Week 1 lecture notes
September 13, 14, 15 (2022)

Topics: Comparing power, log, and exponential functions; trigonometric functions; parse trees

Instructor notes:

- Because this is the first class, we may wish to spend some time introducing ourselves and the course.
- Desmos is a good option to use at the end of certain problems — for example, problem 4 in the first section.
- In the second section, we may not necessarily wish to have students sketch all the functions on paper. For example, we might ask one or all of them to trace some of the functions in the air.

Learning Objectives (Power functions):

- Sketch basic power functions.
- Determine which term in a polynomial function will dominate for small x and for large x.
- Sketch two-term polynomial functions by determining which term dominates for small x and for large x.

Problems and takeaways (Power functions):

1. Which of the following are power functions?
$$x^3,\ \pi x^{102},\ \sqrt{x},\ 1/x.$$
2. **Definition**: A *power function* takes the form $f(x) = Kx^n$ where K is a nonzero constant. For now, our focus will be on power functions where n is a positive integer
3. Sketch the functions x^2, x^3, x^4, x^5, x^6 and compare their behaviour for small x and for large x.
4. What function is a good approximation for $f(x) = x^3 + x^5 + x^7$ for small x? For large x?

   Multiple choice: (A) 1 (B) x^3 (C) x^5 (D) x^7 (E) Not sure
5. Draw a rough sketch of the two term polynomial $f(x) = x^2 - x^4$.
6. **Takeaway**: We can understand power functions and combinations of power functions by investigating their behaviour for both small and large x.

Learning Objectives (Beyond power functions):

- Sketch familiar functions such as e^x, $\log x$, $\sin x$, $\cos x$, $\tan x$, $1/x$, $\sqrt{x}$, and $|x|$.
- Demonstrate that e^x eventually dominates any given power function.

Problems and takeaways (Beyond power functions):

1. Sketch the following functions:
$$e^x,\ \log x,\ \sin x,\ \cos x,\ \tan x,\ 1/x,\ \sqrt{x},\ |x|\,.$$

2. **Note**: You should be familiar with the definitions of trigonometric functions. These definitions will also be reviewed briefly in a small class.
3. Consider e^x and x^2 on the interval $[2, 9]$. Where is e^x bigger? What if x^2 is replaced with x^3? x^4?
4. Draw a rough sketch of $f(x) = e^x - x^4$.
5. **Takeaway**: The function e^x eventually dominates x^n. In other words, for large enough x, we will always have $e^x > x^n$. (We will be able to prove this by the end of the course.)

Learning Objectives (Parse trees):

- Given a complicated function, describe it using a parse tree.
- Given a complicated function, describe in words the order in which to apply operations.

Problems and takeaways (Parse trees):

1. Develop parse trees for the following functions.
   (a) $e^{|x-5|^2}$
   (b) $\frac{1+x}{1+2x-x^2}$
   (c) $\left(\frac{t+\pi}{t-\pi}\right)\sin\left(\frac{t+\pi}{2}\right)$
2. **Takeaway**: Parse trees are a useful way to organize the order of operations when dealing with complicated functions. You will need to be able to break functions down like this implicitly throughout the course. This is covered in **CLP section 0.5**.

Additional problems:

1. The Lennard-Jones potential

$$V(r) = \varepsilon\left(\left(\frac{R}{r}\right)^{12} - 2\left(\frac{R}{r}\right)^{6}\right), \quad \varepsilon, R > 0$$

   describes the potential energy of a diatomic molecule where the atoms are a distance $r > 0$ apart.
   (a) Write $V(r)$ as a rational function.
   Definition: A *rational function* is the quotient of two polynomials.
   (b) What function is a good approximation for $V(r)$ for small r? For large r?
   (c) Draw a rough sketch of $V(r)$.

2. The Morse potential

$$M(r) = \varepsilon\left(1 - e^{-a(r-R)}\right)^2, \quad \varepsilon, a, R > 0$$

   is an alternative to the Lennard-Jones potential.
   (a) What function is a good approximation for $M(r)$ for small r? For large r?
   (b) Draw a rough sketch of $M(r)$.