Electroosmotic Flow through a Cylindrical Nanopore in a Charged Membrane of Finite Thickness

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Nanopore of zero thickness
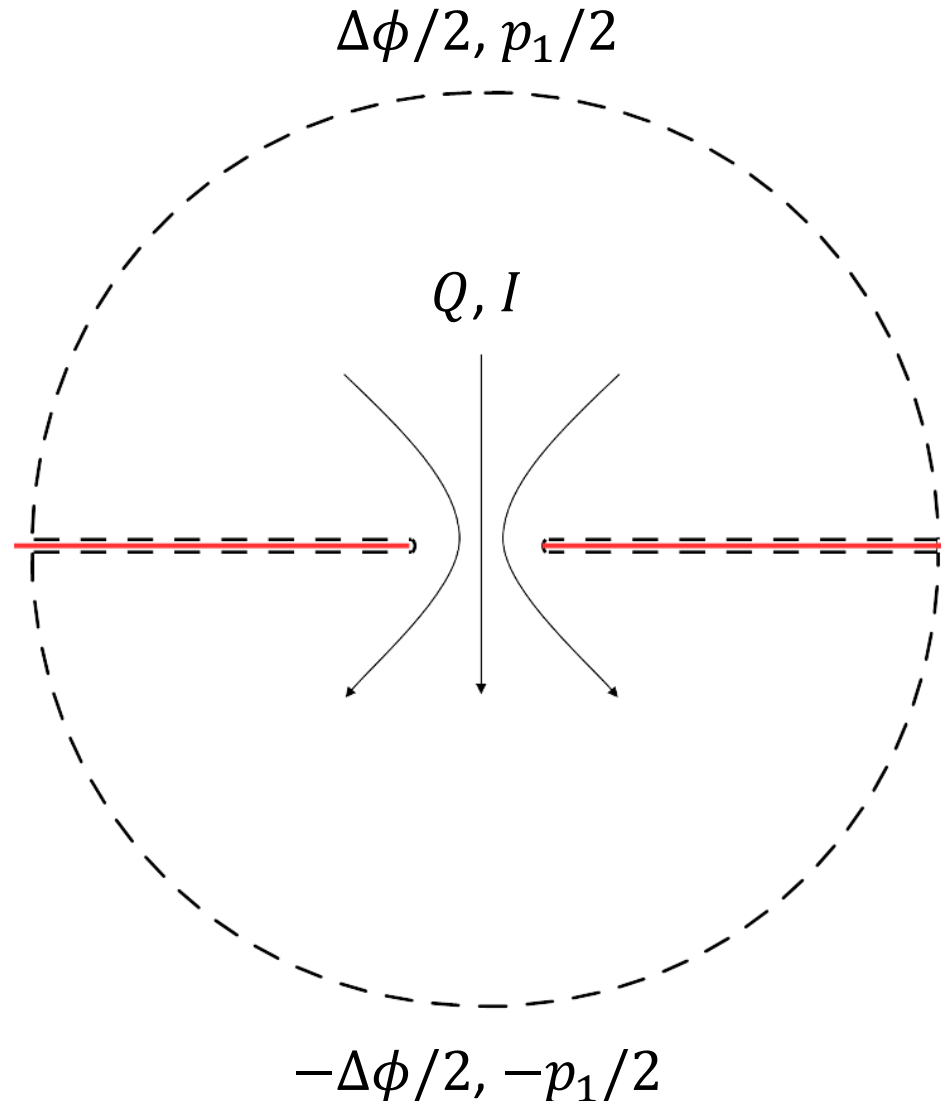
$$\Delta\phi/2, p_1/2$$

