University of Colorado
Boulder

ECEN 5458 Sampled-Data and Digital Control Systems

# Digital Control Using State Variables

Simulation Lab 2 Report

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CONTENTS

## Abstract

**In this lab, we design discrete compensatorss in state-space for a mass-spring-damper-mass system model. First part involves designing a simple compensator by assuming perfect coupling to simplify the plant model. In the second part, the designed simple compensator is applied to the not simplified full plant. The final part will involve designing a compensator for the full plant directly. Their performances are compared in a simulation while also accounting for the non-linear characteristics of real systems with a saturation block.**


## I. Introduction

The system of interest is a mass $M$ that can move only a horizontal direction. A force $u$ is applied to $M$ to control its displacement $y$. $y$ and its time derivatives cannot be measured directly so we use a sensor $m$ coupled to $M$ by spring coefficient $k$ and damper $b$. The displacement $d$ of the sensor $m$ is used as input to the compensator which computes $u$ to drive $M$ to zero displacement given an initial displacement. See Figure 1 for the system setup. Equation 1 shows the dynamic equations of this system. Equation 2 shows the transfer function of the full fourth order plant and its state-space equivalent in continuous domain. Equation 3 show the transfer function of the simplified second order plant assuming perfect coupling ($k$ is $\infty$). The state-space formulation is given as: $\dot{\mathbf{x}} = \mathbf{F}\mathbf{x}+\mathbf{G}u$; $d = \mathbf{H}\mathbf{x}+Ju$ (or simply $\{\mathbf{F}, \mathbf{G}, \mathbf{H}, J\}$) in continuous time-domain where $\mathbf{x} = [\dot{y}\, y\, \dot{d}\, d]^T$ and $\mathbf{x} = [\dot{y}\ y]^T$ for the full $4^{th}$ order and simple $2^{nd}$ order plant respectively.

$$\begin{cases} M\ddot{y} + b(\dot{y} - \dot{d}) + k(y - d) = u \\ m\ddot{d} + b(\dot{d} - \dot{y}) + k(d - y) = 0 \end{cases} \tag{1}$$

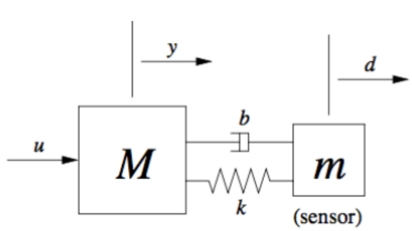



Figure 1: Mass-Spring-Damper-Mass System

$$\begin{cases} G_4(s) = \dfrac{d(s)}{u(s)} = \dfrac{b}{Mm}\dfrac{s + k/b}{s^2(s^2 + (1/m + 1/M)(bs + k))} \\ \mathbf{F} = \begin{bmatrix} -b/M & -k/M & b/M & k/M \\ 1 & 0 & 0 & 0 \\ b/m & k/m & -b/m & -k/m \\ 0 & 0 & 1 & 0 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} \frac{1}{M} \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ \mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}, J = 0 \end{cases} \tag{2}$$

$$\begin{cases} G_2(s) = \dfrac{d(s)}{u(s)} = \dfrac{y(s)}{u(s)} = \dfrac{1}{(M+m)s^2} \\ \mathbf{F} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \mathbf{G} = \begin{bmatrix} \frac{1}{M+m} \\ 0 \end{bmatrix}, \mathbf{H} = \begin{bmatrix} 0 & 1 \end{bmatrix}, J = 0 \end{cases} \tag{3}$$

The discrete equivalent of the above state-space equations with ZOH (zero-order hold) is given by:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{\Phi}\mathbf{x}(k) + \mathbf{\Gamma}u(k) \\ d(k) &= \mathbf{H}\mathbf{x}(k) + Ju(k) \end{aligned} \tag{4}$$

(or simply $\{\mathbf{\Phi}, \mathbf{\Gamma}, \mathbf{H}, J\}$). $\mathbf{\Phi}$ and $\mathbf{\Gamma}$ are calculated using:

$$\Phi = e^{\mathbf{F}T}, \ \ \Gamma = \int_0^T e^{\mathbf{F}\eta}\mathbf{G}\, d\eta$$

where $T$ is the sample period. The transfer function for the state-space system $\{\mathbf{\Phi}, \mathbf{\Gamma}, \mathbf{H}, J\}$ is given in Equation 5.

$$G = \mathbf{H}(z\mathbf{I} - \mathbf{\Phi})^{-1}\mathbf{\Gamma} + J \tag{5}$$

For this lab, $T$ = 0.2s, $M$=20kg, $m$ = 1kg, $k$ = 32N/m, $b$ = 0.3N-sec/m. Dimension-wise, $\mathbf{\Phi}$ is $n \times n$, $\mathbf{\Gamma}$ is $n \times 1$, $\mathbf{H}$ is $1 \times n$ and $J$ is $1 \times 1$ (where in our case, $n = 2$ or 4).

In the first part, we design full-state feedback controller and second order estimator for the second order plant. We use the combined compensator (i.e. controller + estimator) on the second order plant and record the performance. In the second part, we apply this compensator on the full fourth order plant and record the performance. In the third and final part, we design and apply a full-state feedback controller and a fourth order estimator for the full fourth order plant.

## II. Supporting Analysis

As we tried to indicate earlier, this 3-part lab focuses on designing state-space controller and estimator for a given plant and finding how the reference input is introduced into the system. The analysis will focus on how to design each and the other sections will deal with each section. (We will take $J = 0$ in all upcoming discussions).

The denominator (characteristic polynomial) of the transfer function $G$ from its state-space representation

$\{\mathbf{\Phi}, \mathbf{\Gamma}, \mathbf{H}, J\}$ is given by $\det(z\mathbf{I} - \mathbf{\Phi})$ and equating this to zero gives us the poles (i.e. eigenvalues of $\mathbf{\Phi}$ = poles). In a second order continuous-time transfer functions, the characteristic polynomial is given by $s^2 + 2\zeta\omega_n s + \omega_n^2$ (with roots $s_i = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$ when equated to 0, and with discrete equivalents given by the mapping $z_i = e^{s_i T}$). Therefore, by specifying the desired transient characteristics in terms of damping ratio $\zeta$ and natural frequency $\omega_n$, we define our desired characteristic polynomial. We will use the approximations $t_r = \frac{1.8}{\omega_n}$, $t_s = \frac{4.6}{\zeta\omega_n}$ and $M_p = \exp\left(\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\right)$ for the rise time, settling time and overshoot respectively to study the step response later on.

### A. Controller Design

We use control law via state feedback to design the controller and assume the reference input $r = 0$. Control law takes $u(k) = -\mathbf{K}\mathbf{x}(k)$ in Equation 4. This gives us the system $\{\mathbf{\Phi} - \mathbf{\Gamma}\mathbf{K}, \mathbf{0}, \mathbf{H}, 0\}$ where $\mathbf{K} = [k_1 \ \cdots \ k_n]$ is a row vector. The characteristic polynomial is thus given by $\det(z\mathbf{I} - (\mathbf{\Phi} - \mathbf{\Gamma}\mathbf{K})) = \det(z\mathbf{I} - \mathbf{\Phi} + \mathbf{\Gamma}\mathbf{K})$. Thus given desired $\zeta$ and $\omega_n$, we have a desired characteristic polynomial given by $\alpha_c(z)$. By equating the characteristic polynomial from the controller to the desired characteristic polynomial and matching coefficients, we can get the values in $\mathbf{K}$ (the only unknown variable). However, to guarantee that such a $\mathbf{K}$ is possible to calculate such that the poles can be moved to any desired location, we check for controllability of the original system $\{\mathbf{\Phi}, \mathbf{\Gamma}, \mathbf{H}, J\}$ via the controllability matrix $\mathcal{C}$. The system is controllable if $\mathcal{C} = [\mathbf{\Gamma} \ \mathbf{\Phi}\mathbf{\Gamma} \ \cdots \ \mathbf{\Phi^{n-1}\Gamma}]$ is nonsingular (i.e. $\det \mathcal{C} \neq 0$). This turns out to be the case in both plants for this lab.

See Figure 2 for the diagram representation of the controller design (in red).

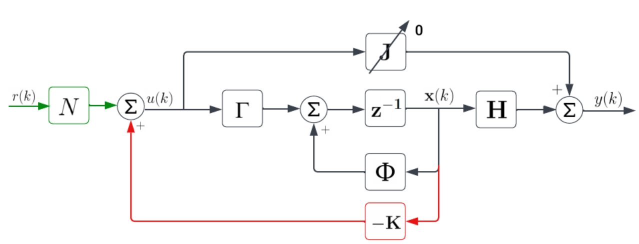



Figure 2: Block Diagram of Control Law Controller system (zooming might help for details)

### B. Estimator Design

In most systems, not all states are measured and thus, some states have to be estimated for state-feedback control. We assume reference input $r = 0$ and use what is called the prediction estimator for all the states. The estimated states for $y(k) = \mathbf{H}\mathbf{x}(k)$ are given by the equation:

$$\hat{\mathbf{x}}(k+1) = \mathbf{\Phi}\hat{\mathbf{x}}(k) + \mathbf{\Gamma}u(k) + \mathbf{L}(y(k) - \mathbf{H}\hat{\mathbf{x}}(k))$$

where $\mathbf{L} = [\ell_1 \ \cdots \ \ell_n]^T$ is a column vector. The error dynamics in estimation is given by

$$\begin{aligned}\tilde{\mathbf{x}}(k+1) &= \mathbf{x}(k+1) - \hat{\mathbf{x}}(k+1) \\ &= \mathbf{\Phi}\mathbf{x}(k) + \mathbf{\Gamma}u(k) \\ &\quad - \mathbf{\Phi}\hat{\mathbf{x}}(k) - \mathbf{\Gamma}u(k) - \mathbf{L}(y(k) - \mathbf{H}\hat{\mathbf{x}}(k)) \\ &= (\mathbf{\Phi} - \mathbf{L}\mathbf{H})\tilde{\mathbf{x}}(k)\end{aligned}$$

The error dynamics system is given by $\{\mathbf{\Phi} - \mathbf{L}\mathbf{H}, \mathbf{0}, \mathbf{H}, 0\}$ and the characteristic polynomial is given by $\det(z\mathbf{I} - (\mathbf{\Phi} - \mathbf{L}\mathbf{H})) = \det(z\mathbf{I} - \mathbf{\Phi} + \mathbf{L}\mathbf{H})$. We want this error to go to zero. So we choose fast stable eigenvalues for $\mathbf{\Phi} - \mathbf{L}\mathbf{H}$, which gives us the desired characteristic polynomial $\alpha_e(z)$. This will lead to $\tilde{\mathbf{x}}(k) \to 0$ for all $\tilde{\mathbf{x}}(0)$ and $u(k)$. Similar to the controller design, we can then equate the two characteristic polynomials and apply matching coefficients method to find the values in $\mathbf{L}$. To guarantee that such an $\mathbf{L}$ is possible to calculate such that the poles can be moved to the desired location, we check for observability of the original system $\{\mathbf{\Phi}, \mathbf{\Gamma}, \mathbf{H}, J\}$ via the observability matrix $\mathcal{O}$. The system is observable if $\mathcal{O} = \begin{bmatrix} \mathbf{H} \\ \mathbf{H}\mathbf{\Phi} \\ \vdots \\ \mathbf{H}\mathbf{\Phi}^{n-1} \end{bmatrix}$ is nonsingular (i.e. $\det \mathcal{O} \neq 0$). This turns out to be the case in both plants for this lab.

