

$$K = \frac{|r' \times r''|}{|r'|^3}$$

(both are parameter invariant)

$$\tau = \frac{(r' \times r'') \cdot r'''}{|r' \times r''|^2}$$

Tangent line: contains  $r(t_0), r'(t_0)$

Osculating plane: contains  $r(t_0), r'(t_0), r''(t_0)$

$$\frac{ds}{dt} = |r'(t)| \rightarrow s(t) = \int |r'(t)| dt, s(\text{initial}) = 0.$$

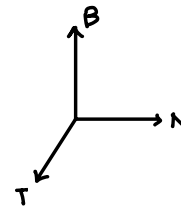
$$\|v\| = \|v \cdot t_0\| = \left\| \frac{ds}{dt} \cdot \frac{dt}{ds} \right\| = 1.$$

$$\frac{d}{dt} \|r'(t)\| = \frac{r'(t) \cdot r''(t)}{\|r'(t)\|} \quad (\|r'\| \neq 0)$$

$$T(s) = \frac{\vec{r}'(s)}{|\vec{r}'(s)|}$$

$$N(s) = \frac{\vec{r}''(s)}{|\vec{r}''(s)|}$$

$$B(s) = T(s) \times N(s)$$



follow rhr for others.

Center of osculating circular:  $r(t_0) + \hat{N}(t_0) \cdot \rho$  /  $y = f(x), x \in [a,b], \int_C g(x,y) ds = \int_a^b g(x, f(x)) \cdot \sqrt{1+f'(x)^2} dx$

$$v = \frac{dr}{dt} = \frac{ds}{dt} \cdot \hat{T}, \quad \frac{d\hat{T}}{ds} = \kappa \hat{N}, \quad \frac{d\hat{T}}{dt} = \kappa \cdot \frac{ds}{dt} \cdot \hat{N}, \quad a = \frac{dv}{dt} = \frac{ds}{dt} \cdot \hat{T} + \kappa \cdot \left(\frac{ds}{dt}\right)^2 \cdot \hat{N}, \quad v \times a = \kappa \left(\frac{ds}{dt}\right)^3 (\hat{T} \times \hat{N})$$

Frenet-Serret Formulae: ①  $\frac{dT}{ds}(s) = \kappa \cdot \hat{N}(s)$ , ②  $\frac{d\hat{N}}{ds}(s) = -\tau(s) \cdot \hat{B}(s) - \kappa(s) \cdot \hat{T}(s)$ , ③  $\frac{d\hat{B}}{ds}(s) = \tau(s) \cdot \hat{N}(s)$

scalar:  $\int_C f dr = \int_a^b f(r(t)) \|r'(t)\| dt$ , Vector Field:  $\int_C F \cdot dr = \int_a^b F(r(t)) \cdot r'(t) dt$

if  $\nabla \times F = \vec{0} \xrightarrow{\text{No}} \text{not conservative}$   
 $\xrightarrow{\text{Yes}} \text{domain simply connected?} \xrightarrow{\text{Yes}} \text{conservative}$   
 $\xrightarrow{\text{No}} \text{inconclusive}$

Potential Exists?  $\xrightarrow{\text{Yes}} \text{Conservative}$   
 $\xrightarrow{\text{No}} \text{Not Conservative}$

For  $\nabla \phi = F$ ,  $\int_C F \cdot dr = \phi(\text{end}) - \phi(\text{start})$

$\nabla \times F = 0 \iff \int_C F \cdot dr \text{ path indep} \iff F \text{ conservative on } U \text{ ( } \phi \text{ defined on } U \text{ )}$   
simple connected path connected

$$\int_C (F_1 dx + F_2 dy + F_3 dz) = \int_C F \cdot dr = \int_{C_1+C_2} F \cdot dr = \int_{C_1} F \cdot dr + \int_{C_2} F \cdot dr, \quad \int_C F \cdot dr = - \int_{-C} F \cdot dr$$

$$m \mathbf{a} = \mathbf{F}, \quad m \cdot \mathbf{v}' = \mathbf{F}$$

$$m \cdot \mathbf{v}' \cdot \mathbf{v} = \mathbf{v} \cdot \nabla \phi$$

$$\frac{d}{dt} \left( \frac{m \cdot \mathbf{v} \cdot \mathbf{v}}{2} \right) = \mathbf{v} \cdot \nabla \phi = \frac{d}{dt} (\phi)$$

$$\frac{1}{2} m |\mathbf{v}|^2 = \phi + \text{constant}$$

$$\frac{1}{2} m |\mathbf{v}|^2 - \phi = \text{constant}$$

$$\phi = \frac{1}{2} m |\mathbf{v}|^2 - E_0 \geq -E_0$$

$$y(x), \kappa = \frac{\left| \frac{d^2 y}{dx^2} \right|}{\left[ 1 + \left( \frac{dy}{dx} \right)^2 \right]^{\frac{3}{2}}}$$