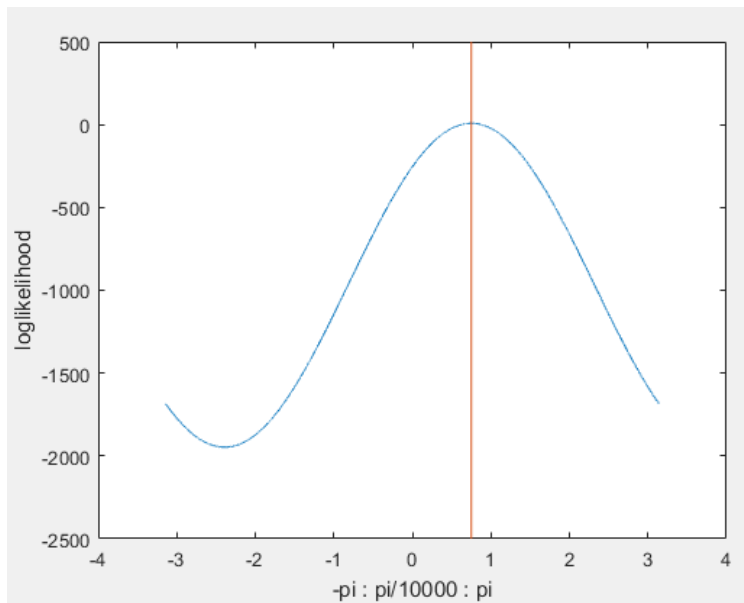
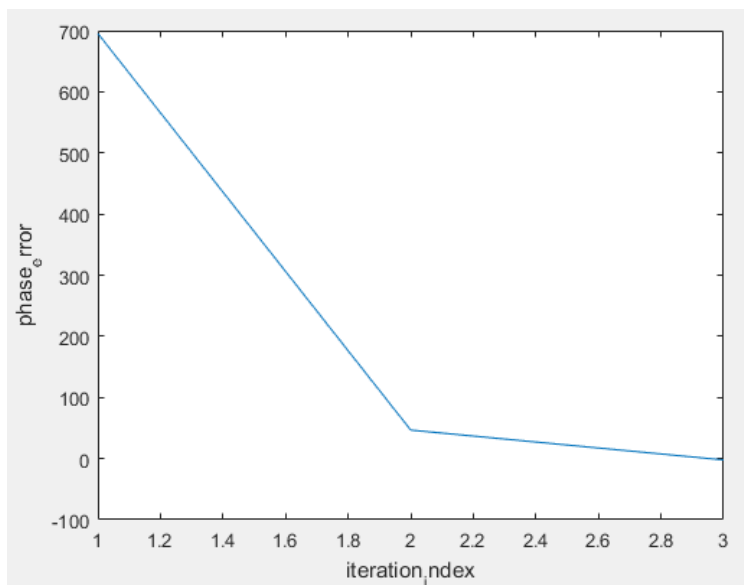


(b)



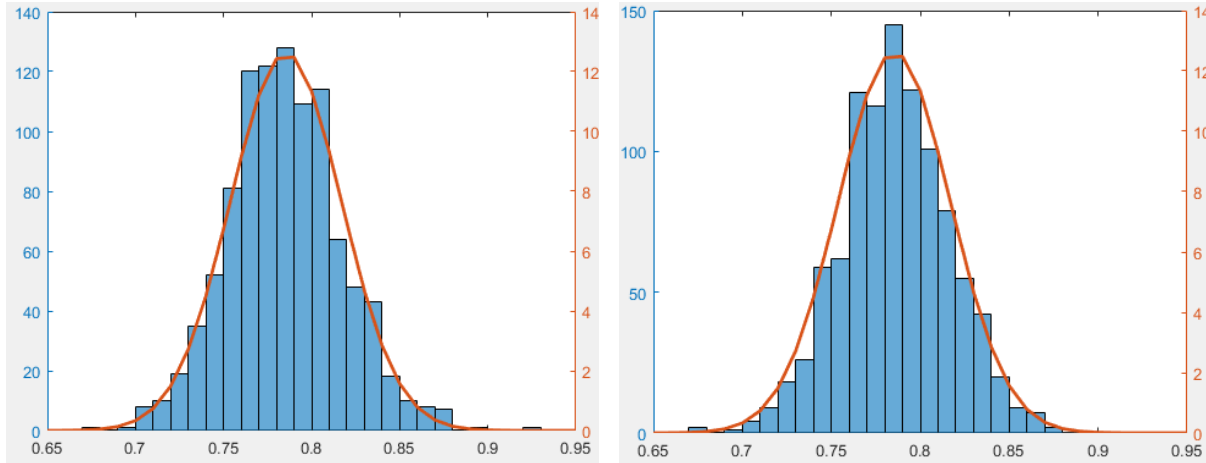
The vertical line is the ϕ_{ML} find by problem(a), which is really close to the maximum of the likelihood function. The phase range I choose is from $-\pi$ to π with stepsize $\pi/10000$.

