INSTRUCTOR NOTES

This is the final small class! Make sure you thank your class and wish them well.

Begin by getting students to sit in their teams. If possible, members should sit facing each other. Do not allow students to change teams.

If only 2 members show up for a team, they may work with another small team, but must submit their own worksheet. If only 1 member shows up for a team, they must work with another team, and may submit a blank worksheet with only their name on it for attendance-taking purposes.

Finally, pass out the handouts and announce the first question. Remember to have a routine to close questions (*e.g.* countdowns).

At the end of the class, remember to collect worksheets.

This week's tip: ***be especially encouraging***. Students are going to be worried about their final exams. Try to give them reason to study, and encouragement to convince them it will make a difference.

NOTES ON QUESTIONS

The large lecture prior to this small class is the second lecture on multivariable functions. This class continues the exploration of multivariable functions with a multivariable optimization problem.

1. **2 minutes**.
   To close the question, have a team with the correct answer draw their graph on the board. Then draw next to it a "tipped over" version of the graph, setting up the three-dimensional scenario described in the remaining questions.

2. **5 minutes**.
   To set up the question, draw an arrow on the graphs on the board indicating the path taken.
   To close the question, have a team describe their answer. Then label the graphs on the board with the answer.
   The path is flat.

3. **3 minutes**.
   To set up the question, draw an arrow on the graphs on the board indicating the path taken.
   To close the question, have a team describe their answer. Then label the graphs on the board with the answer.
   The path is flat.

4. **5 minutes**.
   This question can be solved both graphically and numerically — encourage teams with enough time to take both approaches.
   To close the question, have a team describe their answer. As they describe their answer, write the key points on the board. Then write down the answer.
   Setting $y - 8 - x$ yields $f(x, y) = f(x) = 5x(8 - x)$. The path is symmetric: it goes uphill from an elevation of 0 to an elevation of 80 at $(4, 4)$, then back downhill.

5. **1 minute** of class discussion.
   To close the question, write down the answer on the board.
   The maximum elevation is 80, and the minimum elevation is 0.

6. **10 minutes**.
   Teams may need reminders to take partial derivatives.
   To close the question, write down the critical points, and ask teams that have not yet found them to use them to check their answers.
   $\dfrac{\partial f}{\partial x} = 2xy + y^2 + 3y$ and $\dfrac{\partial f}{\partial y} = x^2 2xy + 3x$. There are four critical points: $(0,0), (0,3), (3,0), (1,1)$.

7. **Remaining time** for the last two questions.
   To close question 7, check in on teams as they complete this question. If they have the right answer, have them complete the final question.
   The maximum elevation is 80, and the minimum elevation is -1.
   To close question 8, ask teams to hand in their last worksheet. As each team hands in their worksheet, thank them and wish them luck on the final exam!

SMALL CLASS: Optimization of multivariable functions

In this class, you will reduce a constrained optimization problem in three dimensions to several single variable calculus problems.

Contributing team members

Student number	Last name	First name

Small class questions

1. You are situated on an island. The boundary of the island is given by the triangle with vertices $(0,0)$, $(8,0)$, and $(0,8)$ on the x-y plane. Draw the region.

Answer:

Scribe:

2. (★☆☆☆) The island is not flat. At point (x, y), its elevation is given by $f(x, y) = xy(x + y - 3)$. Suppose you start your survey at $(8, 0)$ and walk along the boundary to $(0, 0)$. Describe the elevation along your journey. For example, are you going uphill? Downhill? First uphill and then downhill?

Answer:
Scribe:

3. (★☆☆☆) Next, you walk along the boundary from $(0, 0)$ to $(0, 8)$. Describe the elevation along this second part of your journey.

Answer:
Scribe:

4. (★★☆☆) Finally, you walk from $(0, 8)$ to $(8, 0)$, arriving back where you started. Describe your elevation during this final leg of the journey.

Answer:
Scribe:

5. What is the maximum value $f(x, y)$ takes on the boundary? What is the minimum value?

Answer:

Scribe:

6. (★★☆☆) Find the x- and y-values of all four critical points of $f(x, y)$.

Answer:

Scribe:

7. (★☆☆☆, key concept) To find the absolute maximum and absolute minimum of a function of two variables on a closed region, you must check all the critical points as well as all the points on the boundary. (Checking the points on the boundary may reduce to a single variable calculus problem.) On your island, what is the maximum elevation, and what is the minimum elevation?

Answer:
Scribe:

8. As a team, write down one or more pieces of advice for other teams of students starting off in MATH 100. What should they make sure to do? What should they make sure to avoid? What hints, hacks and encouragements can you pass on?

Answer:
Scribe:

Practice questions

The questions below are for practice. They do not contribute to your grade, and it is not expected that you complete them during your small class. However, you are strongly encouraged to work through them.

9. (★★☆☆) Repeat questions 1-7 above, but this time for the island defined by the square $0 \le x \le 8$, $0 \le y \le 8$. What is the maximum elevation, and what is the minimum elevation?

10. (★★☆☆, from OIL Section 2.4, Q4) Find the maximum and minimum values of $f(x, y) = xy - x^3 y^2$ on the square $0 \le x \le 1$, $0 \le y \le 1$.

11. (★★☆☆) Find the maximum and minimum values of $f(x, y) = (x - 2)^2 + (y - 2)^2$ on the square $0 \le x \le 4$, $0 \le y \le 4$. Try to do this first without taking partial derivatives.