Electroosmotic Flow through a Cylindrical Nanopore in a Charged Membrane of Finite Thickness

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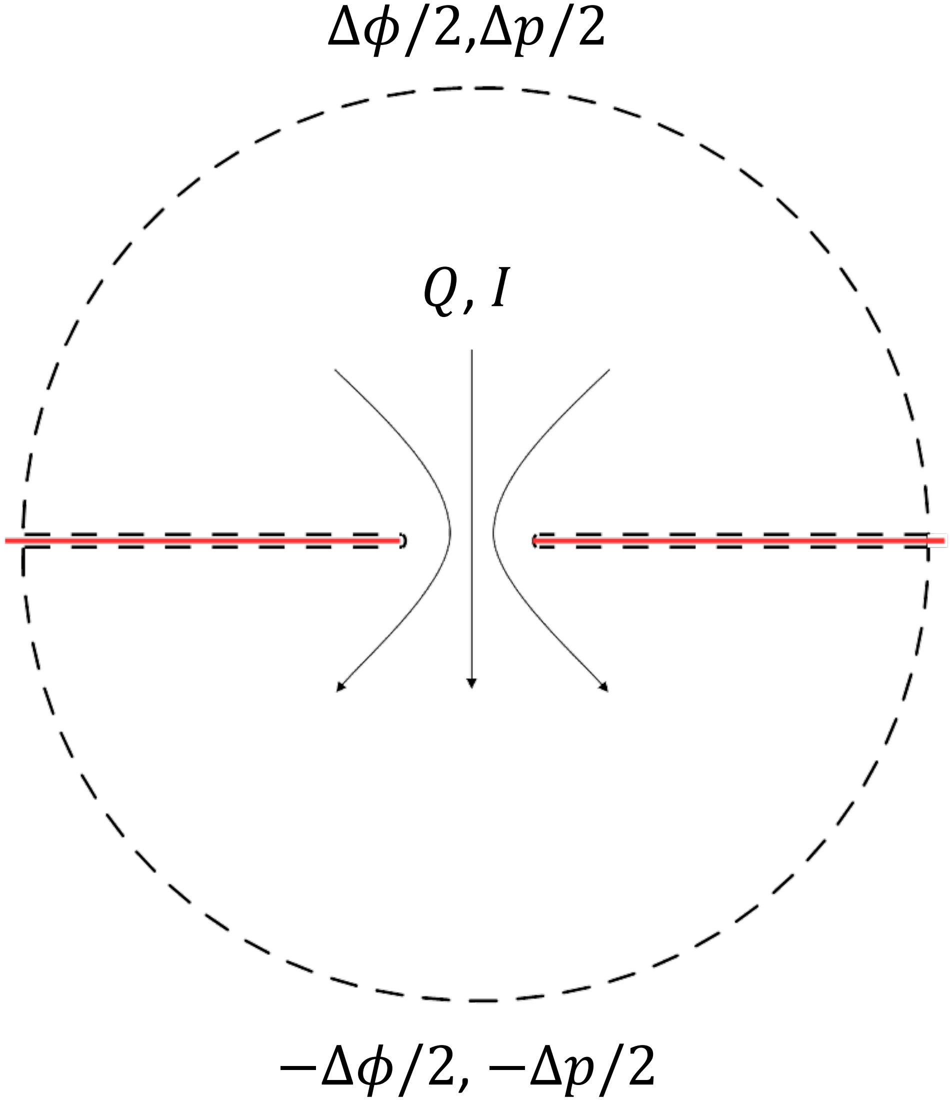
Nanopore in a membrane of zero thickness