(d)



We have the phase error ($\phi_{ML} - \phi$) converge to zero with ONLY “3” iteration times by Method of iterative procedure, which is much faster than grid-search.

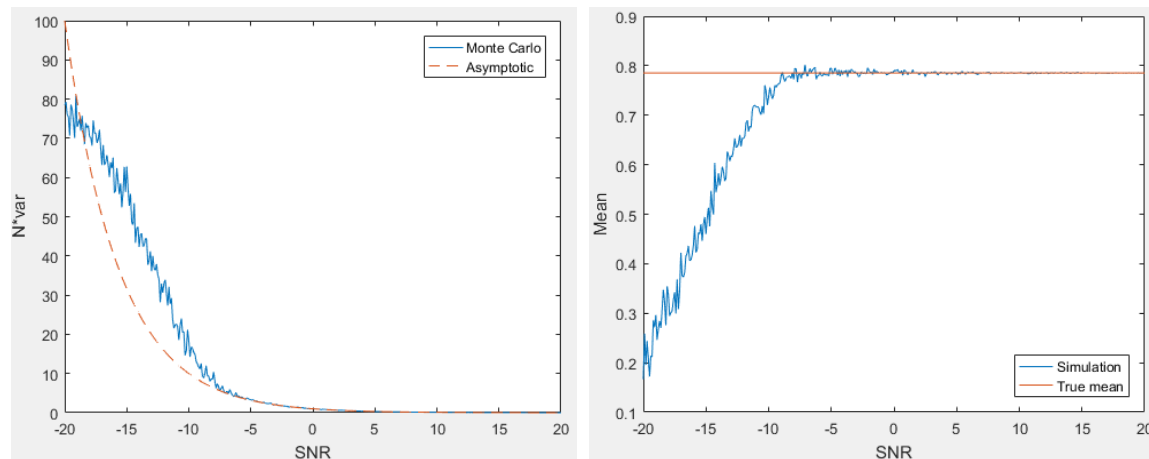
(e)



I plot two histogram to show the 2 different experiment shows similar result. The curve line is the asymptotical PDF(right side y-axis), and the histogram is the histogram of ϕ_{ML} obtained in (b) (left side y-axis). The two figure shows that the simulation result is close to the theoretical result.

The asymptotical PDF is normal distribution with mean = $\phi(\pi/4)$, variance = $(2\sigma^2)/(N A^2) = 0.001$

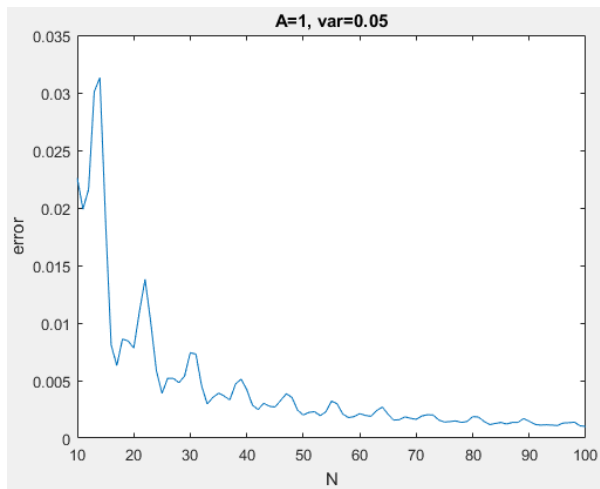
(f)



