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1-(a)

First, $\frac{dT}{dt} = \lambda(T_a - T)$ we know $T_a = 25°C$, $T_0 = 77°C$ and $T_1 = 70°C$

$\frac{dT}{dt} = \lambda(25 - T) \longrightarrow \lambda T + \frac{dT}{dt} = 25\lambda$ we can multiply both sides with $e^{\lambda t}$ and get

$$e^{\lambda t} \cdot \lambda T + e^{\lambda t} \cdot \frac{dT}{dt} = 25\lambda e^{\lambda t} \Rightarrow \frac{d}{dt}(e^{\lambda t} \cdot T) = 25\lambda e^{\lambda t}$$

thus $e^{\lambda t} \cdot T = 25e^{\lambda t} + C$ (C is a constant) making $T = 25 + C \cdot e^{-\lambda t}$

at $t=0$, $T_0 = 77 = 25 + C \cdot e^0 = 25 + C$ $\therefore C = 52$

at $t = \underset{\substack{\uparrow \\ T_1}}{\tau} =$ 100 seconds, $70 = 25 + C \cdot e^{-100\lambda} = 25 + 52 \cdot e^{-100\lambda}$

$$\Rightarrow \frac{45}{52} = e^{-100\lambda} \Rightarrow \lambda = \frac{-1}{100} \ln\left(\frac{45}{52}\right) \approx 0.001446\ s^{-1}$$

1-(b) Now $T_a = 15°C$, therefore $\frac{dT}{dt} = \lambda(15 - T) \Rightarrow \frac{dT}{dt} + \lambda T = 15\lambda \Rightarrow e^{\lambda t}\left(\frac{dT}{dt} + \lambda T\right) = 15\lambda \cdot e^{\lambda t} \Rightarrow e^{\lambda t} \cdot T = 15e^{\lambda t} + D$ (D is a constant)

$$\Rightarrow T = 15 + D \cdot e^{-\lambda t}.$$

Now the new initial temperature is $T(0) = T_1 = 70$ hence $D = 55$.

$T_{sip} = T(360 - 100) = T(260) = 15 + 55 \cdot e^{-260\lambda} = 52.7665°C$

$\therefore 52.7665°C$

1-(c) $T_a = T_b(1 + at)$, $a = 0.0038462\ sec^{-1}$ and $T_b = 10°C$

$\frac{dT}{dt} = \lambda(T_a - T) = \lambda(10 + 10at - T) \Rightarrow \frac{dT}{dt} + \lambda T = \lambda(10 + 10at)$

$T_h(t) = E \cdot e^{-\lambda t}$, $\tilde{T_p}(t) = At + B \Rightarrow A + \lambda(At + B) = \lambda At + (A + \lambda B) = 10a\lambda t + 10\lambda \Rightarrow A = 10a$, $B = 10 - \frac{10}{\lambda}a$ hence $T_p(t) = 10at + 10\left(1 - \frac{a}{\lambda}\right)$

Hence, $T(t) = E \cdot e^{-\lambda t} + 10at + 10\left(1 - \frac{a}{\lambda}\right)$

As 1-(b), $T(0) = 70 = E + 10\left(1 - \frac{a}{\lambda}\right)$ $\therefore E = 60 + 10 \cdot \frac{a}{\lambda}$

Now $T_{sip} = T(260) = \left(60 + 10 \cdot \frac{a}{\lambda}\right)e^{-260\lambda} + 260 \cdot (10a) + 10\left(1 - \frac{a}{\lambda}\right) = 52.8644°C$ $\therefore 52.8644°C$

1-(d)

as both (b), (c) are in the bounds $44°C < T < 60°C$, both are uncomfortable situations.