1-10

由对称性

$$E_x = E_y = 0$$

对于 E_z

$$\begin{aligned}
E_z &= \frac{\sigma_e}{4\pi\varepsilon_0}\int_0^{2\pi} d\theta \int_{\frac{\pi}{2}}^{\pi} d\phi \left[\sin\phi \frac{\cos\phi}{R^3}\right] \\
&= \frac{\sigma_e}{2\varepsilon_0}\int_{\frac{\pi}{2}}^{\pi} d\phi \left[\sin\phi \frac{\cos\phi}{R^3}\right] \\
&= \frac{\sigma_e}{4\varepsilon_0}\int_{\frac{\pi}{2}}^{\pi} d\phi \left[\sin 2\phi \frac{1}{R^3}\right] \\
&= \frac{\sigma_e}{4\varepsilon_0 R^3}
\end{aligned}$$

1-12

我们记

$$\begin{gathered}
R = 0.5\mathrm{m} \quad l = 2\mathrm{cm} \quad q = 3.12\times 10^{-9}\mathrm{C} \\
\sigma = q/(2\pi R - l) \sim q/2\pi R
\end{gathered}$$

我们假定圆环无缺，则又对称性，中心电场强度为 0

而缺少部分若存在，由对称性对圆心的电场强度近似为

$$\begin{aligned}
E_y &= \frac{\sigma}{4\pi R}\int_{-\frac{l}{2\pi R}}^{\frac{l}{2\pi R}} R d\theta \frac{\cos\theta}{R^2} \\
&= \frac{\sigma}{4\pi R^2}(\sin\theta)\bigg|_{-\frac{l}{2\pi R}}^{\frac{l}{2\pi R}} \\
&\approx \frac{\sigma l}{4\pi^2 R^3} \\
&\approx \frac{ql}{8\pi^3 R^4} \\
&\approx 4.02499\times 10^{-12} N/C
\end{aligned}$$

方向指向缺口对面，而缺失则导致剩余部分有一个强度为 E_y 指向缺口处的场强

1-13

由对称性 $E_x = E_y = 0$

对于 E_z

$$
\begin{aligned}
E_z &= \frac{Q/(2\pi RL)}{4\pi\varepsilon_0}\int_0^L 2\pi R dz \frac{L+a-z}{\left(\sqrt{(L+a-z)^2+R^2}\right)^3} \\
&= -\frac{Q}{8\varepsilon_0\pi L}\int_0^L \frac{d(L+a-z)^2}{\left(\sqrt{(L+a-z)^2+R^2}\right)^3} \\
&= \frac{Q}{4\varepsilon_0\pi L}\int_0^L d\frac{1}{\sqrt{(L+a-z)^2+R^2}} \\
&= \frac{Q}{4\varepsilon_0\pi L}\left(\frac{1}{\sqrt{a^2+R^2}}-\frac{1}{\sqrt{(L+a)^2+R^2}}\right)
\end{aligned}
$$

1-14

由对称性 $E_y = E_z = 0$

对于 E_x

$$
\begin{gathered}
E_x = \sigma_e \int_0^\pi R\mathrm{d}\theta\left(-\frac{\sin\theta}{2\pi R\varepsilon_0}\right) \\
= \frac{\sigma_e}{\pi R\varepsilon_0}
\end{gathered}
$$

1-15

考虑对称性，$E_y = E_z = 0$

对于 E_x 有

$$
\begin{aligned}
&E_x = \frac{\sigma_e}{4\pi\varepsilon_0}\left(\int_{_0}^\pi a\mathrm{d}\theta\left(-\frac{\sin\theta}{a^2}\right)+2\int_0^{\frac{\pi}{2}}\frac{a}{\cos^2\theta}d\theta\left(-\frac{\sin\theta}{(a/\cos\theta)^2}\right)\right) \\
&= \frac{\sigma_e}{4\pi\varepsilon_0}\left(\int_{_0}^\pi \mathrm{d}\theta\left(-\frac{\sin\theta}{a}\right)+2\int_0^{\frac{\pi}{2}}d\theta\left(\frac{\sin\theta}{a}\right)\right) \\
&= \frac{\sigma_e}{4\pi\varepsilon_0}\left(\int_{_0}^\pi \mathrm{d}\theta\left(-\frac{\sin\theta}{a}\right)+\int_0^{\pi}d\theta\left(\frac{\sin\theta}{a}\right)\right) \\
&= 0
\end{aligned}
$$

1-18

由高斯定理，有

$$
\nabla\cdot\vec{E} = 0 \Rightarrow \rho = 0 \Rightarrow q = 0
$$

1-20

由高斯定理，以及氢原子的球对称性，距原子核 r 处的电场方向为原子核指向此处，大小为

$$
\begin{aligned}
E &= \frac{1}{4\pi r^2\varepsilon_0}\left(q+\int_0^r \mathrm{d}R\int_0^{2\pi}\mathrm{d}\theta\int_0^{\pi}\mathrm{d}\phi R^2\sin\phi\left(\rho_e\left(r\right)\right)\right)\\
&= \frac{1}{4\pi r^2\varepsilon_0}\left(q+\int_0^r \mathrm{d}R\int_0^{2\pi}\mathrm{d}\theta\int_0^{\pi}\mathrm{d}\phi R^2\sin\phi\left(-\frac{q}{\pi a_0^3}e^{-2R/a_0}\right)\right)\\
&= \frac{1}{4\pi r^2\varepsilon_0}\left(q+4\int_0^r \mathrm{d}RR^2\left(-\frac{q}{a_0^3}e^{-2R/a_0}\right)\right)\\
&= \frac{1}{4\pi r^2\varepsilon_0}\left(q+\frac{2r^2q}{a_0^2}e^{-2r/a_0}-4\int_0^r \mathrm{d}R\frac{qR}{a_0^2}e^{-2R/a_0}\right)\\
&= \frac{1}{4\pi r^2\varepsilon_0}\left(q+\frac{2r^2q}{a_0^2}e^{-2r/a_0}+2r\frac{q}{a_0}e^{-2r/a_0}-2\int_0^r \mathrm{d}R\frac{q}{a_0}e^{-2R/a_0}\right)\\
&= \frac{q}{4\pi r^2\varepsilon_0}\left(\frac{2r^2}{a_0^2}e^{-2r/a_0}+\frac{2r}{a_0}e^{-2r/a_0}+e^{-2r/a_0}\right)
\end{aligned}
$$

