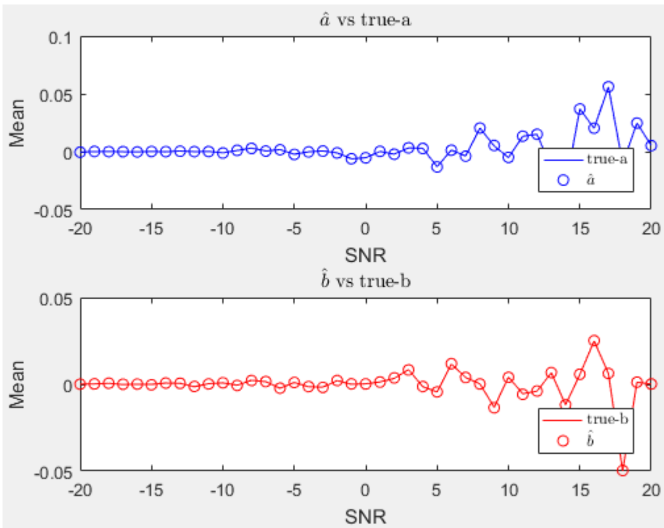
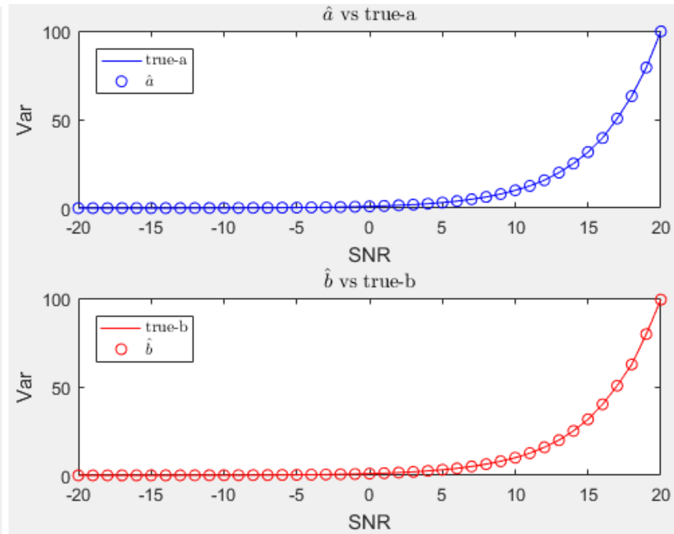
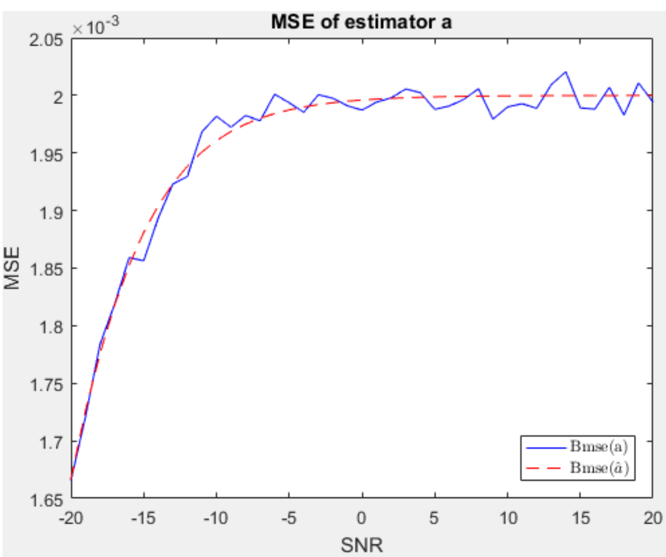
1.(c) N = 1000

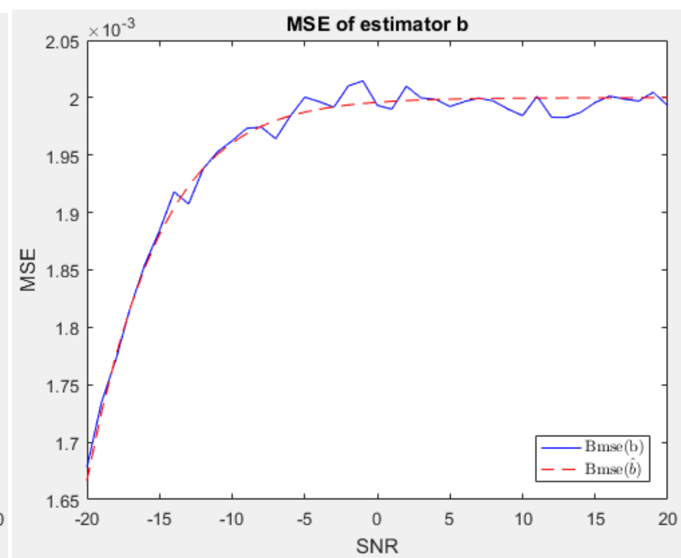




(Fig.1) Mean of estimation VS true θ by SNR

(Fig.2) Variance of estimation VS true θ by SNR





(Fig.3) Bmse(a) vs Bmse(â)by SNR

(Fig.4) Bmse(b) vs Bmse($\hat{b}$)by SNR

1.(d)

