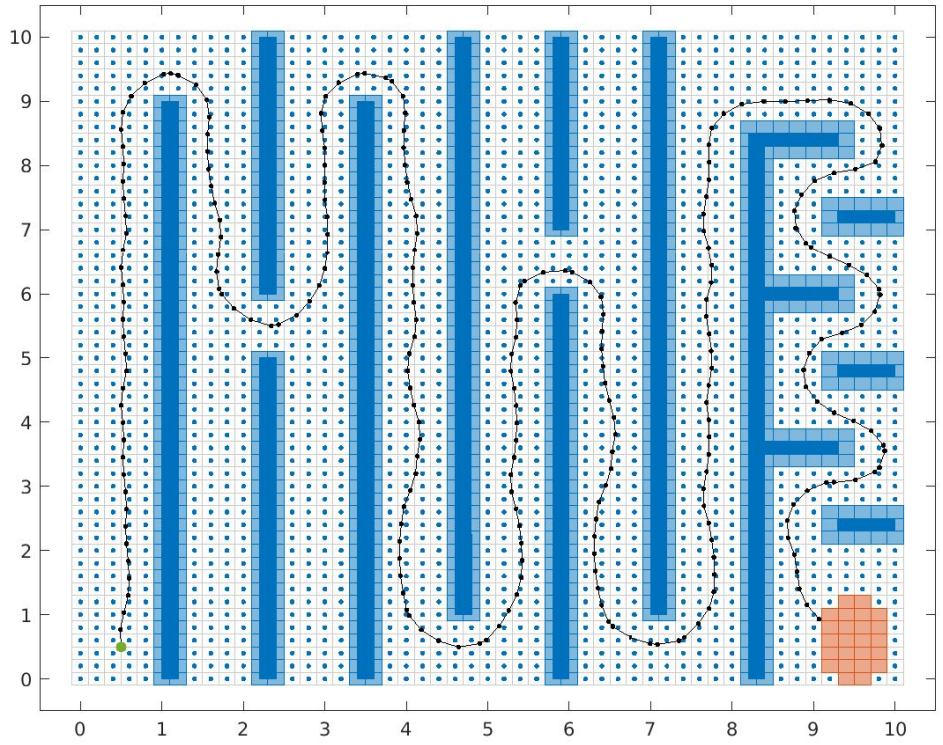
- Reach {F($target$)}
  - given target(s), eventually reach the target(s)
    - No obstacles

- Reach Avoid {G(!$obstacles$) /\ F($target$)}
  - given target(s) and obstacles, reach the target(s) while avoiding obstacles
    - 3 kinds of obstacles - experiment dependent

