Part A: Multiple Part True/False Questions. For each question in Part A, indicate which of the statements, (A)–(D), are **true** and which are **false** (multiple statements may be true)?

Question 1: [6 marks]

Which of the following statements are **true** in the context of the image formation process:

(A) __F__ Spectral distribution of the light source does not impact the image formation.

(B) __F__ A surface point reflects a particular single wavelength of light we perceive as color.

(C) __T__ Albedo constant in BRDF dictates the fraction of the light being reflected from surface.

(D) __F__ Mirror surface reflects all of incident light equally in all directions.

(E) __F__ Viewing direction impacts the amount of observed light for any surface.

(F) __F__ Unlike pinhole camera, human eye images the world right-side up.

Question 2: [6 marks]

$\frac{1}{f} = \frac{1}{z'} - \frac{1}{z}$

Which of the following statements are **true** of the focal length, f, of a lens? Which are **false**?

(A) __T__ The focal length depends on the geometry (i.e., shape) of the lens.

(B) __T__ The focal length depends on the index of refraction of material used to make lens.

(C) __F__ The focal length depends on the wavelength (i.e., colour) of the light imaged.

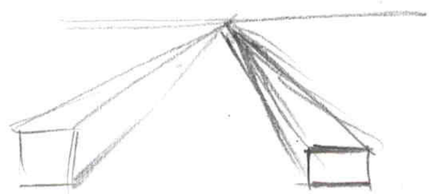
(D) __F__ The focal length depends on the distance from the centre of the lens to the image plane.

(E) __F__ It can be useful to make a lens from the material with refraction index equal to air.

(F) __F__ It is not possible to image an object that is at or closer than f distance to the lens.

Question 3: [5 marks]

Under standard perspective projection, which of the following properties are **true** and which are **false**, in general?



(A) __T__ Straight lines project to straight lines.

(B) __F__ Parallel lines project to parallel lines.

(C) __T__ Distant objects appear smaller.

(D) __F__ Angles are preserved.

(E) __T__ Horizon line does not need to be horizontal, could be at any orientation.

Question 4: [5 marks]

Which of the following process could cause aliasing in a sampled representation?

(A) __F__ Oversampling a signal.

(B) __T__ Using a sampling frequency less than twice the maximum signal frequency.

(C) __T__ Quantizing a signal with too few levels.

(D) __T__ Resizing/rescaling by discarding adjacent samples without low-pass prefiltering.

(E) __T__ Sampling a signal that is not bandlimited.

Question 5: [5 marks]

Which of the statements about pyramid representations are **true** and which are **false**?

(A) T The reason we construct a Gaussian pyramid for template matching is that it allows us to search over scale more robustly (better detection) as compared to creating a pyramid of progressively larger templates and correlating them with the original image.

(B) T The smallest (lowest resolution) level of the Laplacian pyramid is a band-pass filtered version of the original image.

(C) T We set the σ in the Gaussian pyramid construction based on the scaling factor, however, it would be better to choose a sigma for each image based on its frequency spectra.

(D) T Gaussian pyramid can be used for blob detection. blob use same resolution.

(E) T Laplacian pyramid can be used for edge detection.

$(\ ) e^{-\frac{x^2+y^2}{\sigma^2}}$ $\quad \frac{-2x}{\sigma^2} e^{-\frac{x^2+y^2}{\sigma^2}}$ $\quad \frac{-2y}{\sigma^2} e$

Question 6: [5 marks]

Which of the following statements are **true** of the 2D Derivative of the Gaussian ($\frac{\partial G}{\partial x}$ and $\frac{\partial G}{\partial y}$)? Which are **false**?

Gaussian linear

(A) T The 2D Derivative of the Gaussian (∂G) is linear. $G(ax) = aG(x)$

(B) F The 2D Derivative of the Gaussian (∂G) is separable. $G'(ax) =$

(C) T The 2D Derivative of the Gaussian (∂G) is rotationally invariant. Gaussian is

(D) F Convolving an image twice in succession with a 2D Derivative of the Gaussian (e.g., $\frac{\partial G}{\partial x}$) with a given σ is mathematically equivalent to convolving an image with a 2D Second Derivative of the Gaussian with another larger σ.