Electric current

$$I = \frac{\Delta\phi}{R_m}$$

$$Q, I$$

Volumetric flow rate
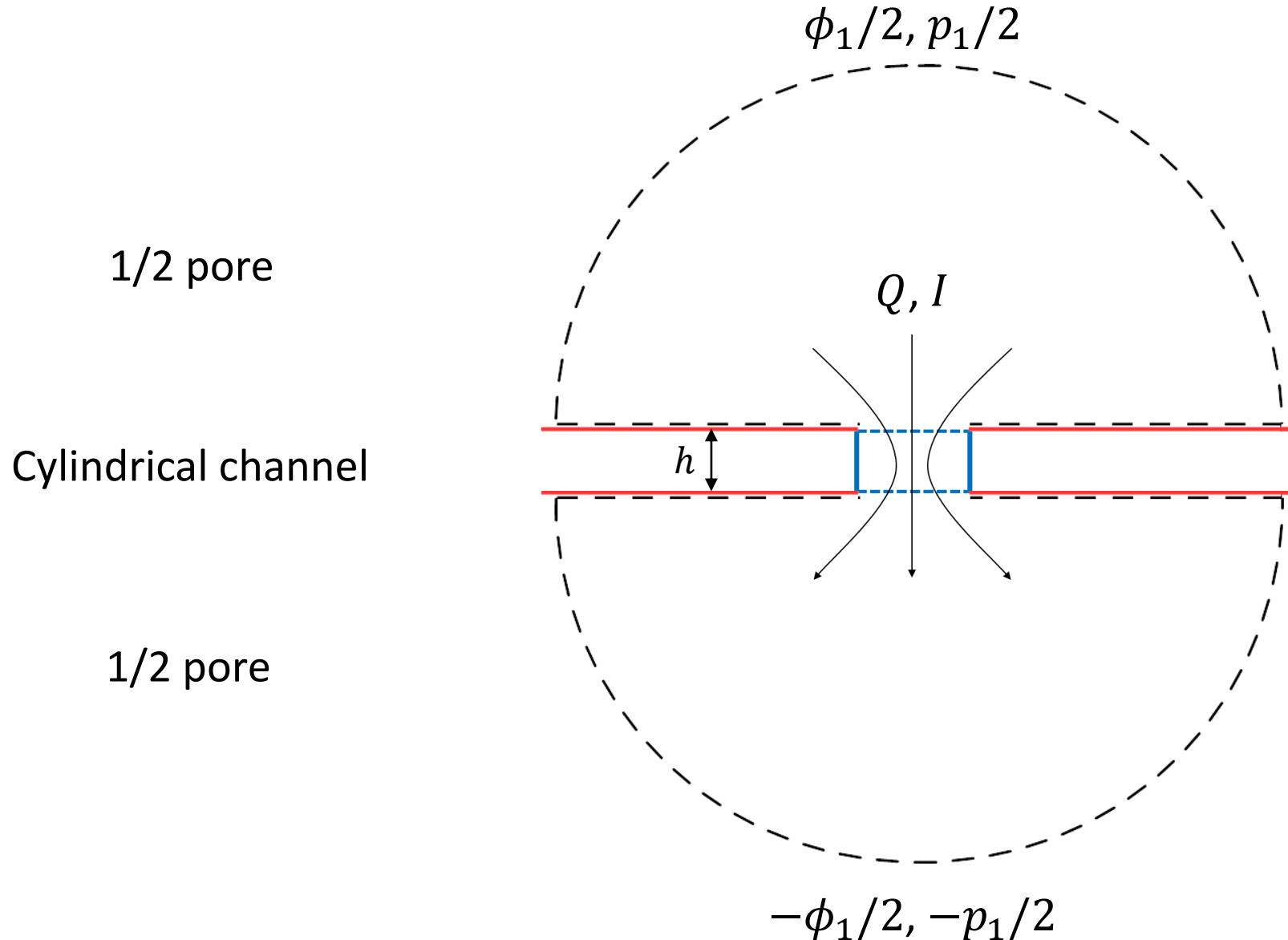
$$Q_{\Delta p} = G_m \Delta p$$
$$Q_{\Delta\phi} = H_m \Delta\phi$$