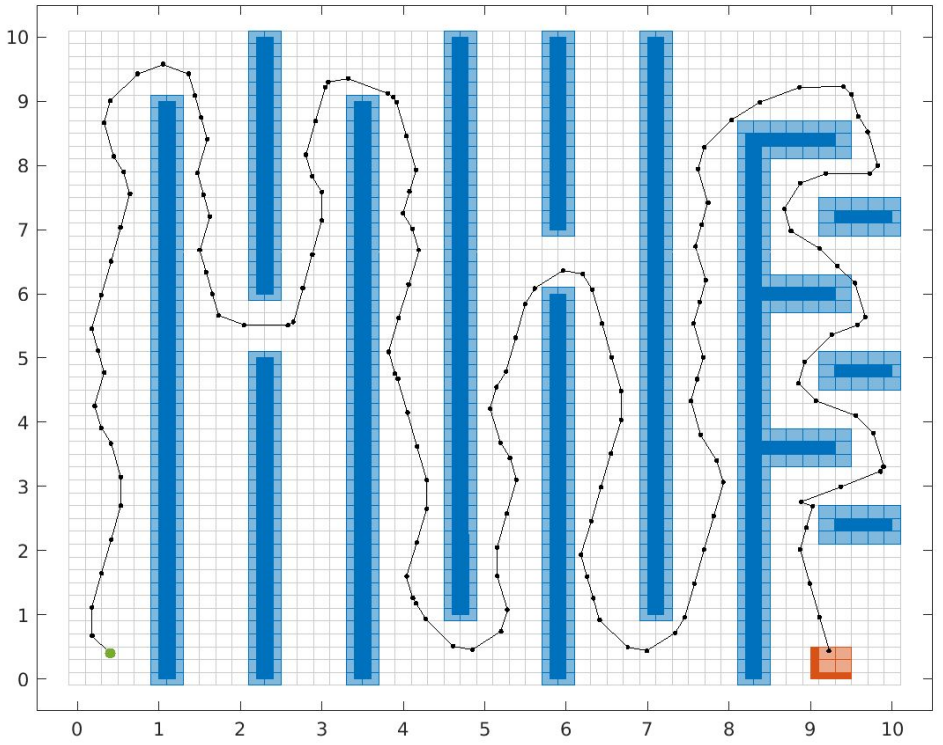
Results

Experiment		Lower Bound	Grid Size	Upper Bound	Abstraction size	Time (s): Abstraction	Time (s): Controller(s)
DD - Obstacle 1 (1 target)	State space	{0,0,-π-0.4}	{0.2,0.2,0.2}	{10,10,π+0.4}	3.29958e+07	125.256	496.152 (291 iterations)
	Input space	{-2/r, -2/r}	{0.6/r, 0.6/r}	{2/r, 2/r}			
DD - Obstacle 2 (1 target)	State space	{0,0,-π-0.4}	{0.2,0.2,0.2}	{10,10,π+0.4}		142.836	327.326 (170 iterations)
	Input space	{-2/r, -2/r}	{0.6/r, 0.6/r}	{2/r, 2/r}			
DD - Obstacle 2 (2 targets)	State space	{0,0,-π-0.4}	{0.2,0.2,0.2}	{10,10,π+0.4}		82.1768	163.101, 194.777 (151, 170 iterations)
	Input space	{-2/r, -2/r}	{0.6/r, 0.6/r}	{2/r, 2/r}			

