Comb filter

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In signal processing, a **comb filter** adds a delayed version of a signal to itself, causing constructive and destructive interference. The frequency response of a comb filter consists of a series of regularly-spaced spikes, giving the appearance of a comb.

Contents

Applications

Comb filters are used in a variety of signal processing applications. These include:

- Cascaded Integrator-Comb (CIC) filters, commonly used for anti-aliasing during interpolation and decimation operations that change the sample rate of a discrete-time system.
- 2D and 3D comb filters implemented in hardware (and occasionally software) for PAL and NTSC television decoders. The filters work to reduce artifacts such as dot crawl.
- Audio effects, including echo, flanging, and digital waveguide synthesis. For instance, if the delay is set to a few milliseconds, a comb filter can be used to model the effect of acoustic standing waves in a cylindrical cavity or in a vibrating string.

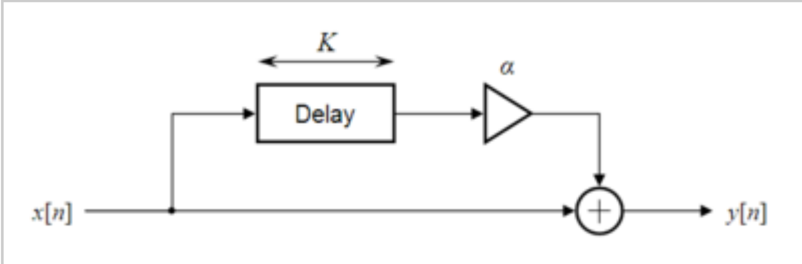
Technical discussion

Comb filters exist in two different forms, *feedforward* and *feedback*; the names refer to the direction in which signals are delayed before they are added to the input.

Comb filters may be implemented in discrete time or continuous time; this article will focus on discrete-time implementations; the properties of the continuous-time comb filter are very similar.

Feedforward form

The general structure of a feedforward comb filter is shown on the right. It may be described by the following difference equation:



Feedforward comb filter structure

$$y[n] = x[n] + \alpha x[n-K]$$

where K is the delay length (measured in samples), and α is a scaling factor applied to the delayed signal. If we take the Z transform of both sides of the equation, we obtain:

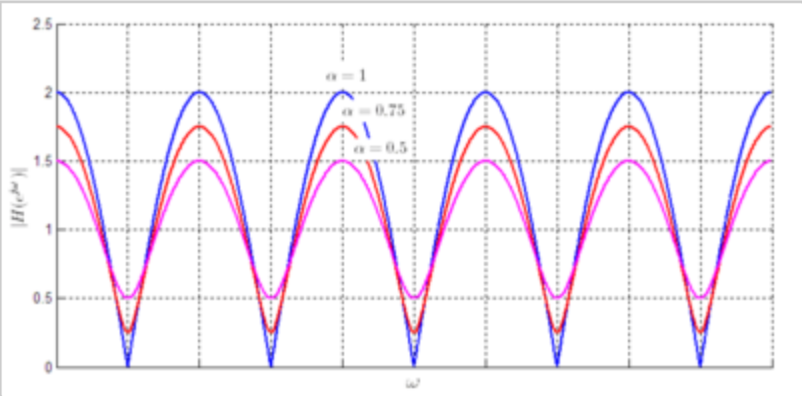
$$Y(z) = (1 + \alpha z^{-K})X(z)$$

We define the transfer function as:

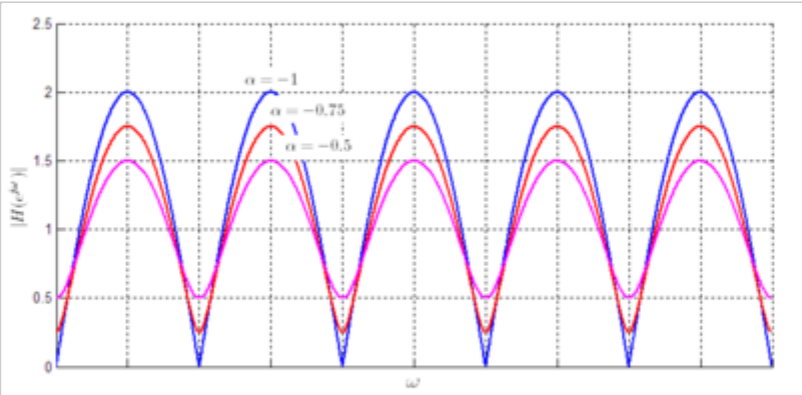
$$H(z) = \frac{Y(z)}{X(z)} = 1 + \alpha z^{-K} = \frac{z^K + \alpha}{z^K}$$