Electric current

$$I = \frac{\Delta\phi}{R_m}$$

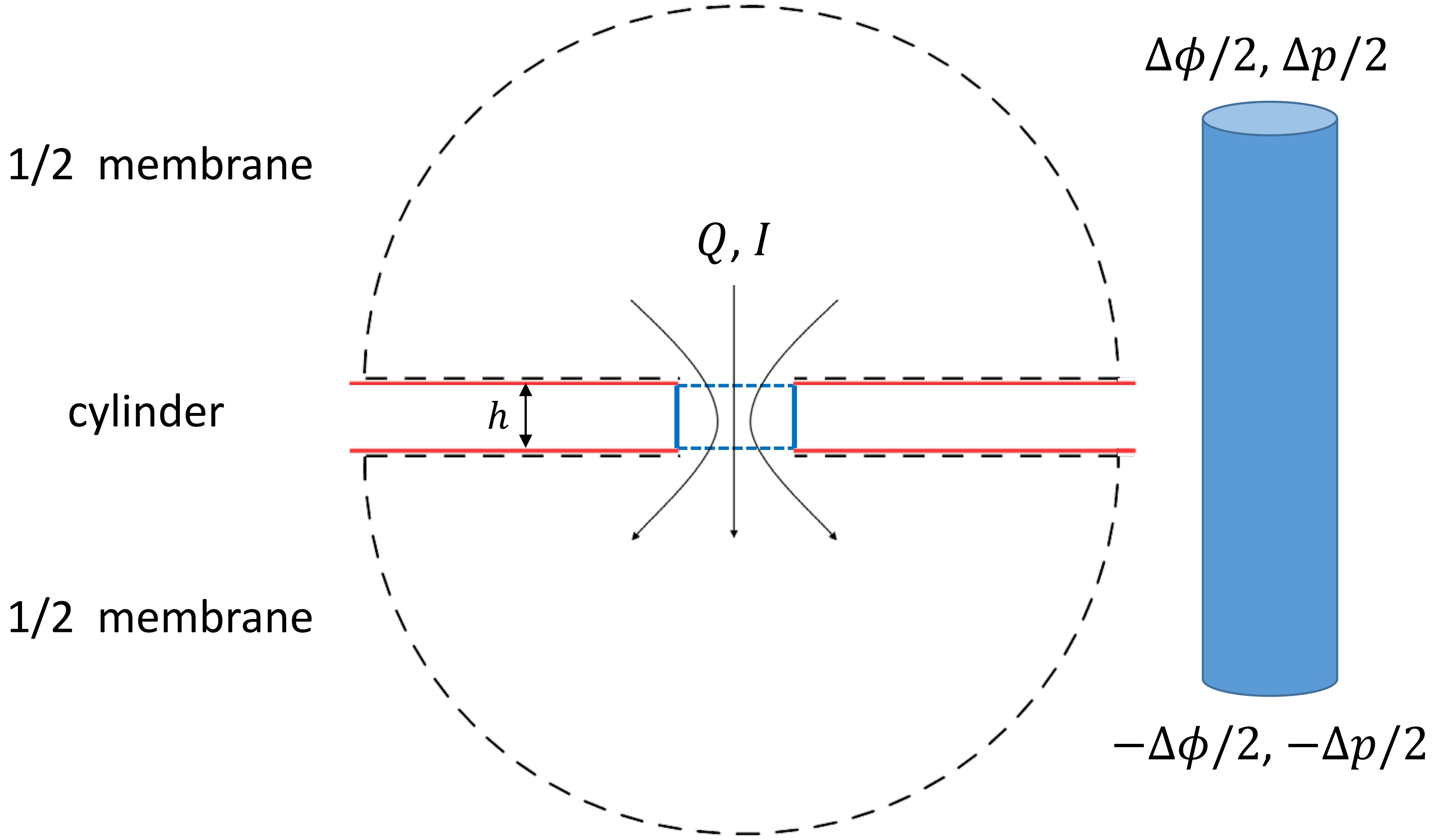


Volumetric flow rate

$$Q_{\Delta\phi} = H_m \Delta\phi$$
$$Q_{\Delta p} = G_m \Delta p$$

$$Q = Q_{\Delta p} + Q_{\Delta\phi}$$
$$= G_m \Delta p + H_m \Delta\phi$$

Zero to finite thickness

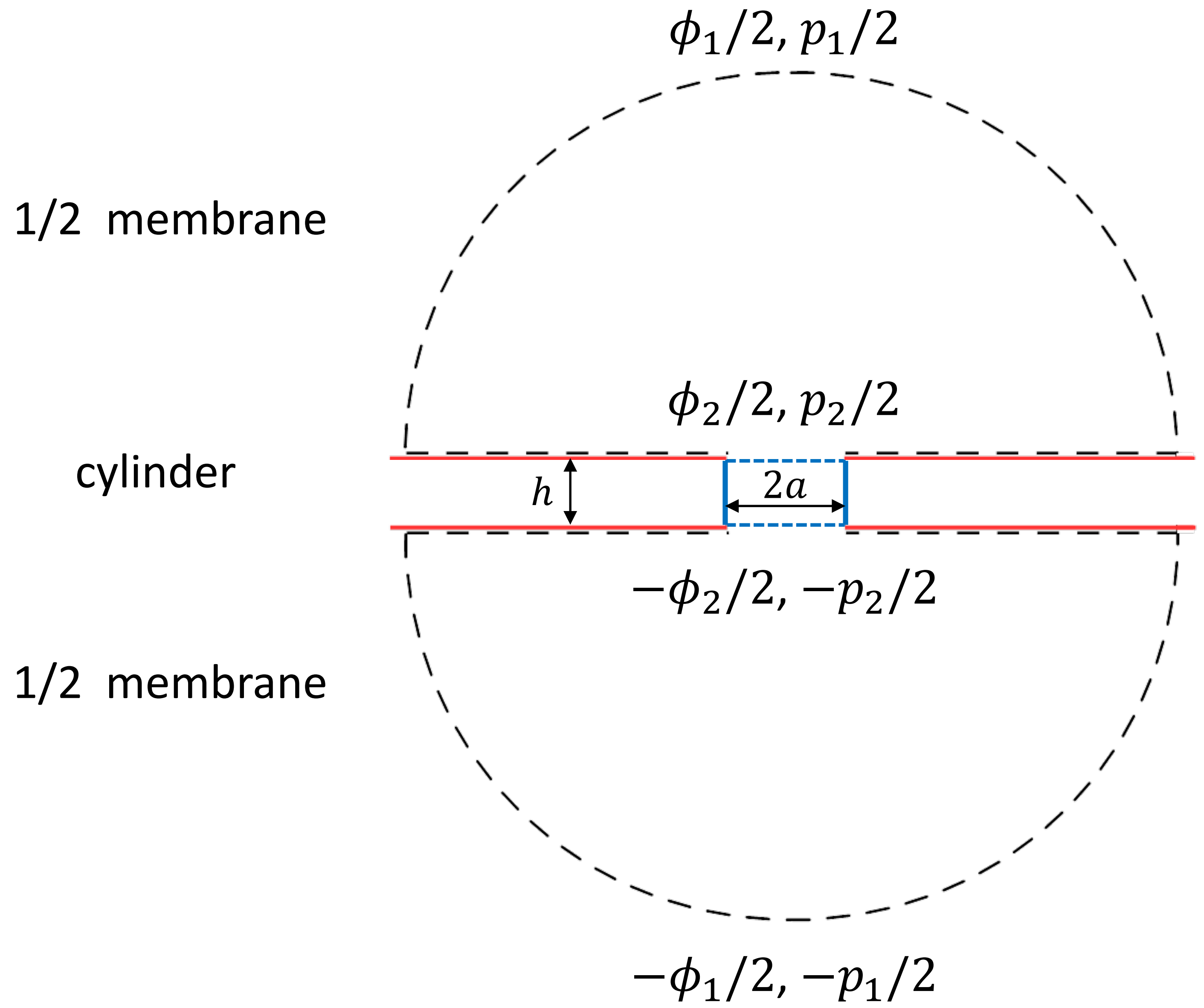


$$I = \frac{\Delta\phi}{R_c}$$

$$Q_{\Delta p} = G_c \Delta p$$
$$Q_{\Delta\phi} = H_c \Delta\phi$$

$$Q = Q_{\Delta p} + Q_{\Delta\phi}$$
$$= G_c \Delta p + H_c \Delta\phi$$

Continuity of I and Q



$$I = \frac{\frac{1}{2}(\phi_1 - \phi_2)}{\frac{1}{2} R_m} = \frac{\phi_2}{R_c}$$

$$Q = 2G_m \frac{(p_1 - p_2)}{2} + 2H_m \frac{(\phi_1 - \phi_2)}{2}$$
$$= G_c p_2 + H_c \phi_2$$

