Compressive Sensing in Single Pixel Cameras

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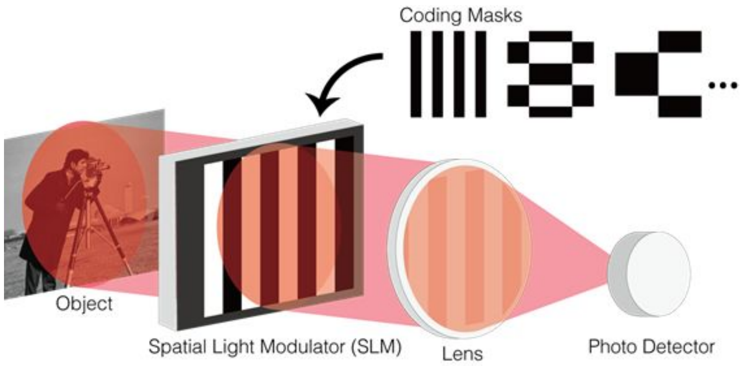
Single Pixel Camera (SPC)

- Uses measurements from a single photosensor
  - Photoresistor, phototransistor or photodiode
- Images are focused onto sensor through a mask
  - Mask is used to select parts of the image to measure
  - Mask can be built with digital micromirror devices (DMD) or LCD
- Images are measured using the mask
  - $\mathbf{b} = \Phi\mathbf{s}$, where $\mathbf{s}$ is a vector of the object



SPC

Simulation of single pixel camera with digital camera

- Simulation steps:
  - Construct masks using MxN sensing matrix Φ ($M << N$)
    - M = # desired measurements, N = number of pixels in image
  - Vectorize original input image $\mathbf{s}$ (from webcam) into 1D array
  - Represent $\mathbf{s} = \Psi\mathbf{x}$
    - Ψ = orthonormal NxN basis matrix made of N Nx1 orthonormal vectors
    - $\mathbf{x}$ = K-sparse coefficient vector
  - Calculate/Obtain measurements using $\mathbf{b} = \Phi\mathbf{s} = \Phi\Psi\mathbf{x} = A\mathbf{x}$
    - $A = \Phi\Psi$ where Φ, Ψ are two incoherent bases
  - Find $\mathbf{x}$ using Compressive Sensing (CS) algorithms
    - BP (Basis Pursuit), BPDN (Basis Pursuit Denoising), LASSO
    - We only have access to $\mathbf{b}, \Phi, \Psi$
  - Recover image $\mathbf{s} = \Psi\mathbf{x}$

Incoherence Property (IP)

- Want Φ, Ψ to be "incoherent" bases:
  - Their vectors are incoherent: less overlap between the set of basis vectors
- Measure of how "different" two bases are:

$$\mu(\bar{\Psi}, \Phi) = \sqrt{n} \max_{i,j} |\psi_i^T \phi_j|$$

μ here ranges between 1 and $\sqrt{n}$

  - Small μ = any vector sparse in one basis cannot be be sparse in the other → important to avoid sensing problems & to capture as much information about our sparse vector **x**

Restricted Isometry Property (RIP)

- Concept was introduced by Emmanuel Candès and Terence Tao
- Important property used in Compressive Sensing (MSP)
- **Definition 2.** $k \in N$, $\epsilon \in (0, 1/3)$, A *is* (k, ϵ) - *Restricted Isometry Property (RIP)*

  if $\forall x \in \mathbb{R}^n$ k-sparse, $||Ax||_2 = (1 \pm \epsilon)||x||_2$
- (simply: want an *M*x*N* matrix *A* that behaves like an orthogonal matrix.)
  - Note: the norm of a vector is invariant under multiplication by an orthogonal matrix
- Having a matrix satisfying RIP → LP problem can be solved → we can recover the signal with much fewer measurements

Choice of Φ, Ψ

- Ψ = sparse basis. Usually, a structured orthonormal basis is chosen
    - ***DCT* (4 types. Type 2 chosen)**
    - *Hadamard*
    - *Identity*
    - DFT
- Φ = want it to be incoherent with Ψ.
    - ***Random Gaussian***
    - *Bernoulli* - not highly likely to be incoherent
- Want $A = \Phi\Psi$ to satisfy RIP.
    - Luckily Gaussian multiplied with structured bases happens to empirically satisfy RIP.