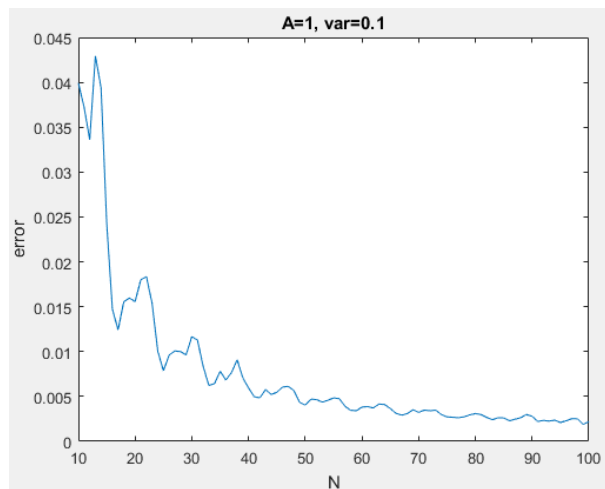
The left figure is about the relation between SNR and variance of noise(σ^2), and the right figure is between SNR and mean. First, we can see that the Monte Carlo simulation has same tendency as asymptotic line. Second, the larger SNR is, the lower variance with same N is. This is because

$\phi_{ML} \sim N(\phi, 1/((N) \cdot \text{SNR}))$. Lastly, we can see from the right figure. The larger SNR is, the lower error between true mean and estimate mean is.

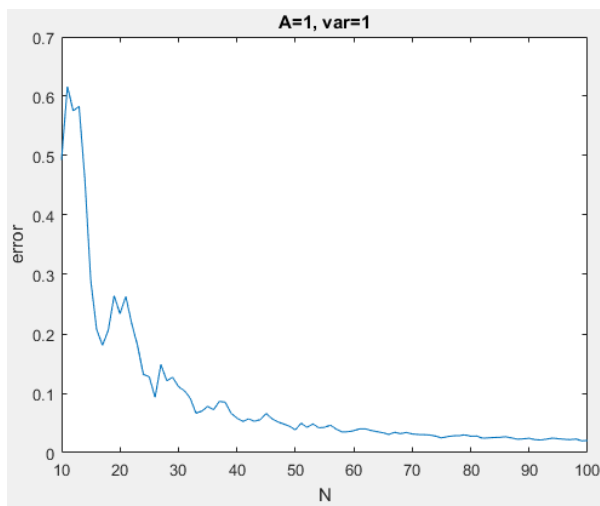
(g)



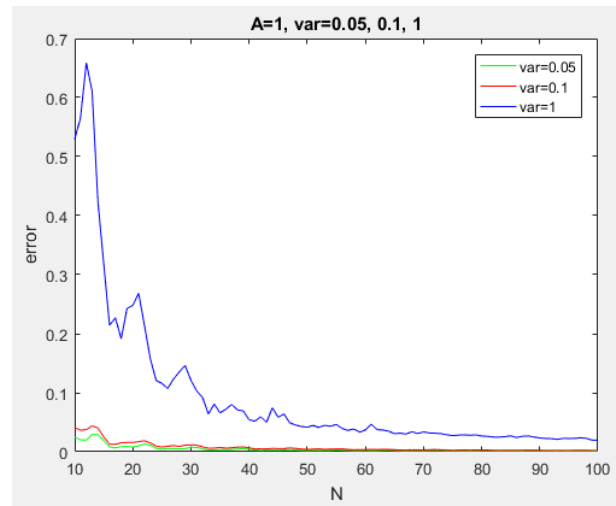
(h)



(i)



(j)



We can see as N increases the error decrease in different variance, which verify the assumption of MLE (with “large N ” we can asymptotically have unbiased estimator)