Physics 200 Homework 2, Part Two

You should complete the questions below using *relativistic* physics.

1. Passing Trains Two trains, A and B, are going in opposite directions on two parallel tracks. Define passing time to be the length of time from the moment when the fronts of the trains are lined up to the moment when the backs of the trains are lined up.

(a) Assume that the two trains are identical (have the same length). Argue, using the Principle of Relativity, that the passing time is the same when viewed from A's frame of reference as it is from B's frame of reference.

(b) Let train A be twice the length of train B. If the passing time in A's frame of reference is 50% longer than the passing time in B's frame of reference, what is the relative speed v of the two trains? Give your answer in the form 'number $\times$ c'.

Hint: solve for γ first, then square it and solve for v.

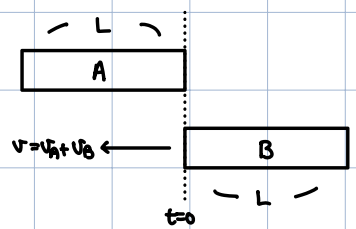
2. Interstellar trips Aidan and Nadia are twins. When they turn 21, they decide to have an adventure. Aidan chooses to travel to a star 10 light years away on a ship whose speed is 0.8c. Nadia chooses to travel to a slightly closer star, 8 light years away. How fast must Nadia's ship travel to and from the closer star so that the two twins are still the same biological age after they both return to Earth? (They won't necessarily return at the same time , but will be the same age as each other for the rest of their lives on Earth).

1-(a)

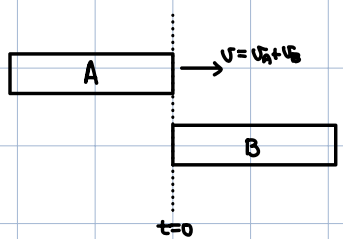
Let's denote each train's velocity as v_A, v_B. Since they are going in opposite directions the relative speed would be $v = v_A + v_B$.

First, let's take a look at each train's reference frame (I will use L to denote the lengths).

A's reference frame:



B's reference frame:



Due to length contraction, each train will observe the moving opponent train as L/γ where $\gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}$.

And in order to pass each other, the moving train will need to cover a distance of $L + \frac{L}{\gamma}$.

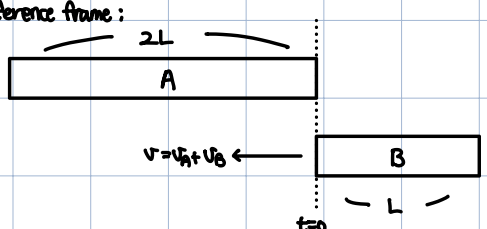
Due to the principal of relativity, the laws of physics are indentical in both frames.

Thus, the passing time is $\frac{L + L/\gamma}{v}$ for each and are equivalent.

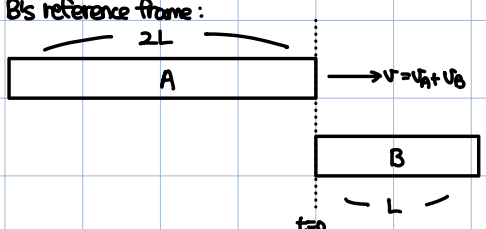
1-(b)

Similar to (a), let's denote the relative speed as $v = v_A + v_B$ and look at each reference frame.

A's reference frame:



B's reference frame:

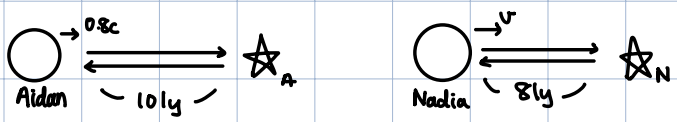


In this case, in A's frame, B has to travel $2L + L/\gamma$ and A has to travel $L + 2L/\gamma$. Therefore, the passing times are $T_A = \frac{2L + L/\gamma}{v}$, $T_B = \frac{L + 2L/\gamma}{v}$.

And since $T_A = 1.5 T_B$, $\frac{2L + \frac{L}{\gamma}}{v} = 1.5 \cdot \frac{L + \frac{2L}{\gamma}}{v} \longrightarrow 2L + \frac{L}{\gamma} = \frac{3}{2}\cdot(L + \frac{2L}{\gamma}) \longrightarrow 2 + \frac{1}{\gamma} = \frac{3}{2} + \frac{3}{\gamma} \longrightarrow \frac{1}{2} = \frac{2}{\gamma}$ $\therefore \gamma = 4$

Since $\gamma = 4$, $4 = \frac{1}{\sqrt{1-(\frac{v}{c})^2}} \longrightarrow 1-(\frac{v}{c})^2 = (\frac{1}{4})^2 \longrightarrow \frac{15}{16} = (\frac{v}{c})^2 \longrightarrow$ $\therefore v = \frac{\sqrt{15}}{4}c$

2.



The total distance Aidan and Nadia have to cover are 20ly, 16ly respectively.

Let's first calculate how much time Aidan has experienced.

$\tilde{T}_A = \frac{20ly}{0.8c} = 25 years$, $\gamma_A = \frac{1}{\sqrt{1-(\frac{0.8c}{c})^2}} = \frac{1}{\sqrt{0.36}} = \frac{1}{0.6}$. $T_A = \frac{\tilde{T}_A}{\gamma_A} = 25 years \cdot 0.6 = 15 years$.

Since we want them to be the same biological age after returning, Nadia would also have to experience 15 years.

$\tilde{T}_N = \frac{16ly}{v_N}$, $\gamma_N = \frac{1}{\sqrt{1-(\frac{v_N}{c})^2}}$. $T_N = 15 years = \frac{\tilde{T}_N}{\gamma_N}$. Let $v_N = xc$ $(x \in \mathbb{R})$, then $15 years = \frac{16\,ly}{xc} \cdot \sqrt{1-(\frac{xc}{c})^2} = \frac{16 years}{x} \cdot \sqrt{1-x^2}$.

$\longrightarrow 15x = 16\sqrt{1-x^2} \longrightarrow 225x^2 = 256 - 256x^2$

$\longrightarrow 481x^2 = 256$

$\longrightarrow x = \sqrt{\frac{256}{481}}$

$\therefore v_N = \frac{16}{\sqrt{481}}c$

So Nadia would need to travel at $\frac{16}{\sqrt{481}}c$.