No.30

# MATH 317 — HOMEWORK 4 — Due date: 1st August - 25 Marks

- Write your name, student number, and signature below.
- Write your answers to each question in the spaces provided. Submit your answers in Canvas on the due date (do not submit any additional pages).

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1. $\boxed{\text{8 marks}}$ Evaluate the flux:

$$\int\int_S F.\hat{n}\, dS$$

Where $F(x, y, z) =< x^2, xy, z >$, and S (surface) is the part of the plane $6x + 3y + 2z = 6$ with $0 \leq x$, $0 \leq y$, and $0 \leq z$, with the normal vector $\hat{n}$ pointing in the positive $z$ direction.

**Solution:**

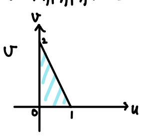
Let $x=u, y=v, z=\frac{6-6u-3v}{2}$ and since $x\geq 0, y\geq 0, z\geq 0$ we can see that $u\geq 0, v\geq 0$ and $2-2u-v\geq 0$.

$r(u,v)=<u,v,\frac{6-6u-3v}{2}>$. $r_u=<1,0,-3>$ and $r_v=<0,1,-\frac{3}{2}>$, $r_u\times r_v=<1,0,-3>\times<0,1,-\frac{3}{2}> = <3,\frac{3}{2},1>$

$\hat{n}=\frac{r_u\times r_v}{|r_u\times r_v|}=<\frac{6}{7},\frac{3}{7},\frac{2}{7}>$.

$\iint_S F\cdot\hat{n}\, dS=\iint_S <u^2,uv,\frac{6-6u-3v}{2}>\cdot<\frac{6}{7},\frac{3}{7},\frac{2}{7}>\cdot\frac{7}{2}\,dudv=\iint_S \left(\frac{6}{7}u^2+\frac{3}{7}uv+\frac{6}{7}-\frac{6}{7}u-\frac{3}{7}v\right)\cdot\frac{7}{2}\,dudv$

If we see S in terms of u and v




the region shaded represents S.

$\frac{7}{2}\iint_S\left(\frac{6}{7}u^2+\frac{3}{7}uv+\frac{6}{7}-\frac{6}{7}u-\frac{3}{7}v\right)dudv=\frac{7}{2}\int_0^1\int_0^{2-2u}\left(\frac{6}{7}u^2+\frac{3}{7}uv+\frac{6}{7}-\frac{6}{7}u-\frac{3}{7}v\right)dvdu=\int_0^1\left(\frac{6}{7}u^2(2-2u)+\frac{3}{14}u(2-2u)^2+\frac{6}{7}(2-2u)-\frac{6}{7}u(2-2u)-\frac{3}{14}(2-2u)^2\right)du\cdot\frac{7}{2}$

$=\left[\frac{4}{7}u^3-\frac{3}{7}u^4+\frac{3}{14}u^4-\frac{4}{7}u^3+\frac{3}{7}u^2+\frac{12}{7}u-\frac{6}{7}u^2-\frac{6}{7}u^2+\frac{4}{7}u^3-\frac{2}{7}u^3+\frac{6}{7}u^2-\frac{6}{7}u\right]_0^1\cdot\frac{7}{2}$

$=\left(\frac{4}{7}-\frac{3}{7}+\frac{3}{14}-\frac{4}{7}+\frac{3}{7}+\frac{12}{7}-\frac{6}{7}-\frac{6}{7}+\frac{4}{7}-\frac{2}{7}+\frac{6}{7}-\frac{6}{7}\right)\cdot\frac{7}{2}$

$=\frac{1}{2}\cdot\frac{7}{2}=\frac{7}{4}$.

$\therefore \frac{7}{4}$

2. $\boxed{\text{12 marks}}$ Evaluate the flux:

$$\int\int_S F.\hat{n}\, dS$$

Where $F(x, y, z) = < x + y, y, z + y >$, and S (surface) is the boundary of right cylinder with base of $x^2 + y^2 = 9$ and height of 2 ($0 \le z \le 2$), with the normal vector $\hat{n}$ pointing outward to the surface (note that the flux across all surfaces should be computed).