$$Q = Q_{\Delta p} + Q_{\Delta\phi}$$
$$= G_m \Delta p + H_m \Delta\phi$$

$$-\Delta\phi/2, -p_1/2$$

Zero to finite thickness

$\phi_1/2, p_1/2$

1/2 pore

Q, I

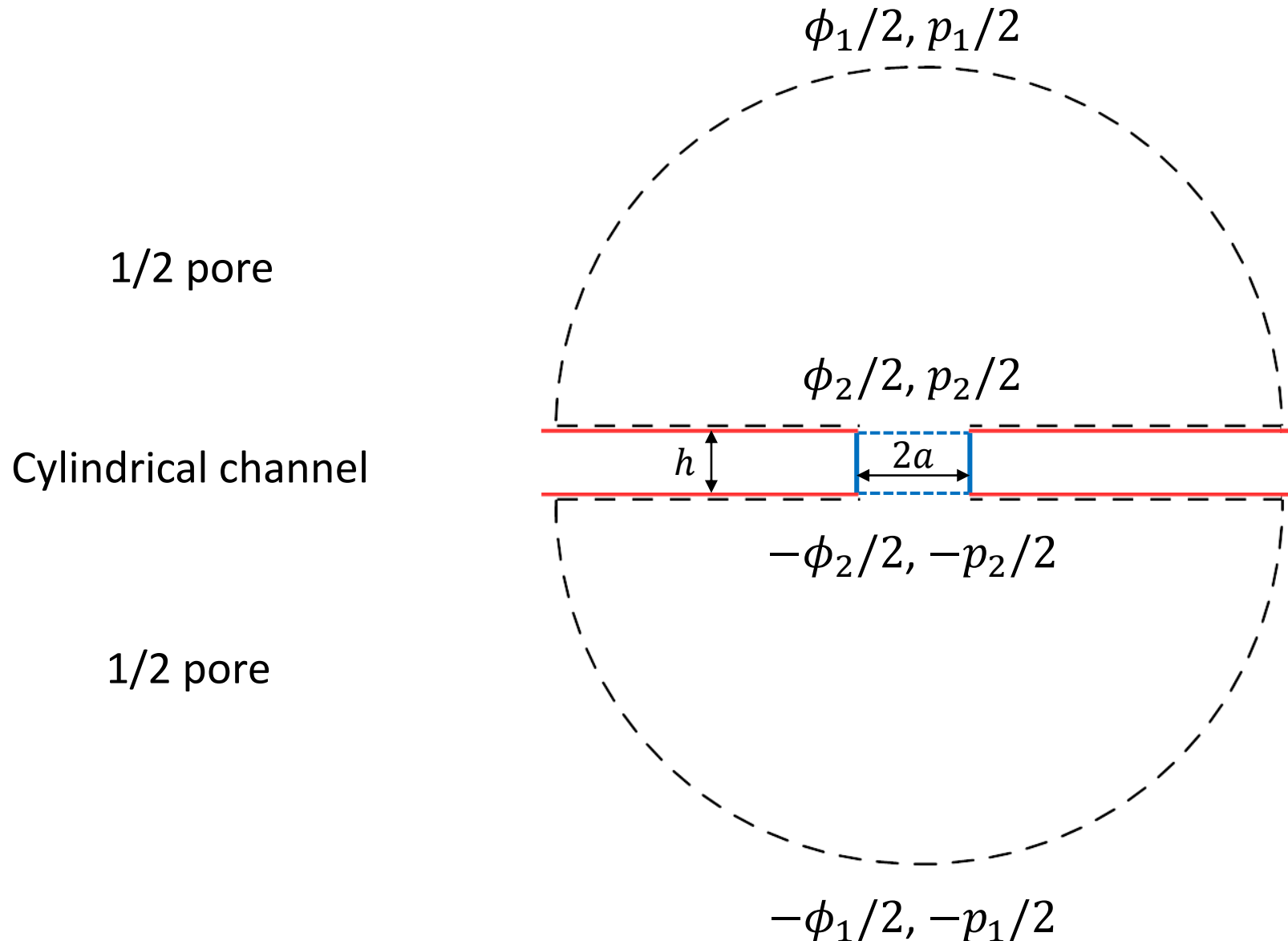Cylindrical channel

h

1/2 pore

$-\phi_1/2, -p_1/2$

Cylindrical channel:

