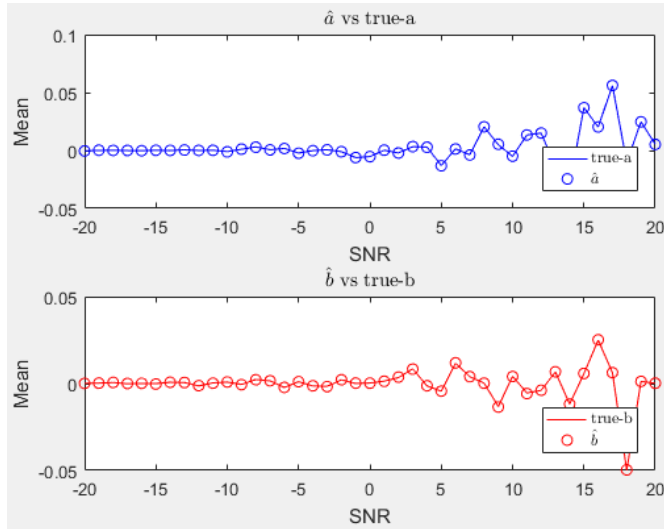**1.(c) N = 1000**
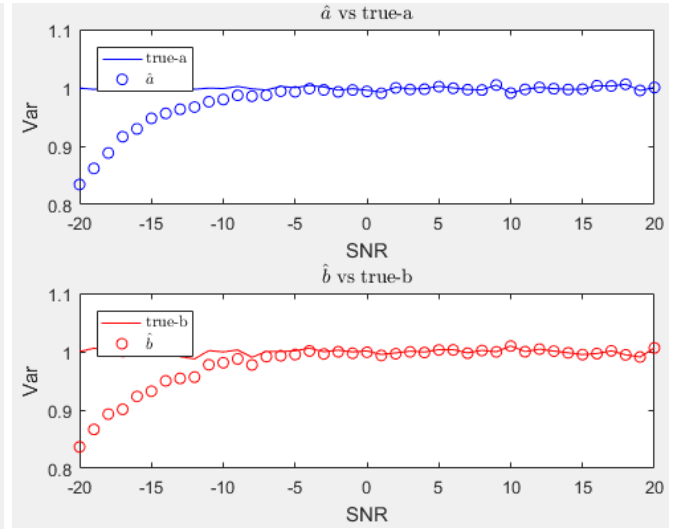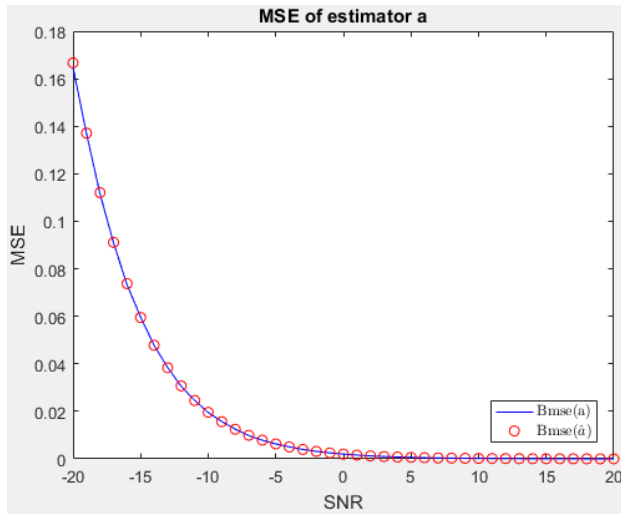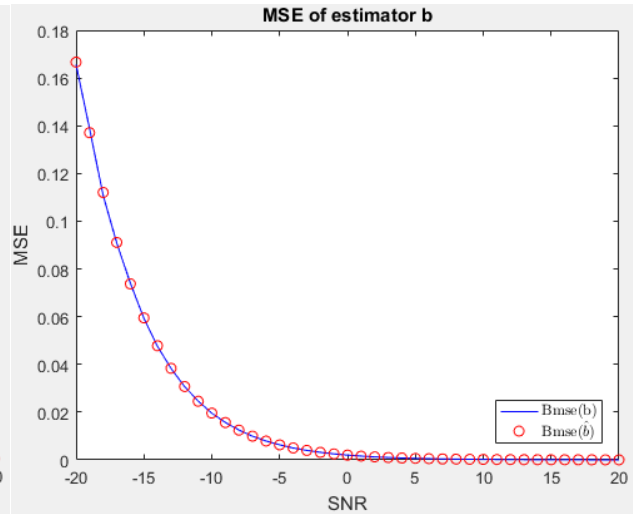

