Minimum Cost Path Determination Using a Simple Heuristic Function

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Abstract

This paper describes the use of heuristics in the determination of a minimum cost path between two points in digital images. The application of four different search methods when applied in two and three dimensional digital images is presented and evaluated. Experiments show that the number of nodes that are being addressed in the search process strongly depends on the discriminative power of the feature used. Furthermore it is shown that for a specific application, the use of a simple heuristic function leads to a considerable reduction in the number of evaluated nodes as compared with the traditional unidirectional approach.

1 Introduction

The determination of the minimum cost path or shortest path between two nodes in a network is a fundamental problem which has received considerable attention from several research communities in the last fourty years. Applications are widespread in *e.g.* artificial intelligence, robotics, Geographical Information Systems, computer vision and fluid dynamics. Most of the research on this topic involves the development and the tuning of special data structures and their performance on real, simulated and special classes of networks [1]. In the majority of these cases the search process is started from a single node and continued until the goal nodes are reached. This approach is referred to as the unidirectional search. Several methods have been suggested to reduce the number of nodes that are being evaluated during the search process, while assuring that the obtained path is still the true minimum cost path. One of these approaches tries to reduce the number of evaluations by starting a search tree from both the start and goal node (bidirectional search). Other approaches use heuristics to limit the evaluation of nodes which are unlikely to be part of the final minimum cost path, based on an estimate of the remaining cost to reach the goal node. The latter approach has received considerable attention in literature as it may lead to a significant reduction in computation time. A few approaches have been reported that combine the potential advantages of both bidirectional and heuristic minimum cost path methods (see *e.g.* [4]). Inspired by these concepts we developed a novel bidirectional heuristic search strategy. The approach is valid for all types of grid like networks, provided that all the transition cost in the network are nonnegative. This paper describes the implementation of the aforementioned approaches for the use in digital images. In section 2 the basic unidirectional approach will be discussed while the extensions will be given in section 3. In section 4 two experiments are performed which enables us to compare the proposed approaches with respect to the unidirectional case in both two- and three dimensional images for two types of features. The results will be discussed in section 5.

2 Basic approach

The approaches that are taken in order to tackle the problem of finding the minimum cost path are generally derived from the algorithm as originally proposed by Dijkstra [2]. In this algorithm, a search process is started in the source node s that iteratively constructs all sub paths of minimum length. This process is continued until the goal node is reached. In most cases a priority queue is applied in order to maintain the leaves of the tree. The nodes in this queue are labeled temporary and can be viewed as front nodes, in analogy with the propagation of a wave front from the source node s. The other nodes in the search tree are considered to be labeled as permanent. It is not trivial to apply the above algorithm for digital images. The problem lies in the determination of the neighbours in the grid and the propagation of the front over the discrete grid. For a detailed discussion on this subject we refer to Sethian [6]. However, since we are interested in reducing the number of evaluated nodes, the selected way of propagation is not relevant. In this paper we use an eight connected neighbourhood in the 2D case and a 26 connected neighbourhood in the 3D case,

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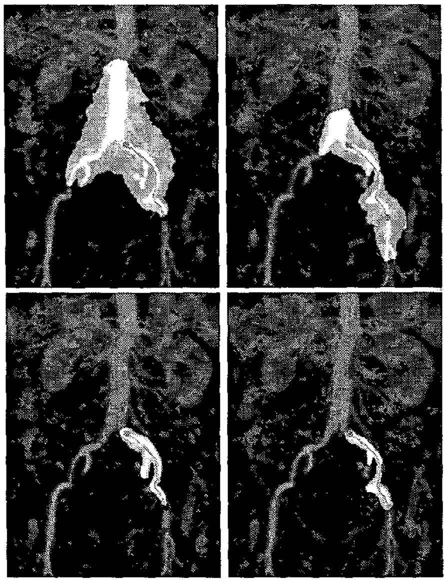

Figure 1. Examples of the different minimum cost path methods in a 2D image using an eight connected neighbourhood and the reciprocal intensities as a feature. In the upper left frame the basic unidirectional search tree is overlayed. In the upper right frame, the bidirectional search tree is depicted. The lower left frame displays the use of heuristics in the unidirectional case, while the lower right frame gives the trees when a bidirectional heuristic search is applied.

merely as an example for the proposed approaches. The costs are properly normalized in order to cope with diagonal transitions and possible anisotropic voxels.

