INSTRUCTOR NOTES

Leave time at the end of the class for students to pick up their tests. These should be handed out by instructors; students should not be able to leaf through a pile of tests. This is also not the time for students to look through or review their tests.

Begin by getting students to sit in their teams. If possible, members should sit facing each other. Do not allow students to change teams.

This small class is an opportunity to encourage teams to think about curves, not values. When trying to determine (for example) where a function is increasing, many teams will be tempted to "plug in" values to check where its derivative is positive. Encourage them instead to think about what the constituents of a function "look like" ("$e^{[\text{anything}]}$ is always positive, so $1 + e^{-kx}$ is positive, so we can ignore $(1 + e^{-kx})^3$..."').

If only 2 members show up for a team, they may work with another small team, but must submit their own worksheet. If only 1 member shows up for a team, they must work with another team, and may submit a blank worksheet with only their name on it for attendance-taking purposes.

Finally, pass out the handouts and announce the first question. Remember to have a routine to close questions (*e.g.* countdowns).

At the end of the class, remember to collect worksheets as you return tests.

This week's tip: ***wait!*** Especially in instructor-led discussions, give plenty of time for students to come up with answers (one tip from a previous instructor: during that time, act as if you are also thinking!). Wait at least 8 actual seconds before providing a hint or a rewording of the question — never an answer.

NOTES ON QUESTIONS

The large lecture prior to this small class is on curve sketching. In this small class, curve sketching techniques are applied to sketch the logistic function.

1. **2 minutes** of instructor-led class discussion. As good answers come up, write them on the board and give teams time to copy them down.
   Possible answers: domain, x- and y-intercepts, vertical and horizontal asymptotes.

2. **6 minutes**.
   Have teams work in order: first find the domain, then the x-intercepts, *etc.*,
   To close the question, ask teams to volunteer their answers for each of the characteristics, and write them down on the board as they come up.

3. **2 minutes**.
   To close the question, ask a team with the correct answer to write the simplified answer on the board.
   $f'(x) = \dfrac{ke^{-kx}}{(1 + e^{-kx})^2}$.

4. **2 minutes** of instructor-led class discussion. As good answers come up, write them on the board and give teams time to copy them down. It is not necessary to convince students *how* these characteristics may be determined.
   Possible answers: intervals of increase and decrease, and extrema.

5. **10 minutes**.
   To close the question, ask a team with the correct answer to write the conclusion on the board.
   $f(x)$ is increasing everywhere.

6. **4 minutes**.
   To close the question, ask a team with the correct answer to write the simplified answer on the board.
   $f''(x) = \frac{ke^{-kx}}{\left(1+e^{-kx}\right)^3}\left(e^{-kx}-1\right)$.

7. **2 minutes** of instructor-led class discussion. As good answers come up, write them on the board and give teams time to copy them down. Again, it is not necessary to convince students how these characteristics may be determined.
   Possible answers: intervals of concavity and inflection points.

8. **10 minutes**.
   To close the question, ask a team with the correct answer to write the conclusion on the board.
   $f(x)$ is concave up on $(-\infty, 0)$ and concave down on $(0, \infty)$.

9. **10 minutes**.
   As soon as half the teams have a reasonable sketch, move on to the next question.

10. **Remaining time** for this optional question.
    To close the question, if enough teams have gotten to this question, use the time for an instructor-led class discussion. Otherwise, teams can work on this question on their own as you circulate.
    There are many examples: for instance, reactant concentration in autocatalytic reactions, population or tumour growth, and uptake of technological innovations.

SMALL CLASS: The logistic model

In this class, you will sketch the graph of a function using characteristics determined from the function, its first derivative, and its second derivative.

Contributing team members

Student number	Last name	First name

Small class questions

Let $f(x) = \dfrac{1}{1 + e^{-kx}}$ where $k > 0$ is a constant. This is a *logistic function*.

1. List characteristics of the graph of $f(x)$ that can be determined from $f(x)$ itself (and not from its derivatives).

   Answer:

   Scribe:

2. (★☆☆☆) Determine the characteristics listed in the previous question.

   Answer:

   Scribe:

3. (★☆☆☆) Find $f'(x)$.

Answer:

Scribe:

4. List characteristics of the graph of $f(x)$ that can be determined from $f'(x)$.

Answer:

Scribe:

5. (★☆☆☆) Determine the characteristics listed in the previous question.

Answer:

Scribe:

6. (★☆☆☆) Find $f''(x)$.

Answer:

Scribe:

7. List characteristics of the graph of $f(x)$ that can be determined from $f''(x)$.

Answer:

Manager: Skeptic: Scribe:

8. (★☆☆☆) Determine the characteristics listed in the previous question.

Answer:

Scribe:

9. (★☆☆☆) Take all the information determined previously and draw a large sketch of $f(x)$.

Answer:

Scribe:

10. Can you think of a function describing a phenomenon in your intended specialization that might have a similar graph?

Answer:

Scribe:

Practice questions

The questions below are for practice. They do not contribute to your grade, and it is not expected that you complete them during your small class. However, you are strongly encouraged to work through them.

11. (★★☆☆) Confirm that the logistic function $f(x)$ satisfies the differential equation

$$f'(x) = kf(x)(1 - f(x)).$$

12. (★★★☆) Draw a set of axes with $f(x)$ as your horizontal axis and $f'(x)$ as your vertical axis. Then sketch the graph of $f'(x) = kf(x)(1 - f(x))$. (Hint: it is a parabola.) What characteristics determined in this small class can you determine using this graph?

13. (★★☆☆) Like the logistic function,

$$g(x) = \frac{x}{\sqrt{1 + x^2}}$$

is an example of a broader class of functions called *sigmoid functions*. Find $g'(x)$ and $g''(x)$, and draw a sketch of the graph of $g(x)$ using information from the function and its derivatives.