Composite electroosmotic conductance

$$I = \frac{\phi_1 - \phi_2}{R_m} = \frac{\phi_2}{R_c}$$

$$Q = G_m(p_1 - p_2) + H_m(\phi_1 - \phi_2) = G_c p_2 + H_c \phi_2$$

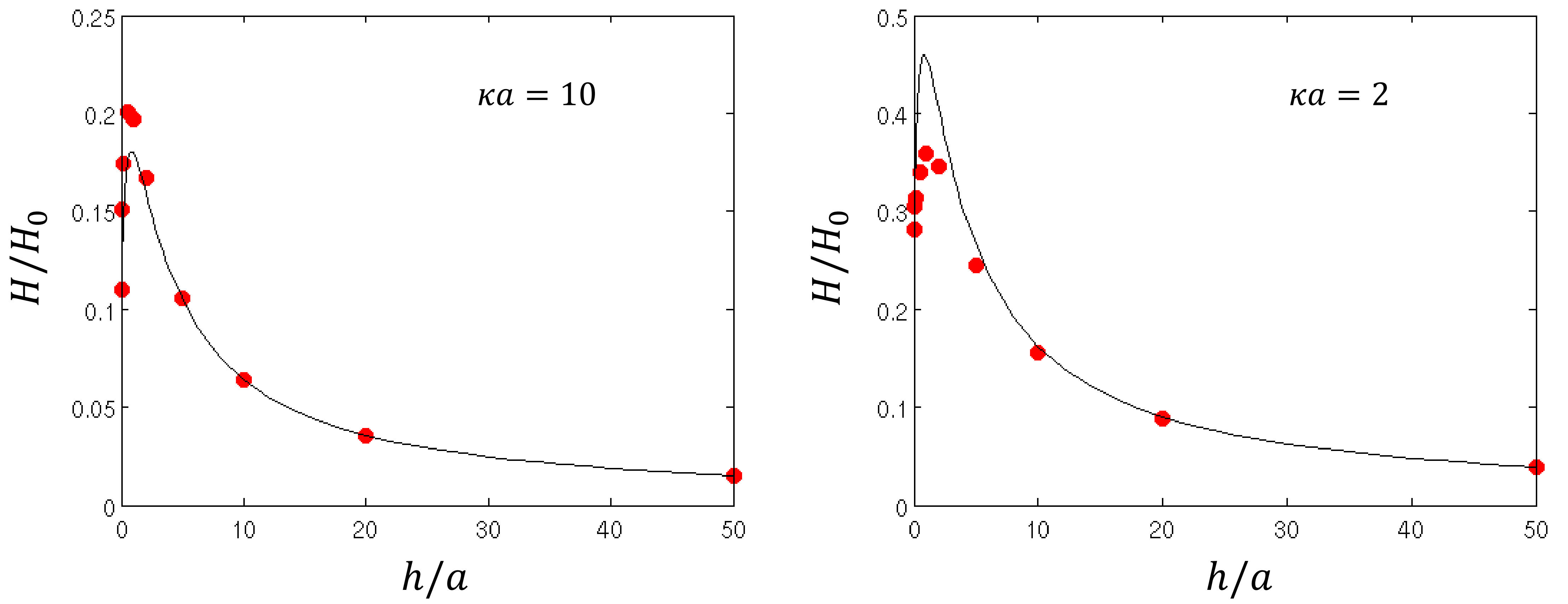
Set $p_1 = 0$ and $\phi_1 = \Delta\phi$

$$H_{comp} = \frac{Q}{\Delta\phi} = \frac{G_m R_c H_c + G_c R_m H_m}{(R_m + R_c)(G_m + G_c)} = \frac{H_m + \frac{16h^2}{3\pi^2 a^2} H_c}{\left(1 + \frac{2h}{\pi a}\right)\left(1 + \frac{8h}{3a}\right)}$$

$$H_m = \frac{2a^3}{\pi\mu}\int_0^{\pi/2} d\eta \int_0^{\infty} \rho_0 \frac{\cos^2\eta \sin\eta}{\cosh\xi} d\xi$$

