Homework 1 and 2
APPM 5630 Spring 2023
Advanced Convex Optimization

Due date: Friday, Feb 3 2023 at midnight (via Gradescope) **Instructor**: Prof. Becker
Theme: Convex sets, convex functions

Instructions Collaboration with your fellow students is allowed and in fact recommended, although direct copying is not allowed. The internet is allowed for basic tasks. Please write down the names of the students that you worked with. An arbitrary subset of these questions will be graded.

Homework submission instructions at github.com/stephenbeckr/convex-optimization-class/tree/master/Homeworks. You'll turn in a PDF (either scanned handwritten work, or typed, or a combination of both) to **gradescope**, using the link from the Canvas assignment.

Note This homework is a bit heavier on theory (HW1) than computation (HW2). As we progress through the semester, the computational part will increase slightly and the theory part will decrease slightly.

Reading Read chapters 1–2 in [BV2004] (and chapter 3 if you have time). The book [BV2004] ("Convex Optimization" by Boyd and Vandenberghe) is freely available at Stephen Boyd's website

Homework 1

Do problems 1–3, and then do problem 4 *or* problem 5.

Problem 1: [BV2004] Problem 2.2: Show that a set is convex if and only if its intersection with any line is convex. Show that a set is affine if and only if its intersection with any line is affine.

Problem 2: [BV2004] Problem 2.8: which of the following sets S (a–d, see book) are polyhedra? If possible, express S in the form $S = \{x | Ax \preceq b, Fx = g\}$.)

Problem 3: [BV2004] Problem 2.12: which of the following sets (a–g, see book) are convex?

Problem 4: [BV2004] Problem 2.13: consider the set of rank-k *outer products*, defined as $\{XX^T \mid X \in \mathbb{R}^{n\times k},\ \operatorname{rank} X = k\}$. Describe its conic hull in simple terms.

Note: the *conic hull* of a set C is the set of all *conic combinations* of points in C, and is also the smallest *convex cone* containing C. It is *not* just the smallest (possibly non-convex) *cone* containing C.

Problem 5: [BV2004] Problem 2.35: a matrix $X \in \mathcal{S}^n$ is called *copositive* if $z^T X z \geq 0$ for all $z \geq 0$. Verify that the set of copositive matrices is a proper cone, and then find its dual cone.

Homework 2

Problem 1: Using the white wine data from the Spanish wine quality database `https://archive.ics.uci.edu/ml/datasets/Wine+Quality` hosted at the UCI machine learning repository, compare ordinary least-squares regression

$$\beta_2 = \operatorname{argmin} \|y - X\beta\|_2$$

with more robust ℓ_1 regression (cf. p. 294 §6.1.1 in [BV2004])

$$\beta_1 = \text{argmin}\, \|y - X\beta\|_1$$

Report on the differences and make observations (e.g., are the estimators significantly different? are there outliers, and if so, does removing the outliers change the estimators?), and probably include a brief plot. You don't need to make a lengthy report, but give at least a few sentences of discussion. You may use any software you wish to solve the regression problems, e.g., `cvx` in Matlab or `cvxpy` in Python. [We'll do a demonstration of these software packages on Friday Jan 27 2023]

What to turn in? Please turn in a short printed document, with plots and text answering the questions above, and print relevant snippets of code (e.g., the part of the code that solves the problems — I don't need to see all of the plotting code, for example). If you have bugs in your code, I won't be able to help you when I grade, but you can always ask at office hours.