

INSTRUCTOR NOTES

This small class has recommended additional materials: one blank sheet of paper per student, to be folded.

Begin by getting students to sit in their teams. If possible, members should sit facing each other. Do not allow students to change teams.

If only 2 members show up for a team, they may work with another small team, but must submit their own worksheet. If only 1 member shows up for a team, they must work with another team, and may submit a blank worksheet with only their name on it for attendance-taking purposes.

Finally, pass out the handouts and announce the first question. Remember to have a routine to close questions (*e.g.* countdowns).

At the end of the class, remember to collect worksheets.

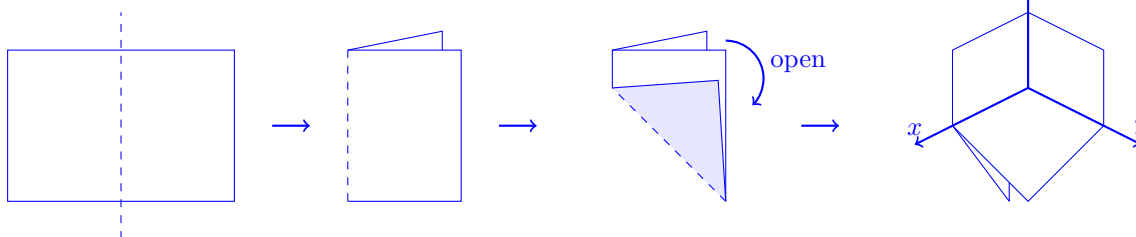
*This week's tip: **gesture, draw, say ... and then write.*** Many students will be unused to thinking about surfaces in space. Gesturing, and encouraging students to gesture (“at $z = 0$ you're at the bottom of the bowl; as z increases, the bowl *widens*”), will help. In fact, in the interests of putting more responsibility on students, it is useful to use more gestures, drawings and spoken explanations, reserving writing for the most important, authoritative statements.

NOTES ON QUESTIONS

The large lecture prior to this small class introduces multivariable functions. In this class, students plot multivariable functions using level curves and traces.

1. **10 minutes** for questions 1, 2 and 3.

The folding directions should be demonstrated:



To close the question, simply make sure most teams can visualize points — there is no need to write anything on the board.

4. **10 minutes.**

To close the question, draw four sets of x - y axes on the board, and ask some teams to draw a level curve on a set of axes as the other teams continue to work. Give the conclusion: we know our surface has no points for $z < 0$, is a single point at $z = 0$, and is a tower of increasingly larger circles for $z > 0$.

5. **10 minutes.**

To close the question, draw four sets of unlabeled axes on the board, and ask some teams to draw a trace on a set of axes, and label the axes, as the other teams continue to work.

The traces are, respectively, the parabolas $z = x^2$, $z = y^2$, $z = y^2 + 1$ and $z = x^2 + 4$.

6. **Remaining time** for the last question.

The shape is a parabolic bowl.

SMALL CLASS: Level curves and traces

In this class, you will plot points on x - y - z axes. Given a function of two variables, you will learn to sketch level curves and traces.

Contributing team members

Student number	Last name	First name

Small class questions

1. Follow the directions of your instructor to fold a sheet of paper to represent the x - y - z axes. Draw lines in the creases to represent the three axes.
2. Plot the points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ by marking them with dots. Then plot the additional points $(2, 2, 0)$, $(2, 0, 2)$, and $(0, 2, 2)$.
3. Use a pen or other small object to indicate the point $(3, 2, 1)$ in space — for example, by holding the base of the pen at the origin and the writing end at the point. Once all your team agrees, test yourselves with different coordinates.

Consider $z = x^2 + y^2$. The goal of the remaining questions is to visualize the surface.

4. (★★★★, key concept) A *level curve* is a “horizontal slice” of a three-dimensional surface. Sketch four level curves on four separate x - y axes by setting: (a) $z = 4$, (b) $z = 1$, (c) $z = 0$ and (d) $z = -1$.

Answer:

Scribe:

5. (★★★★, key concept) A *trace* is like a level curve, but with any variable set equal to a constant. Sketch, on appropriate axes, the traces corresponding to: $y = 0$, $x = 0$, $x = 1$, and $y = 2$.

Answer:

Scribe:

6. (★★☆☆) Use your answers to the previous questions to sketch, on x - y - z axes, the surface $z = x^2 + y^2$. Incorporate at least one level curve from question 4 and one trace from question 5 in your sketch.

Answer:

Scribe:

Practice questions

The questions below are for practice. They do not contribute to your grade, and it is not expected that you complete them during your small class. However, you are strongly encouraged to work through them.

7. (★☆☆☆) Sketch the surface $y = x^2 + z^2$.
8. (★★☆☆) Find the level curves of $x^2 + y^2 = z^2 - 1$ corresponding to $z = 0, \pm 1, \pm 2, \pm 3$, and the traces corresponding to $x = \pm 1$. Then sketch the surface, incorporating the traces in your sketch.
9. (★★☆☆) Find the level curves of $x + y + z = 1$ corresponding to $z = -1, 0, 1$, and the traces corresponding to $x = -1, 0, 1$ and $y = -1, 0, 1$. Then sketch the surface, incorporating the level curves and traces in your sketch.