Plots

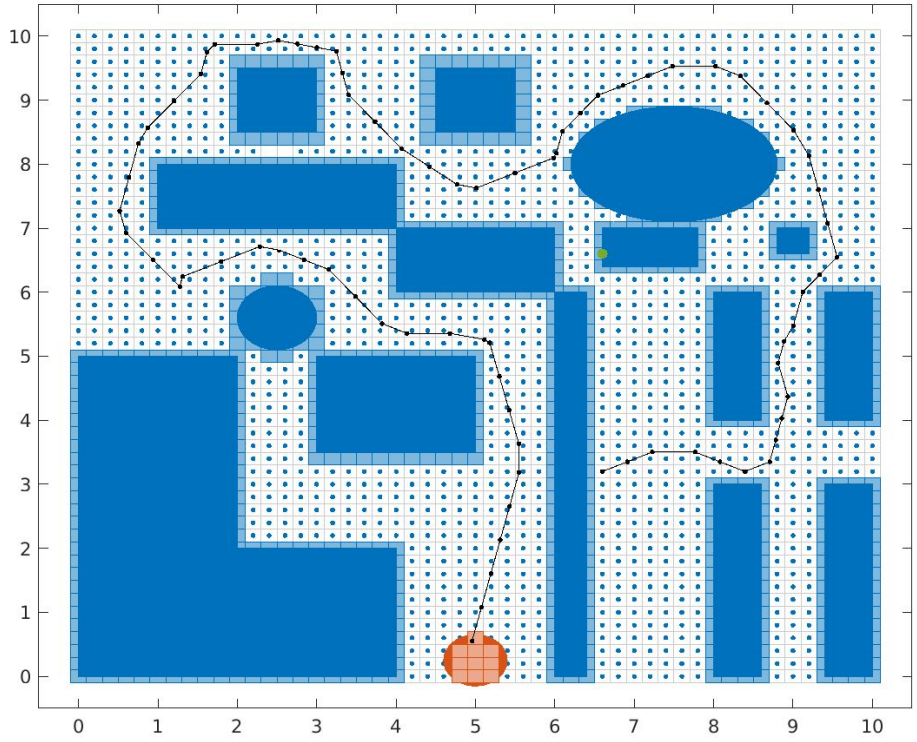


Unicycle

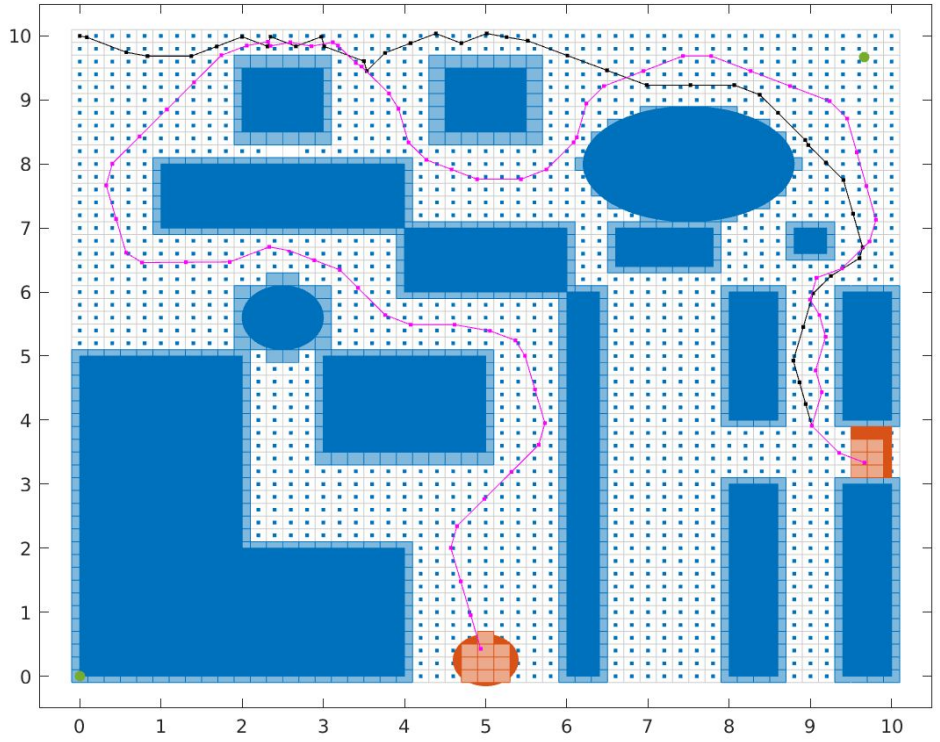


DD - Obstacle 1

Plots

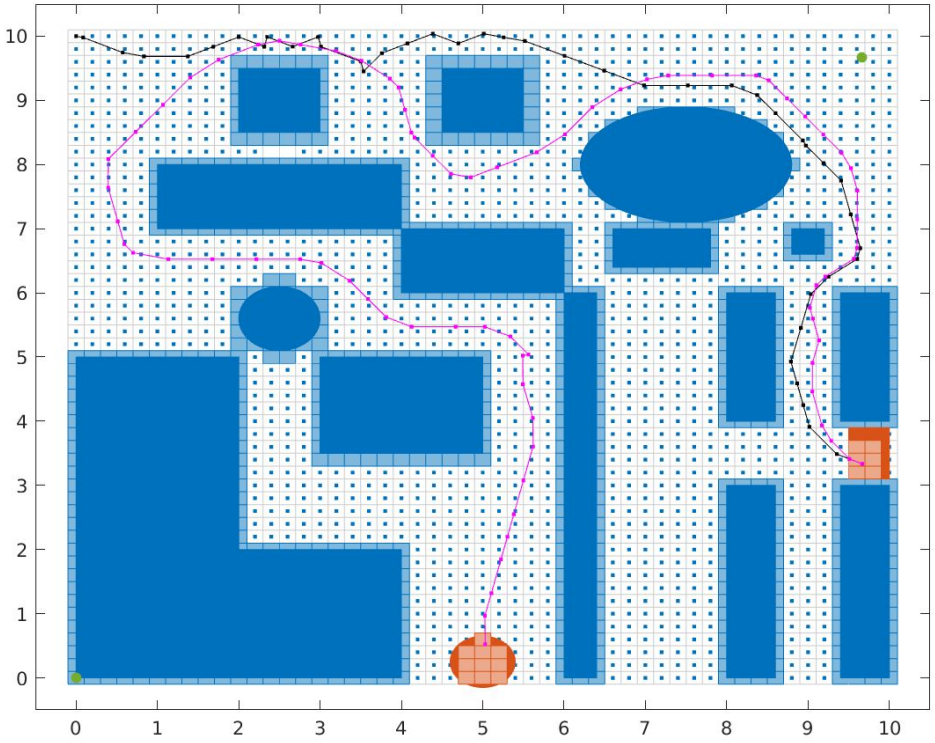


DD - Obstacle 2 (1 target)

