PHYS 200 Final Cheat Sheet

Relativistic Dynamics

$$E = \gamma mc^2, |\vec{p}| = \gamma mv \rightarrow E^2 = m^2c^4 + p^2c^2$$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

LMatrix :

$$\begin{bmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Photons

1. massless
2. no rest frame → if had one, $E = 0, p = 0$ which is nothing.
3. always move at c (in all frames)
4. $E = pc = hc/\lambda$, definite energy and momentum, related to **wavelength**

Mass

1. Energy, momentum is always conserved → mass doens't have to be, can convert mass into KE
2. Fusion: 2 light things → heavy thing + E
3. Fission: 1 heavy thign → 2 (or more) lighter things + E
4. Since stable is less energy state, we exert energy to go to stable state
5. In hydrogen atom $m_H = m_e + m_p - BE \rightarrow m_H < m_e + m_p$

Particle decay, how to solve for diff frames

1. Move to center of mass frame (where stationary object decays).
2. Compute momentum & energy, using LT revert back to original frame (be careful of sign).

Compton Scattering

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

1. m is dependent on the object we are striking the photon to.
2. Don't forget unit conversions.
3. $hc = 1240\text{nm} \cdot \text{eV}$

Classical view

Classical view continued

1. $c = hf$
2. Energy is proportional to E_0^2, B_0^2
3. Intensity $\propto (E_0)^2$ a.k.a Probability $\propto (E_0)^2$ and I
4. Intensity = energy / (area * time), not dependent on frequency of light
5. If intensity increase, it is related to the number of photons

Photo Electric Effect

1. $E_\gamma = hf$
2. $KE_{\max} = hf - W$
3. W is same unless metal changes
4. $eV_{\text{stop}} = KE_{\max} = hf - W$
5. If retarding potential applied, it is not that electrons are not ejected, it is because some ejected electrons can't make through

QM effect of light

1. Compton Scattering
2. Photoelectric: not existence of effect, rather it is effect of retarding potential, nature that it's a hit or a miss and photon disappers after hitting
3. Blackbody radiation: When wavelength is low, radiation is not infinite. (related to cost of ejecting a high energy particle)

Diffraction pattern

1. Photon / electron has a probability of hitting part of screen
2. Somehow each particle knows diffraction pattern. Somehow each particle passes through both slits at the same time
3. We don't know till measure
4. hit screen at certain point (particle behav.), interference (with itself) pattern (wave behaviour)
5. After measurement (position checked), position collapses and has definite position. 6. Before measurement, no position is known (only a probability distribution is known)

Probability

1. Intensity $\propto$ E-field squared, thus probability $\propto$ wave amplitude squared
2. $P(x) = |\psi(x)|^2 = \psi(x)\bar{\psi(x)}$

Terminlogies

1. Measurement : process through which observales is determined / recorded (position, momentum recorded)
2. Eigenstate : A quantum state with 100 percent certainty
3. Quamtum superposition : Combination of different eigenstates with complex coefficients
4. State : complete description of properties at some moment in time

QM / Measurements

1. $\sum_{\forall x} c(x) |x\rangle \equiv \int_{-\infty}^{\infty} c(x) |x\rangle \, dx$
2. $|\psi(x)\rangle = \int_{\infty}^{\infty} \psi(x) |x\rangle \, dx$
3. After measurement, state changes to appropriate eigen state
4. If repeated measurement right after, get same result (system still in that eigenstate)
5. If asked about diff measurement, we have to change basis i.e. new eigenstate
6. Given $a_1 |x_1\rangle + a_2 |x_2\rangle$, a particle DOES NOT have a definite state.

Polarization

1. $|\theta\rangle = \cos\theta |0\rangle + \sin\theta |90\rangle$
2. probability transmission : $\cos^2\theta$
3. probability absorbed : $\sin^2\theta$
4. In case of $|\phi\rangle = a|x\rangle + b|y\rangle$, when getting probability need to normalize $|a|^2 + |b|^2 = 1$
5. Energy is still maintained.