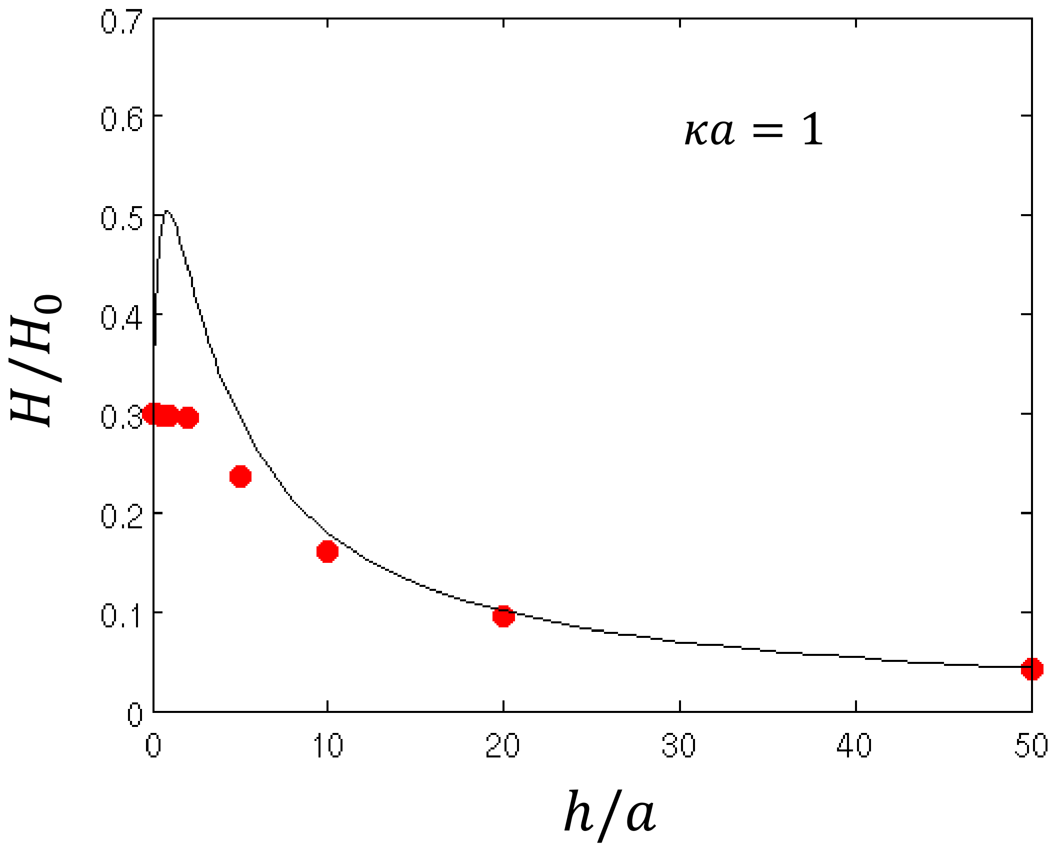
$$H_c = \frac{2\pi a^3 \sigma_c}{\mu h}\left[\frac{1}{2\kappa a}\frac{I_0(\kappa a)}{I_1(\kappa a)} - \frac{1}{(\kappa a)^2}\right]$$