$$I = \frac{\Delta\phi}{R_c}$$

$$Q_{\Delta p} = G_c \Delta p$$
$$Q_{\Delta\phi} = H_c \Delta\phi$$

$$Q = Q_{\Delta p} + Q_{\Delta\phi}$$
$$= G_c \Delta p + H_c \Delta\phi$$

Continuity of I and Q

$\phi_1/2, p_1/2$

$\phi_2/2, p_2/2$

$-\phi_2/2, -p_2/2$

$-\phi_1/2, -p_1/2$

1/2 pore

Cylindrical channel

1/2 pore

h

$2a$

Assuming $\pm\dfrac{\phi_2}{2}$ and $\pm\dfrac{p_2}{2}$ at both ends

$$I = \frac{\phi_1 - \phi_2}{R_m} = \frac{\phi_2}{R_c}$$

$$\begin{aligned}Q &= G_m(p_1 - p_2) + H_m(\phi_1 - \phi_2) \\ &= G_c p_2 + H_c \phi_2\end{aligned}$$
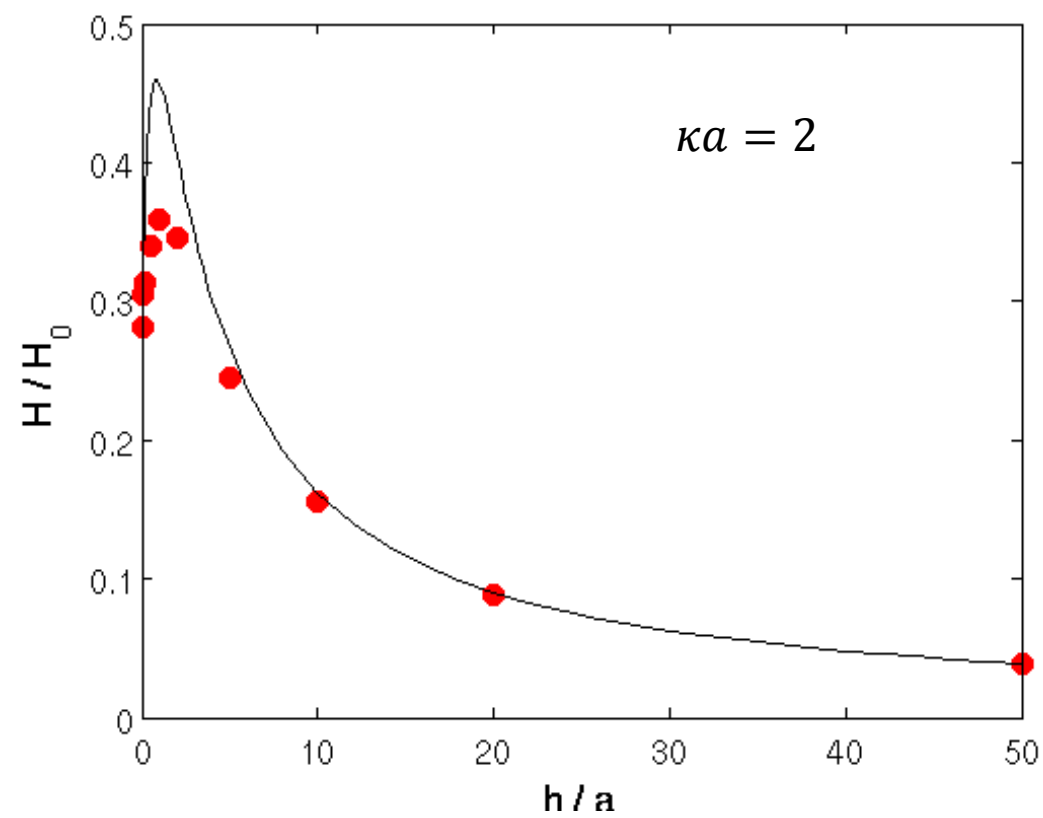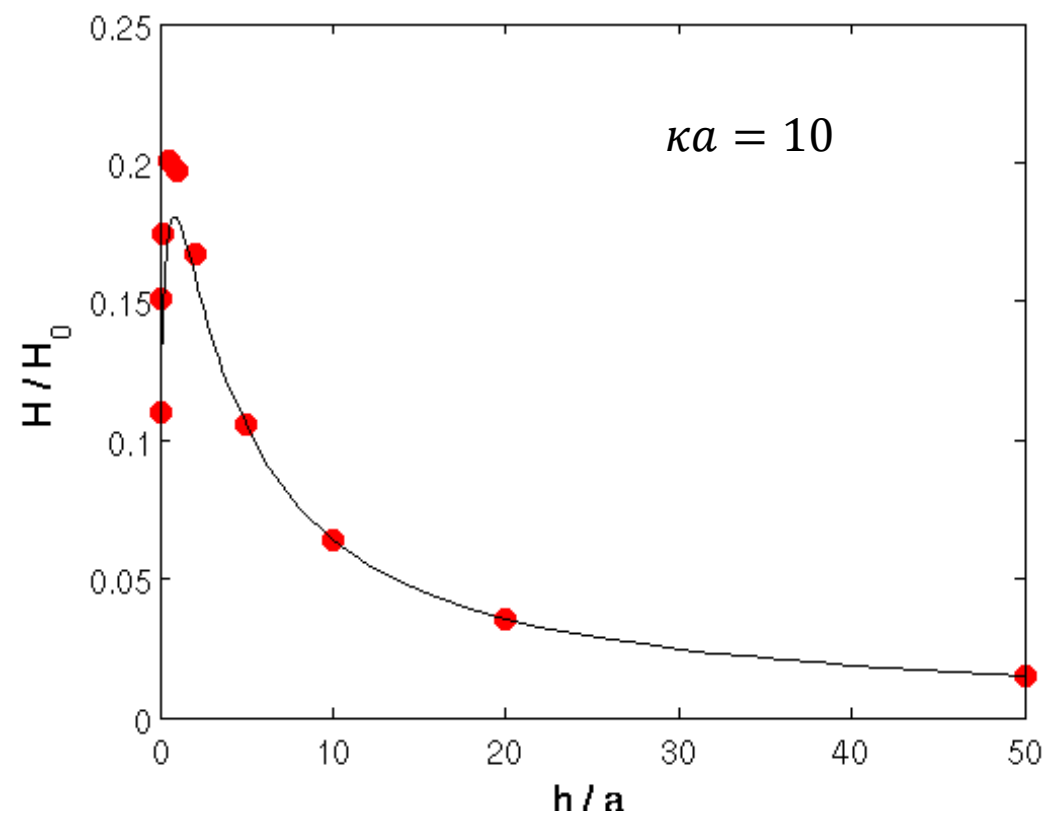
Conjugate electroosmotic conductance