CS (Compressive Sensing) Algorithms

Basis Pursuit (BP):

$$\min \ ||\mathbf{x}||_1$$
$$\text{s.t. } A\mathbf{x} = \mathbf{b}$$

Basis Pursuit Denoising (BPDN):

$$\min \ ||\mathbf{x}||_1 \quad \text{s.t. } ||A\mathbf{x} - \mathbf{b}||_2 \leq \sigma \xrightarrow{\ell 1\text{-equivalent}} \min \ ||\mathbf{x}||_1 \quad \text{s.t. } ||A\mathbf{x} - \mathbf{b}||_1 \leq \sigma \ (\sigma = 0.01)$$

LASSO:

$$\min \ ||A\mathbf{x} - \mathbf{b}||_2 \quad \text{s.t. } ||\mathbf{x}||_1 \leq \tau \xrightarrow{\ell 1\text{-equivalent}} \min \ ||A\mathbf{x} - \mathbf{b}||_1 \quad \text{s.t. } ||\mathbf{x}||_1 \leq \tau \ (\tau \approx 2)$$

ℓ1-norm to LP formulation

$$||\mathbf{x}||_1 = \sum_i |x_i|$$
$$\Rightarrow \sum_i t_i,\ \ |x_i| \le t_i,\ \ t_i \ge 0$$
$$\Rightarrow \mathbf{1}^T\mathbf{t},\ \ \mathbf{x} - \mathbf{t} \le 0,\ \ -\mathbf{x} - \mathbf{t} \le 0,\ \ \mathbf{t} \ge 0 \text{ (dimension } n).$$

$$||A\mathbf{x} - \mathbf{b}||_1 = \sum_i |A_{i,:}\mathbf{x} - b_i|$$
$$\Rightarrow \sum_i s_i,\ \ |A_{i,:}\mathbf{x} - b_i| \le s_i,\ \ s_i \ge 0$$
$$\Rightarrow \mathbf{1}^T\mathbf{s},\ \ -\mathbf{s} \le A\mathbf{x} - \mathbf{b} \le \mathbf{s},\ \ \mathbf{s} \ge 0$$
$$\Rightarrow \mathbf{1}^T\mathbf{s},\ \ -A\mathbf{x} - \mathbf{s} \le -\mathbf{b},\ \ A\mathbf{x} - \mathbf{s} \le \mathbf{b},\ \ \mathbf{s} \ge 0 \text{ (dimension } m).$$

Conversions to LP

$$\begin{array}{l}\min\ ||\mathbf{x}||_1\\ \text{s.t. } A\mathbf{x}=\mathbf{b}\end{array} \xrightarrow[\mathbf{x}\to(\mathbf{x}\ \mathbf{t})^T]{LP} \min \begin{bmatrix}\mathbf{0}\\ \mathbf{1}\end{bmatrix}^T \begin{bmatrix}\mathbf{x}\\ \mathbf{t}\end{bmatrix} \text{ s.t. } \begin{bmatrix}A & 0\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \mathbf{t}\end{bmatrix}=\mathbf{b},\ \begin{bmatrix}I_n & -I_n\\ -I & -I_n\\ 0 & -I_n\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \mathbf{t}\end{bmatrix}\leq 0$$