See Figure 3 for the diagram representation of the estimator design (in red).

### C. Combined Controller and Estimator

We now combine the aforementioned two designs. The major design decision here is that we now take $u(k) = -\mathbf{K}\hat{\mathbf{x}}(k)$ ($\hat{\mathbf{x}}(k)$ instead of $\mathbf{x}(k)$ as in the control law case). We have designed $\mathbf{K}$ with the assumption of having access to $\mathbf{x}(k)$ but it turns out we can use the same $\mathbf{K}$ on $\hat{\mathbf{x}}(k)$ by the separation principle. The separation principle basically states that the controller and estimator designs can be carried out independently. That is shown as follows:

Control Law with estimated states:

$$\mathbf{x}(k+1) = \mathbf{\Phi}\mathbf{x}(k) - \mathbf{\Gamma}\mathbf{K}(\mathbf{x}(k) - \tilde{\mathbf{x}}(k))$$

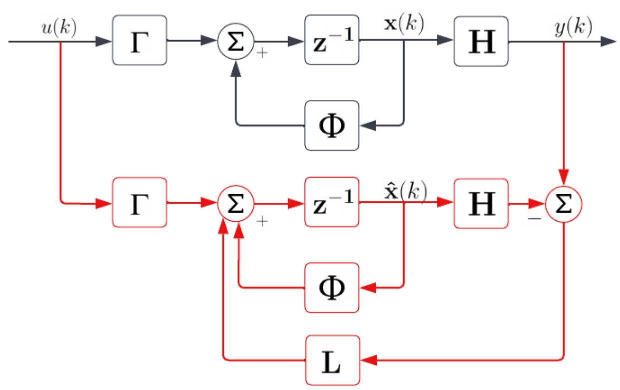



Figure 3: Block Diagram of Prediction Estimator System

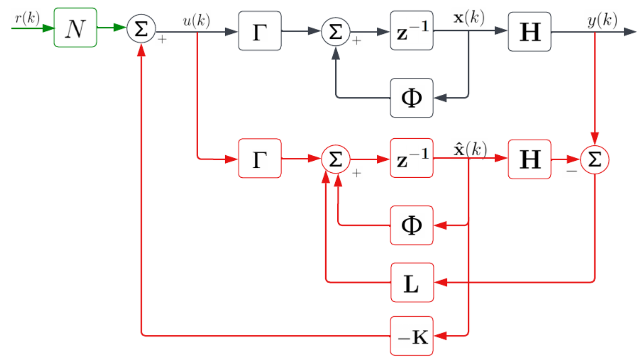



Figure 4: Block Diagram of Combined Controller and Estimator System

Estimator Error:

$$\tilde{\mathbf{x}}(k+1) = (\mathbf{\Phi} - \mathbf{LH})\tilde{\mathbf{x}}(k)$$

Combined:

$$\begin{bmatrix} \mathbf{x}(k+1) \\ \tilde{\mathbf{x}}(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi} - \mathbf{\Gamma K} & \mathbf{\Gamma K} \\ \mathbf{0} & \mathbf{\Phi} - \mathbf{LH} \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \tilde{\mathbf{x}}(k) \end{bmatrix} = \mathcal{F} \begin{bmatrix} \mathbf{x}(k) \\ \tilde{\mathbf{x}}(k) \end{bmatrix}$$

The closed-loop characteristic equation of this combined system is given by:

$$\begin{aligned} \det(z\mathbf{I} - \mathcal{F}) &= \det \begin{bmatrix} z\mathbf{I} - \mathbf{\Phi} + \mathbf{\Gamma K} & \mathbf{\Gamma K} \\ \mathbf{0} & z\mathbf{I} - \mathbf{\Phi} + \mathbf{LH} \end{bmatrix} \\ &= \det(z\mathbf{I} - \mathbf{\Phi} + \mathbf{\Gamma K}) \det(z\mathbf{I} - \mathbf{\Phi} + \mathbf{LH}) \end{aligned}$$

Therefore, the poles of the overall system are combination of the control poles and estimator poles. They can be separately designed.

After forming the combined system, we can look at the transfer function of the compensator $D(z) = \frac{U(z)}{Y(z)}$.

$$\begin{aligned} \hat{\mathbf{x}}(k+1) &= \mathbf{\Phi}\hat{\mathbf{x}}(k) + \mathbf{\Gamma}u(k) + \mathbf{L}(y(k) - \mathbf{H}\hat{\mathbf{x}}(k)) \\ &= (\mathbf{\Phi} - \mathbf{\Gamma K} - \mathbf{LH})\hat{\mathbf{x}}(k) + \mathbf{L}y(k) \\ u(k) &= -\mathbf{K}\hat{\mathbf{x}}(k) \end{aligned}$$

This is essentially $\{\mathbf{\Phi} - \mathbf{\Gamma K} - \mathbf{LH}, \mathbf{L}, -\mathbf{K}, 0\}$. We can then use Equation 5 to find $D(z)$ as a transfer function. $D(z)$ designed for an $n^{th}$ order plant $G(z)$ will be $n^{th}$ order. $K$ is simply applied as feedback so we don't consider it to have any order. However, the estimator block is a system by itself that resides in the computer and is considered to be $n^{th}$ order. Therefore, we refer to the estimator and combined compensator as $n^{th}$ order.

See Figure 4 for the diagram representation of the combined compensator system (in red).

### D. *Nonzero Reference Input*

The general way of introducing the external reference input is shown in Equation 6. Since $r$ is external, the poles are not affected by the introduction of $\mathbf{M}$ and $N$. However, the zeros (and therefore, the transient response) and steady state error are affected.

$$\begin{aligned} \hat{\mathbf{x}}(k+1) &= (\mathbf{\Phi} - \mathbf{\Gamma K} - \mathbf{LH})\hat{\mathbf{x}}(k) + \mathbf{L}y(k) + \mathbf{M}r(k) \\ u(k) &= -\mathbf{K}\hat{\mathbf{x}}(k) + Nr(k) \end{aligned} \tag{6}$$

For this lab, $\mathbf{M}$ and $N$ are designed such that $\tilde{\mathbf{x}}$ is independent of $r$ ($\tilde{\mathbf{x}} \perp r$). The analysis goes as follows:

$$\begin{aligned} u(k) &= -\mathbf{K}\hat{\mathbf{x}}(k) + Nr(k) \\ \mathbf{x}(k+1) &= \mathbf{\Phi}\mathbf{x}(k) + \mathbf{\Gamma}u(k) = \mathbf{\Phi}\mathbf{x}(k) - \mathbf{\Gamma K}\hat{\mathbf{x}}(k) + \mathbf{\Gamma}Nr(k) \\ \hat{\mathbf{x}}(k+1) &= (\mathbf{\Phi} - \mathbf{\Gamma K} - \mathbf{LH})\hat{\mathbf{x}}(k) + \mathbf{LH}\mathbf{x}(k) + \mathbf{M}r(k) \\ \tilde{\mathbf{x}}(k+1) &= \mathbf{x}(k+1) - \hat{\mathbf{x}}(k+1) \\ &= (\mathbf{\Phi} - \mathbf{LH})\tilde{\mathbf{x}}(k) + (\mathbf{\Gamma}N - \mathbf{M})r(k) \\ \Rightarrow \mathbf{M} &= \mathbf{\Gamma}N \text{ (since } \tilde{\mathbf{x}} \perp r) \end{aligned}$$

Therefore, we only need to design $N$. We want zero steady state error (set $e_{ss} = 0$, $r = r_{ss} = y_{ss}$, $u_{ss} = N_u r$, $\mathbf{x}_{ss} = \mathbf{N}_x r$). The steady state equations can be written as follows (and analysis follows):

$$\begin{cases} \mathbf{x}_{ss} &= \mathbf{\Phi}\mathbf{x}_{ss} + \mathbf{\Gamma}u_{ss} \\ y_{ss} &= \mathbf{H}\mathbf{x}_{ss} + Ju_{ss} \end{cases} \Rightarrow \begin{cases} \mathbf{N}_x r &= \mathbf{\Phi}\mathbf{N}_x r + \mathbf{\Gamma}\mathbf{N}_u r \\ r &= \mathbf{H}\mathbf{N}_x r + JN_u r \end{cases}$$

$$\begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi} - \mathbf{I} & \mathbf{\Gamma} \\ \mathbf{H} & J \end{bmatrix} \begin{bmatrix} \mathbf{N}_x \\ N_u \end{bmatrix} \Rightarrow \begin{bmatrix} \mathbf{N}_x \\ N_u \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi} - \mathbf{I} & \mathbf{\Gamma} \\ \mathbf{H} & J \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix}$$

$$\begin{aligned} &\text{But, } \mathrm{u}_{ss} = -\mathbf{K}\mathbf{x}_{ss} + Nr_{ss} \Rightarrow N_u r = -\mathbf{K}\mathbf{N}_x r + Nr \\ &\Rightarrow N = N_u + \mathbf{K}\mathbf{N}_x \end{aligned}$$

Based on the above, we find $\mathbf{N}_x$ ($n$-length column vector) and $N_u$ (scalar) from the matrix equation and apply the last formula to find $N$. Note that it only depends on $\mathbf{K}$ and not $\mathbf{L}$. This completes all the variables to complete the block diagram shown in Figure 4. The setups for the several components in SIMULINK are shown in Figures 23-28 in the Appendix. The irrelevant components for each part are deactivated by code from the full setup. The simulation considers the non-linear characteristic of the D/A hardware via a saturation block at ±10V as well as another saturation block that resides in the computer to simulate saturation (so that the estimator performs well). Most of the plots will include cases of both with and without saturation block for comparison sake (otherwise, it is a system with the saturation block).