An example of the basic minimum cost path algorithm in a 2D image using an eight connected neighbourhood with the reciprocal intensity[1] as feature f is given in the upper left frame of Fig. 1.

3 Extensions to the basic approach

In this section two extensions of the basic approach as discussed in section 2 will be given. Furthermore it is outlined how these two approaches can be combined.

[1] In order to cope with zero intensities, a small value is added to the intensities in the image before the reciprocal value is computed, potentially influencing the resulting path.

3.1 Bidirectional search

Instead of using a unidirectional search as proposed by Dijkstra [2], it is possible to use a bidirectional search process where a wave front is initiated from both the source node s as well as from the goal node g. The evolution of the search trees is continued until the two trees meet. For a proper result, the wave front from the goal node has to be propagated in a backward manner as is commonly performed in robotic path planning applications.

3.2 Unidirectional heuristic search

The A* algorithm [5] makes use of heuristics to steer the search process. Next to the computation of the cumulative costs $c(s,n)$ that were needed to arrive at node n starting from node s, a heuristic estimate $h(n,g)$ of the cost from this node to the goal node g has to be provided. It is important that the heuristic estimate is *admissible*, that is, lower than or equal to the actual cost between the current node and the goal node and nonnegative. A heuristic h_1 is considered to be *more informed* than a heuristic h_2 if both are admissible and $h_1 > h_2$. As a result, less nodes will generally be visited before the goal node is actually reached.

The development of a heuristic function very much depends on the specific network and the extent of knowledge that can be put in the search process. An example of a simple heuristic function that can be applied in the grid like network as present in digital images is given in equation 1.

$$h(n,g) = D(n,g) * a_{min} \quad (1)$$

Here D denotes the distance between the node n and the goal node g while a_{min} contains the minimum transition costs in the entire network. The underlying idea is that in order to arrive at the goal node g a specific amount of transitions have to made which can be estimated by the distance D between the nodes. Each of these transitions will have a cost that is at least a_{min} which leads to cumulative costs that will be the lower limit of the actual costs.

In this paper the Euclidean distance is used to compute the distance D. If a four connected neighbourhood is used, the computationally less expensive 'Manhattan' distance would already suffice. An example of this approach is given in the lower left frame of Fig. 1.

3.3 Bidirectional heuristic search

The combination of the previously discussed approaches is not straightforward since the node where the two trees initially meet is not necessarily situated on the minimum cost path. This is due to the presence of non zero heuristics at this node. There may very well exist another path that will have a lower cumulative cost but which will be found in a later stage of the search process. The challenge is to determine when to terminate the evolution of the trees.

This paper introduces the concept of a *mutual zero heuristic region* for this purpose. In this area both the nodes from tree A as the nodes from tree B are supposed to have a heuristic estimate that is exactly zero. Although this region can have virtually any shape, we use the line (or plane) perpendicular to the line connecting the source node s and the target node g and which is positioned in the middle of these nodes.

During the evolution of the trees there will exist a node for both tree A and B which has just been labeled permanent and which is the first to enter the mutual zero heuristic region. This node is called the *pioneer* node. It is guaranteed that every permanent node that enters this mutual zero heuristic region will have a cumulative cost that is not lower than the cost of the pioneer node of the corresponding tree. Suppose that the pioneer of tree A has a cost c_{pA} and the pioneer of tree B has a cost c_{pB}. If the two trees meet in the mutual zero heuristic region at node n with a cumulative cost c_n the following end conditions are set:

1. Continue with the evolution of tree A until there exist no node on the temporary list that has a value lower then $c_n - c_{pB}$ or the pioneer of tree B is reached.