(Fig.1) Mean of estimation VS true θ by SNR (Fig.2) Variance of estimation VS true θ by SNR

(Fig.3) Bmse(a) vs Bmse(â)by SNR (Fig.4) Bmse(b) vs Bmse(b̂)by SNR

1.(d)

By Fig.1. The accuracy of the "mean" of the estimator is always high, which isn't affected by SNR. The result shows that the error of MMSE estimator is zero on the average.

$$\begin{aligned} E_{x,\theta}(\epsilon) &= E_{x,\theta}\left[\theta - E(\theta|\mathbf{x})\right] \\ &= E_x\left[E_{\theta|x}(\theta) - E_{\theta|x}(\theta|\mathbf{x})\right] \\ &= E_x\left[E(\theta|\mathbf{x}) - E(\theta|\mathbf{x})\right] = 0 \end{aligned}$$
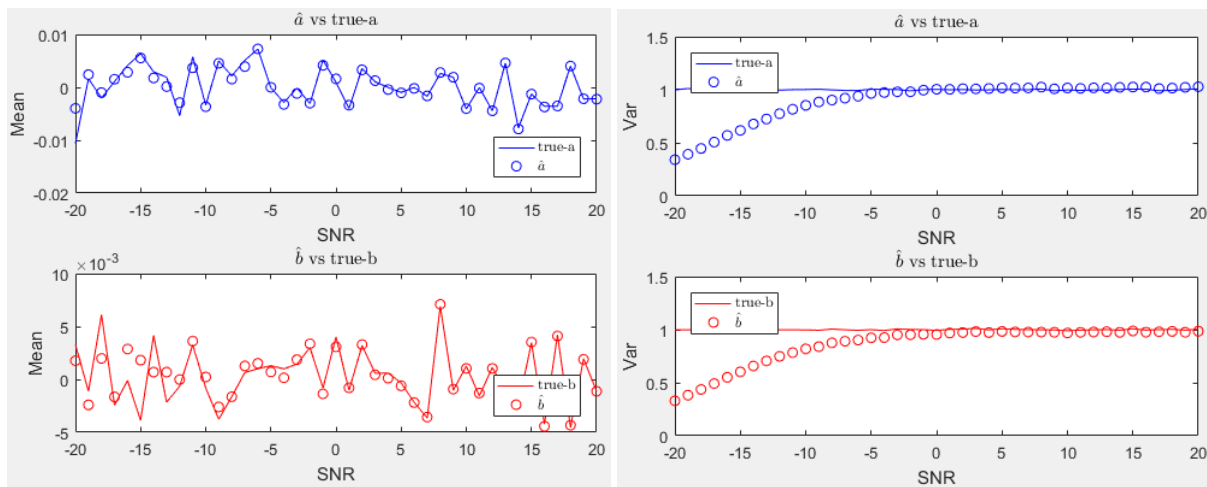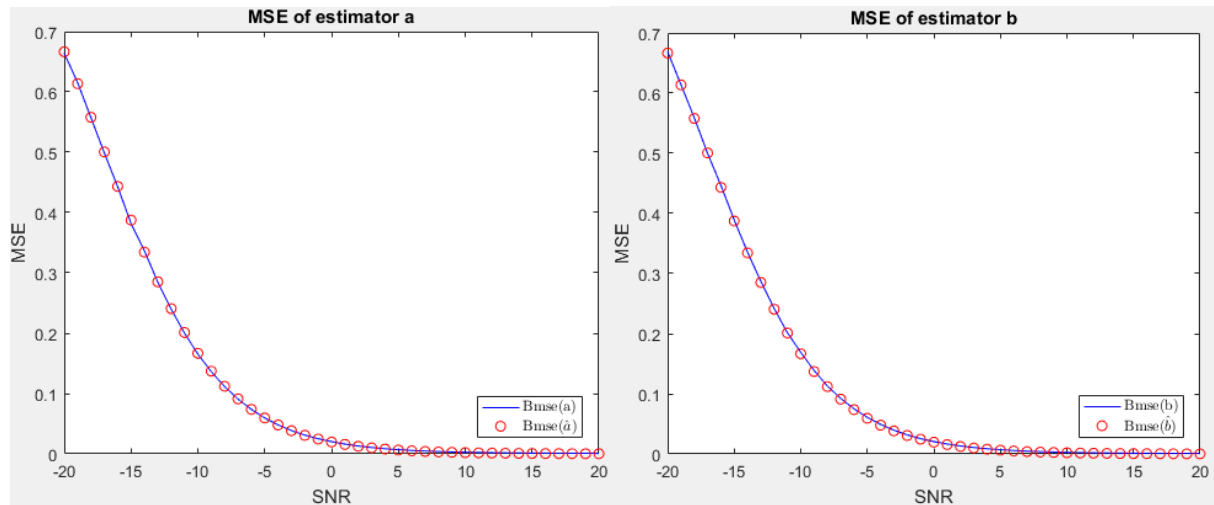
By Fig.3, Fig.4.

