INSTRUCTOR NOTES

Begin by getting students to sit in their teams. If possible, members should sit facing each other. Put unassigned students into separate teams of 3-5 or add them to undersized teams. Make a note of where they are assigned. Do not allow students to change teams.

If only 2 members show up for a team, they may work with another small team, but must submit their own worksheet. If only 1 member shows up for a team, they must work with another team, and may submit a blank worksheet with only their name on it for attendance-taking purposes.

After the teams are assembled, remind them of the small class structure. In particular:

- Small classes are participatory. Active engagement is a key to success.
- Use of phones and other electronic devices is strongly discouraged. Please encourage others on your team to stay focused.
- Fill in the list of contributing team members on your worksheet at the *end* of the small class. Remember that roles (*manager*, *skeptic* and *scribe*) should rotate from question to question.

Finally, pass out the handouts and announce the first question. Remember to have a routine to close questions — for example, give countdowns, and give an indication when there are 10 seconds left.

At the end of the class, remember to collect worksheets.

This week's tip: ***get away from the board***. The front of the room is its natural "centre of gravity". Try to shift that centre. Plan to write on the board only when closing questions 3, 4 and 7, and facilitate discussions from the side or centre of the room.

NOTES ON QUESTIONS

The large lecture prior to this small class is on rational functions, limits and asymptotes. The topic of limits leads to this class's topic of continuity.

1. **6 minutes** for questions 1, 2 and 3. The goal of these questions is to introduce a definition of continuity. Read the first question out loud.
   To close the question, run a class discussion after teams come up with some ideas.
   There are many acceptable answers, all centred around the "niceness" of $f(x)$ around $x = 2$ — "because x^3 has no problems", "because we know what x^3 looks like", "because there are no jumps or holes", "because $f(x)$ is continuous there".

2. **To close the question**, run a class discussion after teams come up with some ideas.
   Again, there are many acceptable answers, all centred around the "non-niceness" of $f(x)$ around $x = 1$ — "because there is a jump", "because the function approaches different things from different sides", "because $f(x)$ is not continuous there".

3. **To close the question**, simply write down this key concept immediately as a "conclusion" to be copied from the previous two questions.
   $f(x)$ is *continuous* at $x = a$ if $\lim_{x\to a} f(x) = f(a)$. Continuous functions are "nice". Informally, where a function is continuous, it can be drawn without lifting your pencil off the page.

4. **10 minutes**. The goal of this question is to identify common functions as continuous.
   When checking in with teams, make sure they are visualizing the functions properly. Students will likely bring up $\tan x$ — a good thing! This function has asymptotes but is continuous on its domain.

To close the question, write down the answer on the board.
All of these "familiar functions" are continuous on their domains.

5. **4 minutes**.
   Make sure that teams attempt to draw a function, not write down an algebraic expression for it (this latter task may be given as a challenge to teams that finish early).
   To close the question, as soon as some teams have viable functions, ask them to sketch their functions on the board. Make sure that jump, infinite and removable discontinuous are represented, and label them.

6. **3 minutes**: 1 minute for teams to write down answers, 2 minutes for class discussion.
   There are many acceptable answers: daily gas prices, pay scales, overtime, on/off switches, *etc.*

7. **10 minutes** for questions 7 (easier) and 8 (harder).
   Assign half the teams to work on each question.
   To close the questions, at the end of the 10 minutes, either have a preselected team give the correct answer or give it yourself, so that other teams can check their work later.
   For question 7, $k = \pm 2$. It's not necessary for teams to evaluate limits of branches. Encourage teams to sketch the functions and observe that the components are continuous.

8. For question 8, $k = 4$. Teams can be given the hint to factor the numerator.

9. **Remaining time** for this question.
   Teams will proceed at different paces. For slow teams, encourage them to consider simple piecewise functions where the pieces are continuous. For fast teams, encourage them to consider subtleties, like piecewise functions where the pieces are glued together but the pieces themselves are discontinuous.
   To close the question, set up head-to-head races between pairs of teams. Each team should write down their function on a scrap of paper and pass it face down to the other team. At the signal, the teams race to find the x-values of both discontinuities.

SMALL CLASS: Continuity

In this class, you will encounter a definition of continuity, classify types of discontinuities, and learn how to select parameters for a function to make it continuous.

Contributing team members

Student number	Last name	First name

Small class questions

1. Let $f(x) = \begin{cases} -x & \text{if } x < 1 \\ x^3 & \text{if } x \geq 1 \end{cases}$. What is $\lim_{x \to 2} f(x)$? Why can we just "plug it in"?

Answer:

Scribe:

2. What is $\lim_{x \to 1} f(x)$? Why can we not just "plug it in"?

Answer:

Scribe:

3. (Key concept) Write down a definition of continuity.

Answer:

Scribe:

4. Which of the following functions are continuous on their domains?

   (a) polynomials
   (b) $\sin(x)$
   (c) $\cos(x)$
   (d) $\tan(x)$
   (e) e^x
   (f) $\log(x)$

Answer:

Scribe:

5. Draw a function whose domain is all real numbers that is *not* continuous.

Answer:

Scribe:

6. What are some real-life examples of discontinuous functions?

Answer:

Scribe:

7. (★☆☆☆) Find all k such that $f(x) = \begin{cases} 8 - kx & \text{if } x < k \\ x^2 & \text{if } x \geq k \end{cases}$ is continuous.

Answer:
Scribe:

8. (★☆☆☆) Find all k such that $g(x) = \begin{cases} \frac{x^3 - 2x^2}{x - 2} & \text{if } x \neq 2 \\ k & \text{if } x = 2 \end{cases}$ is continuous.

Answer:
Scribe:

9. (★☆☆☆) Come up with an algebraic expression for a function with exactly two discontinuities. It can contain familiar functions, and it can be a piecewise function. Your task will be to find the discontinuities in *another* team's function.

Answer:
Scribe:

Practice questions

The questions below are for practice. They do not contribute to your grade, and it is not expected that you complete them during your small class. However, you are strongly encouraged to work through them.

10. (★☆☆☆) Find all k such that $f(x) = \begin{cases} x^3 & \text{if } x < k \\ x + k & \text{if } x \geq k \end{cases}$ is continuous.

11. (★☆☆☆) If $|f(x)|$ is continuous, is $f(x)$ also continuous? Why or why not?

    Note: Showing that a claim is false requires a single *counterexample*. Showing that a claim is true requires a full explanation, sometimes called a *proof*.

12. (★★★☆) If $f(x)$ is continuous, is $|f(x)|$ also continuous? Why or why not?