Results

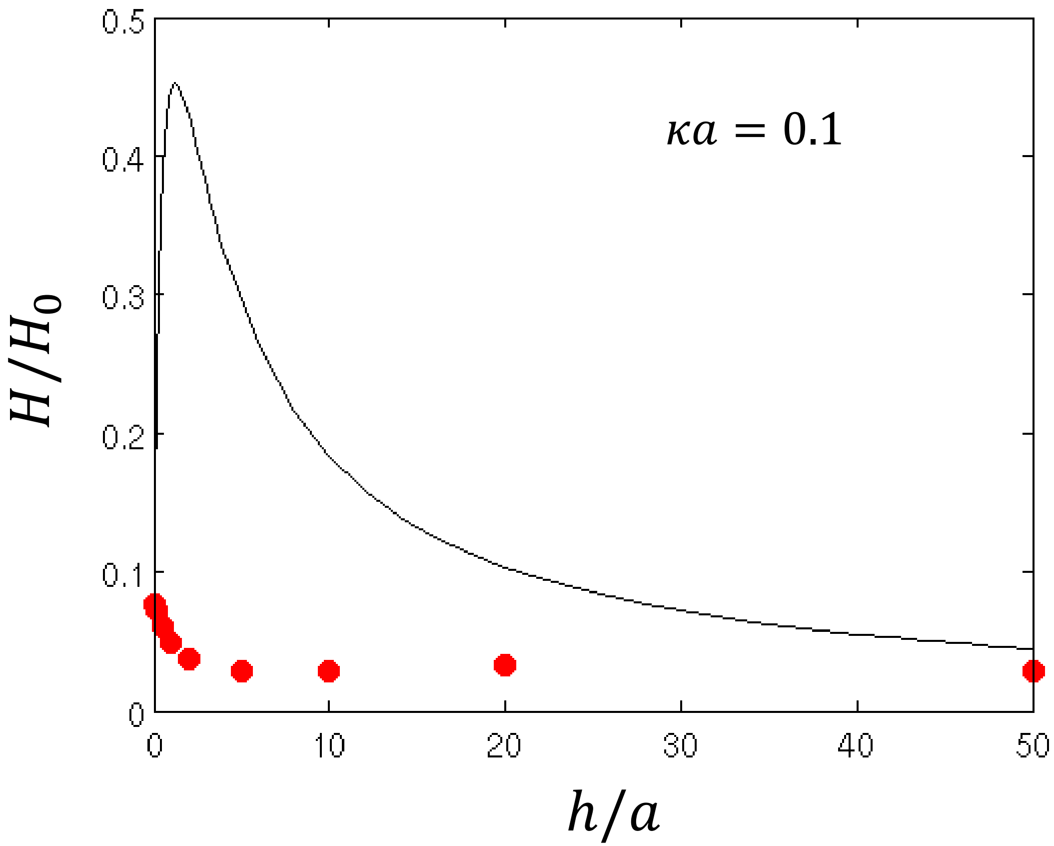


$\kappa a = 10$
H/H_0
h/a
0.25
0.2
0.15
0.1
0.05
0
0
10
20
30
40
50
$\kappa a = 2$
H/H_0
h/a
0.5
0.4
0.3
0.2
0.1
0
0
10
20
30
40
50

Results

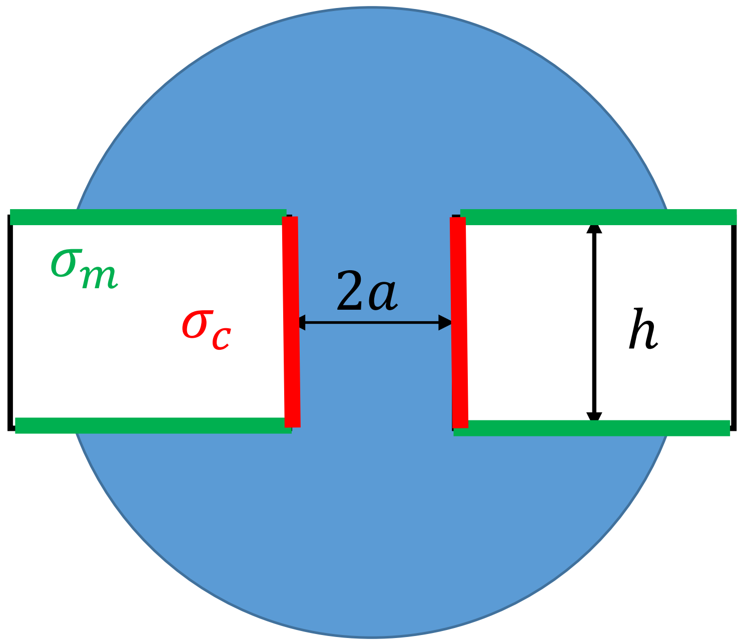


$\kappa a = 1$
H/H_0
h/a
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
10
20
30
40
50