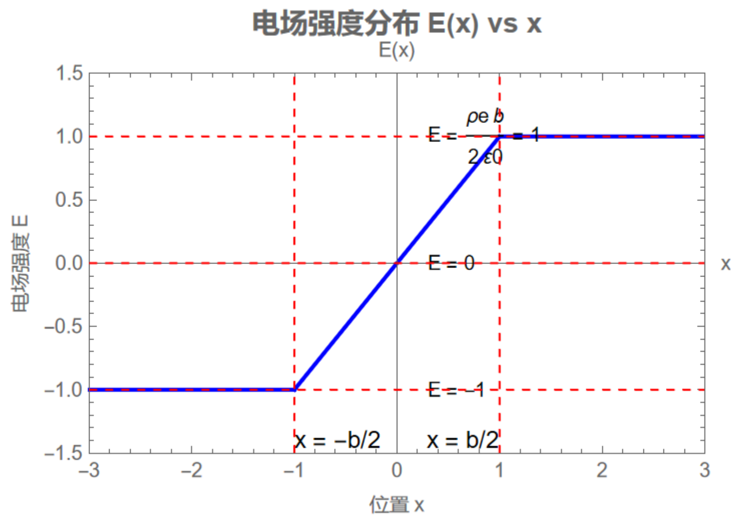
1-25

由高斯定理和对称性，有

$$
\begin{aligned}
E &= \frac{1}{2\pi r\varepsilon_0}\int_0^r \mathrm{d}R\int_0^{2\pi}\mathrm{d}\theta R\left(\rho_e\left(R\right)\right)\\
&= \frac{1}{2\pi r\varepsilon_0}\int_0^r \mathrm{d}R\int_0^{2\pi}\mathrm{d}\theta\frac{R\rho_0}{\left[1+\left(R/a\right)^2\right]^2}\\
&= \frac{1}{r\varepsilon_0}\int_0^r \mathrm{d}R\frac{R\rho_0}{\left[1+\left(R/a\right)^2\right]^2}\\
&= \frac{a^2\rho_0}{2r\varepsilon_0}\left(1-\frac{1}{1+r^2/a^2}\right)
\end{aligned}
$$

1-27

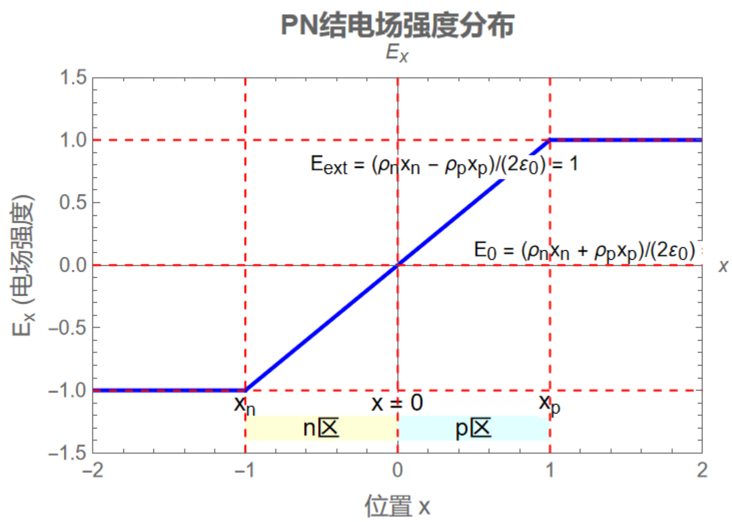
$$
\begin{aligned}
|x| \geqslant \frac{b}{2} &: E = \frac{\rho_e b}{2\varepsilon_0}.\frac{x}{|x|}\\
|x| < \frac{b}{2} &: E = \frac{\rho_e x}{\varepsilon_0}
\end{aligned}
$$



1-28

由 $1-27$ 可知

$$x \geqslant x_p \ or \ x \leqslant x_n: E=\left(\frac{\rho_n x_n}{2\varepsilon_0}-\frac{\rho_p x_p}{2\varepsilon_0}\right)\cdot\frac{x}{|x|}$$
$$-x_n < x \leqslant 0: E=\frac{\rho_n}{\varepsilon_0}\left(x+\frac{x_n}{2}\right)+\frac{\rho_p x_p}{2\varepsilon_0}$$
$$0 < x < x_p: E=-\frac{\rho_p}{\varepsilon_0}\left(x-\frac{x_p}{2}\right)+\frac{\rho_n x_n}{2\varepsilon_0}$$



1-29

$$E=\left(\frac{\sigma_e}{\varepsilon_0}-\frac{1}{\varepsilon_0}\int_0^R \mathrm{d}r\int_0^{2\pi}\mathrm{d}\theta r\sigma_e\right)\cdot\frac{x}{|x|}$$
$$=\frac{\sigma_e}{\varepsilon_0}\left(1-\pi R^2\right)\cdot\frac{x}{|x|}$$