### E. Disturbance/Noise

In the previous design approaches, we assumed no disturbances. However, in this lab, we investigate the effect of disturbances. We considered two locations where disturbances might occur and two forms of disturbances, namely a step disturbance of amplitude 0.1 and a random Gaussian noise of amplitude 0.01 sampled from the standard normal distribution (amplitude values decided after some try and errors in simulation). The condensed model of the state-space compensator design accounting for disturbances is shown in Figure 5 (setting $w_1 = 0$ and $w_2 = 0$ gives us the model that assumes no disturbance). $w_1$ corresponds to disturbance attributed to the plant process while $w_2$ corresponds to disturbance related to the sensor measurement.

### F. Root Locus

Looking at Figure 4, we can consider the red part as a filter or transfer function $\bar{D}(z)$ in the feedback path, and the green part with $N$ as a feedforward filter. Therefore, the configuration reduces to a cascade of a feedforward filter on the input and a positive non-unity feedback

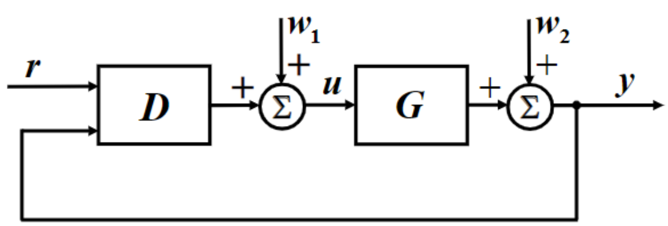



Figure 5: Block Diagram of Condensed State-Space Feedback System (with disturbance)

loop architecture. Therefore, the overall transfer function $H(z)$ can be described as follows:

$$H(z) = \frac{NG(z)}{1 - \bar{D}(z)G(z)} \tag{7}$$

For the root locus plot, we take the denominator. We will cascade a proportional control $\bar{K}$ to $\bar{D}(z)$ to give us the root locus equation $1 - \bar{K}\bar{D}(z)G(z) = 0$ in terms of $\bar{K}$. This looks like the equation of a 0° root locus, but we notice that $\bar{D}$ has a negative factor due to $-\mathbf{K}$. If we factor out the negative sign in the block diagram (or from $\bar{D}$), we will essentially have a negative feedback loop resulting in the root locus equation becoming a 180° root locus. For $\bar{K} = 1$, we get the same roots as we designed.

## III. Results

In this section, we highlight some of the important results for each part of the lab based on designs through the analyses provided in the previous section.

### A. Part 1 - $2^{nd}$ Order Plant, $2^{nd}$ Order Compensator

In this part, we designed the control law controller (and the gain $N$ for the reference input) and prediction estimator for the $2^{nd}$ order plant shown in Equation 3 using the state vector $\mathbf{x} = [\dot{y}\ y]^T = [\dot{d}\ d]^T$. We combined them. The desired location of poles for the closed-loop full state feedback system was set to one corresponding to $\zeta = 0.5$ and $\omega_n = 1.0$ rad/sec. The desired location of poles for the estimator/observer were set at $\zeta = 0.7$ and $\omega_n = 4.0$ rad/sec. The estimator poles are faster than the controller poles so the overall transient response is expected to be dominated by the controller poles when combined. The corresponding predicted/desired transient response characteristics and the measured transient response characteristics are shown in Table 1. The predicted values are based on the dominant poles we selected.

*1) Part 1(a) - Control Law Controller:* The performance of the controller was assessed through the step response of the feedback system shown in Figure 2 and by checking whether the output $y$ tracks the reference input $r$ perfectly. The step response plot is shown in Figure 6. $\mathbf{K}$ and $N$ are shown in Table 1.

*2) Part 1(b) - Prediction Estimator:* The input to the estimator was $y$ from the plant, which is the same as $d$ because we assumed perfect coupling. The performance of the estimator was assessed by noticing if $\tilde{\mathbf{x}}$, the difference between the actual states and estimated states of the open loop estimator system shown in Figure 3

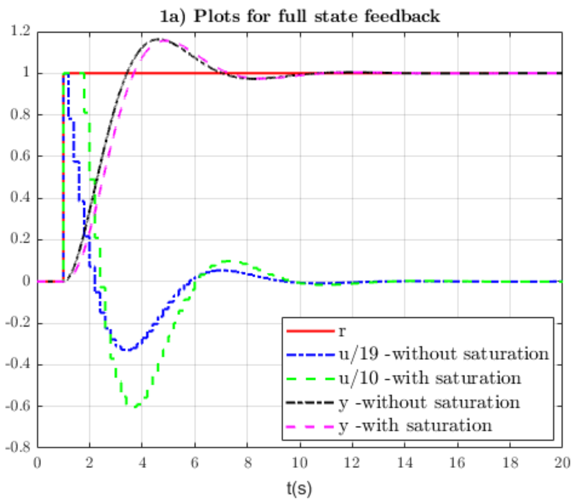



Figure 6: Step Response for Controller

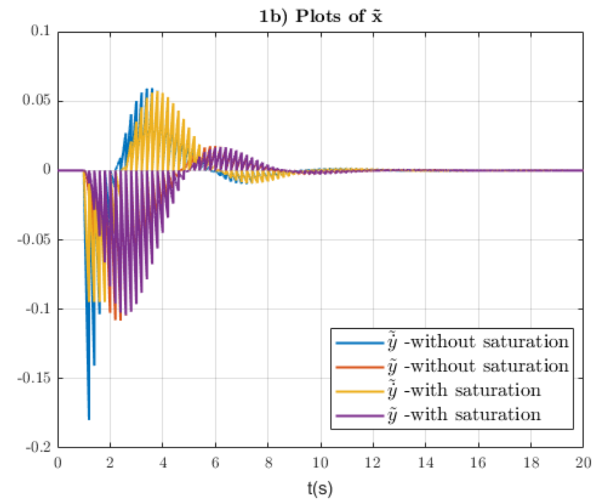



Figure 7: Plot of $\tilde{\mathbf{x}}(k)$ for Estimator

goes to 0. The $\tilde{\mathbf{x}}$ plot is shown in Figure 7. $\mathbf{L}$ is shown in Table 1.

*3) Part 1(c) - Combined Compensator:* The performance of the combined compensator was assessed through the step response of the feedback system shown in Figure 4 and by checking whether the output $y$ tracks the reference input $r$ perfectly. The step response plot is shown in Figure 8. $D(z)$ is shown in Table 1.

*4) Part 1(d) - Combined Compensator with Disturbance:* The performance of the combined compensator with added disturbances was assessed through the step response of the feedback system shown in Figure 4 (with noise blocks added according to Figure 5) and by checking how the output $y$ is affected compared to the reference input $r$ perfectly. The step response plot is shown in Figure 9.

### *B. Part 2 - $4^{th}$ Order Plant, $2^{nd}$ Order Compensator*

For this part, we applied the designed compensator $D(z)$ from the previous part onto the full $4^{th}$ order plant given by Equation 2. We used $d$ output (the sensor measurement) from the plant as input to the estimator used to control $y$. The step responses $y$ (the state we want to control) with and without saturation block are shown in Figure 10 without adding disturbances. We then accounted for disturbances and the resulting step responses are shown in Figure 11. Tabulated results are shown in Table 2.

### *C. Part 3 - $4^{th}$ Order Plant, $4^{th}$ Order Compensator*

This part is mostly similar to Part 1. We designed the control law controller (and the gain $N$ for the reference input) and prediction estimator for the $4^{th}$

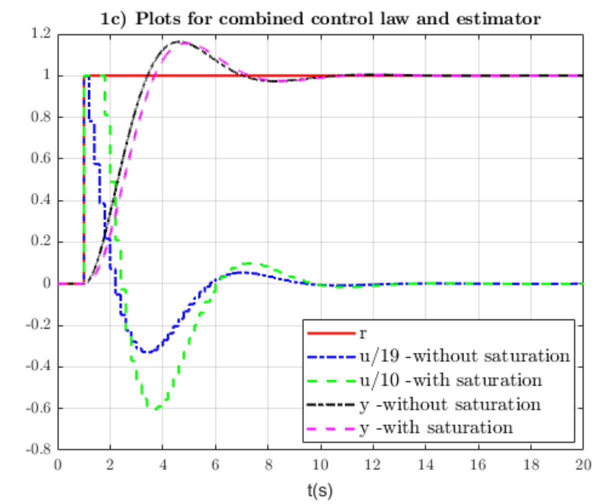



Figure 8: Step Response for Combined Compensator on $2^{nd}$ Order Plant

order plant shown in Equation 2 using the state vector $\mathbf{x} = [\dot{y}\ y\ \dot{d}\ d]^T$. We combined them. The desired location of poles for the closed-loop full state feedback system was set to a pair corresponding to $\zeta = 0.5$ and $\omega_n = 1.0$ rad/sec (same as Part 1) and another pair corresponding to $\zeta = 0.3$ and $\omega_n = 5.80$ rad/sec (the resonant frequency: looking at Equation 2, $\omega_n^2 = k(1/m + 1/M)$ from the canonical form of $2^{nd}$ order transfer function). The latter pair are faster so we expect the transient responses to be dominated by the former pair. The desired location of poles for the estimator/observer were set to be 2.5 times faster than the controller poles (i.e. smaller by a factor of 2.5). Since the estimator poles are faster than the controller poles, the overall transient

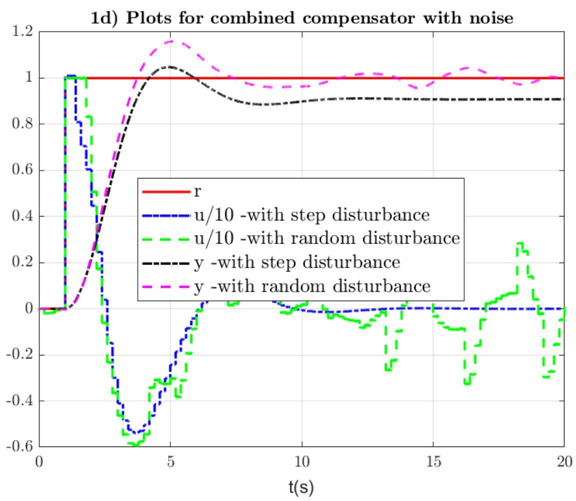



Figure 9: Step Response for Combined Compensator on $2^{nd}$ Order Plant with Disturbance

