

Math 257/316 Assignment 2, 2024  
**Due Monday January 29 Submit online in a PDF document on Canvas  
by 11:59 pm of the due date**

**Problem 1 (Submit):** Consider the ODE:

$$(1 + x^3)y'' - 6xy = 0. \quad (1)$$

- (a) Compute the first 3 nonzero terms of power series expansion about  $x = 0$  for two linearly independent solutions.
- (b) Use the ratio test to determine the radius of convergence of the series. Could your result have been predicted by inspection?

**Problem 2 (Submit):** Consider the Differential equation

$$(1 + x^2)y'' + xy' - y = 0. \quad (2)$$

- (a) Find the first 3 nonzero terms of the power series expansion of the general solution about  $x = 0$ .
- (b) Use the ratio test to determine the radius of convergence of the series. What can you say about the radius of convergence without solving the ODE?
- (c) Determine the solution that satisfies the initial conditions  $y(0) = 1$  and  $y'(0) = 0$ .

**Problem 3 (Do not submit):** Compute the first 3 nonzero terms of the power series expansion about  $x = 0$  of two linearly independent solutions of the ODE:  $y'' - (\sin x)y = 0$ .

**Problem 4 (Do not submit):** Find the power series expansion about  $x = 0$  of two linearly independent solutions of the ODE  $y'' + x^2y = 0$ . What can you say about the radius of convergence of this power series?

**Problem 5 (Do not submit):** Find all the singular points of the following ODEs and determine whether each one is regular or irregular. If the singular point is regular, determine the indicial equation and determine a lower bound for the radius of convergence of the Frobenius series. You must not try to compute the series solution itself.

- a.  $(x^2 - 1)y'' + x^2y' + \cot(x)y = 0$ .
- b.  $\sinh(x)y'' + xy' + y = 0$ .
- c.  $(x^2 + 4)y'' + xy' + y = 0$ .
- d.  $y'' + x^{1/2}y' - y = 0$ .