Estimation and Detection (Fall 2016)

*** Total points: 100 with an additional PLUS 40**

*** Due 10/31/2016**

1. (100 points) Given the following system model:

$$x[n] = A + Bn + Cn^2 + Dn^3 + w[n], n = 0, 1, ..., N-1$$

   (a) (30 points) Obtain the MVUEs for A, B, C and D. Indicate the minimum variance of these estimators.

   (b) (10 points) Let the noise be i.i.d and $w[n] \sim \mathcal{N}(0, 0.01)$. Let $N = 20$, $A = 0.5$, $B = -0.5$, $C = 0.07$, $D = -0.005$. Randomly generate N observations and calculate MVUEs for A, B, C and D using the N observations.

   (c) (20 points) Followed (b) for $M = 1000$ realizations. Let $\mu_A = \frac{1}{M}\sum_{i=1}^{M}\hat{A}_i$, $\mu_B = \frac{1}{M}\sum_{i=1}^{M}\hat{B}_i$, $\mu_C = \frac{1}{M}\sum_{i=1}^{M}\hat{C}_i$, $\mu_D = \frac{1}{M}\sum_{i=1}^{M}\hat{D}_i$, $\sigma_A^2 = \frac{1}{M}\sum_{i=1}^{M}(\hat{A}_i - \mu_A)^2$, $\sigma_B^2 = \frac{1}{M}\sum_{i=1}^{M}(\hat{B}_i - \mu_B)^2$, $\sigma_C^2 = \frac{1}{M}\sum_{i=1}^{M}(\hat{C}_i - \mu_C)^2$, $\sigma_D^2 = \frac{1}{M}\sum_{i=1}^{M}(\hat{D}_i - \mu_D)^2$, where $\hat{A}_i$, $\hat{B}_i$, $\hat{C}_i$, $\hat{D}_i$ are the MVUEs obtained at the ith realization. Calculate σ_A^2, σ_B^2, σ_C^2, σ_D^2 and compare them to the CRLBs of A, B, C and D.

   (d) (10 points) Let $N = 100$, repeat (b)-(c).

   (e) (30 points) Plot the CRLBs of A, B, C, D and σ_A^2, σ_B^2, σ_C^2, σ_D^2 as functions of N, where $N = 20 : 1 : 100$.

2. (40 points) In Problem 1, we assume that the order of the system model is known in advance. That is, the above problem is an order 3 estimation. In this problem set, we assume that the order is NOT known in advance. Thus, the order shall be "guessed".

   (a) (20 points) Assume that the guess is order 2. That is, we assume that only A, B and C are to be estimated but actually there exists D in the model. Please repeat 1(b)-1(e). (This is an underestimate)

   (b) (20 points) Assume that the guess is order 4. Please repeat 1(b)-1(e). (This is an overestimate)