2. Continue with the evolution of tree B until there exist no node on the temporary list that has a value lower then $c_n - c_{pA}$ or the pioneer of tree A is reached.

These end conditions can be updated if another path is found in the mutual zero heuristic region that has a minimum cumulative cost lower than c_n. An example of this approach in a 2D case is given in the lower right frame of Fig. 1.

4 Experiments and results

In this section the unidirectional search (*uni*), bidirectional search (*bi*), unidirectional heuristic search (*uni h*) and bidirectional heuristic search (*bi h*) methods are evaluated in a task to find an initial estimate of the vessel axis from 2D and 3D contrast enhanced magnetic resonance angiography (CE-MRA) images. First a feature image is constructed, expressing the likeliness of a pixel/voxel to belong to the vessel. Hereto two features *viz.* the reciprocal intensity and the reciprocal of the multi scale vesselness filter as developed by Frangi *et al.* [3] are used (see Fig. 2). Point pairs which act as start and goal nodes in the determination of the minimum cost path are randomly selected from a user defined central vessel axis. The number of (pixels/voxels) that are addressed at least once in the search process is used to measure the efficiency of the method. The results are put in a histogram of 20 bins, holding the average cumulative cost as a function of the distance between the source and goal node.

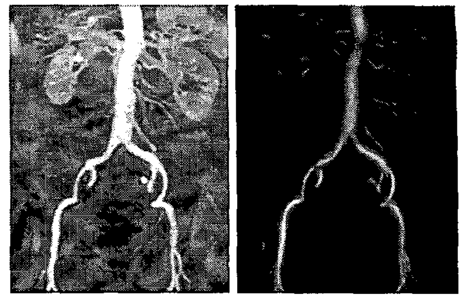

Figure 2. Impression of the two features which reciprocal values are used in the experiments. In the left frame a MIP of the original CE-MRA dataset is shown. The right frame displays the result of the multi scale vesselness filter applied on the MIP.

In the 2D experiment a 770 x 1024 MIP of a 3D CE-MRA dataset of the aorta is used. The results are given in Fig. 3. The results in the upper graph correspond to the use of the reciprocal intensity feature in the determination of the minimum cost path for the four approaches, while the results in the lower graph correspond to the use of the reciprocal vesselness feature.

In the 3D experiment the original 256 x 256 x 30 CE-MRA dataset with an anisotropy factor of 1.10 is used. The results are given in Fig. 4.

The graphs in Fig. 3 and Fig. 4 show that the vesselness feature has better discriminative power than the intensity feature. Fewer nodes are evaluated irrespective of the search method used. In case of a unidirectional search with the less distinctive feature, a quadratic (cubic) increase is expected in the 2D (3D) case. The figures show this trend initially. However at a given point the slope is altered because the borders of the image form a natural barrier for the propagating wave. The front is forced in the direction of the other node, which is very similar to what the heuristic search aims to achieve. This effect is maximal for the largest distances where the source and goal points are put near the corners of the images.

On average, the use of heuristics and the bidirectional search will reduce the number of nodes that is needed in order to obtain a minimum cost path. This is especially the case if the reciprocal intensities are used.

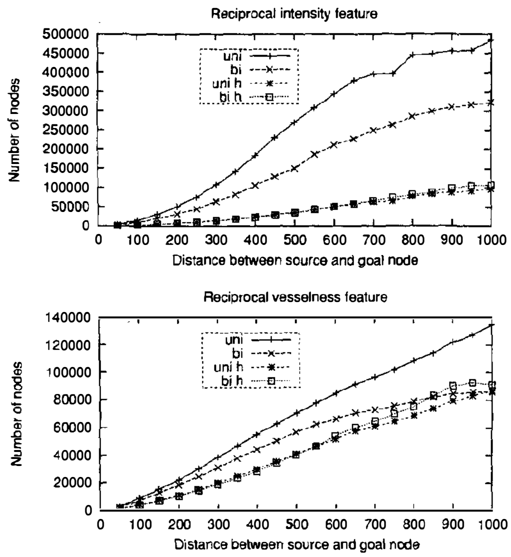

Figure 3. Results of the 2D experiment, displaying the number of visited nodes as a function of the distance between randomly chosen pairs of points for the reciprocal intensity feature and the reciprocal vesselness feature.