By Fig.2. With SNR increasing, variance of both BMSE(θ) and true θ would increase exponentially.

By Fig.1. The accuracy of the “mean” of the estimator is always high, which isn’t affected by SNR. The result shows that the error of MMSE estimator is zero on the average.

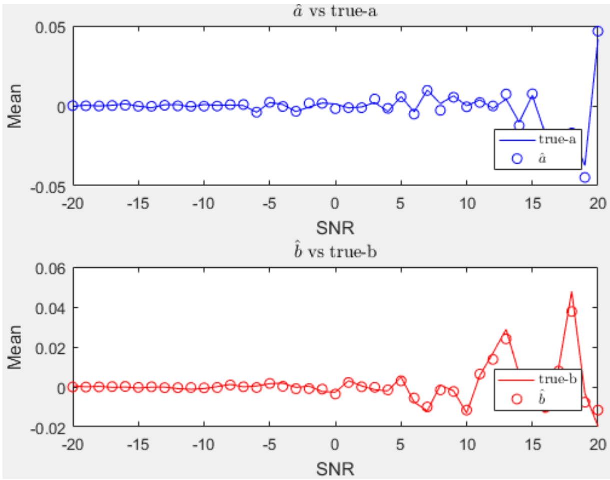
$$
\begin{aligned}
E_{x,\theta}(\epsilon) &= E_{x,\theta}\left[\theta - E(\theta|\mathbf{x})\right] \\
&= E_x\left[E_{\theta|x}(\theta) - E_{\theta|x}(\theta|\mathbf{x})\right] \\
&= E_x\left[E(\theta|\mathbf{x}) - E(\theta|\mathbf{x})\right] = 0
\end{aligned}
$$

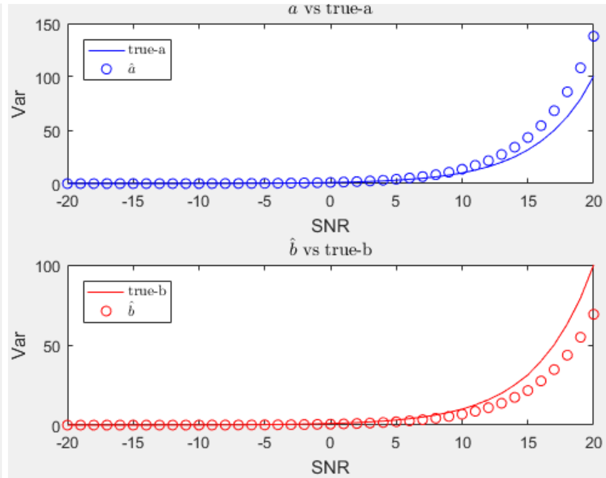
By Fig.3, Fig.4.

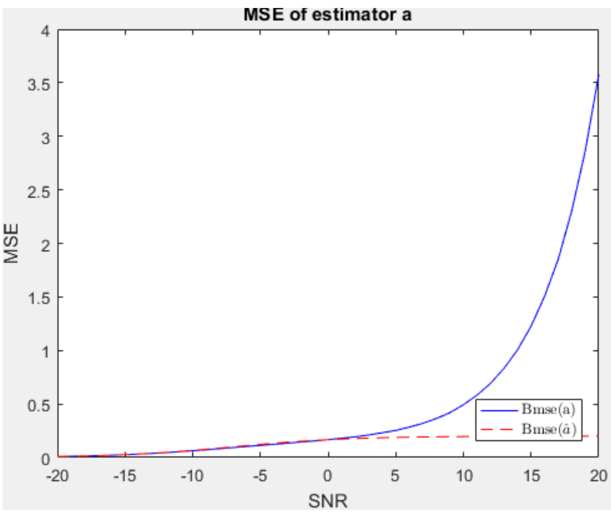
1. Due to the increasing variance of θ and bias^2. The mean square error of both estimators increase as SNR increase logarithmically.
2. The trends of simulation Bmse and theroretical are the same. It shows that the proof in problem(b) is correct!!

