

Problem 1

(a)

$$\begin{aligned} \$10000000 \cdot \left(\frac{6.4}{4}\right)^{0.72} &= \$14027051.3714 \\ &\approx \$14000000 \end{aligned}$$

(b)

$$\begin{aligned} \$10000000 \cdot \left(\frac{6.4}{4}\right)^{0.72} \cdot \frac{170}{105} &= \$14027051.3714 \cdot \frac{170}{105} = \$22710464.1251 \\ &\approx \$22700000 \end{aligned}$$

Problem 2

Excel File Attached for q2.

Problem 3

$$i' = \frac{1+i}{1+f} - 1$$

Given that $i = 0.15$ and $f = 0.1$,

$$i' = \frac{1.15}{1.1} - 1 = 0.045 = 4.5\%$$

Problem 4

Filterco:

$$-\$5000 - \$5000 \cdot \left(\frac{1.08}{1.05}\right)^5 - \$1300 \cdot \frac{1 - 1.04^{10} 1.05^{-10}}{0.05 - 0.04} = -\$22619.88 \approx -\$22620$$

Duro:

$$-\$15000 - \$1000 \cdot \frac{1 - 1.04^{10} 1.05^{-10}}{0.05 - 0.04} + 500 \cdot \frac{1}{1.05^{10}} = -\$23818.88 \approx -\$23819$$

Based on NPW analysis, Filterco unit should be purchased.

Problem 5

(a)

$$\begin{aligned} a &= \frac{94 - 87}{87} = 0.0805 \\ 5.90\% &= \frac{b - 94}{94} \end{aligned}$$

$$\begin{aligned}
b &= 0.059 \cdot 94 + 94 = 99.546 \\
c &= \frac{104 - b}{b} = 0.0447431338 \\
d &= \frac{108 - 104}{104} = 0.0384615385 \\
\frac{118 - e}{e} &= 0.038 \\
e &= \frac{118}{1.038} = 113.6801541426
\end{aligned}$$

$$\therefore a = 8.05\%, b = 100, c = 4.47\%, d = 3.85\%, e = 114$$

(b) The base year of the PSI is 2015. The reason is because the index at this year is 100.

(c) 2012~2017:

$$\left(\frac{108}{82}\right)^{\frac{1}{5}} - 1 = 0.056627673 \approx 5.7\%$$

2015~2019:

$$\left(\frac{118}{99.546}\right)^{\frac{1}{4}} - 1 = 0.0422466355 - 1 = 0.0434329535 \approx 4.3\%$$

Problem 6

(a) The interest rate of 7% would be best for the discount rate, it is also the most common lowest value.

(b) **Case 1: Rent duplex** Let's first determine the yearly costs for rent fees and utilities,

$$\text{rent} = \$460 \cdot 12 = \$5520$$

$$\text{utilities} = \$100 \cdot 12 = \$1200$$

$$\begin{aligned}
NPW &= \$5520 \cdot \frac{1 - (1.025)^{15}(1.07)^{-15}}{0.07 - 0.025} + \$1200 \cdot \frac{1 - (1.05)^{15}(1.07)^{-15}}{0.07 - 0.05} \\
&= \$58275.265 + \$14790.045 = \$73065.31 \approx \$73065
\end{aligned}$$

is the amount it will cost the couple.

Case 2: Buying the house The down payment + sales commission is

$$0.05 \cdot \$94000 \cdot 2 = \$9400$$

Leaving us $\$94000 - \$4700 = \$89300$ to borrow.

Given that we are borrowing for 25 years at a 7% interest rate,

$$n = 25, \quad i = 7\%$$

$$A = \$89300 \cdot (A/P, 7\%, 25) = \$7662.88/\text{year}$$

The utilities, home insurance and maintenance costs can be calculated as,

$$12 \cdot \$300 \cdot \frac{1 - 1.04^{15} \cdot 1.07^{-15}}{0.07 - 0.04} = \$41670.62$$

Let's now get the appreciation when selling the house, in 15 years the house price becomes,

$$\$94000 \cdot (1.035)^{15} = \$157482.79$$

Then the present price is

$$\$157482.79 \cdot \frac{1}{1.07^{15}} = \$57079.01$$

However, we want to subtract the left over mortgage at this point,

$$\$57079.01 - (\$89300 - \$7662.88 \frac{1.07^{15} - 1}{0.07 \cdot 1.07^{15}}) = \$57079.01 - \$19507.148 = \$37571.86$$

Will be the eventual present value of the revenue from the selling the house.

The final costs of buying the house is therefore,

$$\begin{aligned} \$9400 + \$41670.62 + \$7662.88 \cdot \frac{1.07^{15} - 1}{0.07 \cdot 1.07^{15}} - \$37571.86 \\ = \$83291.61 \approx \$83291 \end{aligned}$$

Renting the one-bedroom duplex would be a cheaper option. **(c)** The saving rate for additional cash flows is not provided.

Problem 7

$$\begin{aligned} P(\text{roll } 7) &= \frac{6}{36}, P(\text{roll } 8) = \frac{5}{36} \\ P(\text{Hard-way } 8) &= \frac{1}{36}, P(\text{roll } 8 \cup \text{Hard-way } 8^c) = \frac{4}{36} \end{aligned}$$

Then the winning probability becomes,

$$P(\text{win}) = \frac{\frac{1}{36}}{\frac{1}{36} + \frac{10}{36}} = \frac{1}{11}$$

Likewise the losing probability is,

$$P(\text{lose}) = \frac{\frac{10}{36}}{\frac{1}{36} + \frac{10}{36}} = \frac{10}{11}$$

Thus,

$$\mathbb{E}[\text{return}] = \$9 \cdot \frac{1}{11} - \$1 \cdot \frac{10}{11} = -0.0909090909$$

The expected return is -9 cents.