Radiative Transfer through Interstellar Medium

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Dust is an important component of the interstellar medium, characterized by more broad opacities across the electromagnetic spectrum compared to other components such as atomic and molecular gas. This property can cause light from a background star to be blocked out from our line of view, or similarly can shield a region of interstellar gas allowing molecular formation. Thus, understanding how light is affected as it passes through a dust cloud is important for understanding galactic observations and processes such as planet formation. In this project, we modeled the propagation of radiation inside a three dimensional dust cloud. We achieved this with a Monte Carlo method that traces the reverse paths of photons from different points within the cloud to the surface of the cloud. By assigning weights to many photon paths to a single point within the cloud, based on the optical depth it travelled, we can approximate the likelihood of a photon reaching that point in the cloud from the surface. Based on the likelihoods of these paths, we can map out the relative light intensity across the whole cloud. This model gives insight on how the radiation field of dust clouds depends on factors such as light wavelength and dust density.

I. INTRODUCTION

While dust only makes up about 1-2% of the mass of the interstellar medium, it plays an important role in galactic processes, as well as our observations of the galaxy. These are a result of the broad opacity across the electromagnetic spectrum of dust clouds. While the opacities of other constituents of the interstellar medium such as atomic and molecular gas are due to absorption lines, which only cover specific wavelengths of the electromagnetic spectrum, dust clouds have the ability to block out large portions of the spectrum, a process known as extinction. This property can hinder our ability to observe objects of interest located behind a dust cloud in our field of view.

Dust clouds are generally more opaque at smaller, higher energy wavelengths. This can be seen in the extinction curve of a carbonaceous-silicate grain model, a typical composition of a Milky Way dust cloud. This gives rise to interstellar reddening, where a background object appears dimmer at shorter wavelengths. This phenomena can be corrected for with an understanding of how light is effected as it passes through a dust cloud.

This property also results in dust clouds shielding regions of interstellar medium from higher energy photons, allowing the formation of molecules in those regions. Molecular formation in protostellar clouds is an important process in the formation of planetary systems.

While there exists semianalytical solutions for the propagation of radiation through an interstellar dust cloud, they rely on the assumption of a uniformly dense cloud. This method ignores the possible effects of clumping within the cloud on extinction. On the other hand, a numerical model can simulate the radiation field across a cloud of varying density. This is a more realistic approach, as it is known that extinction can be highly variable in a dust cloud due to non-uniformity (Chandrasekhar & Munch 1950).

Our first task was to model a non-uniform dust cloud.

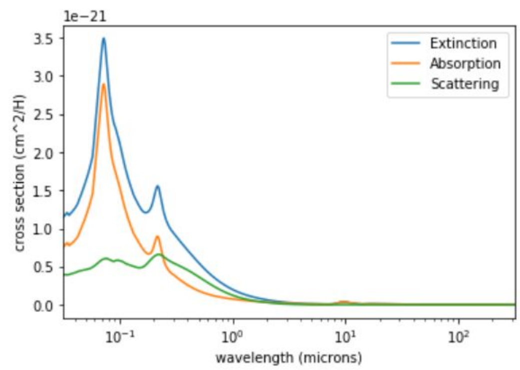

Figure 1: The extinction, absorption, and scattering of a carbonaceous-silicate grain model (Weingartner & Draine 2001) as a function of wavelength on the electromagnetic spectrum.

We use a hierarchical model, which randomly places new generations of dust particles near each particle of the previous generation. This effectively approximates the fractal characteristics of a clumping dust cloud. From this modeled cloud, we can determine the local densities at a given internal resolution of the cloud.

With local dust densities determined, we moved on to the core of the model, a Monte Carlo method for tracing the possible paths a photon could take through the cloud to a given point within the cloud. Our approach is to calculate the possible reverse paths of a photon in the cloud, starting from an extinction point within the cloud and tracing backwards to the edge of the cloud. As the photon's trajectory is mapped out, it collects a weight based on the optical path length of its trajectory, which is used to calculate the mean relative intensity (J_λ) at the initial point. In other words, J_λ at each point within the cloud is dependent on the probability of the photons that enter the cloud reaching that point. By carrying out this process across a large amount of points within the cloud, the radiation field can be effectively mapped out in a given cloud.