$$\begin{array}{l}\min\ ||\mathbf{x}||_1\\ \text{s.t. } ||A\mathbf{x}-\mathbf{b}||_1\leq\sigma\end{array} \xrightarrow[\mathbf{x}\to(\mathbf{x}\ \mathbf{t}\ \mathbf{s})^T]{LP} \min \begin{bmatrix}\mathbf{0}\\ \mathbf{1}\\ \mathbf{0}\end{bmatrix}^T \begin{bmatrix}\mathbf{x}\\ \mathbf{t}\\ \mathbf{s}\end{bmatrix} \text{ s.t. } \begin{bmatrix}I_n & -I_n & 0\\ -I_n & -I_n & 0\\ A & 0 & -I_m\\ -A & 0 & -I_m\\ \mathbf{0}^T & \mathbf{0}^T & \mathbf{1}^T\\ 0 & -I_n & 0\\ 0 & 0 & -I_m\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \mathbf{t}\\ \mathbf{s}\end{bmatrix}\leq\begin{bmatrix}0\\ 0\\ \mathbf{b}\\ -\mathbf{b}\\ \sigma\\ 0\\ 0\end{bmatrix}$$

$$\begin{array}{l}\min\ ||A\mathbf{x}-\mathbf{b}||_1\\ \text{s.t. } ||\mathbf{x}||_1\leq\tau\end{array} \xrightarrow[\mathbf{x}\to(\mathbf{x}\ \mathbf{s}\ \mathbf{t})^T]{LP} \min \begin{bmatrix}\mathbf{0}\\ \mathbf{1}\\ \mathbf{0}\end{bmatrix}^T \begin{bmatrix}\mathbf{x}\\ \mathbf{s}\\ \mathbf{t}\end{bmatrix} \text{ s.t. } \begin{bmatrix}A & -I_m & 0\\ -A & -I_m & 0\\ I_n & 0 & -I_n\\ -I_n & 0 & -I_n\\ \mathbf{0}^T & \mathbf{0}^T & \mathbf{1}^T\\ 0 & -I_m & 0\\ 0 & 0 & -I_n\end{bmatrix}\begin{bmatrix}\mathbf{x}\\ \mathbf{s}\\ \mathbf{t}\end{bmatrix}\leq\begin{bmatrix}\mathbf{b}\\ -\mathbf{b}\\ 0\\ 0\\ \tau\\ 0\\ 0\end{bmatrix}$$

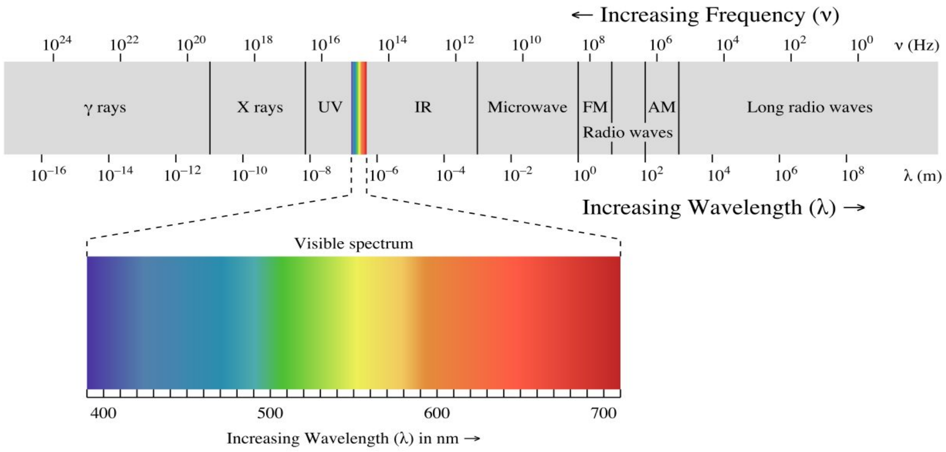
Why compressive sensing?

- Underdetermined linear system (M<<N for A MxN)
    - Multiple solutions possible but able to narrow it down further if we insist on sparse solutions

- Takes advantage of the redundancy
    - many coefficients for images are close to zero that barely affect signal (in this case image quality)

Why compressive sensing?

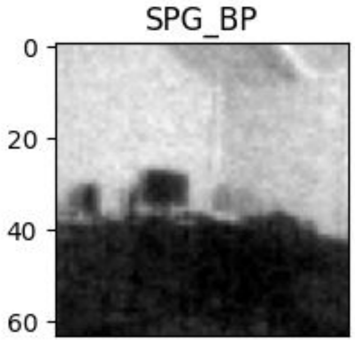
- wrt SPC:
  - SPC can be built without a lens
  - Compressing while imaging
  - Photon detector can be modified
  - Wider range of ER spectrum – UV, Infrared (eg: [SWIR camera]).



