Estimation and Detection (Fall 2016)

* Total points: 100

* Due 1/9/2016

1. Given the system model:

$$x[n] = a\cos(2\pi f_0 n) + b\sin(2\pi f_0 n) + \omega[n],\ n = 0, 1, \ldots, N-1.$$

Let $N = 1000$ and $f_0 = \dfrac{1}{16}$. Let $\omega[n] \sim \mathcal{N}(0, \sigma^2)$. We would like to estimate $\boldsymbol{\theta} = [a\ b]^T$ by Bayesian philosophy, where $\boldsymbol{\theta}$ has prior PDF $\boldsymbol{\theta} \sim \mathcal{N}(\mathbf{0}, \sigma_\theta^2 \mathbf{I})$. Define SNR be $\dfrac{\sigma_\theta^2}{\sigma^2}$.

(a) (5 points) Obtain the MMSE estimator for $\boldsymbol{\theta}$.

(b) (5 points) Obtain the closed-form expressions for $B_{MSE}(\hat{a})$ and $B_{MSE}(\hat{b})$.

(c) (50 points) Plot theoretical and simulation results for $B_{MSE}(\hat{a})$ and $B_{MSE}(\hat{b})$ for SNR from -20 to 20 dB. (Let the number of realizations be $M = 10^5$)

(d) (10 points) How would the value of SNR affects the estimator and the B_{MSEs}.

(e) (30 points) Let N=10 and N=100, repeat (a)-(d). How would the value of N affect the estimator and the B_{MSEs}. The solution for (c) needs plot to get points.