Convergence rates

Sunday, March 21, 2021   8:50 AM

Error	#iter to ε-sol'n	#iter to $\varepsilon=10^{-2}$	#iter to $\varepsilon=10^{-4}$		#iter to 10^{-6}
(Sublinear) $\sqrt{\frac{10^{-2}}{K}}$	$10^{-2}/\varepsilon^2$	100	10^6	$10^4\times$ more	10^{10}
$\frac{1}{K}$	$\frac{1}{\varepsilon}$	100	10^4	$10^2\times$ more	10^6
$\left(\frac{10}{K}\right)^2$	$\frac{10}{\sqrt{\varepsilon}}$	100	10^3	$10\times$ more	10^4
(linear) $\left(10^{-1/50}\right)^K$ ≈.95	$50\cdot\log_{10}(1/\varepsilon)$	100	200	$2\times$ more	300
(quadratic) $\left(\frac{1}{10}\right)^{2^{K-99}}$	$\log_2\left(2^{99}\cdot\log_{10}(1/\varepsilon)\right)$	100	101	$1+$ more	101.6

Gradient descent / proximal gradient descent, ∇f is $L-$Lipschitz

	Not Strongly cvx	Strongly cvx, $\kappa = L/\mu < \infty$
Regular	$O(1/K)$	$O\left((1-1/\kappa)^K\right)$
Nesterov Accelerated	$O(1/K^2)$	$O\left((1-1/\sqrt{\kappa})^K\right)$

Newton's method is (locally) quadratic