INSTRUCTOR NOTES

Begin by getting students to sit in their teams. If possible, members should sit facing each other. Do not allow students to change teams.

If only 2 members show up for a team, they may work with another small team, but must submit their own worksheet. If only 1 member shows up for a team, they must work with another team, and may submit a blank worksheet with only their name on it for attendance-taking purposes.

Finally, pass out the handouts and announce the first question. Remember to have a routine to close questions (*e.g.* countdowns).

At the end of the class, remember to collect worksheets.

This week's tip: ***provide support, not answers***. Resist the temptation to provide answers or to give comprehensive definitions, especially for questions 4 and 8. Instead, provide support. One way is to encourage teams to "try something" — for example, to pick specific examples and see what happens ("pick a point on the plane — what's the y value there? what's the slope?").

NOTES ON QUESTIONS

The large lecture prior to this small class is on differential equations. In this small class, students look at two qualitative methods of solving differential equations.

1. **1 minute**.
   To close the question, ask teams to volunteer answers, and write them on the board as they come up.
   (a) $\frac{dy}{dt} = \frac{7}{64}$ (b) $\frac{dy}{dt} = \frac{1}{4}$ (c) $\frac{dy}{dt} = \frac{7}{64}$ (d) $\frac{dy}{dt} = -\frac{3}{4}$ (e) $\frac{dy}{dt} = -2$.
2. **1 minute** of instructor-led class discussion.
   To close the question, write the answers on the board as they come up.
   There are two answers: $y = 0$ and $y = 1$.
3. **15 minutes** for questions 3, 4 and 5.
   Visit teams frequently to make sure they are on the right track.
   To close the question, draw the axes on the board, and ask some teams to draw slopes for various y-values as the other teams continue to work.

6. **1 minute** of instructor-led class discussion.
   To close the question, write the answers on the board as they come up.
   There are two answers: $y = 0$ and $y = 1$. An important point to take away: we find steady states by setting $\frac{dy}{dt} = 0$.
7. **15 minutes** for questions 8, 9 and 10.
   Visit teams frequently to make sure they are on the right track. It will help teams to consider the function $f(y) = y(1 - y)$. What is the shape of the graph? Where is it positive, and where is it negative?
   To close the question, draw the y-axis on the board, and ask some teams to label steady states as the other teams continue to work.

SMALL CLASS: The logistic model revisited

In this class, you will learn to sketch slope fields and phase lines for differential equations, and to find and analyze steady state solutions.

Contributing team members

Student number	Last name	First name

Small class questions

Consider the differential equation

$$\frac{dy}{dt} = y(1-y).$$

(In a previous small class, you sketched a solution $y = \frac{1}{1+e^{-t}}$.)

1. What is $\frac{dy}{dt}$ for: (a) $y = \frac{1}{8}$, (b) $y = \frac{1}{2}$, (c) $y = \frac{7}{8}$, (d) $y = \frac{3}{2}$ and (e) $y = 2$?

Answer:

Scribe:

2. For what y-value(s) is $\dfrac{dy}{dt} = 0$?

Answer:

Scribe:

3. (Key concept) A *slope field* records values for $\frac{dy}{dt}$ on y vs t axes. Draw the axes for a slope field for the differential equation, and draw slopes for $y = 0$ and $y = 1$, as well as for the other y-values examined in previous questions.

Answer:

Scribe:

4. On the slope field above, sketch some solutions to the differential equation.

5. Suppose you are given the initial condition $y(0) = \dfrac{1}{4}$. On the slope field above, sketch the solution satisfying that condition.

6. (Key concept) A *steady state* is a state in which a system is not changing. For example, a solution $y(t)$ to a differential equation is a steady-state solution if y remains constant for all t. What are the steady states for the given differential equation?

Answer: Scribe:

7. (Key concept) A *phase line* records values for $\frac{dy}{dt}$ on a horizontal y-axis. Draw a horizontal y-axis, and draw dots on the axis at the steady states.

Answer: Scribe:

8. Determine the sign of $\frac{dy}{dt}$ in the intervals between the steady states, and indicate the signs above using arrows either pointing in the positive y direction or in the negative y direction.

9. (Key concept) A steady state solution is *stable* if all nearby solutions tend to it, *unstable* if all nearby solutions tend away from it, and *semistable* otherwise. Label the steady states above as stable, unstable or semistable.

Practice questions

The questions below are for practice. They do not contribute to your grade, and it is not expected that you complete them during your small class. However, you are strongly encouraged to work through them.

10. (★★☆☆) Draw the slope field and phase line for the differential equation $\frac{dy}{dt} = y - y^3$.
Note: you should be able to use your phase lines to confirm your slope fields, and vice versa.

11. (★★☆☆) Draw the slope field and phase line for the differential equation $\frac{dy}{dt} = y^2 + y^3$.

12. (★★★☆) Consider the augmented differential equation

$$\frac{dy}{dt} = y(1 - y) - H.$$

Draw the slope field and phase line for three different values of H: $H = \frac{1}{8}$, $H = \frac{1}{4}$ and $H = \frac{1}{2}$. If the differential equation describes a population y (with respect to time t) and H is a term that represents a "constant harvest", describe in a few sentence what the overall effect of H on the population.