(E) F Convolving an image twice in succession with a 2D Derivative of the Gaussian in x-direction ($\frac{\partial G}{\partial x}$) and then in y-direction ($\frac{\partial G}{\partial y}$) is equivalent to convolving the image with 2D Laplacian of the Gaussian filter.

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Question 7: [4 marks]

Which of the following statements are **true** of corner and edge detection? Which are **false**?

(A) F Eigenvalues of the covariance matrix in Harris can provide orientation for the corner.

(B) T Eigenvalues of the covariance matrix in Harris can provide information about the strength of an edge.

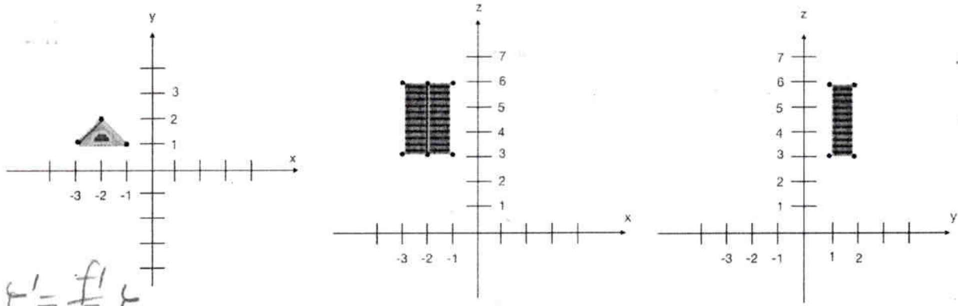
(C) T If one uses the same threshold value for Sobel edge detector as for k_{low} in Canny (assume $k_{high} > k_{low}$), Canny will typically return fewer edge pixels.

(D) F Marr/Hildreth edge detection has poor edge localization Sobel

Part B: Short Answer Questions. Answer each question clearly and concisely. Answers do NOT need to be in complete sentences. Marks will be deducted for long or unclear answers.

Question 8: Projection [13 marks]

A rooftop of the house can be modeled as a triangular prism in 3D with six points in total. The illustration of the roof in the three planes (with $a = 3$) and the corresponding 3D points are given below:



$X' = \frac{f'}{Z}X, \; Y' = \frac{f'}{Z}Y.$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 3+a \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 3+a \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 3+a \end{bmatrix}$$

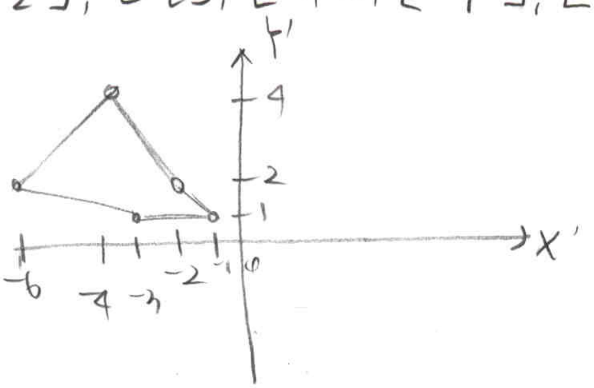
where a is a variable controlling the length of the prism (and, by extension, depth of the house).

(a) [3 marks] Compute the perspective projection of the prizm in the image plane (i.e. give numerical expression for the projected points in terms of a and focal length f where needed).

$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} -f \\ \frac{1}{3}f \end{bmatrix}, \begin{bmatrix} -\frac{1}{3}f \\ \frac{1}{3}f \end{bmatrix}, \begin{bmatrix} -\frac{2}{3}f \\ \frac{2}{3}f \end{bmatrix}, \begin{bmatrix} -\frac{3}{3+a}f \\ \frac{1}{3+a}f \end{bmatrix}, \begin{bmatrix} -\frac{1}{3+a}f \\ \frac{1}{3+a}f \end{bmatrix}, \begin{bmatrix} -\frac{2}{3+a}f \\ \frac{2}{3+a}f \end{bmatrix}$$

(b) [2 marks] Sketch the projection in the imaging plane for $f = 6$ and $a = 3$.