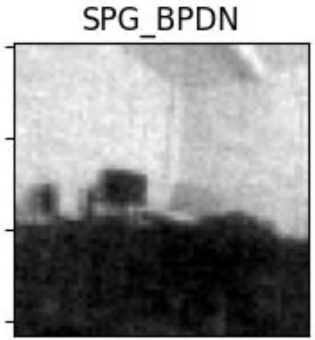
Implementation

- Used custom implementations
  - LP versions of the algorithms
- Used dedicated solver: SPGL1
  - For comparison. Solves the original L2 and L1 norm formulation
- Assessment Metrics:
  - Computation Time
  - Sparsity of coefficient vector
  - $$MSE = \frac{\sum_{M,N} [I_1(m,n) - I_2(m,n)]^2}{M * N}$$
  - $$PSNR = 10 \log_{10}\left(\frac{R^2}{MSE}\right)$$ *R* = 1 (for pixel values [0,1]), or 255 (for pixel values [0,255])
- Better compression/reconstruction = small MSE, large PSNR

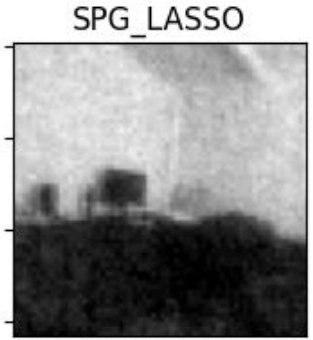
64x64 images
M/N = 0.5



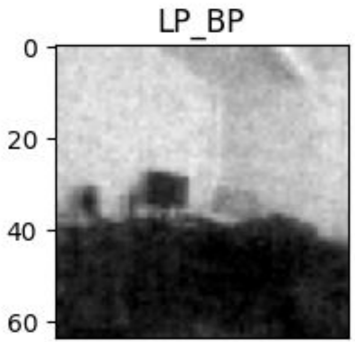
Time: 9.73s
MSE: 0.472
