INSTRUCTOR NOTES

Begin by getting students to sit in teams of 3-5. In this first class, they can join to form teams however they like (or you can direct them, if you would prefer). Students must work in teams, not as individuals. Let students know that fixed teams will be announced next class.

(Looking ahead: Students will be grouped into fixed teams starting with the second small class. You'll want to be prepared for this and print off a copy of the teams for your class (as provided by Jessica through the google sheet in OneDrive) and plan how to communicate the teams to your class in SC2. Warning: it will be chaotic, and it will take a while. Best to view this process as a messy icebreaker rather than as a quick task to be accomplished efficiently.)

After the teams are assembled, brief them on the small class structure. In particular:

- Small classes introduce NEW material. They do not review what is done in large class.
- Small classes are participatory. Active engagement is a key to success.
- Use of phones and other electronic devices is strongly discouraged. Please encourage others on your team to stay focused.
- Fill in the list of contributing team members on your worksheet at the *end* of the small class. Remember that the scribe (the person writing, but not the only person doing the work!) should rotate from question to question.
- All team members should be working on the same problem at the same time. Splitting up problems is a bad strategy for understanding.

You should also introduce yourself! This is your first class of the entire semester with these students. Let them know your name, why you're here (what mathematics do you do?), and if you'd like, tell them something interesting or boring or otherwise human about yourself (favourite hobby?).

Finally, take 1 minute to introduce the topic by connecting it to what they did in large class, and summarizing what they can expect to learn through the worksheet today. Pass out the handouts and instruct students to begin with the first question.

At the end of the class, remember to collect worksheets. Let the students know you'll return them in the next small class but you can encourage them to take pictures of the worksheet once they're done.

NOTES ON QUESTIONS

Here is a basic suggested introduction:

> Prior to this small class, students learned about area functions.
> Sketch a general $A(x) = \int_a^x f(t)\, dt$ and remark that today we will understand what the derivative of the area function is. Indeed, today is a *fundamental day.*

Here are notes on guiding each question:

1. **10 minutes** for getting settled into groups, question 1.
2. **15 minutes**. Students should be able to dig right into this question. Let them know that this question is a good example that illustrates the main idea of the day. Ask them to start on (a) and continue onto (b) and (c) and (d) when they're done. Circulate with your UTA and provide hints and encouragement.

Watch part (b) that they answer every sub-question here. The area function will be new to students, so asking basic questions about what A is at different x values can be helpful.
To close the question, have a student (who you asked about while circulating) draw $A(x)$ on the board. Failing that, have students suggest what function $A(x)$ looks like. Write the answer to part (d) on the board.

3. **10 minutes**. The goal of this questions is to give students the idea of the proof of the fundamental theorem. Coming up with the idea out of nothing is difficult, but walk everyone through it together at the board. Have a lively whole class discussion. Feel free to use colour on your diagram and ask questions like “which colour is our $E(x)$?” Other questions include “how can we approximate this quantity?” or “what does this quotient resemble?” or “what happens when h is very small?”
To close the question, remark that together you've produced a compelling argument. You could even draw a box. Make sure the Theorem is visible on the board somewhere.

4. **10 minutes**. It's OK if this question needs to be shortened or left as homework. They'll have lots of opportunity to practice in the WeBWorK. Students who are fast can move on to the practice problem on the last page.
To close the questions, time permitting, you can have students shout out the answers for parts (a) and (b). For part (c) you might ask a student to identify what the outer and inner functions are here. Either way, encourage students to try the practice problem and let them know you're looking forward to an enjoyable term together.

SMALL CLASS: The Fundamental Theorem of Calculus Part 1

In this class, you will make explicit the relationship between derivatives and integrals using the area function.

Contributing team members

Student number	Last name	First name

Small class questions

1. The *scribe*'s role is to do the writing and record the answers.

   Nominate a team member to be the scribe. As you work through a worksheet, the scribe role *must* rotate from question to question, repeating only after every team member has taken a turn.

