INSTRUCTOR NOTES

Begin by getting students to sit in their teams. If possible, members should sit facing each other. Do not allow students to change teams.

If only 2 members show up for a team, they may work with another small team, but must submit their own worksheet. If only 1 member shows up for a team, they must work with another team, and may submit a blank worksheet with only their name on it for attendance-taking purposes.

Finally, pass out the handouts and announce the first question. Remember to have a routine to close questions (*e.g.* countdowns).

At the end of the class, remember to collect worksheets.

This week's tip: ***limit lecturing***. Remember the 80-20 rule: students should be working, mainly in their teams, at least 80% of the time. Resist the urge to lecture; lecturing is fast and comprehensive, but not the point of small classes.

NOTES ON QUESTIONS

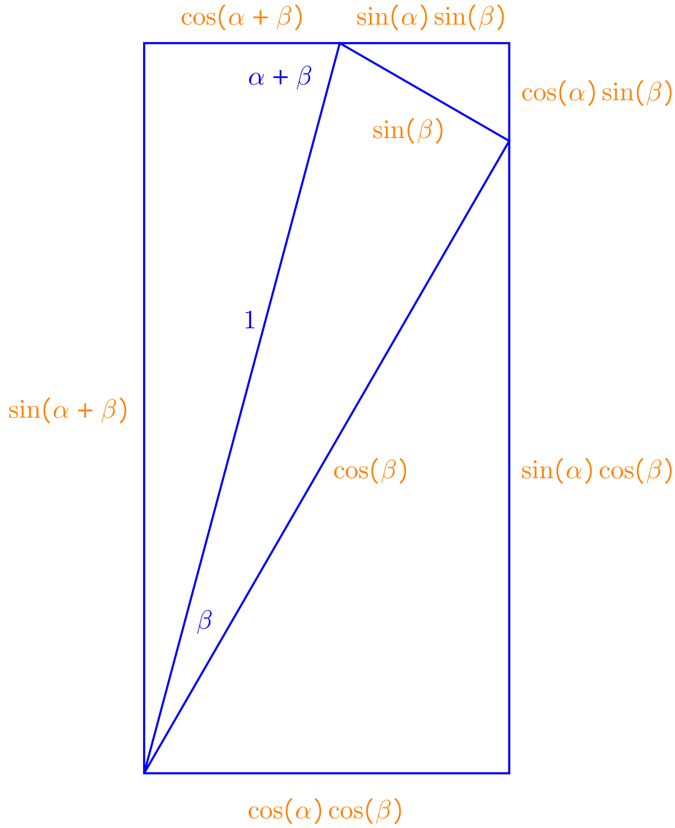
The large lecture prior to this small class is on differentiation rules – the Power Rule, Product Rule and Quotient Rule. In this class, derivatives of trigonometric functions are calculated using those differentiation rules, as well as with some addition formulas which are also derived in this class.

1. **10 minutes**.
   This is a review of trigonometric functions. Read the first question out loud. Visit teams quickly to make sure they are getting started. The unit circle definition will be the least familiar, and hints may be needed. If a team is stuck, encourage a nearby team to help them.
   To close the question, draw the three graphs on the board. There is no need to put the ratio and unit circle descriptions on the board, though you may bring them up as they come up later in the class.

2. **5 minutes**.
   Split the class into three groups of teams, with one group working on each function. Encourage teams to sketch derivatives directly on the graphs in the previous question.
   To close the question, ask a team for each function to sketch their answer on the same axes as you drew in the previous question. Give the conclusions after the team sketches have been drawn. Emphasize that they have not yet been proven, and that the first two will be proven in the small class.
   $\frac{d}{d\theta}\sin(\theta) = \cos(\theta)$, $\frac{d}{d\theta}\cos(\theta) = -\sin(\theta)$ and $\frac{d}{d\theta}\tan(\theta) = \frac{1}{\cos^2(\theta)}$.

3. **2 minutes**.
   To close the question, write down the answers on the board. Then point out that teams will proof and then use addition formulas on $\sin(\theta + h)$ and $\cos(\theta + h)$.
   $\frac{d}{d\theta}\sin(\theta) = \lim_{h\to 0}\frac{\sin(\theta+h)-\sin(\theta)}{h}$ and $\frac{d}{d\theta}\cos(\theta) = \lim_{h\to 0}\frac{\cos(\theta+h)-\cos(\theta)}{h}$.

4. **10 minutes**.
   To set up the question, draw a very large version of the figure below for teams to copy. (The orange labels are the answers, and should not be drawn.)



To close the question, ask some teams to label side lengths as they determine them, and to describe briefly how they found the length. Interrupt the class for each labelling.