PSNR: 3.262



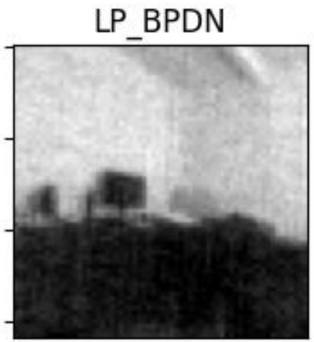
Time: 6.71s
MSE: 0.471
PSNR: 3.274



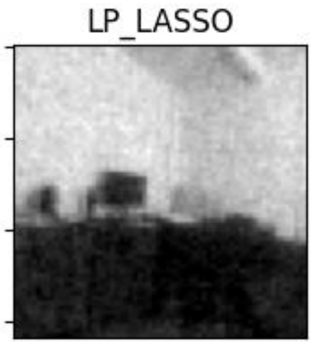
Time: 7.16s
MSE: 0.470
PSNR: 3.281



Time: 4m,23s
MSE: 0.475
PSNR: 3.229

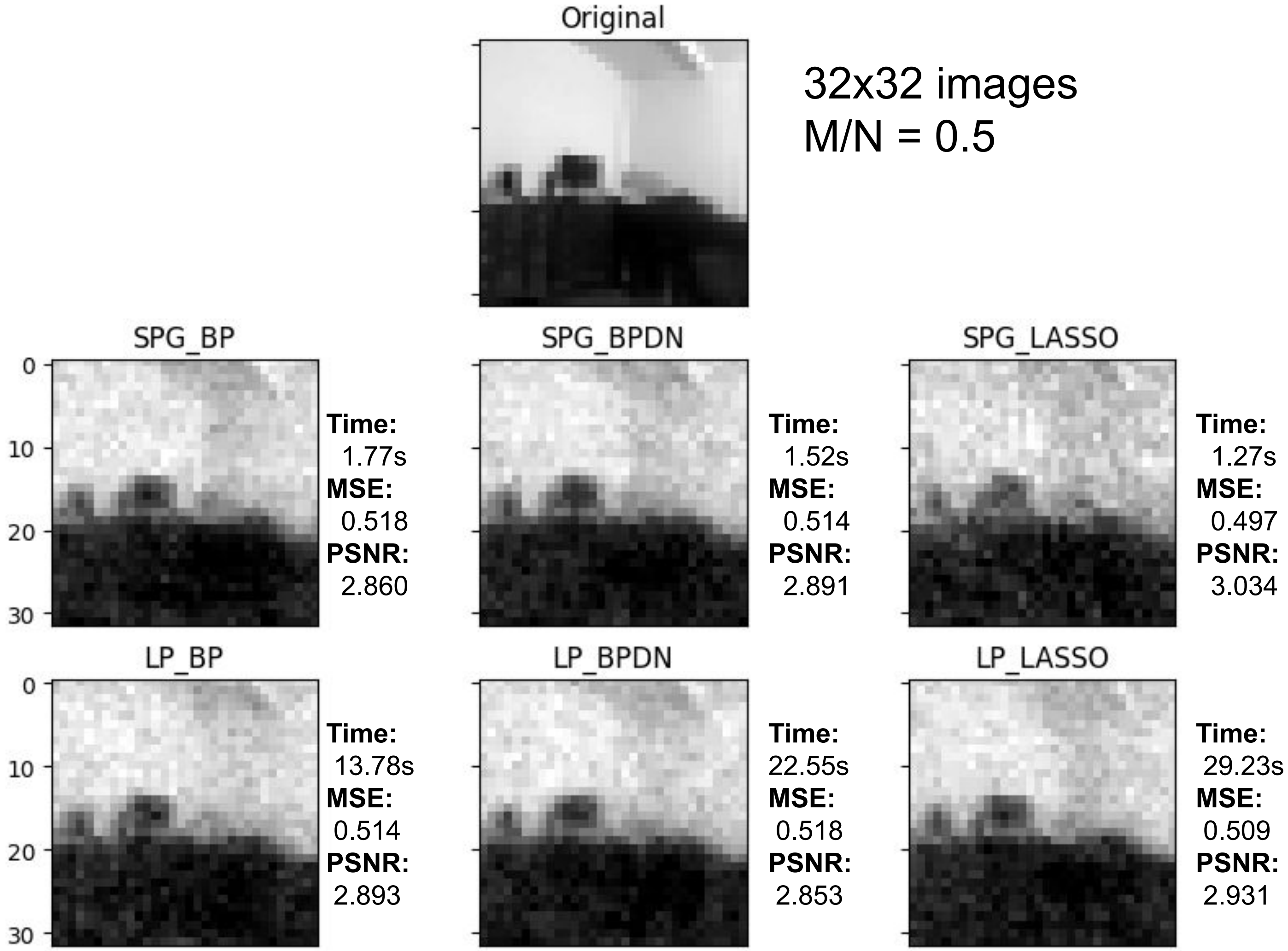


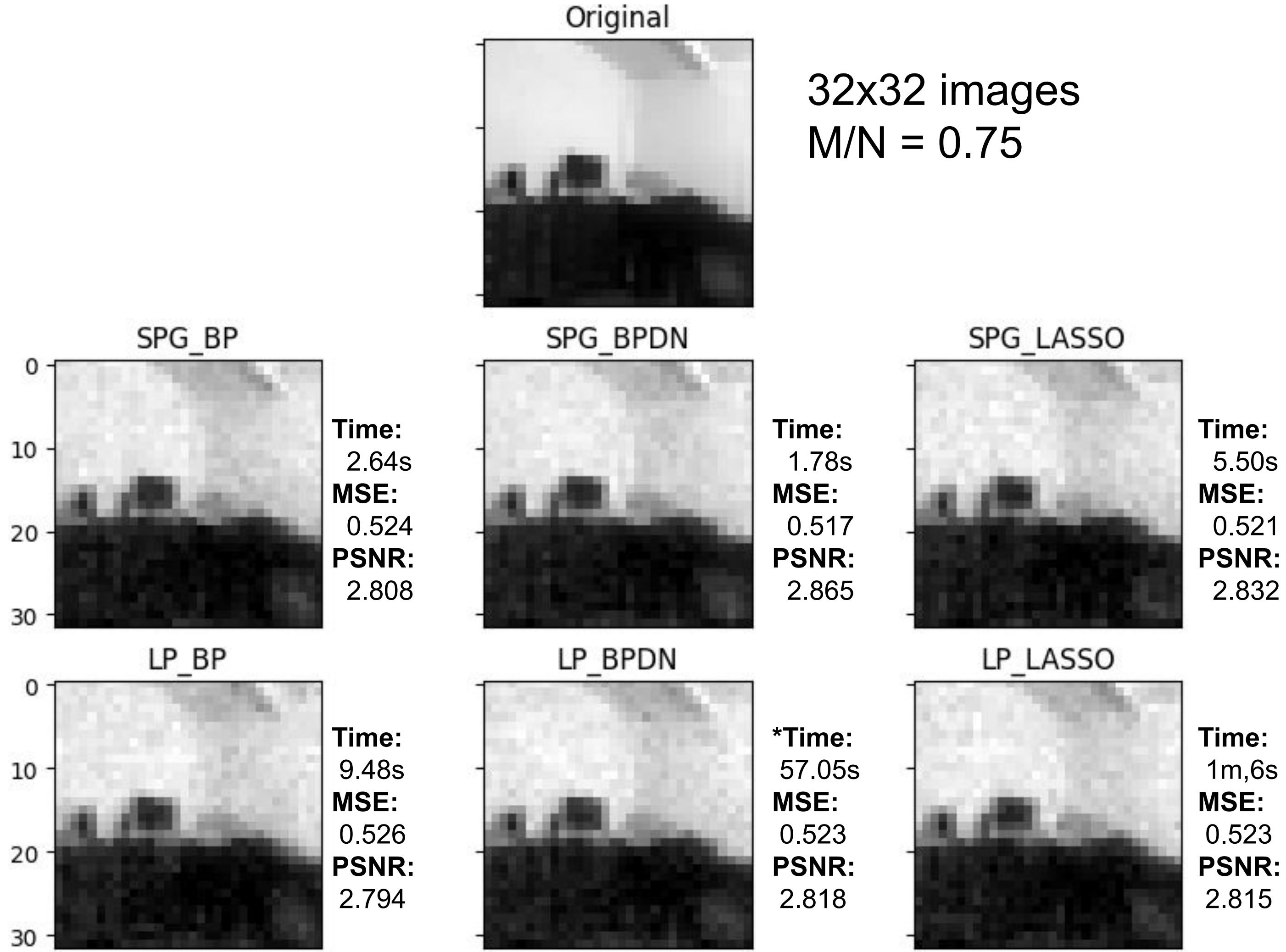
***Time:** 17m,30s
MSE: 0.469
PSNR: 3.285