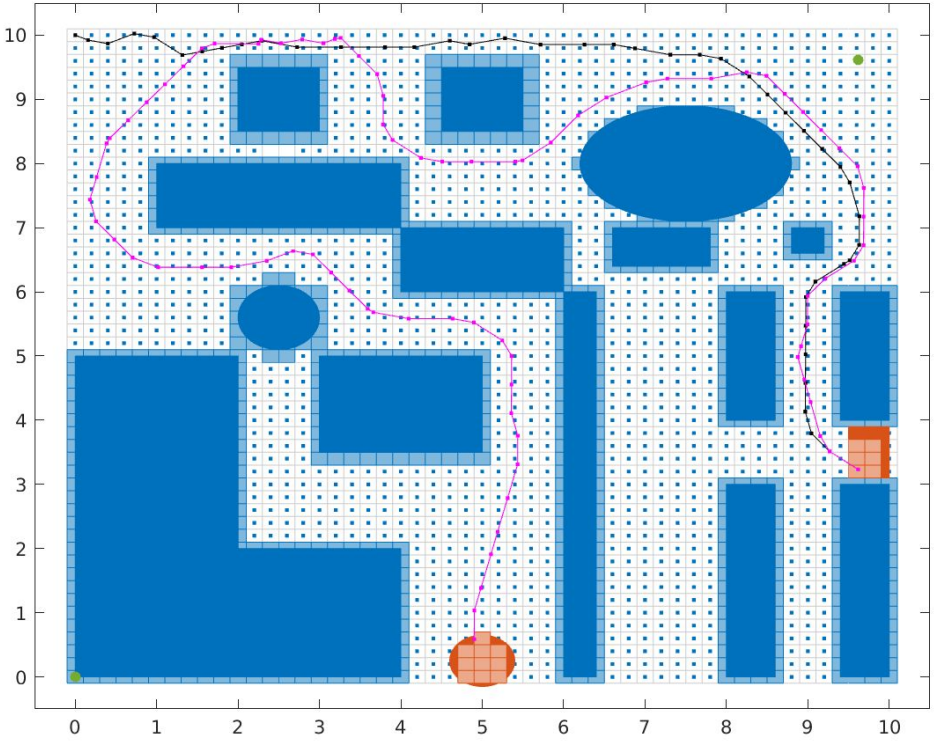


DD - Obstacle 2 (2 targets)

Plots



DD - Obstacle 2 (2 targets)