Our model allows us to visualize in three dimensions how the radiation field is effected by not only the density of the dust within a cloud, but also the wavelength of radiation. Since the opacity of dust particles is different at different wavelengths (Figure 1), the optical path length of a photon's trajectory and thus J_λ will depend on wavelength. For our tests, we used the typical Milky Way dust cloud model from Weingartner & Draine (2001) at 550 nm.

II. METHODS AND PROCEDURES

A. Creation of Dust Clouds

The procedure to generate the hierarchical cloud model is discussed in Mathis et al. (2002). This procedure is as follows. (1) Create a "supercube" with side-lengths L consisting of R^3 evenly sized cubic cells. (2) Place N points randomly inside this supercube. (3) Randomly place another N points within a distance $L/(2\Delta)$ at each point cast in the step before. The length Δ and the fractal dimension of the cloud D are related by $D = \log(N)/\log(\Delta)$. (4) Repeat this procedure i times. The total number of points will be on the order of N^i. (5) Shift any points that landed outside the cube by L until they all lie within the cube. (6) Calculate the point density of each cubic cell.

The dimension is the parameter that describes how "clumpy" the cloud is. Observed D values are between 2.3 and 2.6. With an N of 32 and D of 2.3, about 15% of the cubes are almost empty. Observations done by Sellgreen et al. (1992) of NGC 2023 and NGC 7023 however show material at all points within the structure. To model this, some constant density can be chosen. The randomly generated points can then be superimposed on this base density. The cloud itself is represented by the largest sphere that fits inside the supercube.

To create our model cloud, we used $N = 32$ and $D = 2.3$. We then multiplied the density of each cube by $1.3 * 10^{53}/a^3$ to match realistic dust cloud densities in a 1 pc wide cloud. Projections along each axis of our full model cloud are shown in Figure 3.

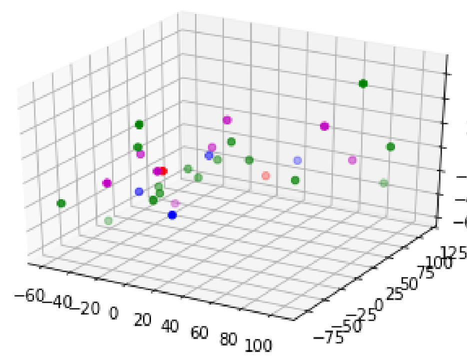

Figure 2: Demonstration of the Cloud Generation Procedure using N = 2 and D = 2.6, before any shifts have been made

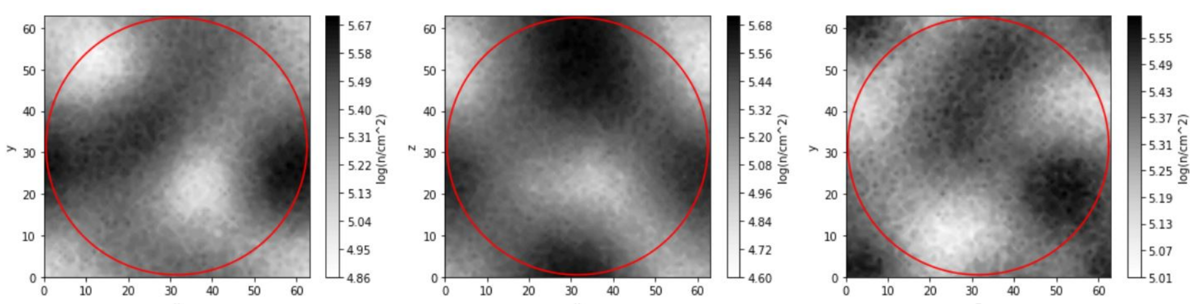

Figure 3: This plot shows the projection of the particle density onto each plane. This density array, generated using the method in Part A, was scaled by a factor of $1.3 * 10^{53}/a^3$ so that the values shown are number densities per cm^3 that are on the order of 1300 which is typical for our galaxy. The red line marks the surface of the cloud.

B. Reverse Monte Carlo for Radiative Transfer

As the photons travel through the gas clouds, there is some probability that the photon survives the course of its trajectory without being absorbed. We used a reverse Monte Carlo method based on this probability distribution to generate the most likely paths for the light to reach the surface of the cloud. We define a weight W that reflects this probability such that,

$$W = \exp(-\tau_a^{tot}), \qquad (1)$$

where τ_a^{tot}, is the total optical depth of absorption along the trajectory. These weights correspond to each path generated through the cloud such that the relative mean intensity can be calculated. We need to generate random paths from an observation point, A, to the external radiation field, a sphere of radius L/2 situated inside our supercube.