**Solution:**

Top: $r(u,v) = \langle u\cdot\cos v, u\cdot\sin v, 2\rangle$ ; $u \in [0,3]$, $v \in [0, 2\pi]$

$r_u = \langle \cos v, \sin v, 0\rangle$, $r_v = \langle -u\cdot\sin v, u\cdot\cos v, 0\rangle \longrightarrow r_u \times r_v = \langle 0, 0, u\rangle$

$$\iint_S F\cdot\hat{n}\, ds = \int_0^{2\pi}\int_0^3 \langle u\cdot(\cos v + \sin v), u\cdot\sin v, u\cdot\sin v + z\rangle \cdot \langle 0,0,u\rangle\, du dv = \int_0^{2\pi}\int_0^3 (u^2\sin v + 2u)\, du dv = \int_0^{2\pi}\int_0^3 (2u)\, du dv = 18\pi.$$

Bottom: $r(u,v) = \langle u\cdot\cos v, u\cdot\sin v, 0\rangle$ ; $u \in [0,3]$, $v \in [0, 2\pi]$

$r_u = \langle \cos v, \sin v, 0\rangle$, $r_v = \langle -u\cdot\sin v, u\cdot\cos v, 0\rangle \longrightarrow r_u \times r_v = \langle 0, 0, -u\rangle$ since we want to point outward.

$$\iint_S F\cdot\hat{n}\, ds = \int_0^{2\pi}\int_0^3 \langle u\cdot(\cos v + \sin v), u\cdot\sin v, u\cdot\sin v\rangle \cdot \langle 0,0,-u\rangle\, du dv = \int_0^{2\pi}\int_0^3 (-u^2\sin v)\, du dv = 0$$

Culinder: $r(u,v) = \langle 3\cos u, 3\sin u, v\rangle$ ; $u \in [0, 2\pi]$, $v \in [0,2]$

$r_u = \langle -3\sin u, 3\cos u, 0\rangle$, $r_v = \langle 0,0,1\rangle \longrightarrow r_u \times r_v = \langle 3\cos u, 3\sin u, 0\rangle$

$$\iint_S F\cdot\hat{n}\, ds = \int_0^2\int_0^{2\pi} \langle 3\cos u + 3\sin u, 3\sin u, 3\sin u + v\rangle \cdot \langle 3\cos u, 3\sin u, 0\rangle\, du dv = \int_0^2\int_0^{2\pi} (9 + 9\sin u\cos u)\, du dv = \int_0^2 (18\pi)\, dv = 36\pi.$$

The total is then $18\pi + 36\pi + 0 = 54\pi$. $\quad \therefore 54\pi$

3. $\boxed{\text{5 marks}}$ Let S be the surface given by the equation $x^2 + z^2 = cos^2 y$ with $0 \leq y \leq \pi/2$. Compute the integral

$$\int\int_S \sin y \, dS$$

**Solution:**

$r(y,\theta) = \langle \cos y \cdot \cos\theta, y, \cos y \cdot \sin\theta \rangle$ then $r_y = \langle -\sin y \cdot \cos\theta, 1, -\sin y \cdot \sin\theta \rangle$ and $r_\theta = \langle -\cos y \cdot \sin\theta, 0, \cos y \cdot \cos\theta \rangle$.

$r_y \times r_\theta = \langle \cos y \cdot \cos\theta, \sin y \cdot \cos y, \cos y \cdot \sin\theta \rangle$ and $|r_y \times r_\theta| = \cos y \cdot \sqrt{1 + \sin^2 y}$.

$\iint_S \sin y \, dS = \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \sin y \cdot \cos y \cdot \sqrt{1+\sin^2 y} \, d\theta \, dy = 2\pi \left[ \frac{1}{3}(1+\sin^2 y)^{\frac{3}{2}} \right]_0^{\frac{\pi}{2}} = 2\pi \cdot \left( \frac{1}{3} \cdot 2\sqrt{2} - \frac{1}{3} \right) = \frac{2\pi}{3}(2\sqrt{2}-1)$.