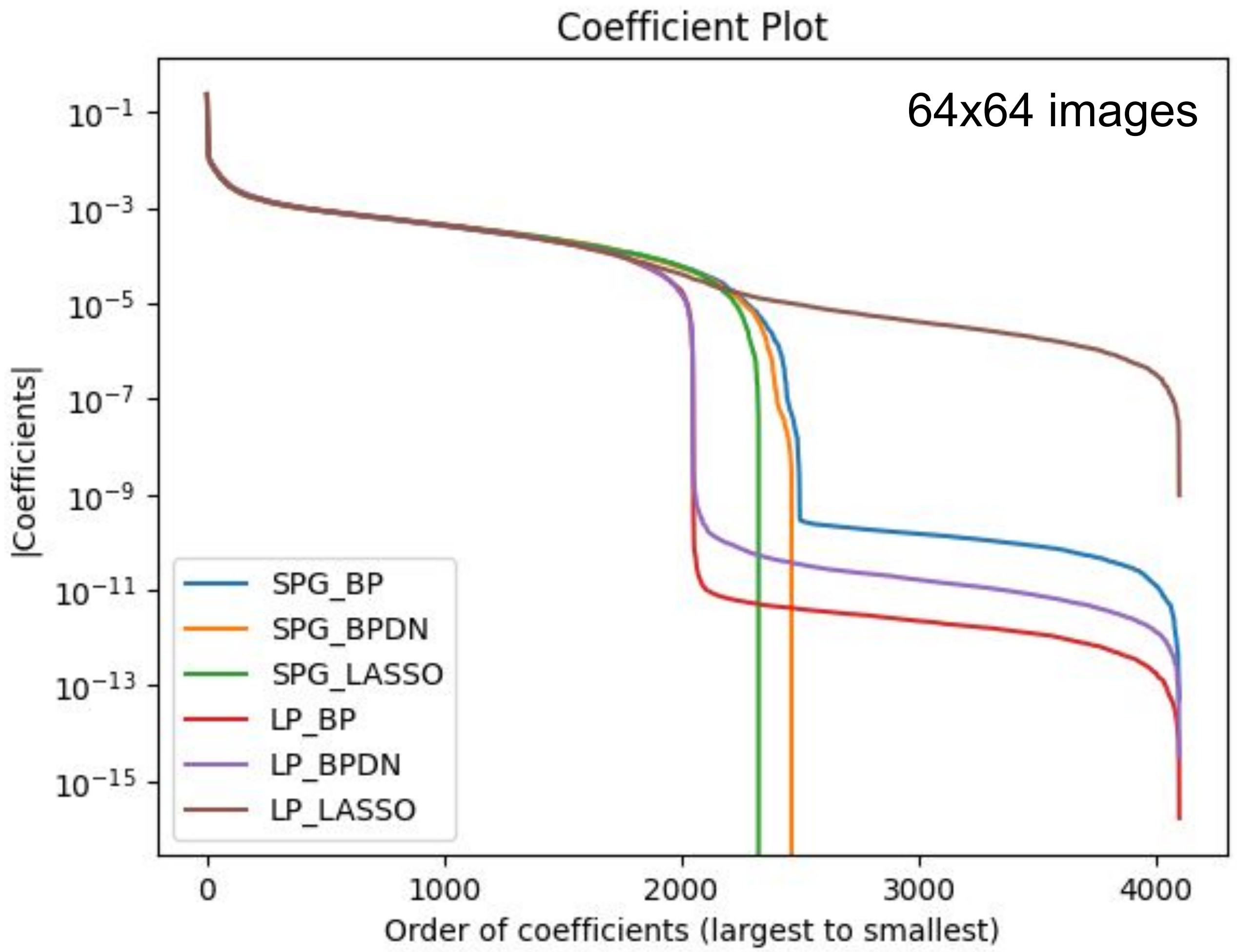
Time: 11m,36s
MSE: 0.467
PSNR: 3.303

32x32 images
M/N = 0.5



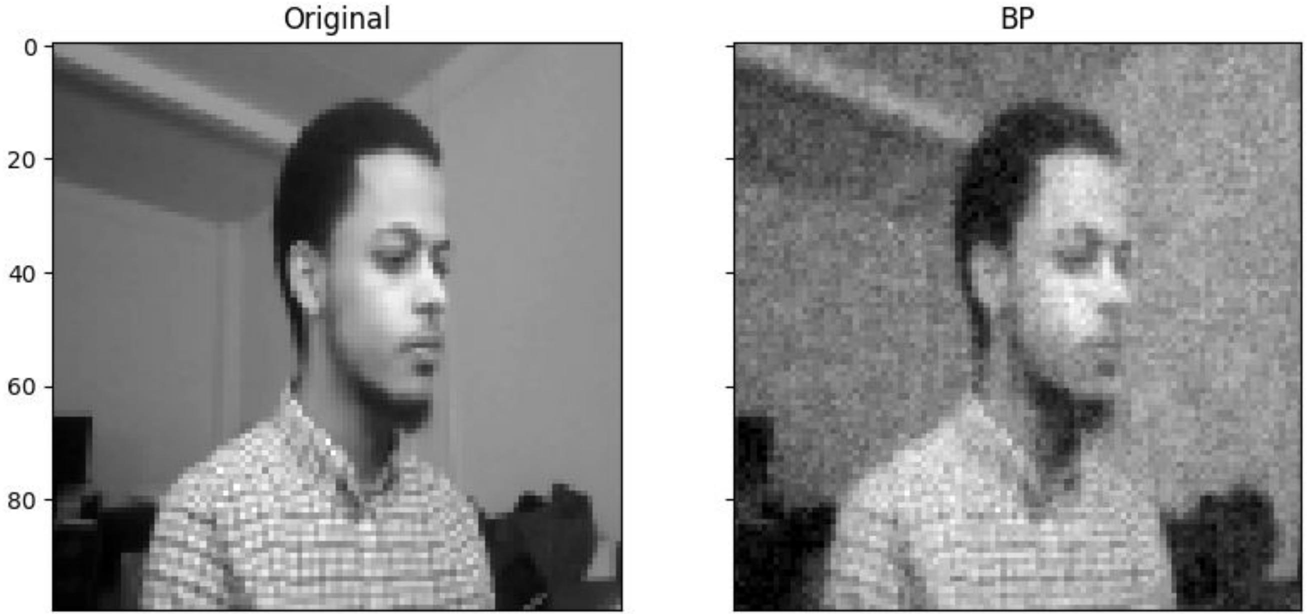


Original
32x32 images
M/N = 0.75
SPG_BP
Time: 2.64s
MSE: 0.524
PSNR: 2.808
SPG_BPDN
Time: 1.78s
MSE: 0.517
PSNR: 2.865
SPG_LASSO
Time: 5.50s
MSE: 0.521
PSNR: 2.832
LP_BP
Time: 9.48s
MSE: 0.526
PSNR: 2.794
LP_BPDN
*Time: 57.05s
MSE: 0.523
PSNR: 2.818
LP_LASSO
Time: 1m,6s
MSE: 0.523
PSNR: 2.815
0
10
20
30



Coefficient Plot
64x64 images
|Coefficients|
Order of coefficients (largest to smallest)
SPG_BP
SPG_BPDN
SPG_LASSO
LP_BP
LP_BPDN
LP_LASSO