We generate K observation directions surrounding A called the $\hat{k}_{obs}$ vectors. These vectors extend to uniformly cover the surface of the cloud. We initialize each path from A to the surface by generating M reverse trajectories. The total length of each propagation is then equal to the optical depth of scattering, τ_s. There is a probability that the photon advances τ_s before scattering is equal to,

$$p = \exp(-\tau_s). \tag{2}$$

Rearranging yields,

$$\tau_s = -\ln(p), \tag{3}$$

where p is a number uniformly distributed between 0 and 1, using the numpy function *random.uniform*. This allows us to generate a probable free path through the cloud in the $\hat{k}_{obs}$ direction. We generate some τ_s' which we define as,

$$\tau_s' \equiv \Sigma \sigma_\lambda n_h \Delta l, \tag{4}$$

where Δl is some displacement along $\hat{k}_{obs}$ and σ_λ is the scattering cross section coefficient, typical values for the Milky Way can be found in Weingartner & Draine (2001). The sum here indicates the crossover of this displacement into a different individual cube with a new density, n_h. We must make this distinction because a cube which has no dust particles generated inside will not cause scattering but a cube who has a higher number of points generated inside will be more likely to scatter the photon. To maximize efficiency, we use a linear interpolation of the eight densities in the current direction allowing us to generate a displacement by solving Eq. 4 for Δl. This method allows for a maximum stepsize of a, where a is the side length of the cubical cell. We then increment in a stepwise fashion until we reach the generated value for τ_s to within a certain tolerance.

After this condition is met we update the trajectory, and τ_a for the step is equal to $(\omega^{-1} - 1)\tau_s$, where ω is the grain albedo from Weingartner & Draine (2001). The total optical depth for absorption of the path is then incremented by this amount.

1. *Parallelization*

Each observation point is independent from the others which allows us to run this program in parallel. The observations list was sliced into smaller arrays depending the amount of CPUs the machine has. We then ran the Monte Carlo algorithm in parallel using these smaller arrays. Longleaf was used to get the results for the test and sample cloud. Using 20 Xeon E5-2680 2.50 GHz CPUs and 40GB of RAM, we estimated a total computing time of about 166 hours.

Now that the next scattering event has been reached we choose a new direction, $\hat{k}_i$, randomly by sampling the phase function,

$$\Phi(\theta) = \frac{1 - g^2}{4\pi(1 + g^2 - 2g\cos\theta)^{3/2}}. \tag{5}$$

Here, θ is the polar angle and g is the asymmetry parameter, another wavelength dependent constant from Weingartner & Draine (2001). We can then integrate and invert equation 4 so that we obtain the following,

$$\theta(p) = \frac{(1 + g^2) - [(1 - g^2)/(1 - g + 2gp)]^2}{2g}, \tag{6}$$

again p is a random number uniformly distributed from 0 to 1. Then, the azumithal angle $\phi = 2\pi d$, where d is a different random number. We then use the Euler transformation to create the new direction vector based on the previous. We continue the above method until we reach the surface of our cloud, with the whole process being repeated M times for a single initial trajectory. We then repeat the process with the next $\hat{k}_{obs}$ vector until there are a total of KM trajectories with weights W_{km}. The mean relative intensity at the point A can then be defined as,

$$\frac{J(A)}{I_0} = \frac{1}{K}\sum_k^K \frac{I(\hat{k}_{obs}, A)}{I_0} = \frac{1}{KM}\sum_k^K \sum_m^M W_{km} \tag{7}$$

where K is set by the resolution of the cloud model, and the number M is determined by the cloud's optical properties but a value of 10 is sufficient for most cases.

We repeat this process for a number of A points within the cloud such that we can sufficiently map the intensity throughout the cloud.

For our test, we selected 10,000 observation points within the cloud. These points were drawn by randomly selecting x,y,z coordinates from the distribution of particles within the cloud along each axis. Thus, areas of higher dust density were weighted higher in our selection of observational points. This random drawing process is highlighted in Figure 4.

III. RESULTS AND CONCLUSIONS

1. *Method Errors*

To measure the success of our program we generated a cloud with uniform density ($n_h = 1300$, equivalent for each cube), an approach taken by Bethell et al. (2004). This case has a well defined mean relative intensity, such that running our algorithm using this example cloud as the test cloud, we can plot the predicted intensities from the algorithm against the plot of the model to see the delineation between the two.