De broglie wave length / others

1. For any particle, $pc = hc/\lambda \implies p = h/\lambda$.
2. Use $e^{i2\pi px/h}$
3. $\psi(x) = \frac{1}{\sqrt{\hbar}} \int \tilde{\psi}(p) e^{ipx/\hbar} dp$
4. $\tilde{\psi(p)} = \frac{1}{\sqrt{\hbar}} \int \psi(x) e^{-ipx/\hbar} dx$
5. For post-measurement, $\psi(x) = e^{ip_0x/\hbar}, \tilde{\psi}(p) = e^{-ipx_0/\hbar}$, cannot be properly normalized
6. $\psi(R,t) = e^{2\pi i(R/\lambda - ft)}, \quad R \approx D + \frac{Y^2}{2D}$
7. $\langle x\rangle = \int_{-\infty}^{\infty} xP(x)dx = \int_{-\infty}^{\infty} x|\Psi(x)|^2 dx$
8. $\hbar = \frac{h}{2\pi}$, heisenberg uncertainty: $\Delta x \Delta p \geq \hbar/2$

Wave Packets

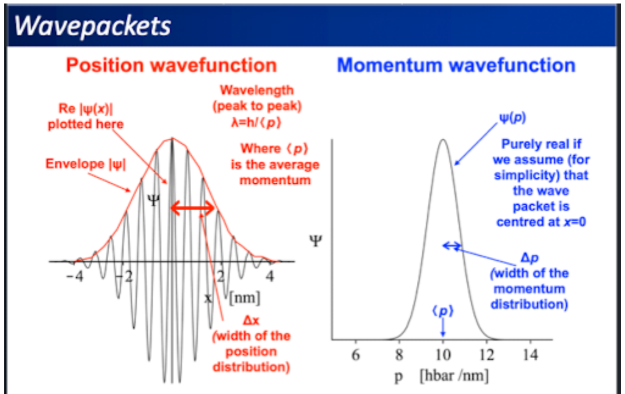

Figure 1: The real part of the position wavefunction for a wavepacket with $\langle p\rangle = 10\ \hbar/\text{nm}$ and width $\Delta x = 1\text{nm}$ [left]; the momentum wavefunction of the same wavepacket [right].

1. Wavepackets are needed to localize real wavefunctions (which are not as shown above)
2. The narrower the wavepacket in position, wider range of frequencies, wider momentum wavefunction
3. The narrower the wavepacket in momentum, the wider position wavefunction.
4. $\langle p\rangle = \int_{-\infty}^{\infty} p|\tilde{\Psi}(p)|^2 dp$
5. $(\Delta p)^2 = \int_{-\infty}^{\infty}(p-\langle p\rangle)^2|\tilde{\Psi}(x)|^2 dp$ (Same for x)

Constants, Formulae and others

$$c = 3.00\times 10^8 \text{ms}^{-1}, \qquad e = 1.60\times 10^{-19}C$$

$$h = 6.626\times 10^{-34}\text{Js}$$

$$m_e = 9.11\times 10^{-31}\text{kg}, \qquad m_p = 1.67\times 10^{-27}\text{kg}$$

$$m_{\pi_0} = 2.406\times 10^{-28}\text{kg}\ , u'_x = \frac{u_x - v}{1 - u_x v/c^2}$$

$$\mathbf{P} = (\gamma(u)mc, \gamma(u)m\vec{u}) = (E/c, \vec{p})$$

$$1J = 6.242\times 10^{18} eV$$

1. Bound system has binding energy. Stable so low E.
2. Bound system's mass is less than the sum of the masses of the components.
3. Unstable bound system has sum of masses heavier than its components.
4. Having half vertical and half horizontal polarized light is different from $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|90\rangle$
5. Bright fringes : $\frac{dY}{D} = m\lambda$, d : slit size, D : Distance to screen, Y : Height