100x100, M/N = 0.5

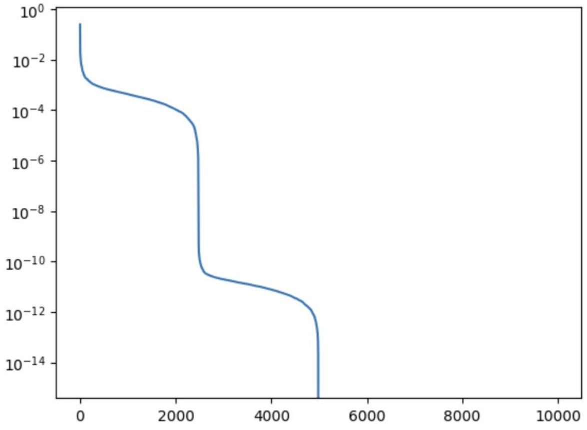


Time = 1hr 20m

MSE = 0.2621554

PSNR = 5.8144114

K = 2475



Challenges of producing real-time images

- Reconstruction is computationally intensive
  - Number of samples M affects computation time

Disadvantages of CS

- $\ell 1$-norm, $\ell 2$-norm minimization are extremely slow and return a not-so-perfect reconstruction of the signal
  - Perhaps good for storage of images (space needed only for coefficients and identifying the orthonormal basis kind)

References

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Thank you!!!