$$I = \frac{\phi_1 - \phi_2}{R_m} = \frac{\phi_2}{R_c}$$

$$Q = G_m(p_1 - p_2) + H_m(\phi_1 - \phi_2) = G_c p_2 + H_c \phi_2$$
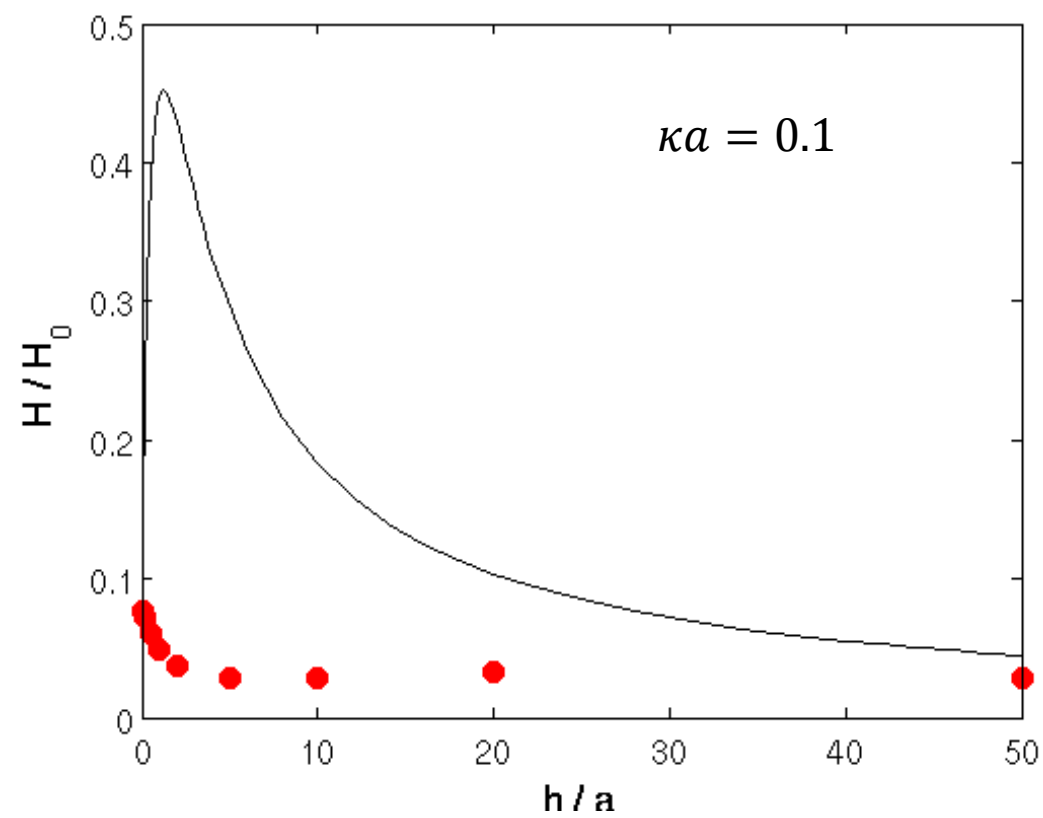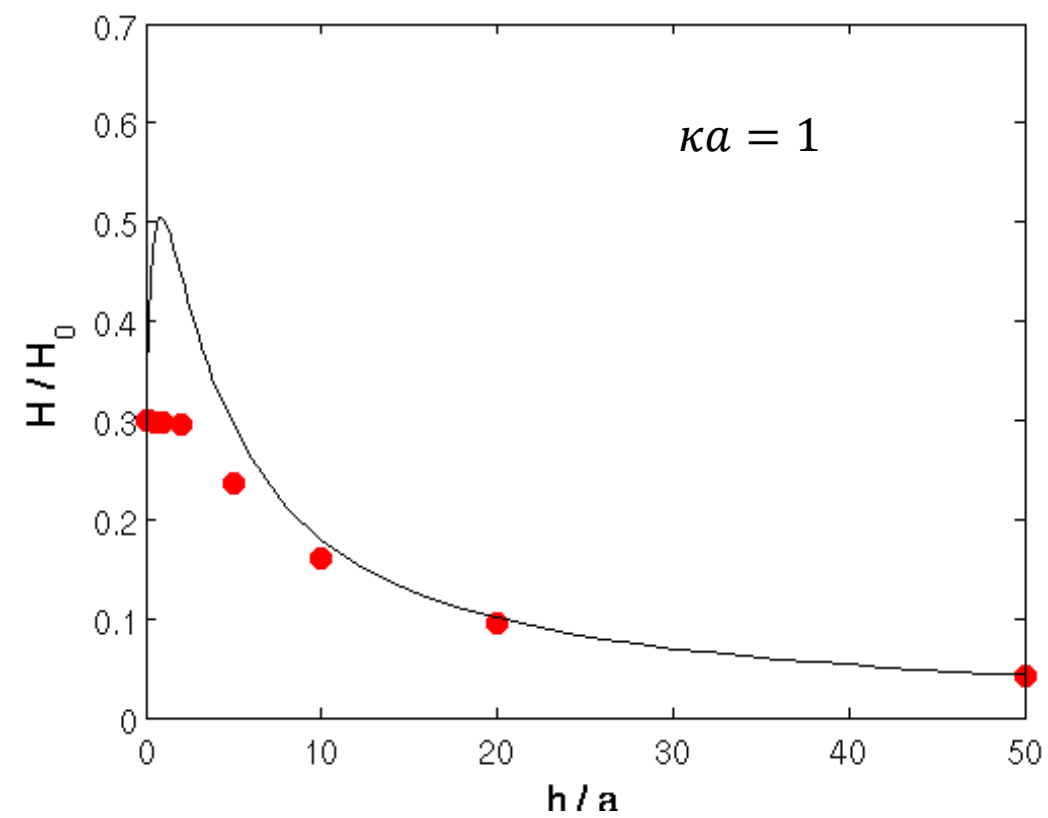
Set $p_1 = 0$ and $\phi_1 = \Delta\phi$