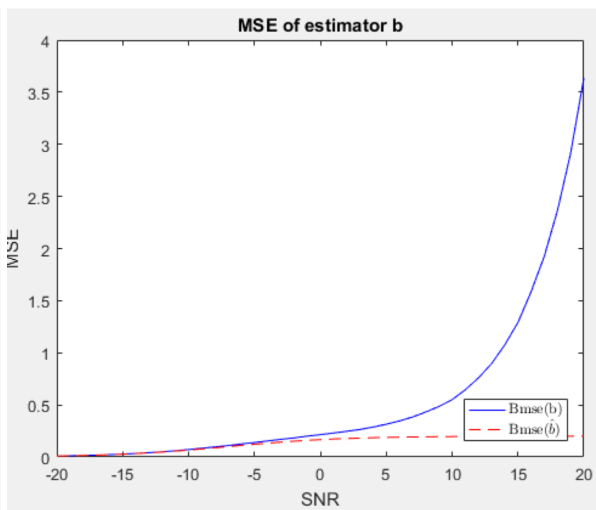
1.(e)

N=10

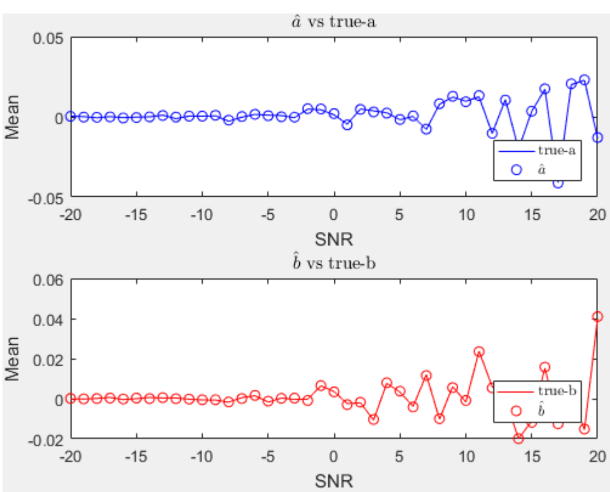

$\hat{a}$ vs true-a
Mean
SNR
true-a
$\hat{a}$
$\hat{b}$ vs true-b
Mean
SNR
true-b
$\hat{b}$

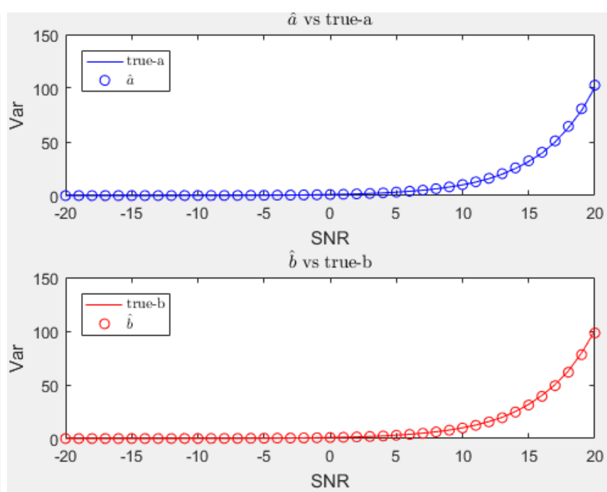

a vs true-a
Var
SNR
true-a
$\hat{a}$
$\hat{b}$ vs true-b
Var
SNR
true-b
$\hat{b}$

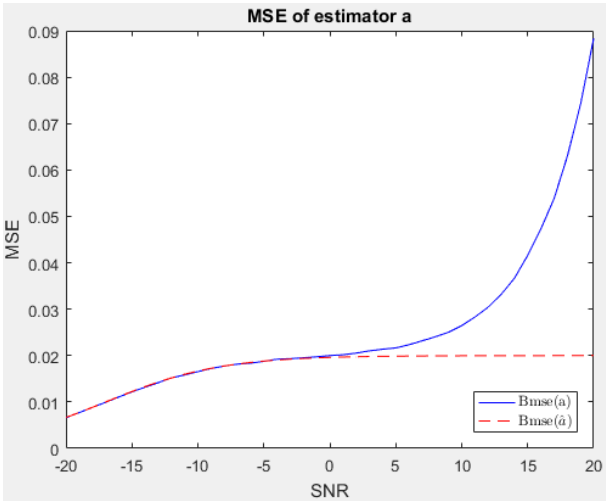

MSE of estimator a
MSE
SNR
Bmse(a)
Bmse($\hat{a}$)