We attempted to employ the analytical solution for a uniform cloud provided by Flannery et al. However, we were not able to compute certain parameters, and so we

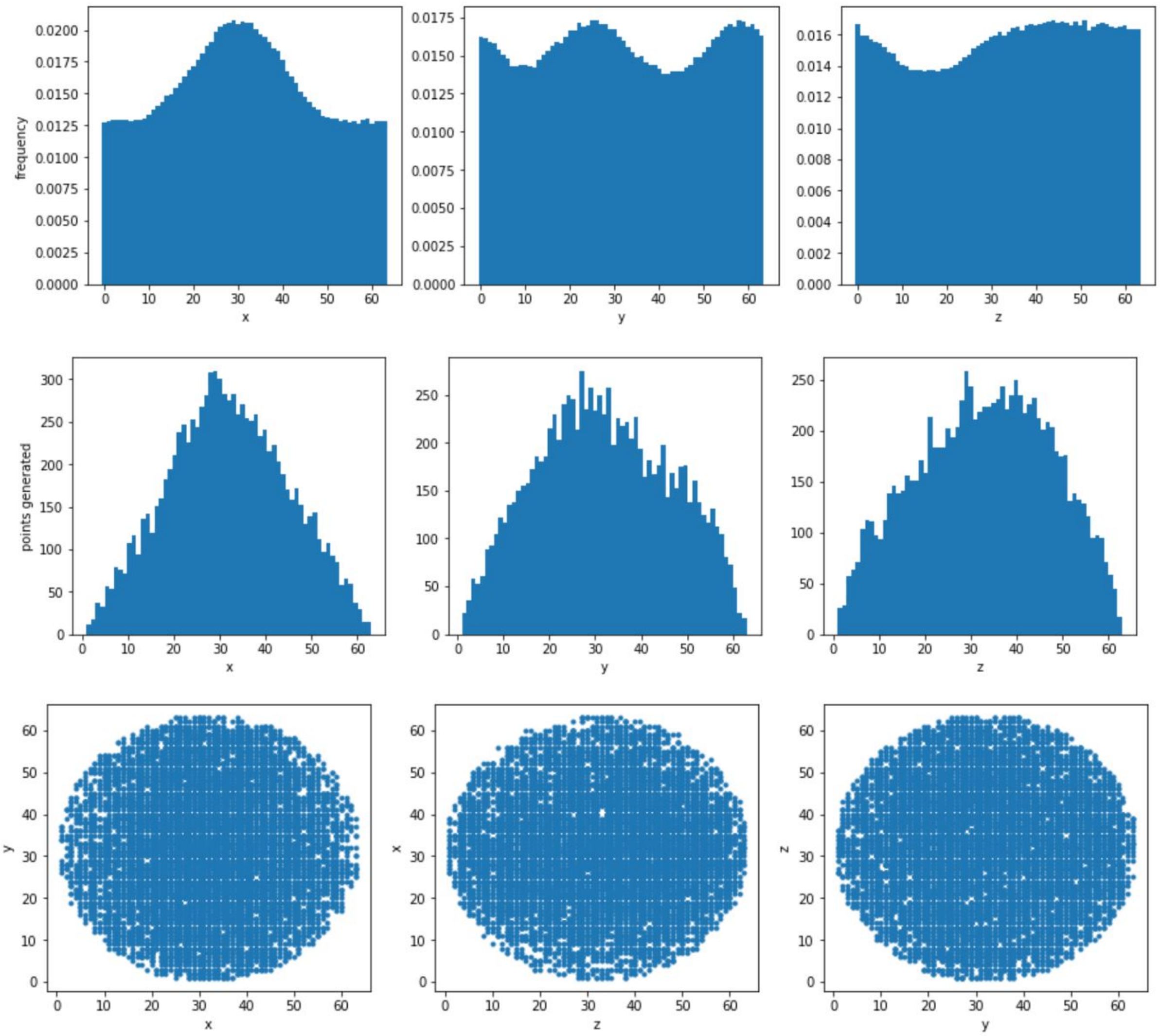

Figure 4: (Top) Distribution of particles along each of the three axes within the full supercube, from which we randomly drew our points from. (Middle) Distribution of points we drew, with points outside of the circular cloud being rejected. (Bottom) Projection maps of observation points one each plane.

could not evaluate the quantitative accuracy of our algorithm. Instead, the accuracy was evaluated qualitatively, with parameters for the analytical solution chosen such that it is on the same scale as our results (Figure 5). Based on the shape of the two we deemed our algorithm to be qualitatively accurate. Also in Figure 5, we see that the light intensity increases further from the cloud's center, which is what is expected given a uniform density.