Time evolution

1. A wavefunction of a moving free particle with a well defined momentum is simply a travelling wave

$$e^{(i/\hbar)(px-Et)}$$

2. $E = 1/2mv^2 = p^2/2m$ (non-relativistic)
3. Time evolution for each component in a quamtum superposition happens independently

Basics

1. Event : something that happens at some place (x) and some time (t)
2. Relativity of Simultaneity: Events in another frame won't be simulatenous.
3. Lengths perpendicular to motion do not change in length.
4. All observers have to agree to some event, whether happening at the same time doesn't matter.

Time Dilation

$$T' = \frac{1}{\sqrt{1-(v/c)^2}}T = \gamma T$$

Where T is proper time, moving clock moves slower. Clock is synchronized from another frame.

Length Contraction

$$L' = \frac{1}{\gamma}L$$

L is proper length, as measured by observer in rest-frame of object.

Lorentz Transformations

Gallelian transformations fail at high speeds.
$t' = \gamma(t - \frac{vx}{c^2}), \quad x' = \gamma(x - vt)$
$t = \gamma(t' + \frac{vx'}{c^2}), \quad x = \gamma(x' + vt')$
" Be careful of v sign!!! Extra terms are for clock synchronization.
Origins have to agree at the same place (think of Thanos!).
$u'_x = \frac{u_x - V}{1-u_xV/c^2}, u_x = \frac{u'_x+V}{1+u'_xV/c^2}$
$u'_y = \frac{u_y}{\gamma_v(1-u_xv/c^2)}, u'_z = \frac{u_z}{\gamma_v(1-u_xv/c^2)}$

Space Time Interval

$$(\Delta s)^2 = -(c\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$

This is lorentz invariant, $(\Delta s)^2 = (\Delta s')^2$
For two given events, any two inertial observers agree on s^2.
Proper time: time measure between two events happening at same place $(\Delta x = 0)$.
Proper length: length measure between two events happening at same time $(\Delta t = 0)$.

Space time diagram, Causality

space-like separated: $(\Delta s)^2 = -(c\Delta t)^2 + (\Delta x)^2 > 0$
time-like separated: $(\Delta s)^2 = -(c\Delta t)^2 + (\Delta x)^2 < 0$
null-like separated: $(\Delta s)^2 = -(c\Delta t)^2 + (\Delta x)^2 = 0$
An event, would be a coordinate in the diagram (axes change per frame).
In time-like separated, some events can happen at same place.
In space-like separated, some evetns can happen at same time (or even reverse time).

Schrodinger Equation

We know that $\frac{\lambda}{p}$ and $f = \frac{E}{h}$ hence,

$$e^{i(2\pi x/\lambda - 2\pi f t)} = e^{i(px-Et)/\hbar}$$

$v_{\text{wave.propagation}} = \lambda f = E/p = (\frac{1}{2}mv^2)/(mv) = \frac{1}{2}$
$v_{wavepacket} = \frac{df}{d(\lambda^{-1})} = \frac{d(E/h)}{d(p/h)} = dE/dp = \frac{d}{dp}(\frac{p^2}{2m}) = \frac{p}{m} = v$
dispersion : shorter wavelengths bunch at the the front, longer ones lag behind.
Schrodinger Equation generates time evolution,

$$\frac{\partial\psi}{\partial t} = \frac{i\hbar}{2m}\frac{\partial^2\psi}{\partial x^2}$$

i.e.

$$\psi(t+\Delta t) \approx \psi(t) + \Delta t\frac{\partial\psi}{\partial t} = \psi(t) + \frac{i\hbar}{2m}\frac{\partial^2\psi}{\partial x^2}\Delta t$$

If we know ψ for all x at some t, we can solve for all t.