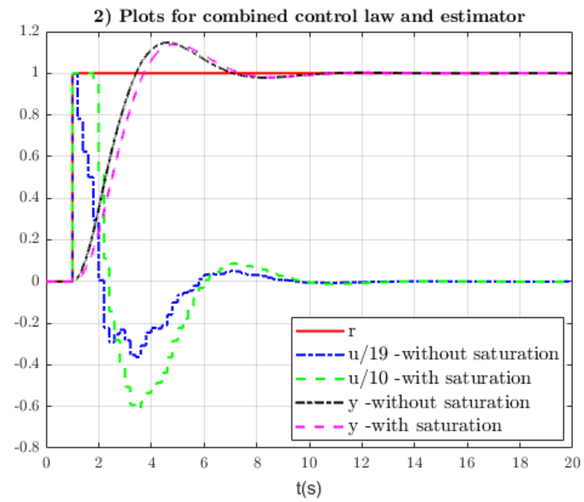



Figure 10: Step Response for Combined Compensator on $4^{th}$ Order Plant

TABLE 1: Key Results from $2^{nd}$ Order Plant, $2^{nd}$ Order Compensator

| | |
|---|---|
| ($\zeta$, $\omega_n$(rad/s)) | (0.5,1.0) |
| **K** | [20.93 18.97] |
| $N$ | 18.97 |
| **L** | $[1.83\ 1.04]^T$ |
| Predicted $M_p$(%) | 16.3 |
| Measured $M_p$(%) | 16.5(●), 16.3(⋆) |
| Predicted $t_r$(s) | 1.8 |
| Measured $t_r$(s) | 1.8(●), 1.9(⋆) |
| Predicted $t_s$(s) | 9.2 |
| Measured $t_s$(s) | 9.2(●), 9.2(⋆) |
| $\lvert e_{ss}\rvert$ | 0 (*), 0.1 (†) |
| $D(z)$ | $\frac{-57.94z+51.01}{z^2-0.74z+0.32}$ |

*: No disturbance, †: Step disturbance.
●: without saturation, ⋆: with saturation

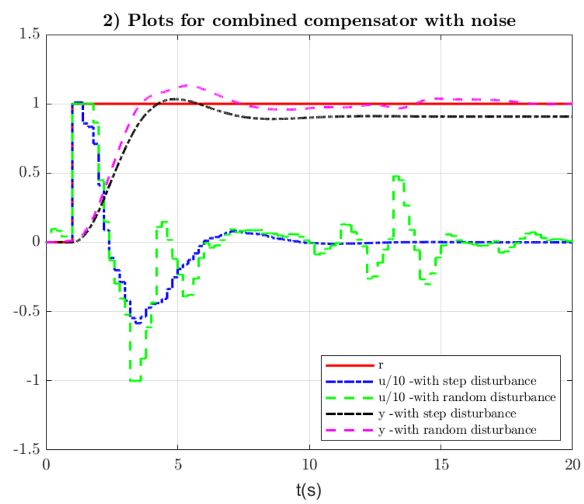



Figure 11: Step Response for Combined Compensator on $4^{th}$ Order Plant with Disturbance

response is expected to be dominated by the controller poles when combined. The corresponding predicted/desired transient response characteristics and the measured transient response characteristics are shown in Table 3. The predicted values are based on the dominant poles we selected.

*1) Part 3(a) - Control Law Controller:* The performance of the controller was assessed through the step response of the feedback system shown in Figure 2 and by checking whether the output $y$ tracks the reference input $r$ perfectly. The step response plot is shown in Figure 12. **K** and $N$ are shown in Table 3.

*2) Part 3(b) - Prediction Estimator:* The input to the estimator was $d$ from the plant. The performance of the estimator was assessed by noticing if $\tilde{\mathbf{x}}$, the difference between the actual states and estimated states of the open loop estimator system shown in Figure 3 goes to 0. The $\tilde{\mathbf{x}}$ plot is shown in Figure 13. **L** is shown in Table 3.

*3) Part 3(c) - Combined Compensator:* The performance of the combined $4^{th}$ order compensator was assessed through the step response of the feedback system shown in Figure 4 and by checking whether the output $y$ tracks the reference input $r$ perfectly. The step response plot is shown in Figure 14. $D(z)$ is shown in Table 3.

*4) Part 3(d) - Combined Compensator with Nonzero Initial Conditions:* So far, we had assumed zero initial conditions. The performance of the combined $4^{th}$ order compensator was evaluated at non-zero initial conditions through the step response of the feedback system shown in Figure 4 and by checking whether the output $y$ tracks the reference input $r$ perfectly and by checking whether the transient responses are affected. The nonzero initial condition situations were considered for the four actual states and their four corresponding estimated states (eight in total). We considered few situations such as initial offsets in all the eight states and initial conditions in each of the four actual states. The step response plots for the situations considered are shown in Figure 18 with the corresponding initial conditions on the title of each of the plots.

*5) Part 3(e) - Combined Compensator Root Loci and with Disturbance:* The performance of the $4^{th}$ order compensator in the presence of disturbance is shown in Figure 15 and compared to Part 2 (Figure 11). The differences between the combined compensators in Part 2 and Part 3 were assessed through their $z$-plane root loci as shown in Figure 16 (since the plant is the same).

*6) Part 3(f) - Combined Compensator Saturation:* As seen from all the step responses above, there is saturation. We tried to create an unsaturated response by reducing the sampling rate (which did not work out) and changing the pole locations. More on this in the next section. The unsaturated step response is shown in Figure 17. The new pole location information and resulting $\mathbf{K}$, $N$ and $\mathbf{L}$ are shown in Table 4.

TABLE 2: Key Results from $4^{th}$ Order Plant, $2^{nd}$ Order Compensator

| | |
|---|---|
| ($\zeta$, $\omega_n$(rad/s)) | (0.5,1.0) |
| $\mathbf{K}$ | [20.93 18.97] |
| $N$ | 18.97 |
| $\mathbf{L}$ | $[1.83\ 1.04]^T$ |
| Predicted $M_p(\%)$ | 16.3 |
| Measured $M_p(\%)$ | 16.5(●), 16.4(⋆) |
| Predicted $t_r$(s) | 1.8 |
| Measured $t_r$(s) | 1.8(●), 1.9(⋆) |
| Predicted $t_s$(s) | 9.2 |
| Measured $t_s$(s) | 9.2(●), 9.2(⋆) |
| $\lvert e_{ss}\rvert$ | 0 (*), 0.1 (†) |
| $D(z)$ | $\frac{-57.94z+51.01}{z^2-0.74z+0.32}$ |

*: No disturbance, †: Step disturbance.
●: without saturation, ⋆: with saturation

TABLE 3: Key Results from $4^{th}$ Order Plant, $4^{th}$ Order Compensator

| | |
|---|---|
| ($\zeta$, $\omega_n$(rad/s)) | (0.5,1.0), (0.3,5.8) |
| $\mathbf{K}$ | [53.90 -105.88 -36.84 119.99] |
| $N$ | 14.12 |
| $\mathbf{L}$ | $[1.43\ 1.20\ 2.52\ 1.81]^T$ |
| Predicted $M_p(\%)$ | 16.3 |
| Measured $M_p(\%)$ | 1.8(●), 1.78(⋆) |
| Predicted $t_r$(s) | 1.8 |
| Measured $t_r$(s) | 1.8(●), 1.9(⋆) |
| Predicted $t_s$(s) | 9.2 |
| Measured $t_s$(s) | 9.2(●), 9.2(⋆) |
| $\lvert e_{ss}\rvert$ | 0 (*), 0.1 (†) |
| $D(z)$ | $\frac{-74.54z^3+7.076z^2+154.7z-92.09}{z^4-0.61z^3-0.01z^2-0.14z-0.0032}$ |

*: No disturbance, †: Step disturbance.
●: without saturation, ⋆: with saturation

TABLE 4: Key Results from Unsaturated Example

| | |
|---|---|
| ($\zeta$, $\omega_n$(rad/s)) | (0.6,1.10), (0.8, 3.48) |
| $\mathbf{K}$ | [41.58 -407.61 -33.33 412.59] |
| $N$ | 4.98 |
| $\mathbf{L}$ | $[1.15\ 1.01\ 1.51\ 1.66]^T$ |
| Predicted $M_p(\%)$ | 9.5 |
| Measured $M_p(\%)$ | 10(●), 10(⋆) |
| Predicted $t_r$(s) | 1.63 |
| Measured $t_r$(s) | 1.65(●), 1.65(⋆) |
| Predicted $t_s$(s) | 6.96 |
| Measured $t_s$(s) | 7(●), 7(⋆) |
| $\lvert e_{ss}\rvert$ | 0 |

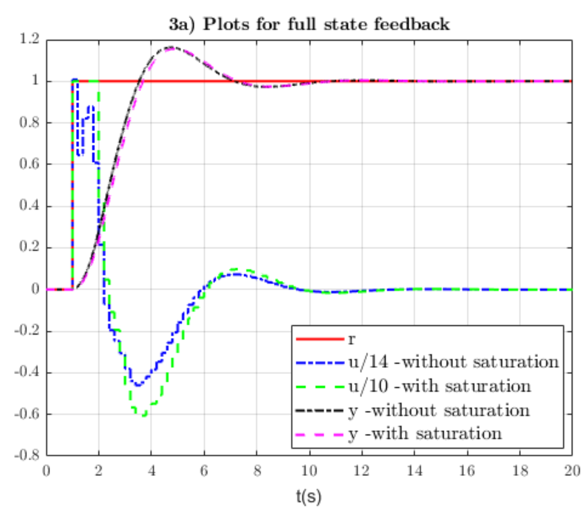



Figure 12: Step Response for Controller

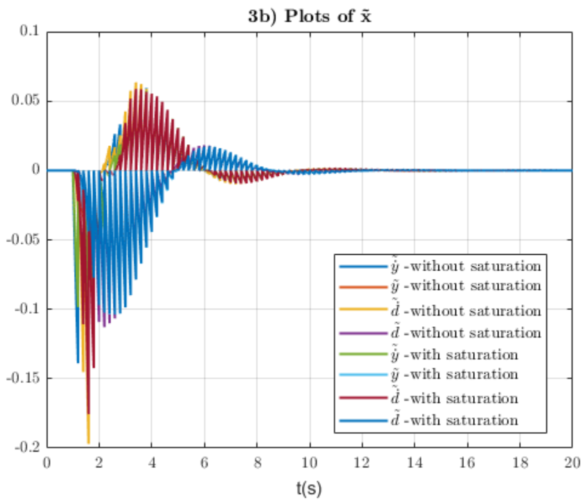



Figure 13: Plot of $\tilde{\mathbf{x}}(k)$ for Estimator

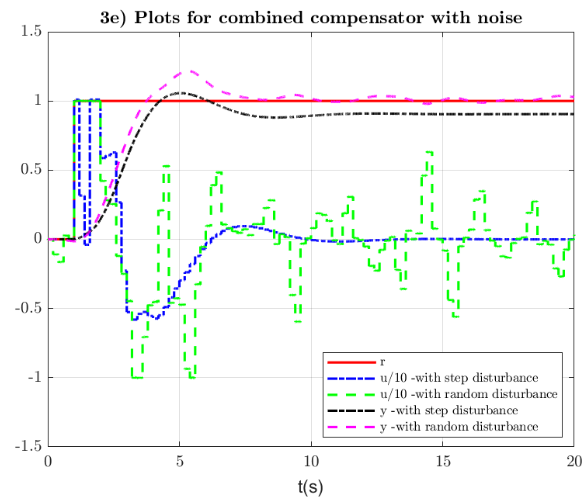



Figure 15: Step Response for Combined Compensator on $4^{th}$ Order Plant with Disturbance

