Problem 1

A new machine comes with 200 free service hours over the first year. Additional time costs \$140 per hour. What are the average and marginal costs per hour for the following quantities of service hours?

(a) 250

$$\text{average cost} = \frac{200 * \$0 + (250 - 200) * \$140}{250} = \frac{\$7000}{250} = \$28/\text{hour}$$

$$\text{marginal cost} = \$140/\text{hour}$$

(b) 350

$$\text{average cost} = \frac{200 * \$0 + (350 - 200) * \$140}{350} = \frac{\$21000}{350} = \$60/\text{hour}$$

$$\text{marginal cost} = \$140/\text{hour}$$

(c) 500

$$\text{average cost} = \frac{200 * \$0 + (500 - 200) * \$140}{500} = \frac{\$42000}{500} = \$84/\text{hour}$$

$$\text{marginal cost} = \$140/\text{hour}$$

Problem 2

A small machine shop purchases electricity under the following rates:

Usage charges: - First 100 kWh at 13 cents per kWh - Next 200 kWh at 10.5 cents per kWh - Any additional kWh at 9 cents per kWh

Demand charges: - First 35 kW without cost - Next 80 kW at \$ 5 per kW - Any additional kW at \$8 per kW

The shop uses 3,800 kWh per month, and has a peak usage of 150 kW

(a) Calculate the total monthly bill, including usage and demand charges, for this shop, rounded to the nearest dollar.

$$\text{usage charge} = \$0.13/\text{kWh}*100\text{kWh}+\$0.105/\text{kWh}*200\text{kWh}+\$0.09/\text{kWh}*3500\text{kWh} = \$349/\text{month}$$

$$\text{demand charge} = \$0/\text{kW}*35\text{kW}+\$5/\text{kW}*80\text{kW}+\$8/\text{kW}*35\text{kW} = \$680/\text{month}$$

$$\text{total bill} = \text{usage charge} + \text{demand charge} = \$349 + \$680 = \$1029/\text{month}$$

(b) What are the average and marginal usage charge costs per kilowatt hour, rounded to the nearest cent? (Ignore demand charges.)

$$\text{average charge} = \frac{\text{usage cost}}{\text{total usage}} = \frac{\$349}{3800\text{kWh}} = 9\ \text{cents/kWh}$$

$$\text{marginal usage charge} = 9 \text{ cents/kWh}$$

(c) What will the blended rate per kilowatt hour be (including demand charges), rounded to the nearest cent?

$$\text{blended rate} = \frac{\text{total usage cost} + \text{demand charge cost}}{\text{usage amount (kWh)}} = \frac{\$1029}{3800\text{kWh}} = 27\text{cents/kWh}$$

(d) If the shop uses 1,200 kWh more energy each month, but doesn't increase its peak usage, how much will the monthly bill increase, rounded to the nearest dollar? Which marginal usage charge applies?
If we use 1200 kWh more for each month the increase in usage cost will be

$$1200\text{kWh} * \$0.09/\text{kWh} = \$108/\text{month}$$

The marginal usage charge that applies in this case is \$0.09/kWh.

(e) If the shop instead adds additional machinery that will be used only occasionally each month, it will increase peak usage by an additional 40 kW and will increase energy usage by 300 kWh each month. How much will the monthly bill increase, rounded to the nearest dollar?
We can apply the marginal usage charge that arises from the extra 300kWh used in usage and the extra 40kW that arises from the peak usage.

$$\text{increase in monthly bill} = 300\text{kWh} * \$0.09/\text{kWh} + 40\text{kW} * \$8/\text{kW} = \$347/\text{month}$$

Problem 3

A student decides to buy a used ukulele. If they paid for it now, it would cost \$100. However, the student is short on cash so instead agrees to pay \$120 for it 6 months from now when they will have the cash. **(a) Assuming semi-annual (every 6 months) compounding, what is the nominal annual interest rate they will be paying for deferring payment?**

$$\text{nominal interest} = 2 * i_s = 2 * \frac{120 - 100}{100} = 0.4 = 40\%$$

(b) What is the effective annual interest rate?

$$\text{effective interest} = (1 + i_s)^m - 1 = (1.2)^2 - 1 = 0.44 = 44\%$$

Problem 4

The treasurer of a firm noted that many invoices were received with the following terms of payment: "2%-10 days, net 30 days". Thus, if he were to pay the bill within 10 days of its date, he could deduct 2%. On the other hand, if he did not pay the bill within 10 days, the full amount would be due 30 days from the invoice date. Assuming a 20-day compounding period, what effective annual interest rate is the 2% deduction for prompt payment equivalent to?

$$i_s = \frac{\$2}{\$98} = 0.0204$$

$$i_e = (1+i_s)^m - 1 = (1+0.0204)^{\frac{365}{20}} - 1 = (1+0.0204)^{18.25} - 1 = 1.44564 - 1 = 0.44564 = 44.56\%$$

Problem 5

A nation recently loaned $1 billion to another foreign nation that needed to borrow money. The loan will pay 2.5% interest, but no money will be paid back until 30 years from now, when the original loan and all the compounded interest will be paid back. One of the nation's leaders objected to the purchase, arguing that the correct interest rate for a loan like this should be 3.5%. The result of loaning the funds at too low of a repayment rate, she said, was a large inappropriate gift to the foreign country without appropriate approval. Assuming the leader's math is correct, how much will the foreign country have saved in interest when it repays the loan? Round your answer to the nearest million dollars.

$$\text{saved interest} = \$1000M \times (1.035^{30} - 1.025^{30}) = \$709.226M \approx \$709M$$