**INSTRUCTOR NOTES**

This small class has recommended additional materials: one measuring tape and one long piece of string.

This small class is an opportunity to defend parameters — often a source of stress for students. Encourage teams to use parameters ($S$ instead of a numerical speed, $L$ instead of a numerical length) until the last possible step. You can justify this by occasionally using the parameters to validate steps ("if $L$ were smaller, note that $\frac{d\theta}{dt}$ would be larger — this makes sense because...").

Begin by getting students to sit in their teams. If possible, members should sit facing each other. Do not allow students to change teams.

If only 2 members show up for a team, they may work with another small team, but must submit their own worksheet. If only 1 member shows up for a team, they must work with another team, and may submit a blank worksheet with only their name on it for attendance-taking purposes.

Finally, pass out the handouts and announce the first question. Remember to have a routine to close questions (*e.g.* countdowns).

At the end of the class, remember to collect worksheets.

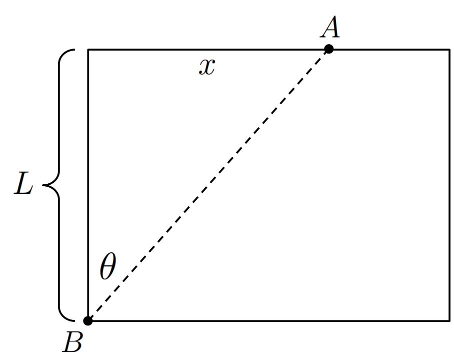
*This week's tip:* ***visit for a minute***. For longer questions, like questions 7, 8 and 9, it will be tempting to visit a team and then stay with them until they puzzle out the answer. Resist the urge to do this; instead, drop in, do a quick assessment, provide some support (a confirmation or an idea), and then move on.

**NOTES ON QUESTIONS**

The large lecture prior to this small class is on the Chain Rule and some of its consequences. This small class is about another consequence, related rates problems.

1. **1 minute** for teams to guess a typical walking speed, **2 minutes** to measure the walking speed of a student volunteer. calculate their speed.
   **To close the question**, first ask teams to volunteer their answers. Then measure the walking speed of a student volunteer (Student $A$) and write it on the board.

2. **6 minutes**.
   Read the question out loud. As you do so, position two volunteers (Student $A$ and Student $B$) with the string taut between them. As $A$ walks, indicate where $\theta$ is. Do not draw the picture at this point.
   **To close the question**, ask a team with a sensible graph to explain it to the rest of the class.



3. **1 minute**.
   At the end of the class, return to these answers to see who was closest.

4. **3 minutes**.
   **To close the question**, draw the sketch above on the board, and ask teams to use the same symbols.

5. **1 minute** of instructor-led class discussion.
   **To close the question**, have a team write the answer on the board.
   We know that $\frac{dx}{dt} = S$.

6. **1 minute** of instructor-led class discussion.
   **To close the question**, have a team write the answer on the board.
   We wish to find $\frac{d\theta}{dt}$ when $\theta = \frac{\pi}{4}$.

7. **5 minutes**.
   Some teams will want to calculate the length of the hypotenuse; encourage this as a second solution after a more direct approach first. Some teams will want to solve for $\theta$; this is an additional problem, below.
   $\tan(\theta) = \frac{x}{L}$.

8. **10 minutes**.
   **To close the question**, have a team put their work on the board.
   $\frac{d\theta}{dt} = \frac{S}{L}\cos^2(\theta)$, which when $\theta = \frac{\pi}{4}$ is equal to $\frac{S}{2L}$.

9. **5 minutes** of instructor-led class discussion. As good answers come up, write them on the board and give teams time to copy them down.
   Examples of good answers: the sign is correct, the answer makes sense for extreme cases (for example when $\theta \to \frac{\pi}{2}$), the answer makes sense if the parameters are increased or decreased (for example if $S$ were much larger).

10. **1 minute**. Simply write down the answer for teams to copy:

    (a) Draw and label a picture.
    (b) Write down what is known.
    (c) Write down what you wish to find.
    (d) Write down an equation that relates the thing whose rate you want to the thing whose rate you know.
    (e) If necessary, reduce the equation to one variable.
    (f) Differentiate (using the Chain Rule) and solve.
    (g) Do a “reality check” to see if your answer makes sense.

## SMALL CLASS: Related rates

*In this class, you will learn and implement a sequence of steps to solve "related rates" problems.*

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### Contributing team members

| Student number | Last name | First name |
|---|---|---|
| | | |

### Small class questions

1. What is your walking speed (in metres per second)?

| Answer: |
|---|
| Scribe: |

2. Suppose Student $A$ starts in one corner of the room, and Student $B$ starts in the other corner closest to $A$. A string is held tight between them. As $A$ walks along the long wall, the angle $\theta$ between the string and the wall changes. Sketch the graph of $\theta$ with respect to time $t$.

| Answer: |
|---|
| Scribe: |

3. At the moment $\theta = \frac{\pi}{4}$, what would you guess is the rate of change of $\theta$?

Answer:

Scribe:

4. Draw and label a picture illustrating the problem. Start by drawing a rectangle representing a bird's eye view of the classroom.

Answer:

Scribe:

5. What do we know? What information are we given?

Answer:

Scribe:

6. What do we wish to find out?

Answer:

Scribe:

7. What is an equation that relates *the thing whose rate we want to find out* to *the thing whose rate we know*?

| Answer: |
| --- |
| Scribe: |

8. Use differentiation — in particular, the Chain Rule — to find the rate we need.

| Answer: |
| --- |
| Scribe: |

9. Does this answer “make sense”? How can we tell?

Answer:

Scribe:

10. What are the general steps for solving related rates problems?

Answer:

Scribe:

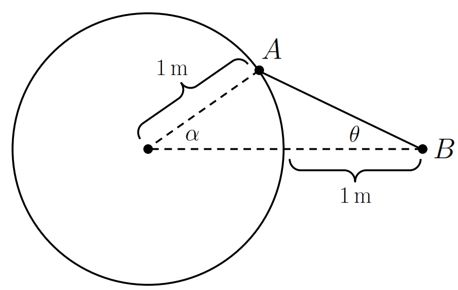
**Practice questions**

*The questions below are for practice. They do not contribute to your grade, and it is not expected that you complete them during your small class. However, you are strongly encouraged to work through them.*

11. (★★☆☆) The radius of a sphere is increasing at a rate of 5 mm/sec. How quickly is the volume increasing when the radius is 20 mm?

12. (★★★☆) Confirm for the related rates problem in the small class that $\frac{d\theta}{dt} = \frac{S}{L}\cos^2(\theta)$, but this time solve for $\theta$ before differentiating.

13. (★★★★) Imagine a disc of radius 1 m spinning anticlockwise at $\pi$ rad/sec. A tight elastic connector joins the point $A$ on the rim of the disc to the stationary point $B$ located 1 m away from the disc, as shown below.




How quickly is the angle $\theta$ changing at the moment $\alpha = \frac{\pi}{3}$?
*Hint:* Begin by writing $\tan(\theta)$ in terms of $\alpha$.