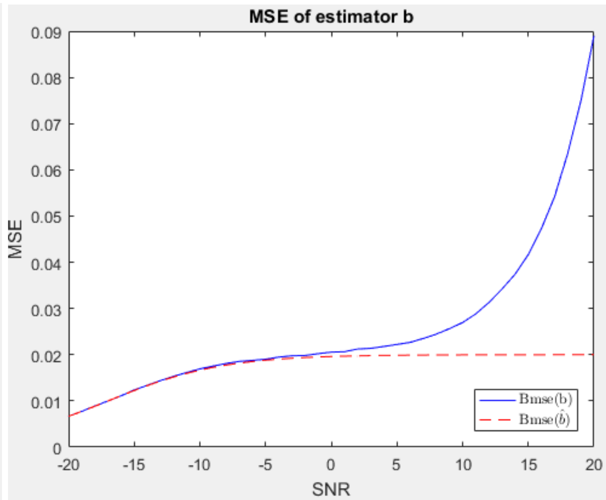

MSE of estimator b
MSE
SNR
Bmse(b)
Bmse($\hat{b}$)

N=100


$\hat{a}$ vs true-a
Mean
SNR
true-a
$\hat{a}$
$\hat{b}$ vs true-b
Mean
SNR
true-b
$\hat{b}$

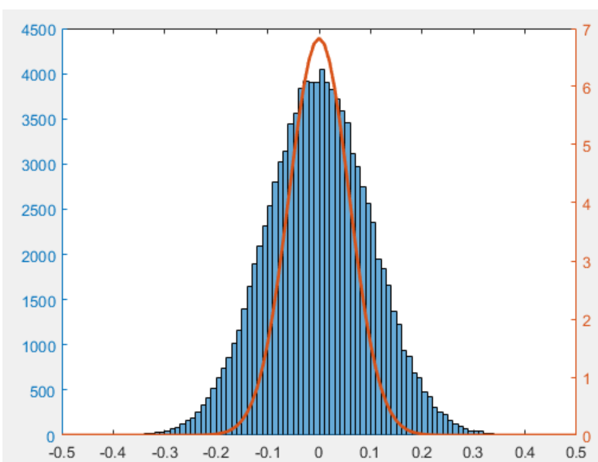

$\hat{a}$ vs true-a
Var
SNR
true-a
$\hat{a}$
$\hat{b}$ vs true-b
Var
SNR
true-b
$\hat{b}$



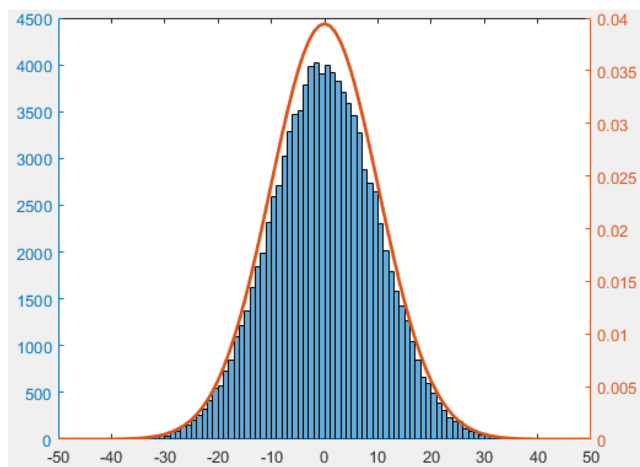


1. With N increasing, MSE deceased(When N=10, MSE =[0,4]. When N=1000, MSE=[$1.6*10^{-3}$,$2*10^{-3}$])
2. If N is small, Bmse(θ) and Bmse($\hat{\theta}$) has same tendency only in small SNR. After tangent of Bmse(θ) is positive, Bmse($\hat{\theta}$) still flat(converge to a constant). This phenomenon is because of the approximation of H^TH. We proof the Bmse by assume N is large enough which would result in H^TH = N/2.
3. Due to MSE growths exponentially as SNR increases when N is small(N=10), but it growths logarithmically when N is large(N=1000)

[OTHER DISCUSSION]



(Fig.5) N=1000, SNR=-20dB, theoretical PDF VS Simulation



(Fig.6) N=1000, SNR=20dB, theoretical PDF VS Simulation

By Fig.5, Fig.6. When SNR is -20dB, the theoretical PDF is more concentrated than simulation data. But when SNR is 20dB(the higher one), the simulation data fit the theoretical PDF.