1. Due to the increasing variance of θ and bias^2. The mean square error of both estimators exponentially decade as SNR increase.
2. The trends of simulation Bmse and theroretical are the same. It shows that the proof in problem(b) is correct!!

1.(e)N=10
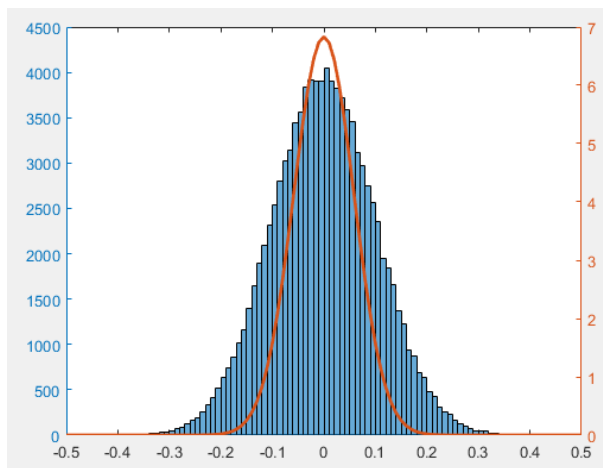

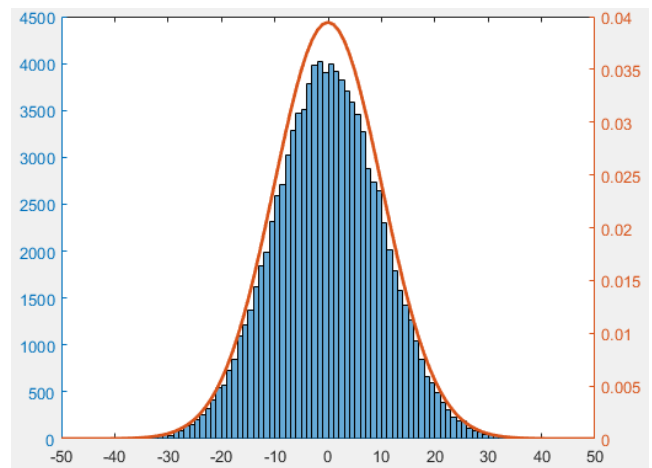## N=100

MSE of estimator a — MSE of estimator b
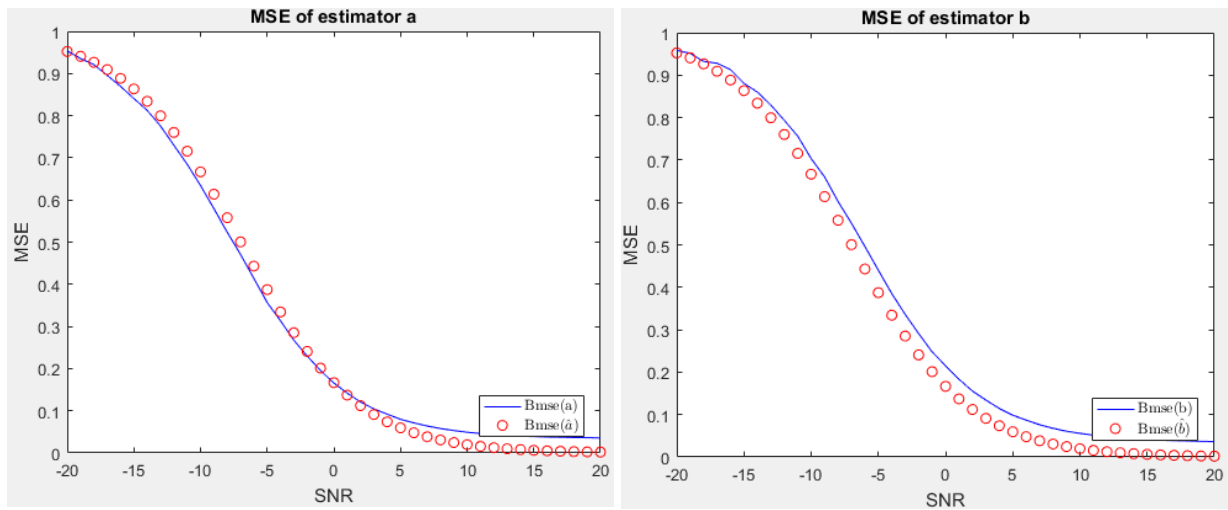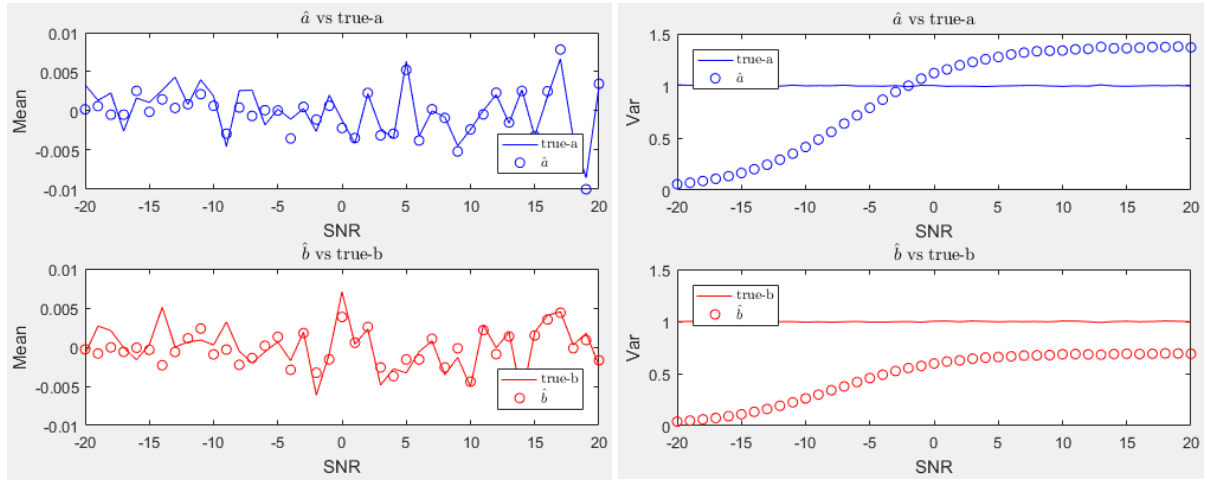
With N increasing, the accuracy of MSE estimator becomes higher(Simulation result is close to theoretical value)

[OTHER DISCUSSION]