$\kappa a = 0.1$
H/H_0
h/a
0
0.1
0.2
0.3
0.4
0.5
10
20
30
40
50

End effects: charge spill when $a\kappa$ small

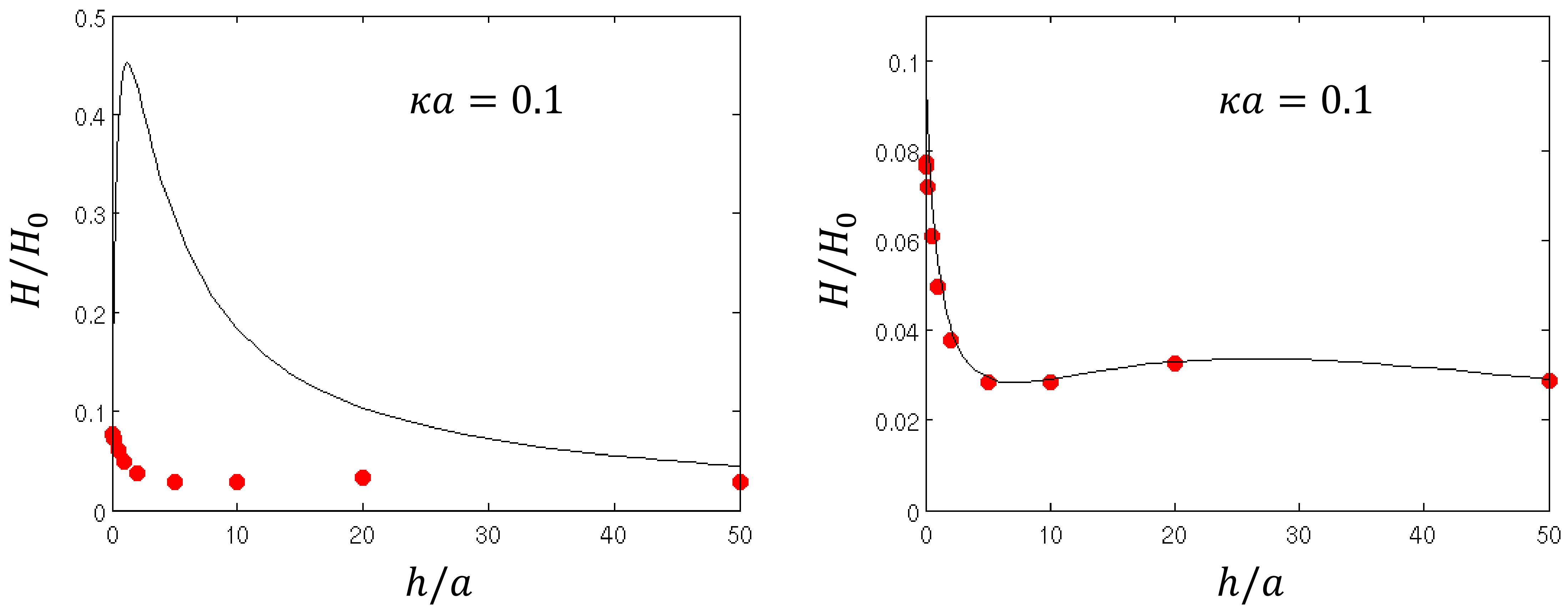


$$H_{comp} = \frac{Q}{\Delta\phi} = \frac{G_m R_c H_c + G_c R_m H_m}{(R_m + R_c)(G_m + G_c)} = \frac{H_m + \frac{16h^2}{3\pi^2 a^2} H_c}{\left(1 + \frac{2h}{\pi a}\right)\left(1 + \frac{8h}{3a}\right)}$$

	H_c	H_m
Spill not considered	$\frac{\pi a^3 \sigma_c}{4\pi h}$	$\frac{a^3 \kappa \sigma_m}{3\mu}$
Spill considered	$\frac{\pi a^3 \sigma_c}{4\pi}\left(\frac{h - h_{lost}}{h^2}\right)$	$\frac{a^3 \kappa}{3\mu}\left[\sigma_m + \frac{\pi \sinh(\kappa h/2)\sigma_c}{4\cosh(\kappa h/2) + \pi a\kappa \sinh(\kappa h/2)}\right]$

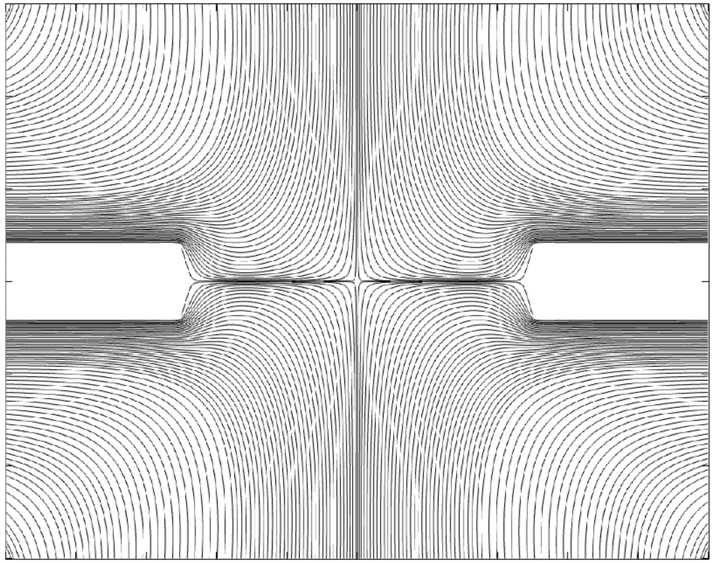
Langmuir **30** (31), 9261-9272;

