Estimation and Detection (Fall 2016)

*** Total points: 100**

*** Due 12/05/2016**

1. (100 points)

$$x[n] = A\cos(2\pi f_0 n + \phi) + \omega[n],\ n = 0, 1, ..., N-1$$

Assume A and f_0 are known. Let $A = 1$, $f_0 = 0.06$, $\phi = \pi/4$, $\sigma^2 = 0.05$ and $N = 100$.

(a) (15 points) Prove the following approximation:

$$\begin{aligned}\hat{\phi} &= -\arctan\frac{\sum_{n=0}^{N-1}[A\cos(2\pi f_0 n + \phi) + \omega[n]]\sin 2\pi f_0 n}{\sum_{n=0}^{N-1}[A\cos(2\pi f_0 n + \phi) + \omega[n]]\cos 2\pi f_0 n} \\ &\approx -\arctan\frac{-\frac{NA}{2}\sin\phi + \sum_{n=0}^{N-1}\omega[n]\sin 2\pi f_0 n}{\frac{NA}{2}\cos\phi + \sum_{n=0}^{N-1}\omega[n]\cos 2\pi f_0 n}\end{aligned}$$

(b) (15 points) Obtain the MLE by grid search. Plot the log-likelihood function vs. ϕ. Indicate $\hat{\phi}_{ML}$ that you find. (Hint: Let the grid step be $\pi/10000$).

(c) (15 points) Using the Method of Scoring iterative procedure, to obtain the MLE for ϕ. Show the mathematical derivation.

(d) (10 points) Followed (b), plot $(\hat{\phi}_{ML} - \phi)$ vs. k, where k is the iteration index.

(e) (10 points) Repeat $M = 1000$ realizations, plot the asymptotical PDF of $\hat{\phi}_{ML}$ in page 20 in the slide, and the histogram of $\hat{\phi}_{ML}$ obtained in (b).

(f) (10 points) Now let σ^2 be a variable. Define SNR be $A^2/2\sigma^2$. Let $M = 1000$. Plot $N\text{var}\{\phi\}$ (both Monte Carlo simulation and asymptotic theoretical PDF) as a function of SNR from -20 dB to 20 dB with step size 0.1 dB.

(g) (10 points) Let the approximation error at the ith realization be e_i. Plot $\frac{1}{M}\sum_{i=1}^{M}|e_i|^2$ as a function of N, where $N = 10$ to 100 with step size 1.

(h) (5 points) Let $A = 1$ and $\sigma^2 = 0.1$. Repeat (g).

(i) (5 points) Let $A = 1$ and $\sigma^2 = 1$. Repeat (g).

(j) (5 points) Conclude your observations in (g)-(i).