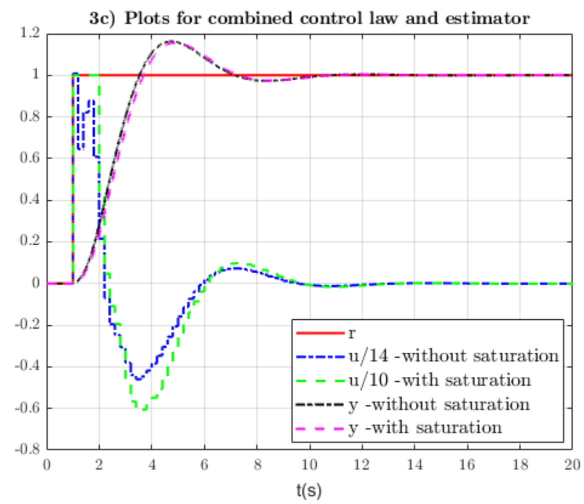
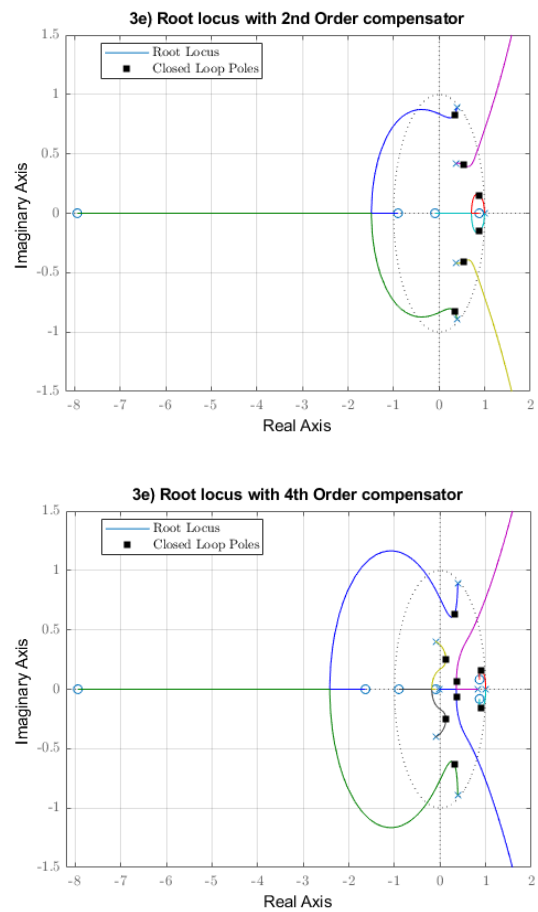



Figure 14: Step Response for Combined Compensator on $4^{th}$ Order Plant




(a)

(b)

Figure 16: Root Locus with (a) $2^{nd}$ Order Compensator (b) $4^{th}$ Order Compensator

## IV. Discussion

In this section, I will give comments related to each part of the Results section sequentially (referencing each other where necessary) and give a more general remark afterwards (including the effect of saturation).The results with and without saturation are very close based on the plots and tables in general so the following discussions hold for both cases.

### A. Part 1

*1) 1(a):* As shown in Figure 6 and Table 1, the controller tracks a step input perfectly and the transient responses are as predicted (with some discrepancy be-

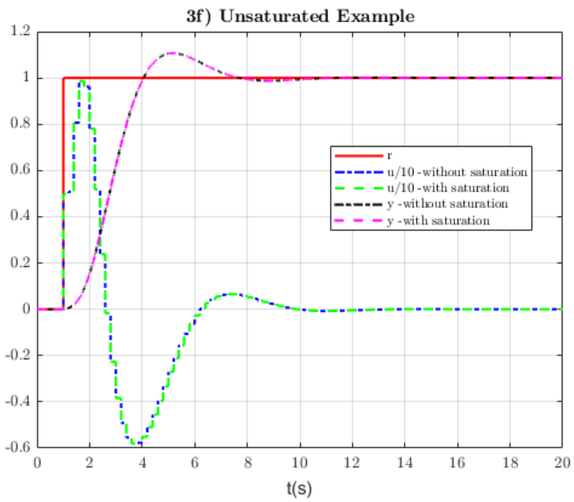



Figure 17: Unsaturated Step Response Example

cause of some numerical approximations). The controller designed seems to work well.

*2) 1(b):* As shown in Figure 7, the estimator designed leads to a $\tilde{\mathbf{x}}(k)$ that goes to 0. This means that our estimator works well in getting the states and can be used in a combined compensator design.

*3) 1(c):* The above controller and estimator combined together were applied and as shown in Figure 8 and Table 1, results are the same as from 1(a). This empirically reinforces the separation principle and agrees with the expected theory.

*4) 1(d):* As shown in Figure 9, the disturbances propagate through the system and the step response seems to be a noisy response. With the random Gaussian noise, it depends on the exact value of the noise at the given time and therefore, we have this "oscillatory" behaviour around the steady state value. With the step disturbance (of amplitude 0.1), it is similar to a constant offset and this offset is seen in the output and the steady state error seems to mirror the amplitude of the noise. As was mentioned in Homework 6, this is a characteristic of using a full estimator and therefore, our results agree with theory. The noisy sensor measurements as well as some of the noise from the plant process are not mitigated by this design and an integral control and/or an anti-aliasing filter seems to be necessary.

### *B. Part 2*

As shown in Figure 10 and Table 2, the combined compensator seems to be working well on the $4^{th}$ order plant with the transient response and steady state error being as desired. It seems that the simplified assumption of the plant (perhaps due to ease or not being aware of structural resonance) is not too off and can still be controlled with the $2^{nd}$ order compensator. The resonances ignored in the simplification do not seem to have a major effect on the performance of the overall closed-loop system.

Regarding the response with added disturbance shown in Figure 11, I used several runs and noticed that the noise is not as pronounced as the result from Part 1d (Figure 9). My guess is that simpler systems are relatively more prone to propagate the disturbance. The transient responses are near identical with less overshoot in Part 2. The steady state response with step disturbance is identical but with random disturbance, the value does not settle at a given value in both cases so there really is not a steady state error. Longer time runs also do not guarantee a steady state value because the random disturbance is unpredictable.

### *C. Part 3*

Some parts (a,b,c and some of e) have same discussion as Part 1 (feel free to skip them, I included them for completeness).

*1) 3(a):* As shown in Figure 12 and Table 3, the controller tracks a step input perfectly and the transient responses are as predicted (with some discrepancy because of some numerical approximations). The controller designed seems to work well.

*2) 3(b):* As shown in Figure 13, the estimator designed leads to a $\tilde{\mathbf{x}}(k)$ that goes to 0. This means that our estimator works well in estimating the states and can be used in a combined compensator design.

*3) 3(c):* The above controller and estimator combined together were applied and as shown in Figure 14 and Table 3, results are the same as from 3(a). This empirically reinforces the separation principle and agrees with the expected theory.

*4) 3(d):* From Figure 18, we can see that non zero initial conditions affect the transient response but not the steady state performance. From (a)-(d), we used non-zero initial conditions on each of the actual states and assumed zero initial conditions on the estimated states. As we can see, the transient response is drastically poor especially with some states over others ($\dot{y}$ vs $\dot{d}$ for example) and saturation is persistent much longer than the zero initial conditions case. I reasoned that this might be because the estimator has to "wait" a bit longer for the actual states to adjust and then converge to those values. This affects the rise time, settling time and depending on the state, the overshoot. the In (e),