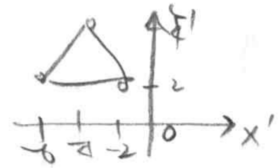
$$\begin{bmatrix} X' \\ Y' \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix}, \begin{bmatrix} -4 \\ 4 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$



(c) [2 marks] Describe both numerically and in terms of concepts we learned about in projection, what happens as $a \to \infty$.

When $a \to \infty$, the three latter points will lead to a projection on the origin since $\lim_{a \to \infty} \frac{f}{a+3} = 0$

Thus, will form only a triangle on the image plane for first three points.



(d) [3 marks] Consider what happens if the projection is not perspective, but rather weak perspective which is governed by a scaling parameter m, i.e.,

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = m \begin{bmatrix} X \\ Y \end{bmatrix}. \tag{1}$$

Compute an appropriate value for m in that case in terms of focal length f and a. Describe what must be true of a and/or f for this to be a good (accurate) approximation.

$m = \frac{f}{Z_0}$ where Z_0 is the average of depths. thus

$m = \dfrac{f}{\frac{3a+18}{9}} = \dfrac{f}{\frac{1}{3}a+2}$. In order for this to be a good approximation, we want the depths to be equal. $3 = a+3 \Rightarrow \underline{a=0}$.

(e) [3 marks] Consider what happens if the projection is orthographic. What would be the projected points be then.

If orthographic, we drop the depths at each point.

$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix}$

0

0

Question 9: Image Filtering [11 marks]

(a) [2 marks] Give a 5×5 filter that when applied as **correlation** shifts the image left by 2 pixel and up by 1 pixel and makes it three times ($3\times$) as bright.

0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	3
0	0	0	0	0

(b) [2 marks] Consider the following filter applied as **correlation**. Provide a filter that when applied as **convolution** will lead to the same output.

0	-1	1
-1	-1	1
1	1	0

Convolution Filter (answer):

0	1	1
1	-1	-1
1	-1	0

(c) [2 marks] Could the above filter be a derivative or gradient filter of sorts? Why or why not?

No, it does not sum up to 0 in the first place.

(d) [3 marks] Suppose we want to first sift and brighten the image using the filter in (a) and subsequently (afterwords) apply filter in (b). We can do this by pre-convolving the two filters before applying a single pre-convolved filter to the image. Compute this pre-convolution filter (which will be applied as **convolution**) .

pre convolved.

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3 & 3 \\ 0 & 0 & 0 & 0 & -3 & -3 & 3 \\ 0 & 0 & 0 & 0 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

0	0	0	0	0	0	0
0	3	3	0	0	0	0
3	-3	-3	0	0	0	0
3	-3	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

(e) [2 marks] Consider designing a filter that would allow you to find local maxima (i.e, if the central value is greater than all its 8 neighbors). Assume we would find a maxima by ~~thresholding the filter response at zero.~~ Design a reasonable **correlation** filter for this task:

-1/8	-1/8	-1/8
-1/8	1	-1/8
-1/8	-1/8	-1/8

To be a local maxima, it should be higher than zero (≥0), when subtracting the average of neighbors.

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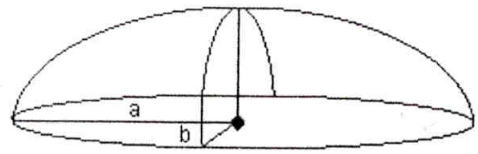
Question 10: Smoothing [14 marks]

Consider making a smoothing filter based on the ellipsoid function defined and illustrated below:

$$P_{a,b}(x,y) = \begin{cases} 0 & \text{if } \frac{x^2}{a^2} + \frac{y^2}{b^2} > 0 \\ \sqrt{1 - \frac{x^2}{a^2} + \frac{y^2}{b^2}} & \text{otherwise} \end{cases}$$



where parameters $a, b > 0$ give an extent (minor and major axes) of the ellipse similar to σ in a Gaussian. These parameters will similarly control the extent and the smoothing of this filter. Notably, non-zero values for the function will only result for $-a < x < a$ and $-b < y < b$.

(a) [2 marks] Would you expect this filter to be rotationally invariant in general (for all values of a and b)? If not, could it be made rotationally invariant and, if so, what condition would need to true with respect to a and b.

No, since in general $a \neq b$ for an ellipse.

By equating a, b thus $a=b$ it can be rotationally invariant.