Results – effects of charge spill

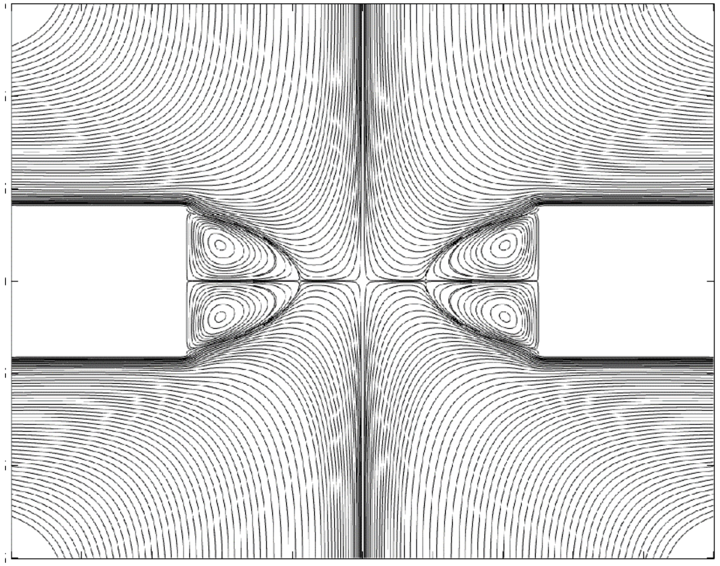


Eddies

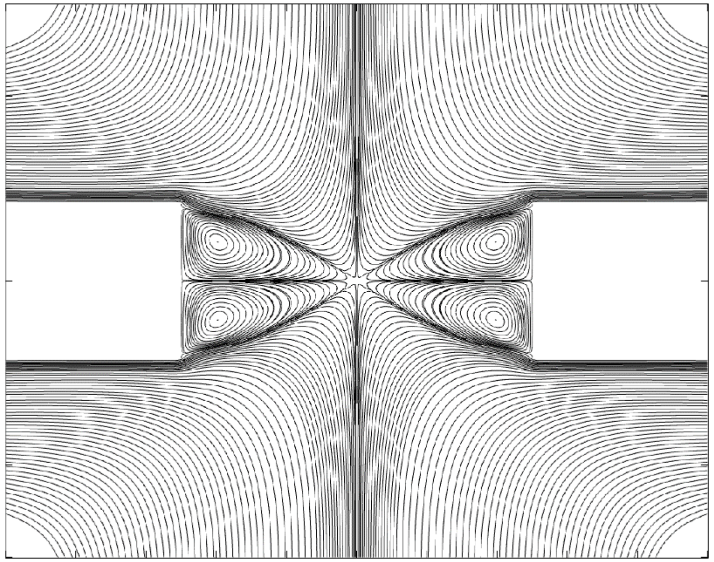
$\frac{h}{a} = 0.4$



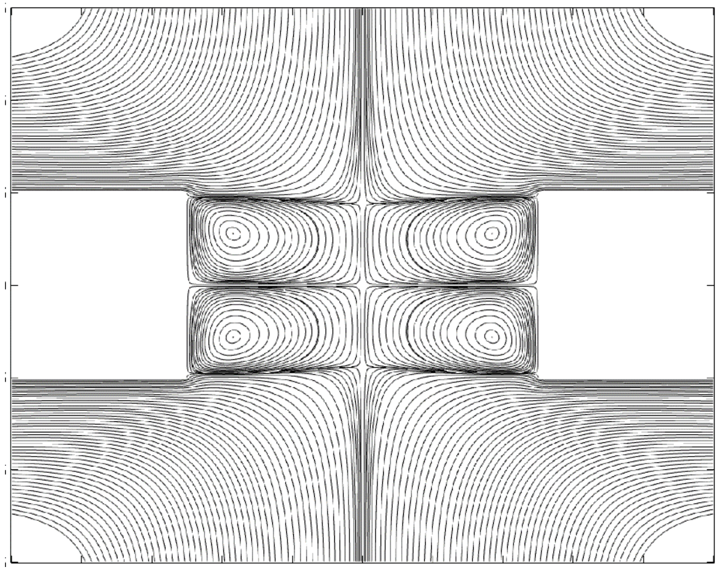
$\frac{h}{a} = 0.8$



$\frac{h}{a} = 0.84$



$\frac{h}{a} = 1$



Conclusion

- Composite model for electroosmotic conductance of nanopore when h is finite
- Model agrees with simulation when $\kappa a \gg 1$
- Charge spill needs to be considered when $\kappa a \ll 1$
- When membrane surfaces is not charged, induced charge electroosmosis generates pairs of toroidal counter rotating eddies

References
Journal of Fluid Mechanics **749**, 167-183;
Langmuir **30** (31), 9261-9272;
Physics of Fluids (in press).