$$H_{comp} = \frac{Q}{\Delta\phi} = \frac{G_m R_c H_c + G_c R_m H_m}{(R_m + R_c)(G_m + G_c)} = \frac{H_m + \frac{16h^2}{3\pi^2 a^2} H_c}{\left(1 + \frac{2h}{\pi a}\right)\left(1 + \frac{8h}{3a}\right)}$$
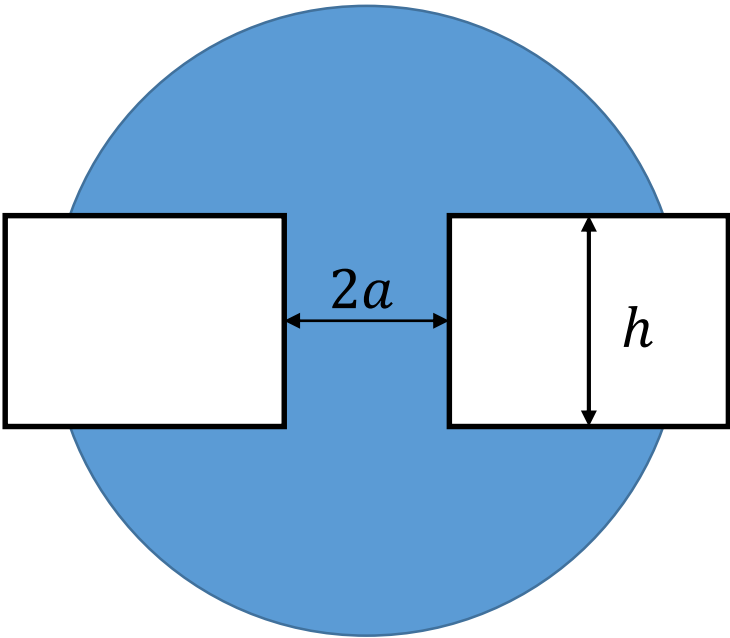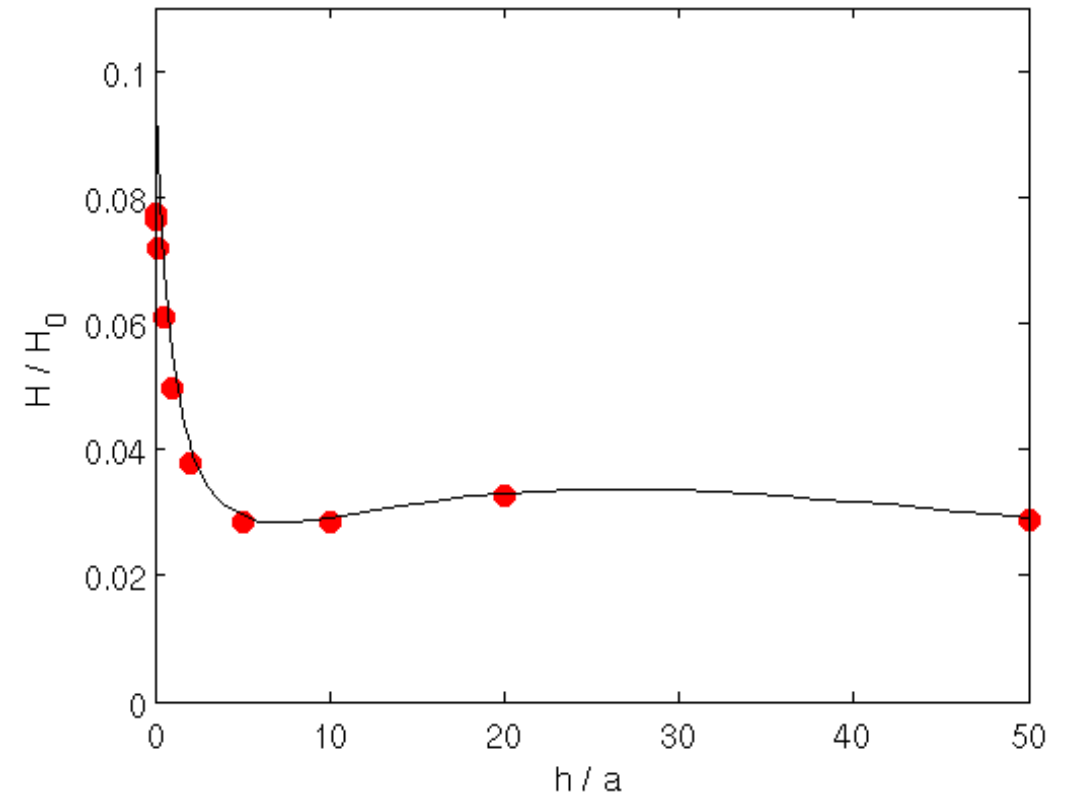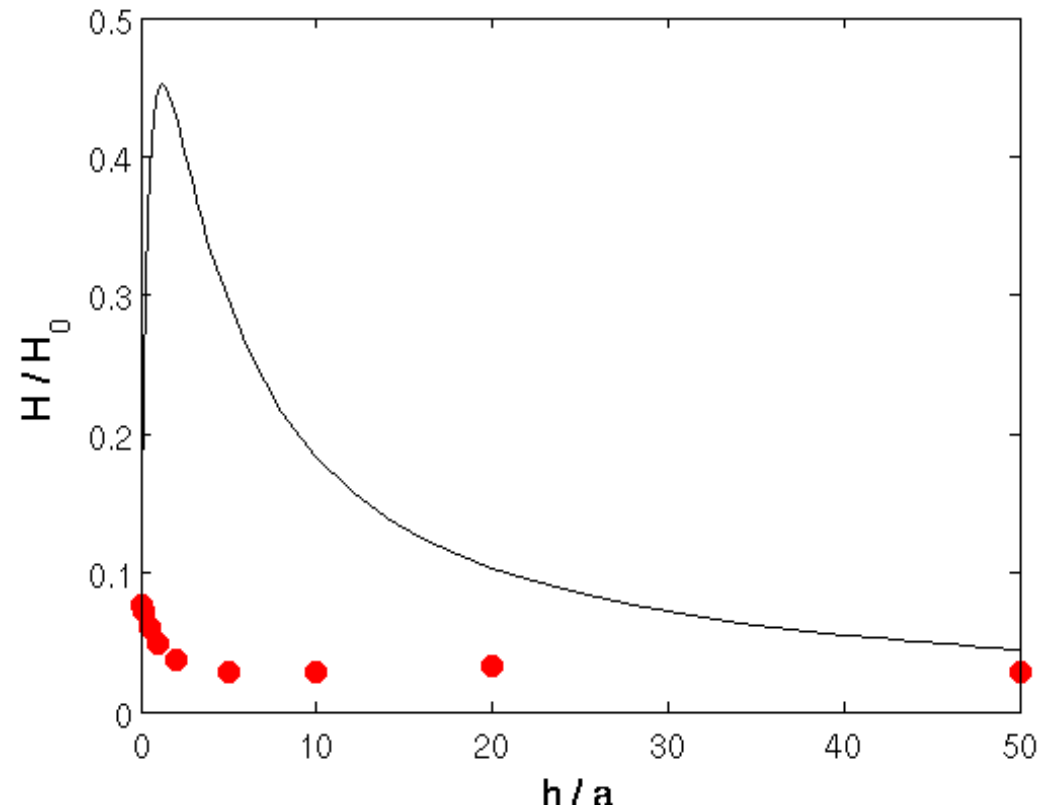
Results