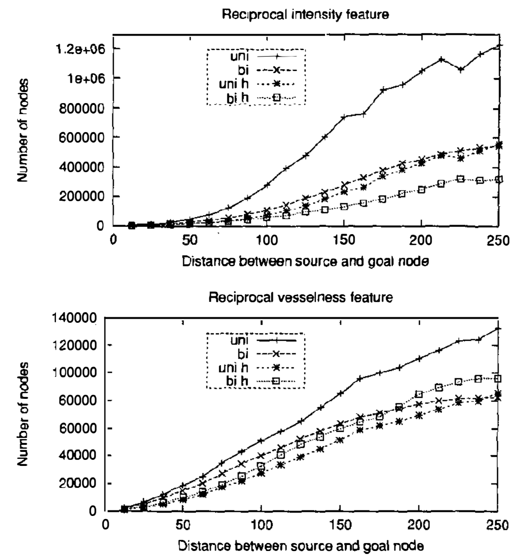

Figure 4. Results of the 3D experiment, displaying the number of visited nodes as a function of the distance between randomly chosen pairs of points for the reciprocal intensity feature and the reciprocal vesselness feature.

The heuristic search methods yield the same performance except in the three dimensional case where the reciprocal intensities are used as a feature. Here the bidirectional heuristic approach performs better than its unidirectional counterpart.

5 Discussion and conclusion

Several extensions to the basic unidirectional minimum cost path approach are discussed and experimentally evaluated. In general, the reduction in the number of evaluated nodes will largely depend on the distance between the source and goal node, the quality of the image, the discriminative power of the feature and the heuristic function used.

It is shown that the feature based on the more discriminative vesselness filter will significantly decrease the number of nodes that are being addressed during the search processes. This phenomenon is independent on the specific search method used. Furthermore it can be concluded that the benefit of the heuristic approaches is inversely related to the discriminative power of the feature.

It is not straightforward to extrapolate the results of the experiment as performed in this paper to the possible gain in overall computation time. This strongly depends on the implementation, *e.g.* whether the features are precomputed or that the features are computed during the search process itself. Eventually, the actual choice for one of the discussed search methods will depend on the interactive nature and the specific application at hand.

Although the methods are evaluated on digital images, they can be applied to any grid like network as long as the transition costs are nonnegative. The best performance of the heuristic approaches will be obtained in huge networks that are not bound by natural borders.

References

[1] B. V. Cherkassky, A. V. Goldberg, and T. Radzik. Shortest paths algorithms: Theory and experimental evaluation. *Mathematical Programming*, 73:129–174, 1996.

[2] E. Dijkstra. A note on two problems in connexion with graphs. *Numerische Mathematic*, 1:269–271, 1959.

[3] A. F. Frangi, W. J. Niessen, K. L. Vincken, and M. A. Viergever. Vessel enhancement filtering. In *Medical Image Conference and Computer Assisted Interventions*, pages 130–137. Springer-Verlag, 1998.

[4] J. B. H. Kwa. BS*: An admissible bidirectional staged heuristic search algorithm. *Artificial Intelligence*, 38:95–109, 1989.

[5] J. Pearl. *Heuristics: Intelligent Strategies for Computer Problem Solving*. Addison Wesley, Reading, Mass, 1984.

[6] J. A. Sethian. *Level Set Methods and Fast Marching Methods*. Cambridge University Press, second edition, 1999.