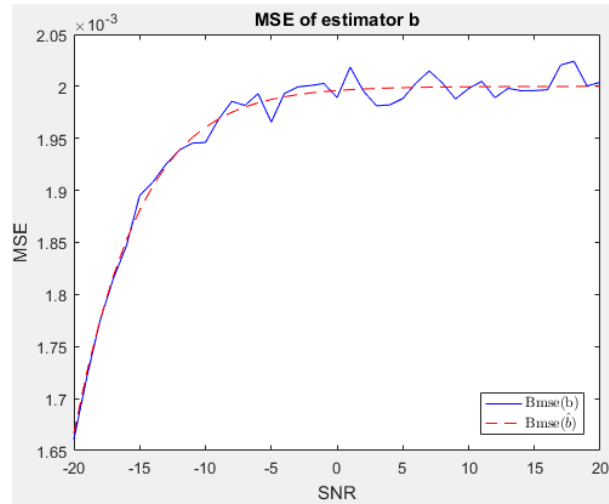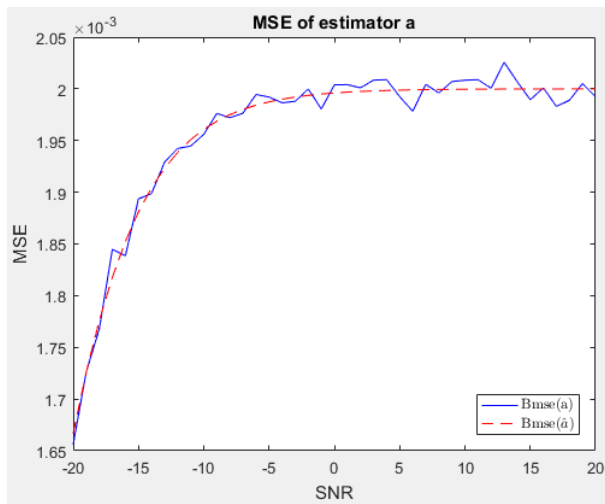(Fig.5) N=1000, SNR=-20dB, theoretical PDF VS Simulation   (Fig.6) N=1000, SNR=20dB, theoretical PDF VS Simulation
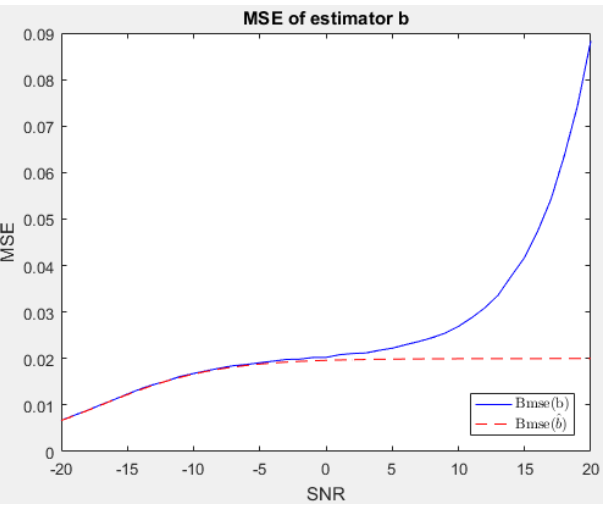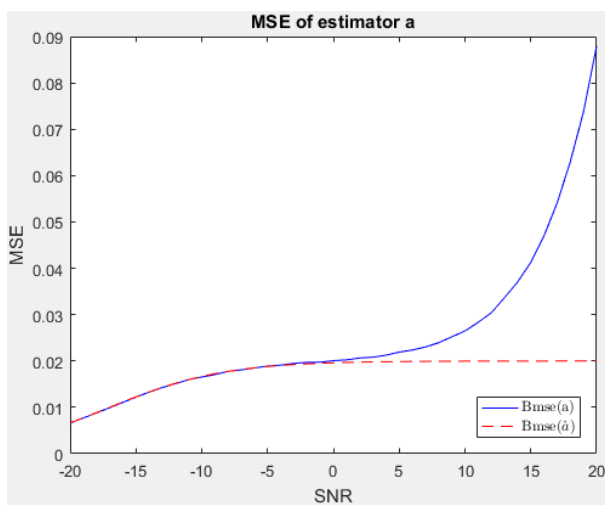
By Fig.5, Fig.6. When SNR is -20dB, the theoretical PDF is more concentrated than simulation data. But when SNR is 20dB(the higher one), the simulation data fit the theoretical PDF.

[Set variance of sigma = 1, M=10^5, and change variance of theta by SNR]

N=1000

N=100

1. We can see the result is different from the initial case, including the curve and accuracy.
2. The accuracy increase a lot when N becomes large.