SE cont'd

For a **free particle**, there is no force, momentum **does not change**.
So if we measure position across some time, it is different (momentum!!)
But if we measure momentum across some time, it is same!
At the same time, energy is the same (if potential is same).
For a free particle, S.E. is,

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t)$$

If we have potential, then,

$$E = \frac{p^2}{2m} + V(x)$$

and,

$$i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(x)\psi$$

Classically, an electron would bounce between bound states, but in QM the electron still would exist outside the bound!
Now with potential, the width of the wavepacket wouldn't always increase.
Boundaries have to be continuous and approach 0 for $\pm\infty$

SE will well defined energy

Let's consider the SE with well-defined energy,

$$i\hbar\frac{\partial}{\partial t}e^{-iEt/\hbar} = Ee^{-iEt/\hbar}$$

Then,

$$i\hbar\frac{\partial}{\partial t}(e^{-iEt/\hbar}\psi_E(x) = Ee^{-iEt/\hbar}\psi_E(x))$$

This is a stationary state: PDF doesn't change with time.

Energy Eigenstates

Energy is also a classical observable, so there must exist states with well-defined energy.
A.k.a **Energy Eigenstates**

Time independent SE

$$[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x)]e^{-iEt/\hbar\psi_E(x)} = Ee^{-iEt/\hbar}$$

$$= Ee^{-iEt/\hbar}\psi_E(x)$$

Hence,

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi_E(x) + V(x)\psi_E(x) = E\psi_E(x)$$

Solutions will only exist for discrete energies $\rightarrow$ energy spectrum. Solutions are called energy eigenstates.

Infinite Square Well

$$\psi_n = \sin\left(\frac{n\pi x}{L}\right)$$

$$E_n = \frac{\hbar^2}{2m}(\frac{n\pi}{L})^2 = \frac{h^2 n^2}{8mL^2}$$

Finite Square Well

$V = 0$ inside well, $V = V_0$ outside well.

$$\frac{\partial^2\psi}{\partial x^2} = \frac{2m}{\hbar^2}[V(x) - E]\psi$$

If $E > V$ (inside well),

$$\psi(x) = C\sin(kx) + D\cos(kx)$$

If $E < V$ (inside barrier),

$$\psi(x) = Ae^{-kx} + Be^{kx}$$

Depending on sign of x either A or B can go to zero. Other constants can be determined by continuity of 0th, 1st derivative.

Barrier Penetration

Outside the well we get a function $e^{\pm\kappa x}$ In anther way,

$$e^{-x/\eta} \rightarrow \eta = \frac{1}{\kappa} = \frac{\hbar}{\sqrt{2m(V-E)}}$$

So now we have a finite chance of finding an electron outside of the well.
Lowest energy is still not zero - electron always has some kinetic energy.

Quantum Tunneling

If the barrier isn't infinite width (would also work with infinite width), the wave would partially reflect and transmit at the same time.
If we recall that $e^{-x/\eta}$ the bigger the difference between the potentail well height and particle energy, the less the wavefunction extends outside well.

$$P_{\text{transmit}} = |\Psi|^2 = (e^{-w/\eta})^2 = e^{-2w/\eta}$$

Hydrogen Atom

time independent SE,

$$-\frac{\hbar}{2m}(\partial_x^2 + \partial_y^2 + \partial_z^2)\psi_E - \frac{1}{4\pi\epsilon_0}\frac{e^2}{|\vec{r}|}\psi_E = E\psi_E$$

$$E_n = -\frac{13.6\text{eV}}{n^2}$$

An Hydrogen Atom is stable!,
1. The electron isn't orbiting the nucleus, has a distributeed PDF centered on nucleus.
2. Existence of lowest energy state implies stability (no lower state to go to).
3. If atom radiates or absorbs photon, the electron jumps from one energy level to another (only certain frequencies are permitted (QM!)).

$$\lambda' = \gamma(1 + \frac{v}{c})\lambda$$