(b) [4 marks] For a particular $a = 1.5$ and $b = 4.5$ we obtain the following 2D smoothing elliptical filter by evaluating the function above at corresponding pixel positions. Briefly describe two things that are problematic with the filter and how they could be fixed.

0	0.60	0.90	0.60	0
0	0.71	0.98	0.71	0
0	0.75	1.00	0.75	0
0	0.71	0.98	0.71	0
0	0.60	0.90	0.60	0

① Doesn't sum to 1, it might change overall intensity.

⇒ need to normalize the sum to 1.

☆ ② Has unnecessary columns of zeros on both sides.

To reduce computation, remove these columns.

(c) [2 marks] Compared to other smoothing filters we studied (e.g., Gaussian filter), what unique behavior would the elliptical filter with $a = 1.5$ and $b = 4.5$ have? (In terms of how it smooths/blurs the image). Give example where this behavior could be useful?

It would smooth the image in a particular direction more than the other (in this case the vertical is smoothed more).

This behavior can be useful if an image has more noise in one direction than the other.. It would keep detail for direction with less smoothing.

(d) [4 marks] Consider the **central pixel** in the following image patches. State whether the the value of this center pixel will increase, decrease or stay the same (**hint:** no computation other than mental should be necessary here) in the case of smoothing with standard filters:

Image Patch 1
$$\begin{bmatrix} 220 & 10 & 10 \\ 10 & 10 & 200 \\ 10 & 240 & 10 \end{bmatrix}$$

Image Patch 2
$$\begin{bmatrix} 5 & 5 & 18 \\ 5 & 17 & 5 \\ 18 & 5 & 5 \end{bmatrix}$$

$\frac{18}{e} + \frac{18}{e} + 17$

$\frac{36}{2e} + \frac{17}{2}$

$\frac{18}{e} + \frac{17}{2}$

$\frac{2(10)}{2e} + \frac{10}{2} + \frac{2 \cdot 10}{2e}$

	Image Patch 1	Image Patch 2
Box filter	Increase	decrease
Median filter	Same	decrease
Gaussian filter (with $\sigma = 1$)	Increase	Decrease
Bilateral filter (with $\sigma_r = \sigma_d = 1$)	Decrease	Increase

(e) [2 marks] Can bilateral filter be implemented as convolution? Why or why not?

No, since it is not linear nor shift invariant.

Question 11: Derivatives and Gradients [9 marks]

Consider the following two filters. We have seen the Sobel (without normalization), but not Prewitt. Both filters are separable, meaning that they can be written as an outer product of 1D row and 1D column filters.

$$\text{Sobel} = \frac{1}{4}\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \qquad \text{Prewitt} = \frac{1}{3}\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

(a) [2 marks] Write each of these two filters (Sobel and Prewitt) in terms of their corresponding 1D filters (S_h/S_v and P_h/P_v respectively).

Sobel =

$$\underbrace{\begin{bmatrix} \frac{1}{4} & \frac{2}{4} & \frac{1}{4} \end{bmatrix}}_{S_h} \otimes \underbrace{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}_{S_v}$$

outerproduct.

Prewitt =

$$\underbrace{\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}}_{P_h} \otimes \underbrace{\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}}_{P_v}$$

(b) [2 marks] Name each of the 1D filters and specifically describe their function.

S_h is Gaussian filter that performs smoothing.

S_v is central difference filter that performs differentiation in vertical direction

P_h is box filter that performs smoothing

P_v is central difference filter that performs differentiation in vertical direction

(c) [1 marks] In terms of concepts we have seen, why might Sobel be preferred to Prewitt?