Results

End effects: charge spill when $a\kappa$ small

$$H_{comp} = \frac{Q}{\Delta\phi} = \frac{G_m R_c H_c + G_c R_m H_m}{(R_m + R_c)(G_m + G_c)} = \frac{H_m + \frac{16h^2}{3\pi^2 a^2} H_c}{\left(1 + \frac{2h}{\pi a}\right)\left(1 + \frac{8h}{3a}\right)}$$
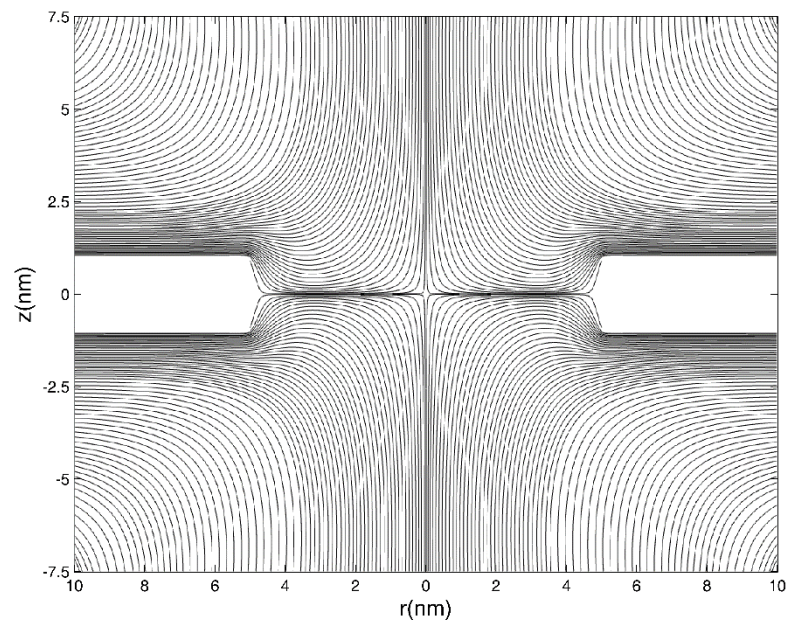
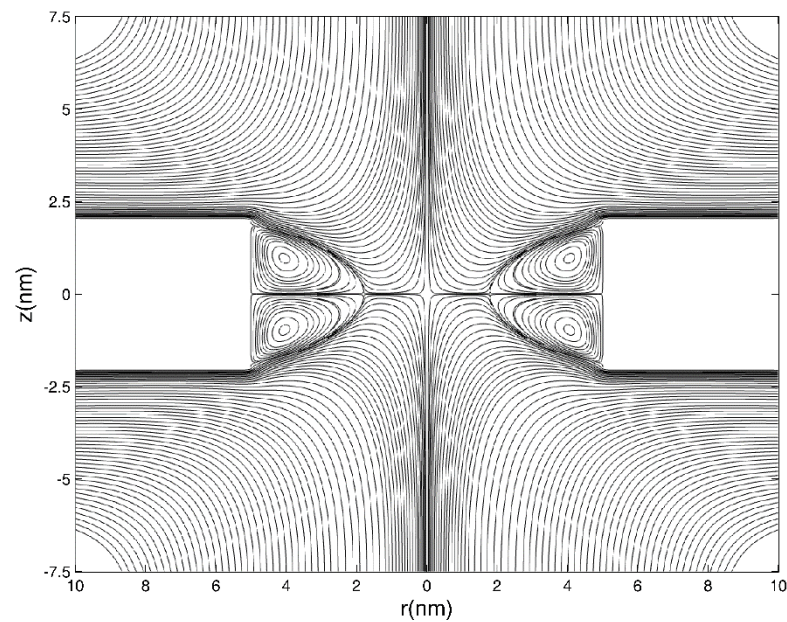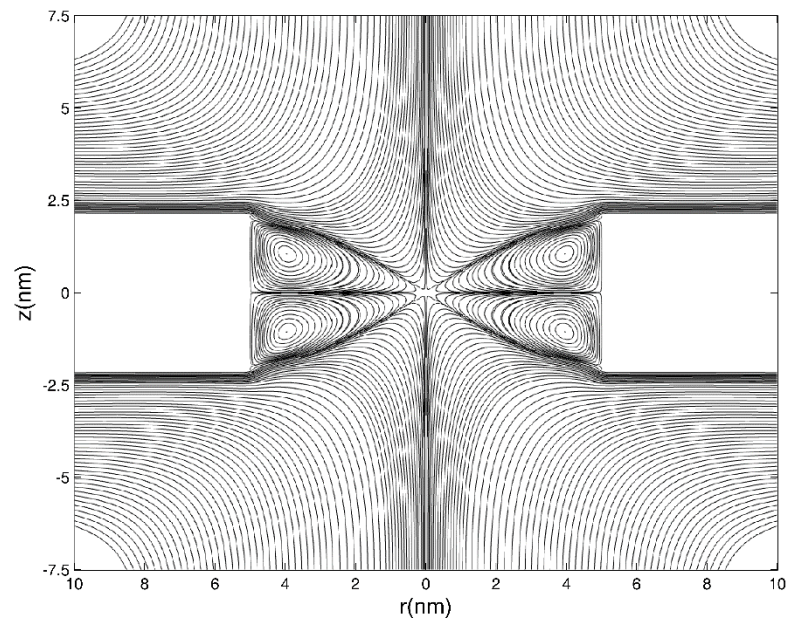|	H_c	H_m
Spill not considered	$\dfrac{\pi a^3 \sigma_c}{4\pi h}$	$\dfrac{a^3 \kappa \sigma_m}{3\mu}$
Spill considered	$\dfrac{\pi a^3 \sigma_c}{4\pi}\left(\dfrac{h - h_{lost}}{h^2}\right)$	$\dfrac{a^3 \kappa}{3\mu}\left[\sigma_m\right.$

