Math 257/316 Assignment 1, 2024

Due Monday January 22, Submit online in a PDF document on Canvas by 11:59 pm of the due date

Problem 1 (Submit): (**ODE Review**) Find the general solutions of the following equations:

a. $y'' - 2y' + 2y = e^x \cos x$
b. $(1-x)\, y' = y \ln y$
c. $y'' - 4y' + 4y = \sin x + x^2$
d. $\left(1+x^2\right) y' + 2xy = 1$
a. $2x^2y'' - xy' + y = x^{1/2}$
b. $x^2y'' - xy' + y = 0$
c. $x^2y'' - xy' + 5y = x^2$

Problem 2 (Submit): (**Power series solution**): Consider the following linear ODEs:

$$y' + (1+x)y = 0 \tag{1}$$
$$y'' + 4y = 0 \tag{2}$$

a. Use the methods you learned for solving ODEs to solve equations (1) and (2).

b. Expand the solutions to (1) and (2) as Taylor series about the point $x_0 = 0$.

c. Now for each of equations (1) and (2) assume a power series solution of the form

$$y(x) = \sum_{n=0}^{\infty} a_n x^n \tag{3}$$

obtain a recursion for the coefficients a_n. Use these recursions to determine the series representation of each solution. Compare these results to the series obtained in part b above.

Problem 3 (Do not submit): (**Power series solution**): Consider the following first order linear ODEs:

$$y' + (1-2x)y = 0 \tag{4}$$
$$xy' + (2-x)y = 0 \tag{5}$$

a. Solve the differential equations (4) and (5) using the appropriate integrating factors.

b. Expand the solution to (4) as Taylor series about the point $x_0 = 0$. Expand the exponential in the solution to (5) as a power series.

c. Now for (4) assume a power series solution of the form

$$y(x) = \sum_{n=0}^{\infty} a_n x^n \tag{6}$$

obtain a recursion for the coefficients a_n. Use these recursions to determine the series representation of the solution. Compare this result to the series obtained in part b above.

d. Try using the same power series expansion (6) to solve (5). What happens?

e. Consider the following recursive strategy to generate an approximate solution to (5). Rewrite (5) as

$$xy' + 2y = xy \tag{7}$$

Now assuming $x \to 0$ and discarding the right hand side of (7), find a first order approximation y_0 as the solution to

$$xy_0' + 2y_0 = 0$$

Now substitute y_0 on the right side of (7) and solve for y_1

$$xy_1' + 2y_1 = xy_0$$

Continue this process till you obtain y_2. How does y_2 compare with the series solution to (5) obtained in b? Can you use this series to motivate a modification to the series expansion (6) that would be appropriate to use to obtain a series solution to (5)?