A. Full Nonuniform Cloud

We ran our program with the generated cloud shown in Figure 3, and the 10,000 observational points shown in Figure 4. The resulting projection maps of the relative light intensity across each plane of the cloud are shown in Figure 6.

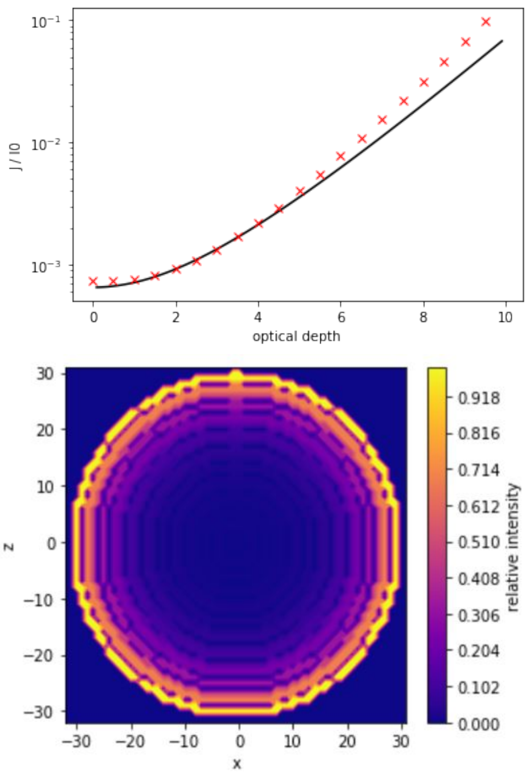

Figure 5: Above: A qualitative plot of the analytical solution (solid line) against the intensities generated by the algorithm. Below: A interpolation of the above plot to visualize the intensities along the cross sectional plane of the cloud.

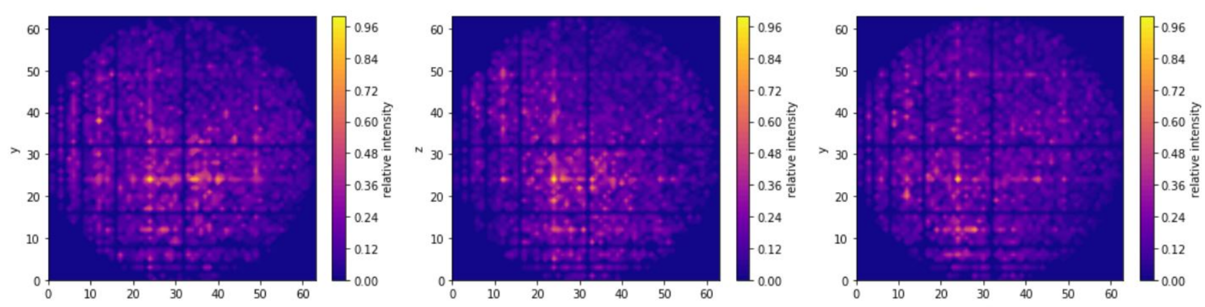

Figure 6: Projection maps of the relative light intensity across each plane of the dust cloud.

Looking at the relative intensities across the cloud in comparison to the dust density map, we can see a correlation between areas of low particle density and higher light intensity. This is expected, as areas of lower dust density should be easier to reach by photons, and thus have a higher light intensity.

Using this model, further tests can be performed to probe the behavior of light moving through dust clouds. Our model allows us to map the radiation field of the cloud for photons of wavelength 10^{-4} - 10^{4} microns. Thus, the radiation field of the cloud at different wavelengths can be modeled and compared.

It is also possible to use a different dust grain model to compare clouds of different composition. Any dust model can be used, so long as it provides the important parameters needed for the Monte Carlo calculations.

[1] Radiative Transfer in Dusty Media `http://www.ita.uni-heidelberg.de/~dullemond/lectures/radtrans\_2012/Chapter\_5.pdf`

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