

$$E_n = \sigma_{e^n}/2 = \begin{cases} 6 * 10^{-5} \\ 1 * 10^{-5} \\ 5.5 * 10^{-5} \end{cases}$$

以 σ_{e^2} 为零电势面

$$\begin{cases} V_A = +5 * 10^{-6} \\ V_B = -7 * 10^{-7} \end{cases} \Rightarrow U_{AB} = -2 * 10^{-7}$$

(2)

$$W = -U_{AB} \cdot q_0 = -2 * 10^{-15} J$$

(1)

$$V = \frac{1}{4\pi\epsilon_0 r} \cdot \frac{4}{3} \rho_e \pi R^3 = \frac{\rho_e R^3}{3\epsilon_0 r}$$

(2)

$$V = \frac{\rho_e R^2}{3\epsilon_0}$$

(3)

$$\vec{E} = \frac{4}{3} \rho_e \pi r^3 \frac{1}{4\pi\epsilon_0 r^2} = \frac{\rho_e r}{3\epsilon_0}$$

$$V = \frac{\rho_e R^2}{3\epsilon_0} - \int_R^r \frac{\rho_e r'}{3\epsilon_0} dr' = \frac{\rho_e R^2}{3\epsilon_0} - \left(\frac{\rho_e r^2}{6\epsilon_0} - \frac{\rho_e R^2}{6\epsilon_0} \right) = \frac{\rho_e R^2}{2\epsilon_0} - \frac{\rho_e r^2}{6\epsilon_0}$$

(1) 令内部圆柱面电荷线密度为 σ

$$E(r) = \frac{1}{2\pi r \epsilon_0} \sigma (R_1 < r < R_2)$$

$$- \int_{R_1}^{R_2} E(r) = 450V$$

$$\Rightarrow \frac{\sigma}{2\pi\epsilon_0} \ln\left(\frac{R_1}{R_2}\right) = 450V$$

$$\Rightarrow \sigma = \frac{900\pi\epsilon_0}{\ln(0.3)} C \cdot m^{-1}$$

(2)

$$E(r) = \frac{1}{2\pi r \epsilon_0} \sigma = \frac{450}{\ln(0.3)r} (V) (R_1 < r < R_2)$$

以无穷远点为势能零点

$$\vec{E} = \begin{cases} \frac{\rho_e r}{2\varepsilon_0} & r \leq R \\ \frac{\rho_e R^2}{2\varepsilon_0 r} & r > R \end{cases}$$

$$V = - \int_0^r \vec{E} \cdot d\vec{r} = \begin{cases} \frac{\rho_e r^2}{4\varepsilon_0} \\ \frac{\rho_e R^2}{4\varepsilon_0} + \frac{\rho_e R^2}{2\varepsilon_0} \ln \frac{r}{R} \end{cases}$$

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$$V_B = \int_{-\frac{l}{2}}^{\frac{l}{2}} \frac{1}{4\pi\varepsilon_0} \frac{l_e dx}{\sqrt{\left(\frac{x}{2}\right)^2 + b^2}}$$

$$= \int_{-\arctan \frac{l}{4b}}^{\arctan \frac{l}{4b}} \frac{1}{2\pi\varepsilon_0} \frac{l_e}{\cos \theta} d\theta$$

$$= - \int_{-\arctan \frac{l}{4b}}^{\arctan \frac{l}{4b}} \frac{1}{2\pi\varepsilon_0} \frac{l_e}{1 - \sin^2 \theta} d\sin \theta$$

$$= - \frac{1}{2\pi\varepsilon_0} \frac{l_e}{2} \ln \left(\frac{1+x}{1-x} \right) \Bigg|_{x=-\frac{l}{\sqrt{16b^2+l^2}}}^{x=\frac{l}{\sqrt{16b^2+l^2}}}$$

$$= - \frac{l_e}{2\pi\varepsilon_0} \ln \left(\frac{1+x}{1-x} \right) \Bigg|_{x=-\frac{l}{\sqrt{16b^2+l^2}}}^{x=\frac{l}{\sqrt{16b^2+l^2}}}$$

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(1)

$$V_P = \int_{R_1}^{R_2} \left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{2\pi r \sigma_e dr}{\sqrt{r^2 + x^2}} \right)$$

$$= \int_{R_1}^{R_2} \left(\frac{r \sigma_e dr}{2\varepsilon_0 \sqrt{r^2 + x^2}} \right)$$

$$= \frac{\sigma_e}{2\varepsilon_0} \left(\sqrt{R_2^2 + x^2} - \sqrt{R_1^2 + x^2} \right)$$

(2)

令原点 O 的电势为

$$V = \frac{\sigma_e}{2\varepsilon_0} (R_2 - R_1)$$

则无穷远处电势为

$$V = \frac{\sigma_e}{2\varepsilon_0} \left(\sqrt{R_2^2 + x^2} - \sqrt{R_1^2 + x^2} \right)$$

$$= \frac{\sigma_e}{2\varepsilon_0} \left(\sqrt{1 + \left(\frac{R_2}{x} \right)^2} - \sqrt{1 + \left(\frac{R_1}{x} \right)^2} \right)$$

$$\approx \frac{\sigma_e (R_2^2 - R_1^2)}{4x\varepsilon_0} \approx 0$$

因而得到

$$\begin{aligned}\frac{1}{2}mv_{min}^2 &= \frac{\sigma_e}{2\varepsilon_0}(R_2 - R_1) \\ \Rightarrow v_{min} &= \sqrt{\frac{\sigma_e}{m\varepsilon_0}(R_2 - R_1)}\end{aligned}$$

(不是，这计算就是以无穷远点为势能零点算的啊，我还再算一次干嘛)

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作法类似上一题（懒得算了）

$$V = \frac{\sigma_e}{2\varepsilon_0} \sqrt{R_2^2 + \left(\frac{R_2}{\tan \theta}\right)^2} + \frac{\sigma_e}{2\varepsilon_0} \sqrt{R_1^2 + \left(\frac{R_1}{\tan \theta}\right)^2}$$

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$$\begin{aligned}V &= \frac{\sigma_e}{2\varepsilon_0} \left(\sqrt{R_2^2 + x^2} - \sqrt{R_1^2 + x^2} \right) \\ E &= -\frac{\partial V}{\partial x} = -\frac{\sigma_e}{2\varepsilon_0} \left(\frac{x}{\sqrt{R_2^2 + x^2}} - \frac{x}{\sqrt{R_1^2 + x^2}} \right)\end{aligned}$$