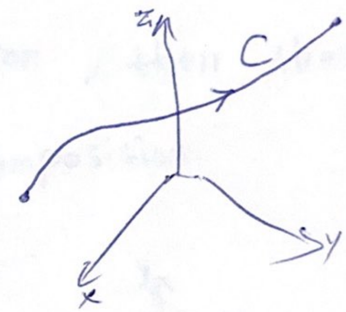
Lecture 2

Re-parametrization:

let $\boxed{\vec{r}(t) = \langle F_1(t), F_2(t), F_3(t) \rangle, \quad t \in [a,b]}$

be parametrization curve C,



If $t(u)$ is a function of new variable "u":

with domain being some interval $[c,d]$

and range $[a,b]$, and suppose $dt/du > 0$

Then $\boxed{\vec{r}(t(u)) = \langle F_1(t(u)), F_2(t(u)), \dots \rangle \quad u \in [c,d]}$

Also parmametrizes C.

$\boxed{\text{why } dt/du > 0}$ $\Longrightarrow$ This ensures that both parametrizations trace over C with same orientation.

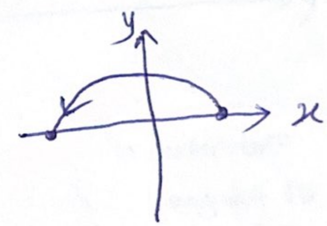
* For simplicity, refer to such a composition $\vec{r}(t(u))$ simply as $\vec{r}(u)$
  (Compositions understood by context)

* If we have a 2 parametrizationof a curve C, are the parameters themselve related togethor?

* In general, given 2 parametrizations of the Same C $(\vec{r}(t), \vec{r}(u))$,
  both traversing C with the Same orientation, then these are
  related as above via $t(u)$ or $u(t)$ & composition.

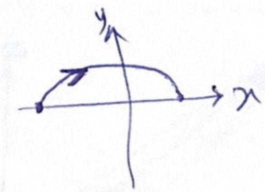
Exp)

$\langle \cos t, \sin t \rangle, \quad t \in [0, \pi]$



$\langle \cos(2u), \sin(2u) \rangle, \quad u \in [0, \pi/2]$

$\langle x, \sqrt{1-x^2} \rangle; \quad x \in [-1, 1] \longrightarrow$



$\Rightarrow \langle -x, \sqrt{1-x^2} \rangle; \quad x \in [-1, 1]$

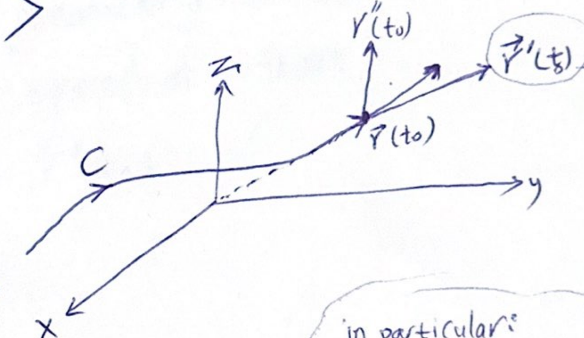
$t(u) = 2u, \quad -x(t) = \cos t$ or $x(t) = -\cos t$ $\longleftarrow$ formal relation between parameters

Diffentiation of $\vec{r}(t)$:

position : $\vec{r}(t) = \langle x(t), y(t), z(t) \rangle \qquad t \in [a,b]$

velocity : $\vec{r}'(t) = \langle x'(t), y'(t), z'(t) \rangle$

Acceler. : $\vec{r}''(t) = \langle x''(t), y''(t), z''(t) \rangle$



$\rightarrow$ points in direction of travel at $t = t_0$

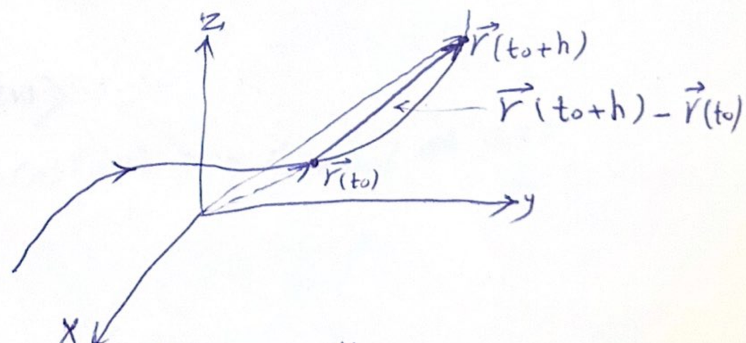
in particular:
tangent to C at $\vec{r}(t_0)$

in particular:

$$\|r'(t_0)\| = \text{speed of travel at } t = t_0$$

Recall:
Definition of derivative:

$$r'(t_0) = \lim_{h \to 0} \frac{\vec{r}(t_0+h) - \vec{r}(t_0)}{h}$$



By diagram: $\vec{r}'(t_0)$ really is tangent to C at point $\vec{r}(t_0)$.

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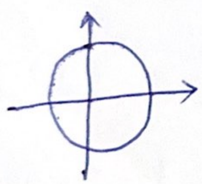
So: $\boxed{C \text{ has a well defined tangent line at } \vec{r}(t_0) \text{ provided } \|\vec{r}'(t_0)\| \neq 0.}$

On the other hand:

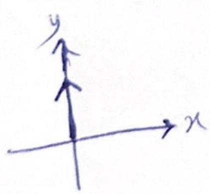
$\vec{r}'(t_0)$, and its components alone, do not reflect anything about geometry of $\underline{C}$ at $\vec{r}(t_0)$.
Just refers to speed of travel.
Similarly for $\vec{r}''(t_0)$ ....

Exp)



$$\begin{cases} \vec{r}(t) = \langle \cos t, \sin t \rangle \\ \vec{r}'(t) = \langle -\sin t, \cos t \rangle = \langle 0, 1 \rangle \text{ at } t = 0 \end{cases}$$

$$\begin{cases} \vec{r}(u) = \langle \cos(2u), \sin(2u) \rangle \\ \vec{r}'(u) = \langle -2\sin(2u), 2\cos(2u) \rangle = \langle 0, 2 \rangle \text{ at } u = 0 \end{cases}$$



$$\begin{cases} \vec{r}(t) = \langle 0, t \rangle \\ \vec{r}'(t) = \langle 0, 1 \rangle \end{cases}$$

Notations:

$\vec{r}(t)$: parametrizes a curve C

$\rightarrow \vec{v}(t) = \frac{d\vec{r}}{dt}(t)$: velocity vector at $\vec{r}(t)$

$\rightarrow a(t) = \frac{d^2\vec{r}}{dt^2}(t)$: accel. vector at $\vec{r}(t)$