5. **2 minutes**.
   To close the question, write down the addition formulas.
   $\sin(\alpha+\beta)=\sin(\alpha)\cos(\beta)+\cos(\alpha)\sin(\beta)$ and $\cos(\alpha+\beta)=\cos(\alpha)\cos(\beta)-\sin(\alpha)\sin(\beta)$.

6. **10 minutes**.
   Teams may need the hint that the "variable" in the limit is h not θ.
   $\frac{d}{d\theta}\sin(\theta)=\lim_{h\to 0}\frac{\sin(\theta)\cos(h)+\cos(\theta)\sin(h)-\sin(\theta)}{h}=\cos(\theta)\lim_{h\to 0}\frac{\sin(h)}{h}-\sin(\theta)\lim_{h\to 0}\frac{\cos(h)-1}{h}$.
   $\frac{d}{d\theta}\cos(\theta)=\lim_{h\to 0}\frac{\cos(\theta)\cos(h)-\sin(\theta)\sin(h)-\cos(\theta)}{h}=\cos(\theta)\lim_{h\to 0}\frac{\cos(h)-1}{h}-\sin(\theta)\lim_{h\to 0}\frac{\sin(h)}{h}$.
   To close the question, write down the answers on the board.

7. **2 minutes** of instructor-led class discussion.
   To close the question, either highlight the correct answer — that $\sin(h)\approx h$ and $\cos(h)\approx 1$ for small h — or give it yourself. The answer does not have to be written down.

8. **Remaining time** for this question.

SMALL CLASS: Trigonometric functions and their derivatives

In this class, you will review the definitions of trigonometric functions, and determine their derivatives using the limit definition of derivative, trigonometric limits, addition formulas, and Product and Quotient Rules.

Contributing team members

Student number	Last name	First name

Small class questions

1. In high school, you should have encountered three descriptions of each of the functions $\sin(\theta)$, $\cos(\theta)$ and $\tan(\theta)$: the graph, the description using a ratio of right triangle side lengths, and the definition using the unit circle. Give all three descriptions for all three functions below. Start by drawing the graphs.

Answer:
Scribe:

2. Use the graphs from the previous question to guess the derivatives of $\sin(\theta)$ and $\cos(\theta)$.

Answer: Scribe:

3. (★☆☆☆) Write the definitions of $\frac{d}{d\theta}\sin(\theta)$ and $\frac{d}{d\theta}\cos(\theta)$ using the limit definition of derivative.

Answer: Scribe:

4. Given the figure drawn by your instructor of a right triangle inscribed in a rectangle, determine the lengths of the missing sides.

Answer:

Scribe:

5. Use the figure from the previous question to find addition formulas for $\sin(\alpha + \beta)$ and $\cos(\alpha + \beta)$.

Answer:

Scribe:

6. Use the addition formulas to write $\frac{d}{d\theta}\sin(\theta)$ and $\frac{d}{d\theta}\cos(\theta)$ in terms of $\sin(\theta)$ and $\cos(\theta)$. (You may still have unresolved limits in your answers.)

Answer:

Scribe:

7. Short of a formal proof, how could you convince someone that $\lim_{h\to 0} \dfrac{\sin(h)}{h} = 1$ and $\lim_{h\to 0} \dfrac{\cos(h) - 1}{h} = 0$?

8. (★★☆☆) Apply the limits above to find $\frac{d}{d\theta}\sin(\theta)$ and $\frac{d}{d\theta}\cos(\theta)$.

Answer:

Scribe:

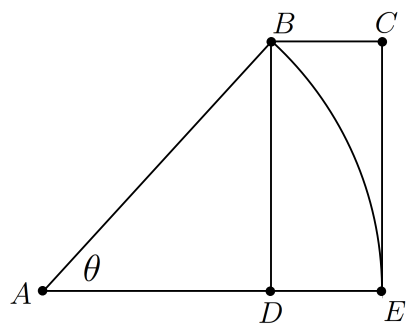
Practice questions

The questions below are for practice. They do not contribute to your grade, and it is not expected that you complete them during your small class. However, you are strongly encouraged to work through them.

9. (★☆☆☆) Find $\dfrac{d}{d\theta}\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$ using the derivatives determined in the small class.

10. (★☆☆☆) Find the derivatives of $\sec(\theta) = \dfrac{1}{\cos(\theta)}$, $\csc(\theta) = \dfrac{1}{\sin(\theta)}$ and $\cot(\theta) = \dfrac{1}{\tan(\theta)}$.

11. (★★★★) Use the graphic below to justify the limit $\displaystyle\lim_{\theta\to 0^+}\frac{\sin(\theta)}{\theta} = 1$ formally.



Hint: Calculate the areas of the triangle ABD, the disc sector ABE, and the quadrilateral $ABCE$. Then set them up in an inequality.