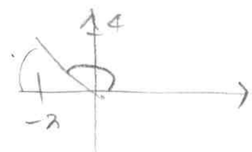
Since sobels horizontal filter will smooth by a Gaussian filter, compared to Prewit which uses a box filter, it will smooth more idealy in a sense that it includes spatial approximity. Hence, the differentiation will perform better by preserving finer details compared to a box filter.

$$\begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \quad \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

(d) [2 marks] Lets assume you want to apply x-directional Sobel followed by y-directional Sobel. What would be the most computationally efficient way to do so? Describe and justify the order of operations. **No actual computation is necessary!**

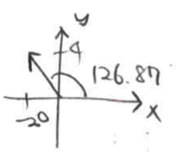
Since a Sobel is separable (and linear), we can separate the total convolutions into a combination of two row and two column convolutions. (separating as in (a))

(e) [2 marks] If the image gradient is $(-3, 4)$, what is the magnitude of the gradient? What is the orientation of the gradient (in degrees)? Recall that Quadrant I spans 0 to 90 degrees; Quadrant II is 90 to 180 and so on. You can leave answers in not-reduced form (e.g., expression in terms of trig functions is perfectly fine).



magnitude $= \sqrt{I_x^2 + I_y^2} = 5$

direction $= \arctan\left(\frac{4}{-3}\right)$ = which will be located in the second quadrant, $126.87°$



Question 12: Corner and Edge Detection [7 marks]

(a) [3 marks] Harris corner detection stems from Autocorrelation which computes local SSD for a patch. Describe the structure of SSD output that you expect to see when computing it for a patch in a (i) region of constant intensity, (ii) a checker board, (iii) a linear gradient region oriented from left-to-right (e.g., rows of pixels that have values 0, 10, 20, 30, 40, etc.)

(i) 0's (SSD = 0) (ii) constant differences. (same SSD)

(iii) constant differences (same SSD)

(b) [4 marks] A location in an image has Harris, or Covariance, matrix:

$$\mathbf{C} = \begin{bmatrix} 1 & 5 \\ 2 & 10 \end{bmatrix}$$

Is this likely to represent a corner? Explain why or why not (numerically). By computing eigenvalues or otherwise (e.g., through linear algebra), deduce what kind of image structure is likely. Note, Harris equation is: $det(\mathbf{C}) - \kappa \cdot trace^2(\mathbf{C})$, where $\kappa = 0.04$.

$$det(C - \lambda I) = det\left(\begin{bmatrix} 1-\lambda & 5 \\ 2 & 10-\lambda \end{bmatrix}\right)$$

$$= \lambda^2 - 11\lambda + 10 - 10 = \lambda^2 - 11\lambda = 0 \qquad \lambda_1 = 0,\ \lambda_2 = 11$$

Since $\lambda_1 = 0$ it is less likely to represent a corner. More of an edge.

Question 13: Texture Synthesis [7 marks]

Consider texture synthesis approach of Efros and Leung for filling in a pixel marked (q) in the texture below. Assume we are using the rest of the image as the source of texture for copying.

230	230	100	230	230	230	230	100	230	100
230	100	230	100	230	230	230	230	100	230
100	230	100	230	230	100	230	230	230	230
230	230	230	230	230	230	230	230	230	230
230	230	230	100	230	230	230	230	230	100
230	230	100	230	230	100	230	230	100	230
230	230	230	230	**q**	230	230	100	230	230
100	230	230	100	230	230	230	230	230	230
230	230	100	230	230	100	230	230	230	230
230	230	230	230	230	100	230	230	230	100

(a) [3 marks] Assuming we only consider exact matches and a 3×3 neighborhood, compute the probability of the pixel q being each color:

$P(q = 230 \mid 3 \times 3 \text{ Neighborhood}(q)) = \frac{6}{8} = 0.75$

$P(q = 100 \mid 3 \times 3 \text{ Neighborhood}(q)) = \frac{2}{8} = 0.25$

$P(q = 0 \mid 3 \times 3 \text{ Neighborhood}(q)) = 0$

(b) [3 marks] Now consider a 5×5 neighborhood. Compute the probability of the pixel q being each color now:

$P(q = 230 \mid 5 \times 5 \text{ Neighborhood}(q)) = \frac{18}{24} = 0.666..$

$P(q = 100 \mid 5 \times 5 \text{ Neighborhood}(q)) = \frac{6}{24} = 0.333..$

$P(q = 0 \mid 5 \times 5 \text{ Neighborhood}(q)) = 0$

(c) [1 marks] How does increasing the size of the neighborhood affect the texture synthesis results?

When increasing the size of the neighborhood, the probability of certain patches that match change. That is, for a larger neighborhood when doing texture synthesis, we would have a larger variety of patterns if the thresholds maintain as the smaller neighborhood.