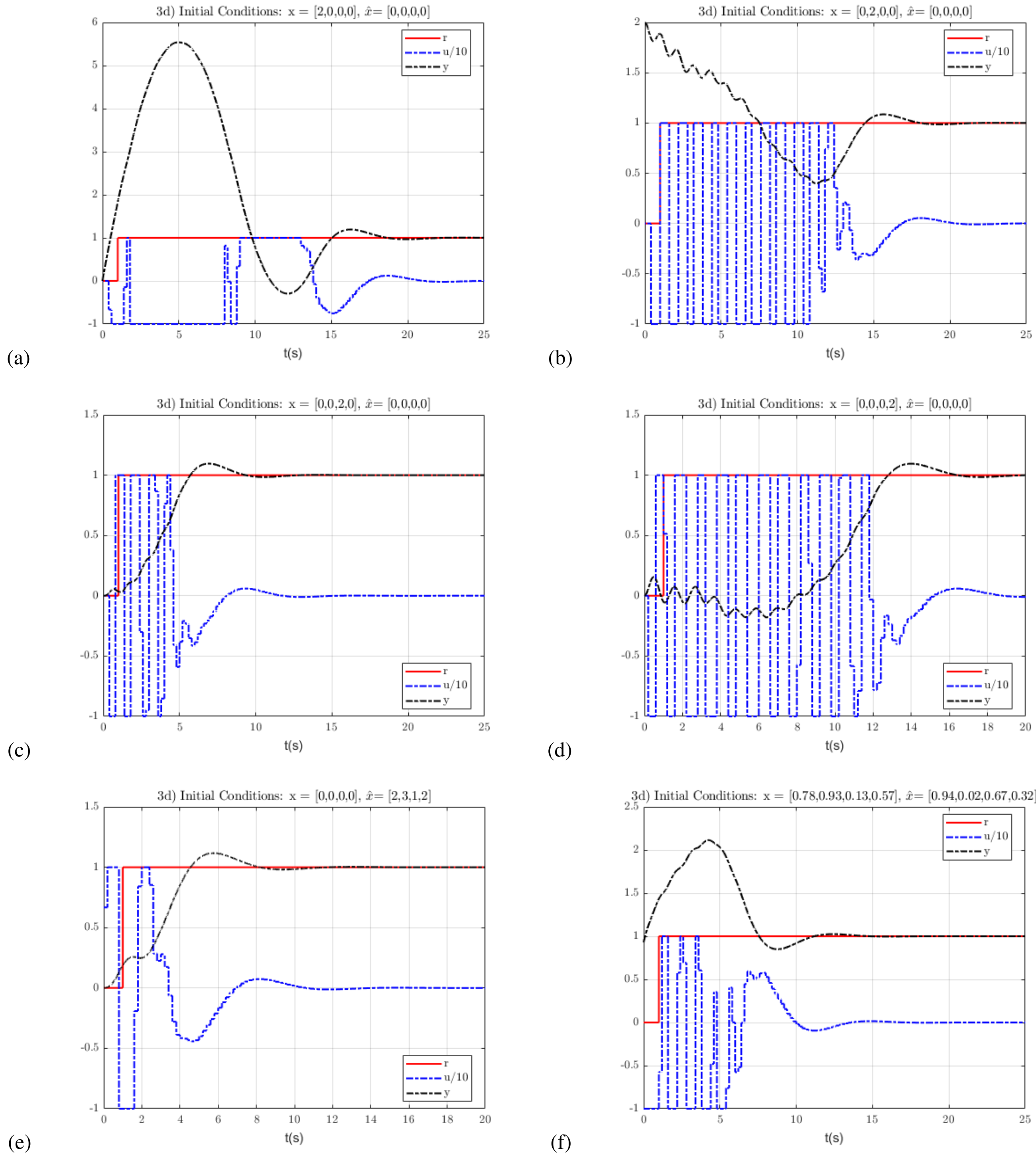



Figure 18: Step Responses for Nonzero Initial Conditions on (a) $\dot{y}$ (b) $y$ (c) $\dot{d}$ (d) $d$ (e) All Estimated States $\hat{\mathbf{x}}$ (f) All 8 states

we randomly initialized the estimated states and found that the performance is not as bad as the previous ones with slight delays in terms of rise time and settling time. According to theory and the empirical evidences we showed of $\tilde{\mathbf{x}}(k)$, we know that the response will converge and the plot seems to agree with that reasoning. Finally in (f), we chose random initial conditions for all the actual and estimated states (perhaps faulty calibrations and inaccurate assumptions of initial conditions) and see a combined effect of the other figures at a smaller scale (because the initial conditions happened to be small).

*5) 3(e):* As shown in Figure 15, the disturbances propagate through the system and the step response seems to be a noisy response. With the random Gaussian noise, it depends on the exact value of the noise at the given time and therefore, we have this "oscillatory" behaviour around the steady state value. With the step disturbance (of amplitude 0.1), it is similar to a constant offset and this offset is seen in the output and the steady state error seems to mirror the amplitude of the noise. As was mentioned in Homework 6, this is a characteristic of using a full estimator and therefore, our results agree with theory. The noisy sensor measurements as well as some of the noise from the plant process are not mitigated by this design and an integral control and/or an anti-aliasing filter seems to be necessary.

The responses in Part 2 and Part 3 with disturbances are very similar, indicating that the disturbance does not affect the response significantly. We ran the simulation several times and the responses were mostly similar with few exceptions in the random disturbance case where occasional large disturbance values lead to larger "oscillatory" behaviour. Again, my guess is that with these higher order systems, the disturbance is more suppressed and not propagated as easily.

In terms of root loci shown in Figure 16, the design in Part 2 is a combination of $2^{nd}$ order compensator and $4^{th}$ order plant leading to a $6^{th}$ order overall system (which is why we see six closed loop poles in Figure 16a for $\bar{K} = 1$). The design in Part 3 is a combination of $4^{th}$ order compensator and $4^{th}$ order plant leading to an $8^{th}$ order overall system (which is why we see eight closed loop poles in Figure 16b for $\bar{K} = 1$). However, we can see that the overall behaviour of both root loci is very similar and can help explain why our performance results are similar for both parts.

*6) 3(f):* As we see in most of the figures for the previous parts, the compensators lead to saturation. Unlike Simulation Lab 1 where we designed lead compensators and had the flexibility to choose the gain given appropriate choices of poles and zeros, in this case, the designs depend on our choices of desired poles and choice of sampling rate. We first tried changing the sampling rate and found that increasing it lead to more saturation (the factor dividing $u$ was getting larger) and decreasing it was leading to very funky, unstable results. We then opted to change the pole locations of the controller (and estimator poles were set to be 2.5x faster than the controller). We found by try and error that the controller pole locations from $\zeta$ and $\omega_n$ shown in Table 4 lead to unsaturated response shown in Figure 17.

### *D. General Remarks*

The effect of saturation was not as pronounced in this lab as Simulation Lab 1 (as seen with most of the factors dividing $u$ being close to the saturation level of 10). Generally saturation lowers the transient performance (longer rise time and settling time, but lower overshoot). Here, the step responses with and without saturation were very similar which made our life easier in analysis and finding the transient response values.

State-space feedback design is easier to setup but also more complicated than a direct design. The intuition of setting the gains and zeros directly is somewhat abstracted in the setup of the controller and estimator so the transition from direct design method does not feel smooth. I think this might be because we discussed a compensator cascaded with the plant in the classic case but we are now putting the compensator (without the $N$ gain) in the feedback path. In addition, the resulting compensators might also feel like an overkill since a state-space compensator designed for an $n^{th}$ order plant leads to an $n^{th}$ order compensator. As we saw above, the full plant was sufficiently controlled with a second order compensator but a direct state-space design on the full plant leads to a higher order compensator that might not really be necessary and lead to more cost. I believe system identification and subsequent reasonable simplifications by ignoring some resonances will be helpful in this design method. On the positive side, in terms of performance, the designs led to more or less exactly the desired/predicted performance. Moreover, the resulting designs of the controller and estimator were scalable. The setups do not change and all it required was switching the plants (which helped a lot in setting up the simulation for example, as seen in Figure 23 in the Appendix).

## V. Conclusions

This lab mainly emphasized the design of digital controllers using state variables. By designing a compensator for a simplified model as well as the full model of the plant, I was able to assess their effects on performance. While a design for the full plant (after identifying it) is straight forward, this lab also introduced me to the idea of structural resonances in the plant. Since the resonances were fast in our case, we were able to ignore them and design for a simplified model that led to a simpler compensator whose performance in the closed-loop system was comparable to the full order compensator. In addition, I was able to get more insight on reduced order estimators and compare them to full state estimators (further highlighted in the next section). Through the design and analysis process, I was able to derive a general intuition of some of the pros and cons in this approach. These intuitions will be valuable when I come across real life implementations.

With more time at my disposal, I could investigate the effects of sampling rates and parameter variations in our design and see how that turns out compared to the theoretical expectations as highlighted in class and in the book.

In conclusion, I appreciate having the chance to work on this well-organized lab. It was exciting to investigate several aspects of modern digital control systems design and still end up with potential future works. Investigating disturbances was an eye-opener on why there exist so many ways to deal with noise and why it is a critical aspect of the design process.

## VI. Part 4 - Bonus

### A. 4(c) Reduced Order Estimator

In the prediction estimator used so far, we estimate all the states. Here we assume we measured $d$ and design a reduced order estimator for the other three states. We do so by rearranging Equation 4 for the unknown states. Take the known state $d(k) = x_a(k)$ and the unknown three states in their discretized form $[\dot{y}\ y\ \dot{d}]^T = \mathbf{x}_b(k)$. We want to estimate $\mathbf{x}_b(k)$ from $d(k)$ and $u$. The math following Equation 4 goes as follows:

$$\begin{cases} \mathbf{x}(k+1) = \begin{bmatrix} \mathbf{x}_b(k+1) \\ x_a(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{\Phi}_{bb} & \mathbf{\Phi}_{ba} \\ \mathbf{\Phi}_{ab} & \Phi_{aa} \end{bmatrix} \begin{bmatrix} \mathbf{x}_b(k) \\ x_a(k) \end{bmatrix} + \begin{bmatrix} \mathbf{\Gamma}_b \\ \Gamma_a \end{bmatrix} u(k) \\ \quad d(k) = \begin{bmatrix} 0 & \cdots & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{x}_b(k) \\ x_a(k) \end{bmatrix} \end{cases}$$

$$\begin{cases} \mathbf{x}_b(k+1) = \mathbf{\Phi}_{bb}\mathbf{x}_b(k) + (\mathbf{\Phi}_{ba}\mathbf{x}_a(k) + \mathbf{\Gamma}_b u(k)) \\ \qquad\qquad\quad = \mathbf{\Phi}_{bb}\mathbf{x}_b(k) + \mathbf{\Gamma}_r u_r(k) \\ \qquad y_r(k) = x_a(k+1) - \Phi_{aa}x_a(k) - \Gamma_a u(k) \\ \qquad\qquad\quad = \mathbf{\Phi}_{ab}\mathbf{x}_b(k) \end{cases}$$

This system is essentially $\{\mathbf{\Phi}_{bb}, \mathbf{\Gamma}_r, \mathbf{\Phi}_{ab}, 0\}$ for input $u_r(k)$, states $\mathbf{x}_b(k)$ and output $y_r(k)$. We can design $\mathbf{\Gamma}_r u_r(k)$ and $y_r(k)$ based on their equivalent formulations above and we already know $\mathbf{\Phi}_{bb}$ and $\mathbf{\Phi}_{ab}$. Thus, we can design a reduced order prediction estimator $\mathbf{L}_r$ using the same setup as Figure 3 and the error dynamics formulation. By parallel with the full estimator error formula, $\tilde{\mathbf{x}}_b(k+1) = (\mathbf{\Phi}_{bb} - \mathbf{L}_r\mathbf{\Phi}_{ab})\tilde{\mathbf{x}}_b(k)$. Thus we can find $\mathbf{L}_r$ by setting $\det(z\mathbf{I} - \mathbf{\Phi}_{bb} + \mathbf{L}_r\mathbf{\Phi}_{ab}) = \alpha_{re}(z) = 0$ where