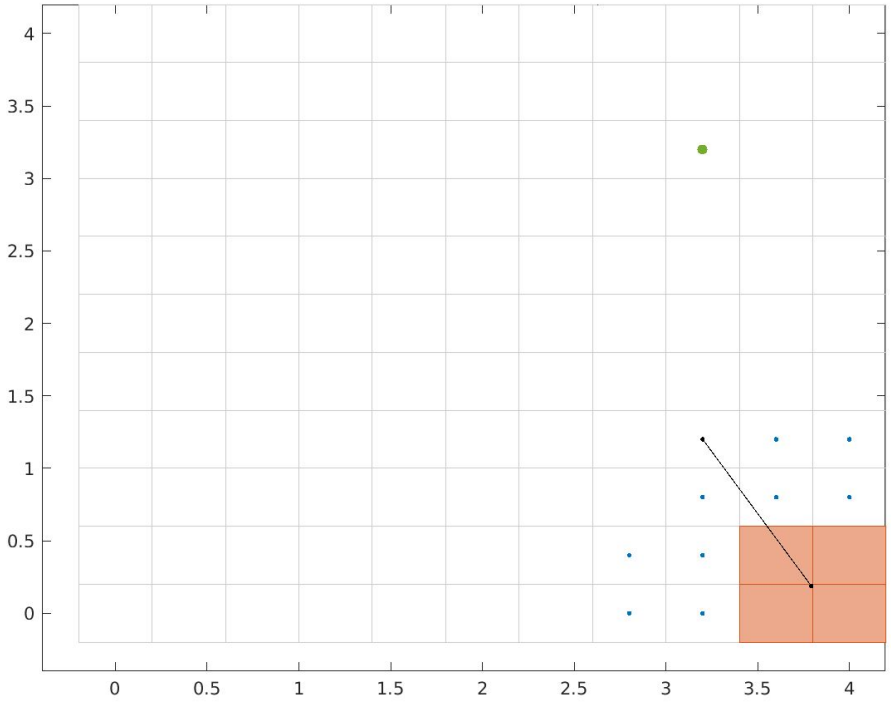


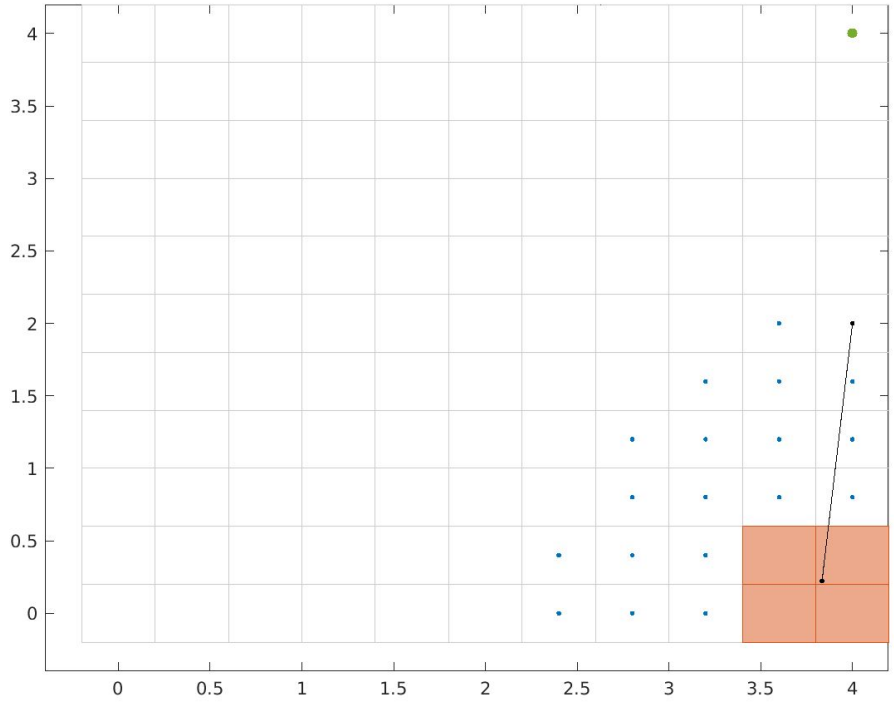
DD - Obstacle 2 (2 targets)

Results

Experiment		Lower Bound	Grid Size	Upper Bound	Abstraction size	Time (s): Abstraction	Time (s): Controller(s)
SDD - No obstacle	State space	{0,0,-π-0.4, -4/r, -4/r}	{0.4,0.4,0.3, 0.5/r,0.5/r}	{4,4,π+0.4, 4/r, 4/r}	4.55304e+09	1211.79	7.11977 (4 iterations)
	Input space	{-30,-30}	{5,5}	{30,30}			
Trailer - No obstacle	State space	{0,0,-π-0.4, -π-0.4}	{0.4,0.4,0.2, 0.2}	{10,10,π+0.4, π+0.4}	1.43612e+09	2937.32	2992.21 (72 iterations)
	Input space	{-2, -1}	{0.5, 0.25}	{2, 1}			
Trailer - Obstacle 3 (1 target)	State space	{0,0,-π-0.4, -π-0.4}	{0.4,0.4,0.2, 0.2}	{10,10,π+0.4, π+0.4}		3068.73	5067.97 (140 iterations)
	Input space	{-2, -1}	{0.5, 0.25}	{2, 1}			

