2.

$F_d = \pi \cdot \rho_a \cdot R \cdot v \cdot \sqrt{a^2v^2 + c^2R^2v^2}$. When we hit the golf ball drag force is opposite to motion, hence $\Sigma F = ma = -F_d$.

$m = \frac{4}{3}\pi R^3 \cdot \rho_b \Rightarrow \left(\frac{4}{3}\pi R^3 \rho_b\right) \cdot v \cdot \frac{dv}{dx} = \pi \cdot \rho_a \cdot R \cdot v \cdot \sqrt{a^2v^2 + c^2R^2v^2} \cdot (-1)$

$\Rightarrow \frac{\frac{4}{3}R \cdot \rho_b}{\sqrt{a^2v^2 + c^2R^2v^2}} dv = -dx \Rightarrow \frac{4R^2\rho_b}{3\rho_a \cdot a \cdot v} \frac{1}{\sqrt{1 + \left(\frac{cR}{av}v\right)^2}} dv = -dx \Rightarrow \int_{v_0}^{0} \frac{1}{\sqrt{1 + \left(\frac{cR}{av}v\right)^2}} dv \cdot \frac{4R^2\rho_b}{3\rho_a \cdot a \cdot v} = -x$

↳ let $\tan\theta = \frac{cR}{av} \cdot v$, then $\sec^2\theta \cdot d\theta = \frac{cR}{av} dv$. So $\int_{v_0}^{0} \frac{1}{\sqrt{1 + \left(\frac{cR}{av}v\right)^2}} dv = \int_{\alpha}^{0} \sec\theta \, d\theta \cdot \frac{av}{cR}$ where $\alpha = \arctan\left(\frac{cR}{av} \cdot v_0\right)$

↳ $\left[\ln(\sec\theta + \tan\theta)\right]_{\alpha}^{0} = -\ln(\sec\alpha + \tan\alpha)$

Therefore, $\ln(\sec\alpha + \tan\alpha) \cdot \frac{4R^2 \cdot \rho_b}{3\rho_a \cdot a \cdot v} \cdot \frac{av}{cR} = \ln(\sec\alpha + \tan\alpha) \cdot \frac{4R\rho_b}{3\rho_a \cdot c}$

$= \ln\left(\frac{cR}{av}v_0 + \sqrt{1 + \left(\frac{cR}{av}v_0\right)^2}\right) \cdot \frac{4R\rho_b}{3\rho_a c} = x$

Let's call the original ball's travelling distance $x = \ln\left(\frac{cR}{av}v_0 + \sqrt{1 + \left(\frac{cR}{av}v_0\right)^2}\right) \cdot \frac{4R\rho_b}{3\rho_a c}$

then professor dimple's ball is $x_{dimp} = f \cdot \frac{4R\rho_b}{3\rho_a c} \cdot \ln\left(\frac{1}{f^2} \cdot \frac{cR}{av}v_0 + \sqrt{1 + \left(\frac{1}{f^2}\frac{cR}{av}v_0\right)^2}\right)$ since a increased to fa, and c decreased to $\frac{c}{f}$

Therefore professor dimple's ball is better if $f > 1$ and $v_0 \gg \frac{av}{cR}$.