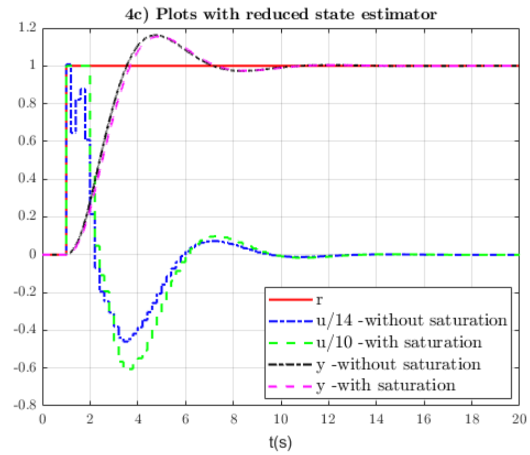



Figure 19: Step Response from using Reduced Order Estimator

TABLE 5: Key Results from $4^{th}$ Order Plant, $4^{th}$ Order Compensator (with Reduced Order Estimator)

| | |
|---|---|
| $(\zeta,\ \omega_n$(rad/s)) | (0.5,1.0), (0.3, 5.80) |
| $\mathbf{K}$ | [53.90 -105.88 -36.84 119.99] |
| $N$ | 14.12 |
| $\mathbf{L}_r$ | $[4.13\ 1.71\ 7.69]^T$ |
| Predicted $M_p(\%)$ | 16.3 |
| Measured $M_p(\%)$ | 18(•), 18($\star$) |
| Predicted $t_r$(s) | 1.8 |
| Measured $t_r$(s) | 1.8(•), 1.84($\star$) |
| Predicted $t_s$(s) | 9.2 |
| Measured $t_s$(s) | 9.2(•), 9.2($\star$) |
| $\lvert e_{ss}\rvert$ | 0 |

•: without saturation, $\star$: with saturation

$\alpha_{re}(z)$ is the desired characteristic polynomial. For this lab, $\mathbf{x}_b(k)$ has three elements and we place all three of our desired poles at $z = 0$ so that $\alpha_{re}(z) = z^3$ (MATLAB doesn't like repeated poles so we had to put 3 poles at $10^{-4}$, 0, $-10^{-4}$). After finding $\mathbf{L}_r$ and estimating the three states using the reduced order estimator, we merge the fourth state $d$ to the rest and feed it to control law controller to complete the combined controller-reduced order estimator configuration. The step response using this setup is shown in Figure 19. $\mathbf{L}_r$ and other important information are tabulated in Table 5. The design is a bit more laborious and complicated than the full estimator but we can see that in this case, the results are near as desired and tracks the input perfectly. It was nice to explore its use.

### B. 4(d) Bode Plots for the Compensators

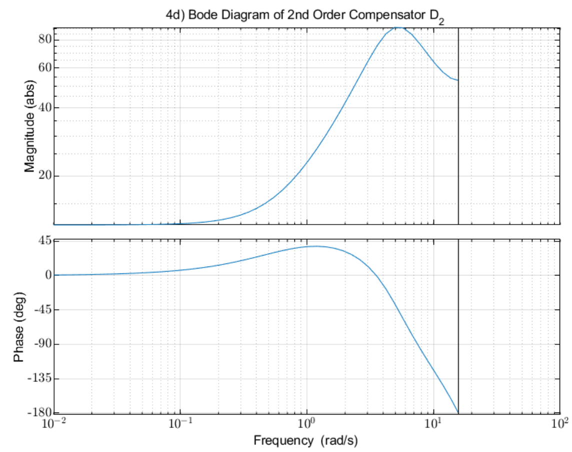
Here we plotted the discrete frequency response of both the compensators independently the way the compensators were applied in each of the parts above. Figures 20 and 21 show each case. The subscripts in $D$ and $G$ indicate their order. Data extracted from the Bode plots is shown in Table 6. The compensators have a lead-like behaviour, which improves the transient response characteristics as desired. Combined with the plants, we can see that they have a steep decline in magnitude as $w \to 0$ indicating at least a Type 1 system. This explains why the steady state error is 0 for a step input. Similarly the closed-loop system (calculated from Equation 7) frequency responses shown in Figure 22 have almost a gain of 1 (some discrepancy perhaps because of numerical calculations) and are (almost) flat at lower frequencies, indicating that we are able to track $r$ perfectly with $y$.

TABLE 6: Bode Plot Information

| | $\|D_2\|$ | $\|D_4\|$ | $\|D_2G_2\|$ | $\|D_2G_4\|$ | $\|D_4G_4\|$ |
|---|---|---|---|---|---|
| $\omega_c = \omega_n$ * | - | - | 0.8 | 0.95 | 1.5 |
| PM (°) * | $\infty$ | $\infty$ | 35 | 30 | 30 |
| $\omega_{180}$ | 15.71 | 11 | 3 | 3 | 3 |
| GM | 0.018 | 0.005 | 10 | 1.33 | 1.33 |
| $K_p$ | 12 | 20 | - | - | - |
| $K_v$ | - | - | 1 | 80 | 10 |

* The $\omega_c$ and PM information for the $|DG|$ is based on the first crossing point

(a)




(b)

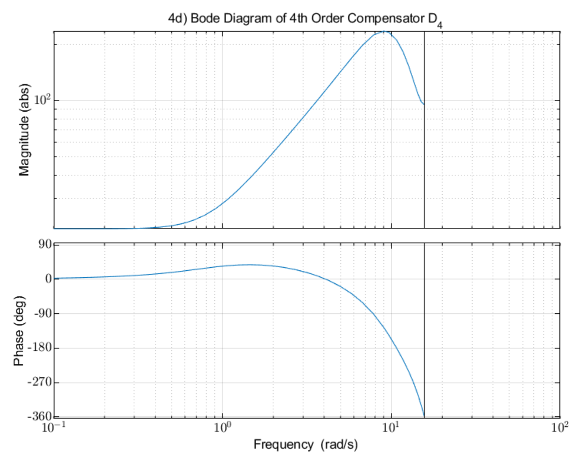
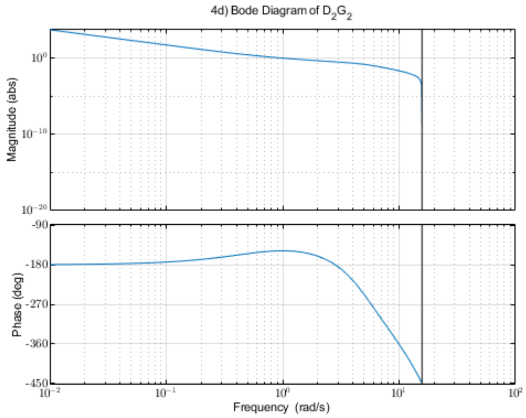
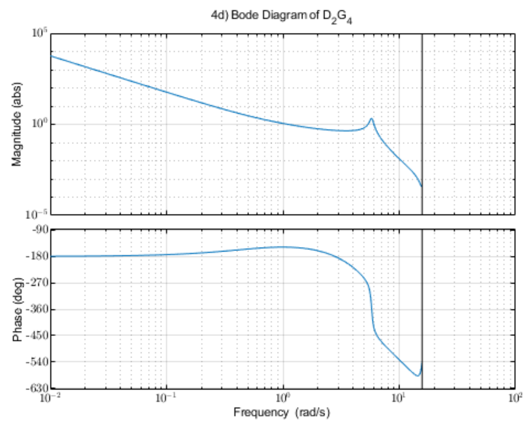


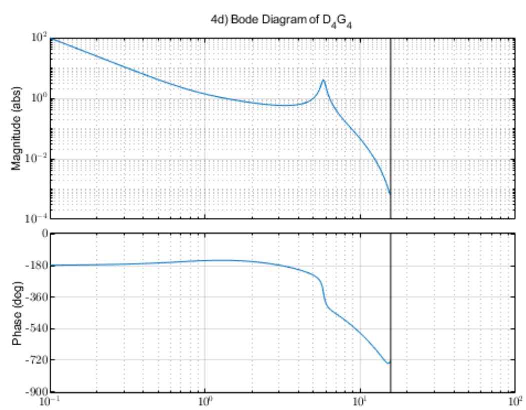

Figure 20: Bode plot of (a) $2^{nd}$ Order Compensator (b) $4^{th}$ Order Compensator




(a)

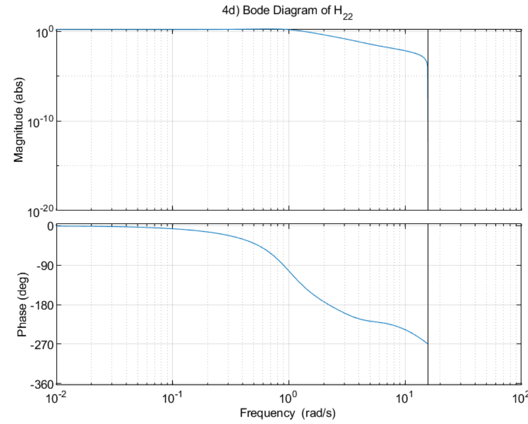



(b)

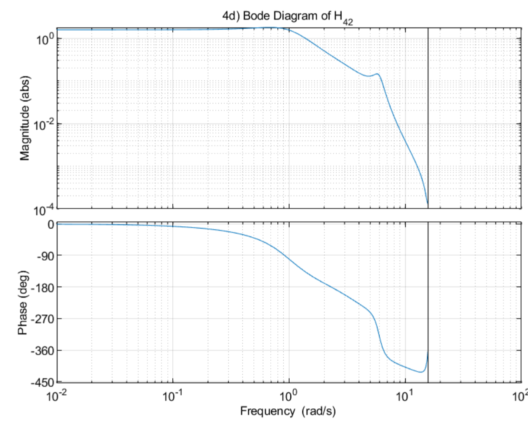



(c)

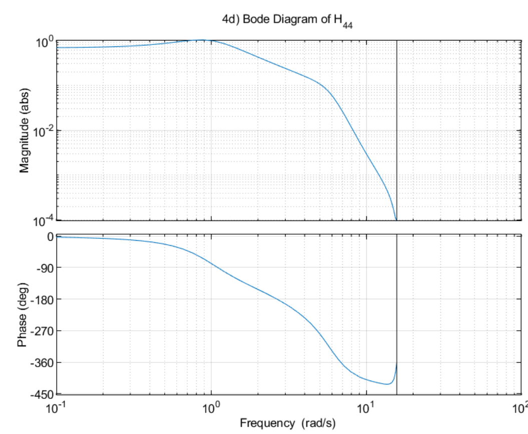
Figure 21: Bode plot of (a) $2^{nd}$ Order Compensator on $2^{nd}$ Order Plant (b) $2^{nd}$ Order Compensator on $4^{th}$ Order Plant (c) $4^{th}$ Order Compensator on $4^{th}$ Order Plant




(a)




(b)




(c)

Figure 22: Bode plot of Closed-Loop Transfer Function for (a) $2^{nd}$ Order Compensator on $2^{nd}$ Order Plant (b) $2^{nd}$ Order Compensator on $4^{th}$ Order Plant (c) $4^{th}$ Order Compensator on $4^{th}$ Order Plant