   Your team may also wish to have other roles. For example, the *manager*'s role is to keep the team on task ("Okay, what's next?"); the *skeptic*'s role is to question the team's answer to make sure it is sound ("How do we *know* that x^3 is small?"). Your team should always be working on the same problem together. The goal isn't speed, it's conversation and understanding.

2. **The area function**

   (a) (★☆☆☆) Consider the area function $A(x) = \displaystyle\int_a^x f(t)\ \mathrm{d}t$. Suppose that $a = 0$ and $f(t) = \cos t$. Find $A(2\pi)$ by drawing a picture.

Answer:
Scribe:

(b) (★★☆☆) Again consider $A(x) = \int_0^x \cos t \, \mathrm{d}t$. Let's get a rough sketch of the function $A(x)$. First, find $A(0)$ and $A(\pi)$. Next, determine the sign of $A(\pi/2)$ and $A(3\pi/2)$. Finally, think about the intervals $(0, \pi/2)$, $(\pi/2, 3\pi/2)$, and $(3\pi/2, 2\pi)$ and determine if $A(x)$ is increasing or decreasing on these intervals. What familiar function does $A(x)$ look like?

Answer:

Scribe:

(c) (★☆☆☆) With your guess for $A(x)$ in part (b) above, what do you think $A'(x)$ is?

Answer:

Scribe:

(d) (★☆☆☆) In general, if $A(x) = \int_0^x f(t) \, \mathrm{d}t$ what do you think $A'(x)$ is? Remember, this is a function of x and not t.

Answer:

Scribe:

3. **The Fundamental Theorem of Calculus Part 1**

(a) (★★★★) The Theorem states that if $A(x) = \int_a^x f(t)\ \mathrm{d}t$, where f is a continuous function, then $A'(x) = f(x)$.

With the help of your small class instructor, understand the proof of the Fundamental Theorem of Calculus Part 1. You can find a helpful picture on page 47 of CLP-2.

Consider $A(x) = \int_a^x f(t)\ \mathrm{d}t$. We want to show that $A'(x) = f(x)$. Draw a picture showing both $A(x)$ and $A(x+h)$. Write $A(x+h) = A(x)+E(x)$ and indicate this difference $E(x) = A(x+h)-A(x)$ clearly. Write a sentence arguing that, based on your picture, $E(x) \approx h \cdot f(x)$ and that this becomes an equality when $h \to 0$. Rearrange everything to find that $f(x) \approx \dfrac{A(x+h)-A(x)}{h}$. What does this resemble? Take the limit as $h \to 0$ to achieve the desired result.

Answer:

Scribe:

4. **Using Part 1 of the Fundamental Theorem of Calculus.**

(a) (★☆☆☆)

Use the Fundamental Theorem to confirm our earlier suspicions and compute $\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_0^x \cos(t)\ \mathrm{d}t\right)$.

Answer:
Scribe:

(b) (★☆☆☆)

Compute $\frac{\mathrm{d}}{\mathrm{d}x}\left(\int_1^x \arctan t\ \mathrm{d}t\right)$.

Answer:
Scribe:

(c) (★☆☆☆)

Compute $\frac{\mathrm{d}}{\mathrm{d}x}A(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\int_1^{x^2} \arctan t\ \mathrm{d}t\right)$. To do so, recognize $A(x) = f(g(x))$ as the composition of two functions. What are f and g here? Once you have them all you need is chain rule.

Answer:
Scribe:

Practice questions

The questions below are for practice. They do not contribute to your grade, and it is not expected that you complete them during your small class. However, you are strongly encouraged to work through them.

5. (★★☆☆) Let $A(x) = \displaystyle\int_{e^x}^{x^2} \sin t \; \mathrm{d}t$. Find $A'(x)$. For this more challenging problem you will need the identity $\displaystyle\int_a^b f(t) \; \mathrm{d}t = -\int_b^a f(t) \; \mathrm{d}t$ which you will see later. Convince yourself it should be true and then solve the problem.