INSTRUCTOR NOTES

The instructors' small class sheet will always begin, as it does here, with notes and a tip.

Begin by introducing yourselves briefly. Provide some personal details; you will be working with these students all term! In addition, when interacting with teams, take the time to address students by their names.

After you have introduced yourselves, assemble the teams using the provided spreadsheet. Put unassigned students into separate teams of 3-5 or add them to undersized teams. Make a note of where they are assigned. Do not allow students to change teams.

After the teams are assembled, introduce the small class structure. In particular:

- Small classes are an opportunity to learn math in a problem-based, team-based setting. This is an *authentic* way to learn math. Research shows that it may take longer, but more is retained.
- Teams have been selected deliberately. You may not change teams.
- Use of phones and other electronic devices is strongly discouraged. Extended focus is a key ingredient of success in this course. Please encourage others on your team to stay focused.
- Teams will be guided by a worksheet. Each team will have one worksheet. The worksheet must be handed in at the end of the small class, and will be returned in the next small class.
- The pace will be set by the instructors. Teams may go ahead, but make sure to check in with an instructor before going too far.

Finally, pass out the handouts and announce the first question. Remember to have a routine to close questions — for example, give countdowns (*e.g.* "2 minutes left!") if a question takes more than 1 minute, and always give an indication when there are 10 seconds left (*e.g.* "10 more seconds to finish up!").

At the end of the class, remember to collect worksheets.

This week's tip: ***use students' names.*** The tone of the small classes is set early. When responding to students, ask them for their names and then use them. When visiting teams, be encouraging, and aim to keep them engaged and off their phones.

NOTES ON QUESTIONS

Notes on questions are always organized as they are below. The numbers correspond to the worksheet questions. Each note begins with a recommended time in bold (*e.g.* "**2 minutes**") and ends with an instruction for how to close the question (*e.g.* "**To close the question**..."), and a brief sketch of a correct answer.

1. **1 minute**. This is to introduce the idea of rotating roles on teams.
   To close the question, ask the scribes to stand up. Remind students that the roles should rotate around their group from question to question.
2. **2 minutes**. This is a quick team-building exercise.
   To close the question, ask a few teams to announce their names.
3. **15 minutes** for questions 3, 4 and 5.
   Remind teams to check in with an instructor after question 3. Remember to give countdowns.
   To close the question, ask a team to provide the answer.
   $9 + x^3 \approx 9$ since x is much smaller than 9.

4. **To close the question**, ask a team to provide the answer.
   $9 + x^3 \approx x^3$.

5. **To close the question**, ask a team with a good sketch to draw their answer on the board. Assign them a bonus engagement mark, record it, and remind the class that bonus marks may be awarded for excellent work.

6. **10 minutes**.
   To close the question, give the answer and sketch the graph on the board. Some teams may observe that this function is even; in that case, sketch the graph for $x < 0$ too.
   $f(x)$ resembles the power function $\frac{6}{16}x^4$ near the origin, and 6 far from the origin.

7. **2 minutes**.
   To close the question, write the answer on the board.
   $f(x)$ resembles the power function $\frac{A}{B}x^n$ near the origin, and the constant A far from the origin.

8. **5 minutes**. This is a wrap-up question.
   To close the question, encourage teams to exchange contact information, and to copy or take pictures of the practice questions if they wish. Ask them to drop off their worksheets at the front of the room as they leave. Then thank them and dismiss the class.

SMALL CLASS: Combinations of familiar functions

In this class, you will establish teams and work on understanding the "asymptotics" of functions. In particular, you will sketch rational functions of the form $\frac{Ax^n}{B+x^n}$ ("Hill functions") by considering behaviour for small x and large x.

Small class questions

1. The *scribe*'s role is to do the writing and record the answers.

   Nominate a team member to be the scribe. As you work through a worksheet, the scribe role *must* rotate from question to question, repeating only after every team member has taken a turn.

   Your team may also wish to have other roles. For example, the *manager*'s role is to keep the team on task ("Okay, what's next?"); the *skeptic*'s role is to question the team's answer to make sure it is sound ("How do we *know* that x^3 is small?").

2. Your team name is an adjective that describes calculus, plus an interesting category your team members are all part of (*e.g.* "Glorious Vegans"). Come up with your team name and write it in the box below.

Team name: Scribe:

   Almost all small class questions will have a box like this. Your "tidied" answer should go in the box, and you can use the margins and other empty space on each page for rough work.

3. Consider the function

$$f(x) = \frac{5x^3}{9 + x^3}, \quad x \geq 0.$$

What function does the denominator behave like for very small x?

Answer:

Scribe:

4. What function does the denominator behave like for very large x?

Answer:

Scribe:

5. Using your two answers above, sketch the graph of $f(x)$.

Answer:

Scribe:

6. (★☆☆☆) Sketch the graph of $f(x) = \dfrac{6x^4}{16 + x^4}, \quad x \geq 0.$

Answer:

Scribe:

The "★☆☆☆" notation indicates that this question is exam-suitable. See the Canvas course page for a description of what the stars mean. (In this case, it means that you must eventually be able to do such questions in under two minutes without expending much mental energy).

7. (Key concept) Consider a rational function of the form $f(x) = \frac{Ax^n}{B+x^n}$ where A and B are nonzero constants. What function does $f(x)$ resemble near the origin? What function does $f(x)$ resemble far from the origin?

Answer: Scribe:

The "Key concept" notation indicates that a question summarizes an important idea. Being able to explain key concepts in plain English is a crucial ingredient of success in this course.

8. The following table will normally be at the top of your small class worksheet. In it, write down the student numbers and names of all *contributing team members* — that is, members who took their turn as scribe, and contributed by participating actively throughout the small class.

Student number	Last name	First name

Practice questions

The questions below are for practice. They do not contribute to your grade, and it is not expected that you complete them during your small class. However, you are strongly encouraged to work through them.

9. (★☆☆☆) Sketch the graph of $f(x) = \dfrac{x}{1+x^2}$.

   Hint: What function does the denominator behave like for very small x? What function does it behave like for very large x? What function does $f(x)$ resemble near the origin? What function does $f(x)$ resemble far from the origin?

10. (★☆☆☆) Sketch the graph of $f(x) = \dfrac{1}{e^x+1}$.

11. (★☆☆☆) Sketch the graph of $f(x) = e^x \cos x$.

    Hint: Sketch the graph of e^x first. What is the effect of multiplying by $\cos(x)$?