# APPENDIX: SIMULINK SETUPS

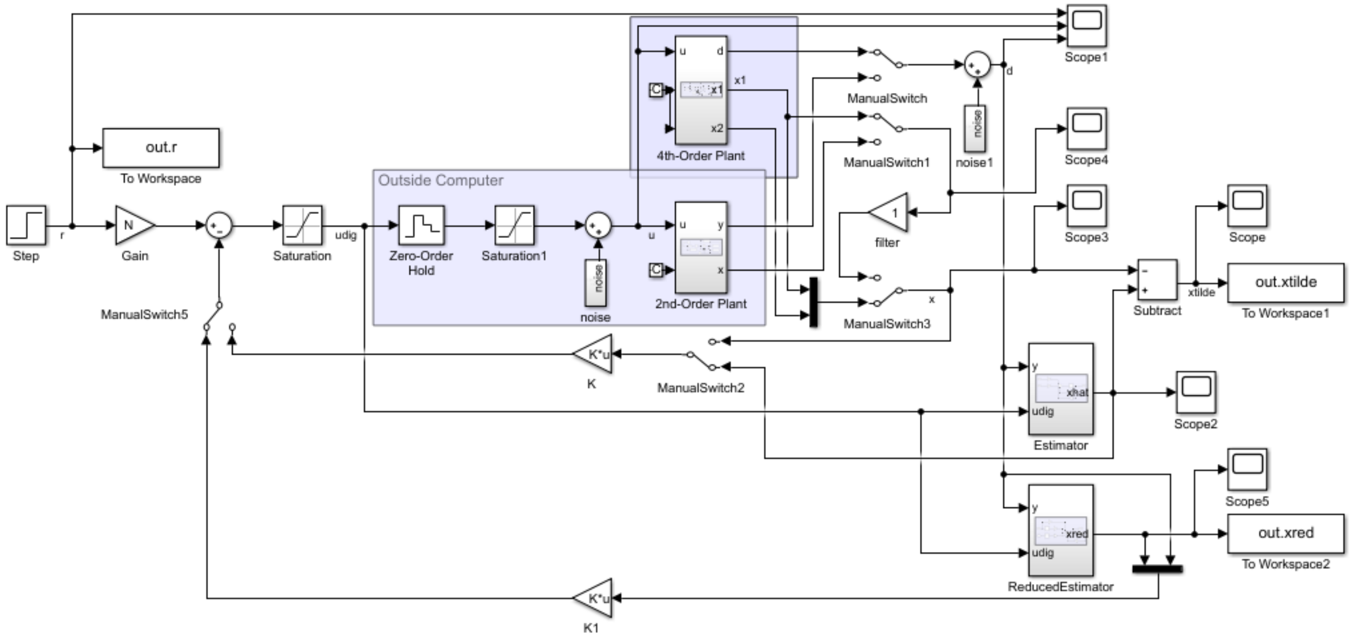



Figure 23: Full Simulink Setup

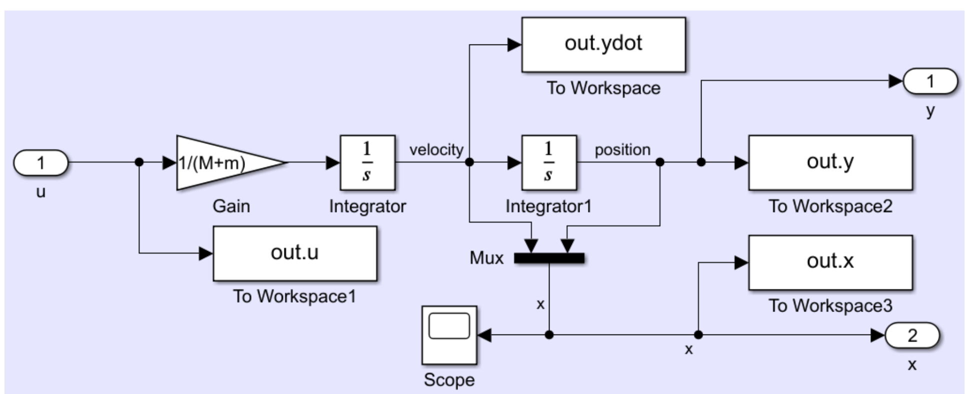



Figure 24: $2^{nd}$ Order Plant

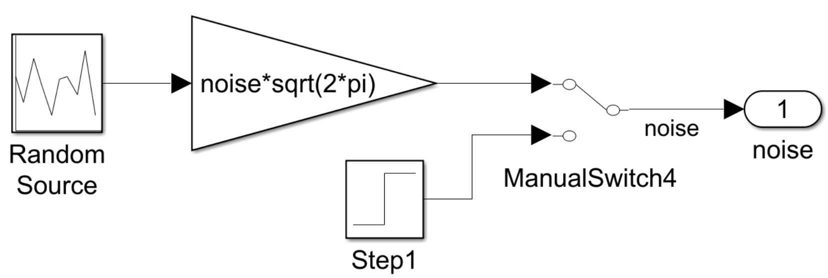



Figure 25: Disturbance Types (random Gaussian or step)

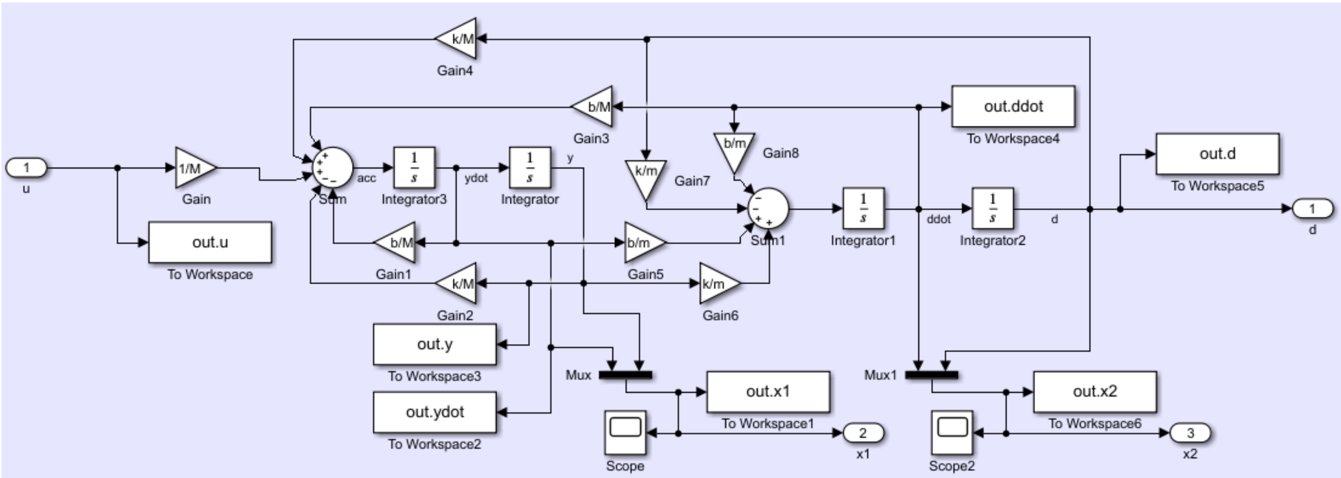



Figure 26: $4^{th}$ Order Plant

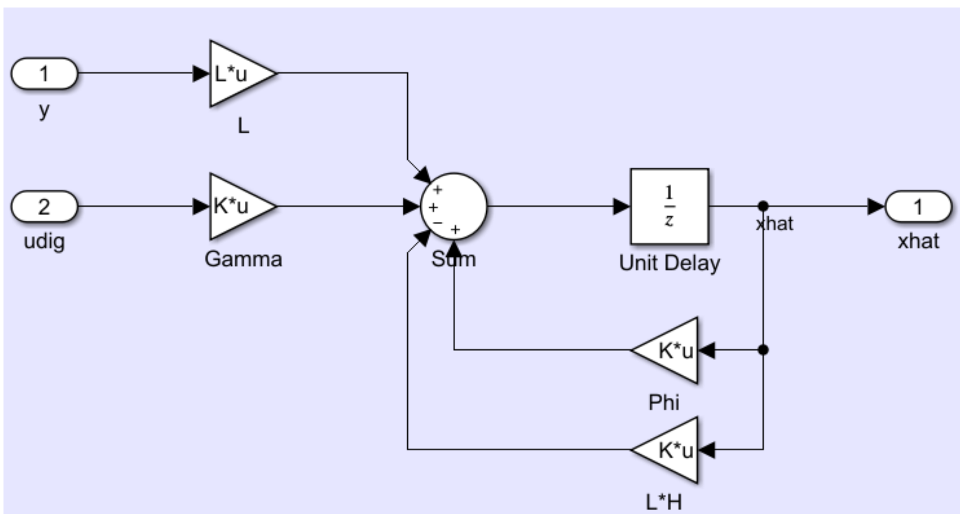



Figure 27: Prediction Estimator

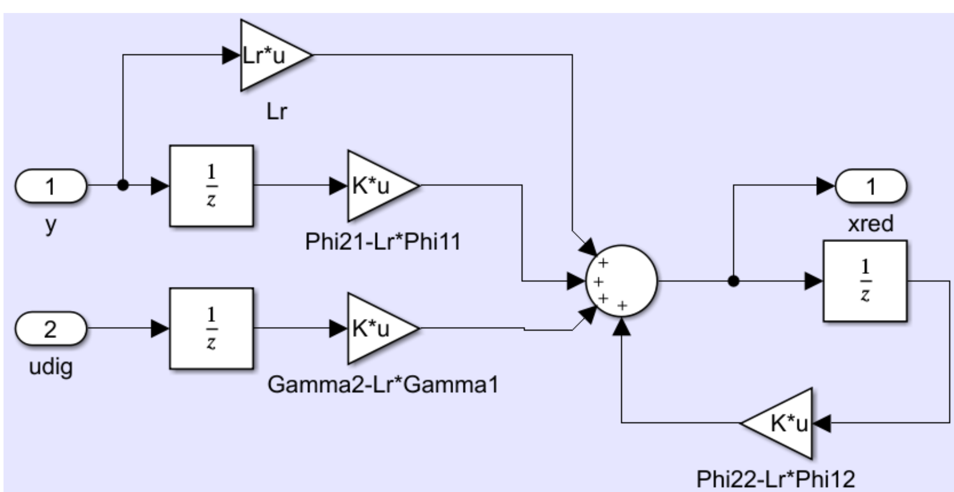



Figure 28: Reduced Order Estimator