Results – spill taken into account

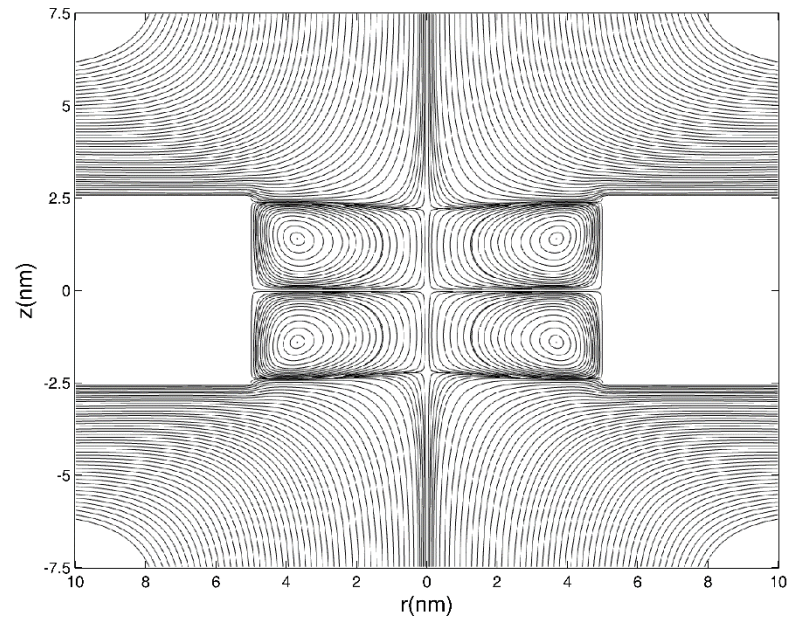# Eddies

$\dfrac{h}{a} = 0.4$

$\dfrac{h}{a} = 0.8$

$\dfrac{h}{a} = 0.84$

$\dfrac{h}{a} = 1$

Conclusion

- Composite model for electroosmotic conductance of nanopore when h is finite

- Model agrees with simulation when $\kappa a \gg 1$

- Charge spill needs to be considered when $\kappa a \ll 1$

- When membrane surfaces is not charged, induced charge electroosmosis generates pairs of toroidal counter rotating eddies

References
Journal of Fluid Mechanics **749**, 167-183;
Langmuir **30** (31), 9261-9272;
Physics of Fluids (in press).