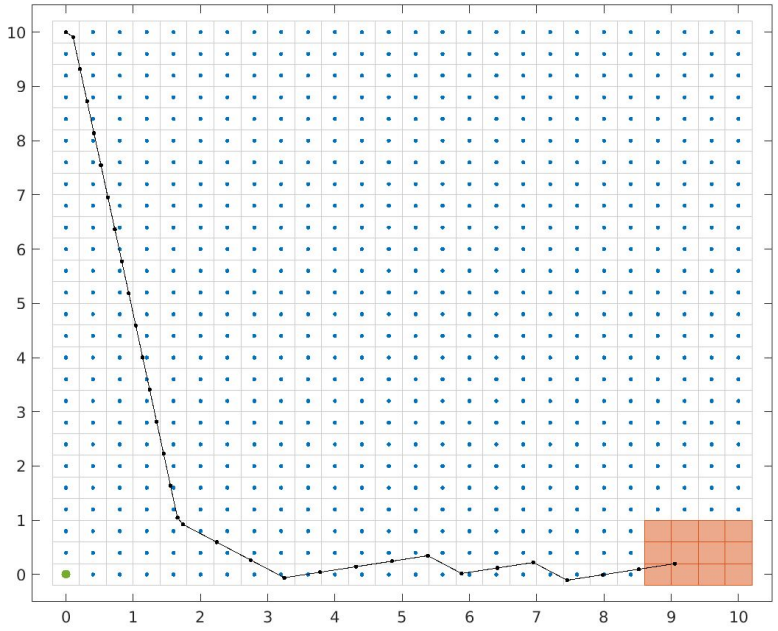
Plots

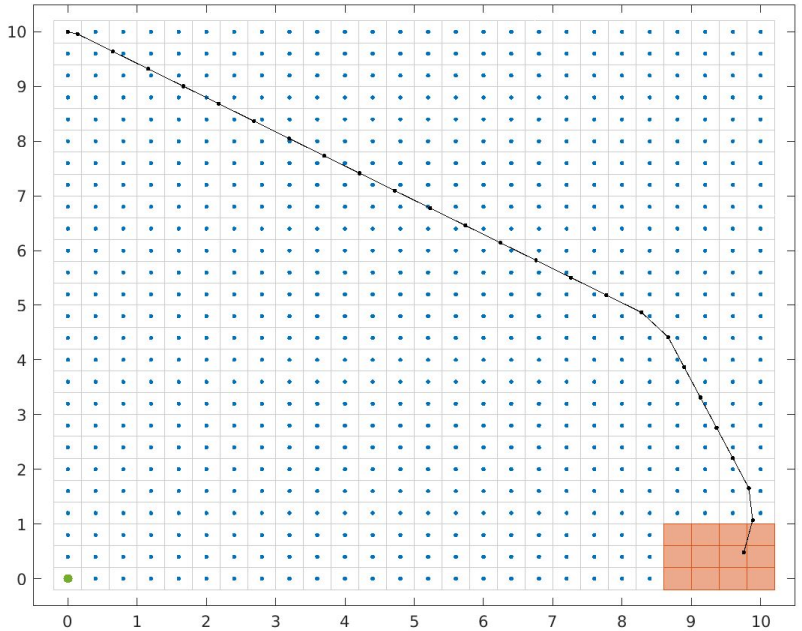


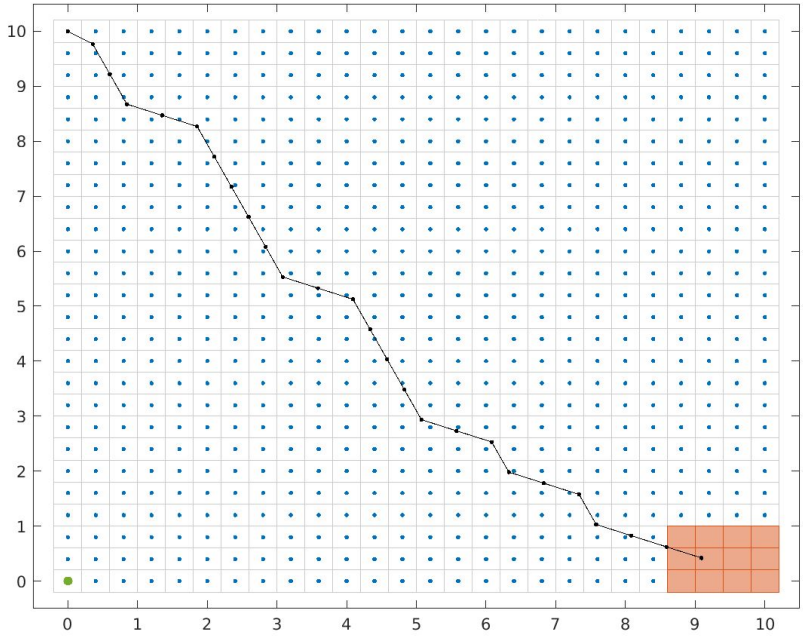


SDD - No Obstacle

Plots

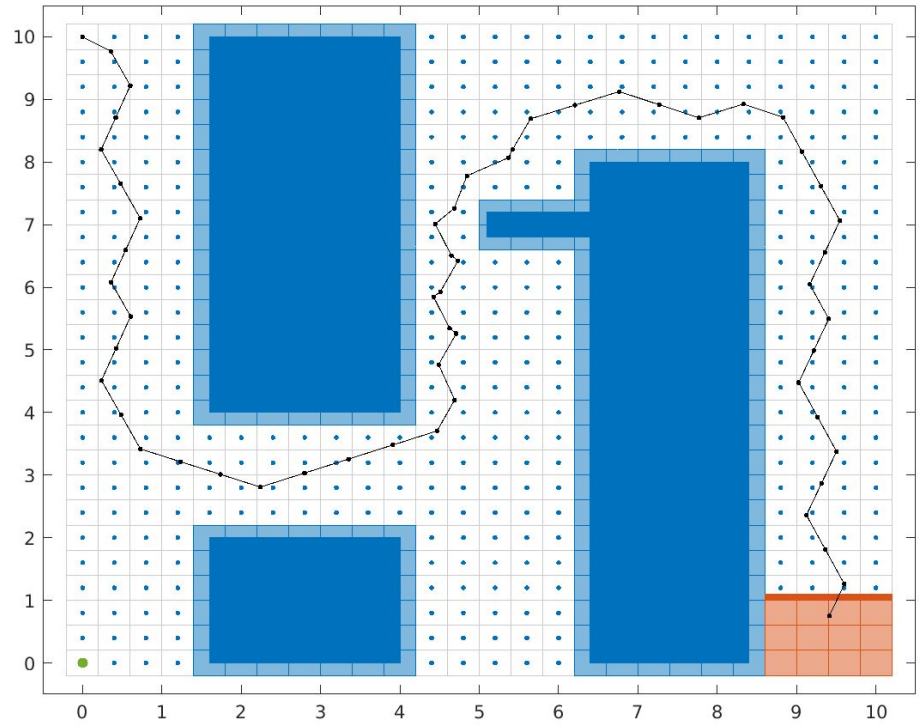


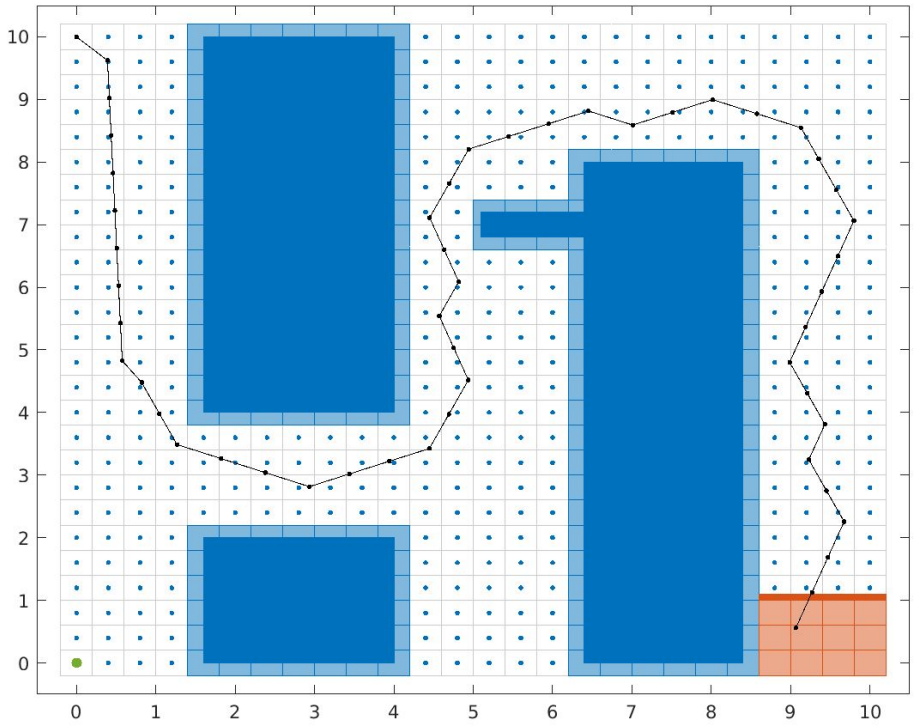




Trailer - No obstacle

Plots





Trailer - Obstacle 3

References/Tools

- SCOTS: Automated Synthesis of Symbolic Controllers for General Nonlinear Systems [static CUDD-lib Version]
- LaValle SM. 2006. Planning Algorithms. Cambridge, UK: Cambridge Univ. Press.

- Extra Images:
  Autonomous Cars