Frequency response

To obtain the frequency response of a discrete-time system expressed in the Z domain, we make the substitution $z = e^{j\omega}$. Therefore, for our feedforward comb filter, we get:



Feedforward magnitude response for various positive values of α



Feedforward magnitude response for various negative values of α

$$H(e^{j\omega}) = 1 + \alpha e^{-j\omega K}$$

Using Euler's formula, we find that the frequency response is also given by

$$H(e^{j\omega}) = \left[1 + \alpha\cos(\omega K)\right] - j\alpha\sin(\omega K)$$

Often of interest is the *magnitude* response, which ignores phase. This is defined as:

$$|H(e^{j\omega})| = \sqrt{\Re\{H(e^{j\omega})\}^2 + \Im\{H(e^{j\omega})\}^2}$$

In the case of the feedforward comb filter, this is:

$$|H(e^{j\omega})| = \sqrt{(1+\alpha^2) + 2\alpha\cos(\omega K)}$$

Notice that the $(1 + \alpha^2)$ term is constant, whereas the $2\alpha\cos(\omega K)$ term varies periodically. Hence the magnitude response of the comb filter is periodic.

The graphs to the right show the magnitude response for various values of α, demonstrating this periodicity. Some important properties:

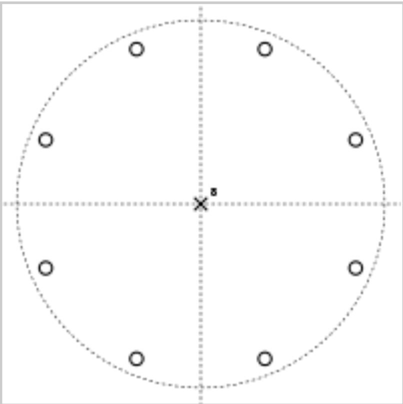
- The response periodically drops to a local minimum (sometimes known as a *notch*), and periodically rises to a local maximum (sometimes known as a *peak*).
- The levels of the maxima and minima are always equidistant from 1.
- When $\alpha = \pm 1$, the minima have zero amplitude. In this case, the minima are sometimes known as *nulls*.
- The maxima for positive values of α coincide with the minima for negative values of α, and vice versa.

Pole-zero interpretation

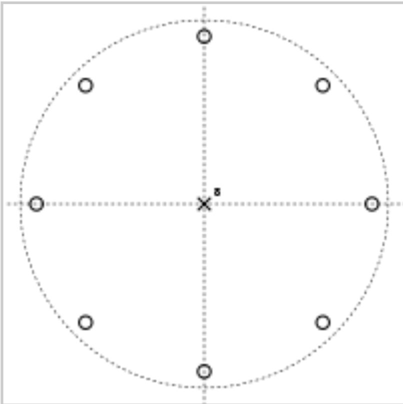
Looking again at the Z-domain transfer function of the feedforward comb filter:

$$H(z) = \frac{z^K + \alpha}{z^K}$$

we see that the numerator is equal to zero whenever $z^K = -\alpha$. This has K solutions, equally spaced around a circle in the complex plane; these are the zeros of the transfer function. The denominator is zero at $z^K = 0$, giving K poles at $z = 0$. This leads to a pole-zero plot like the ones shown below.



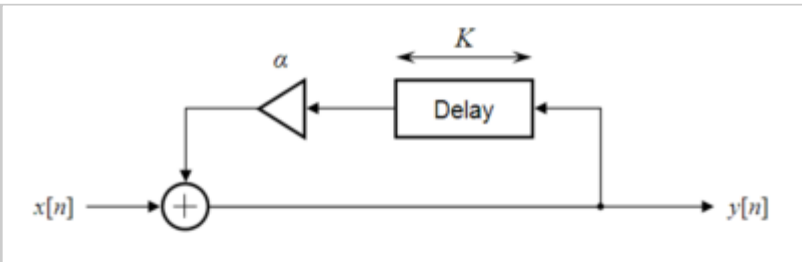
Pole-zero plot of feedfoward comb filter with $K = 8$ and $\alpha = 0.5$



Pole-zero plot of feedfoward comb filter with $K = 8$ and $\alpha = -0.5$

Feedback form

Similarly, the general structure of a feedback comb filter is shown on the right. It may be described by the following difference equation:



Feedback comb filter structure

$$y[n] = x[n] + \alpha y[n-K]$$

If we rearrange this equation so that all terms in y are on the left-hand side, and then take the Z transform, we obtain:

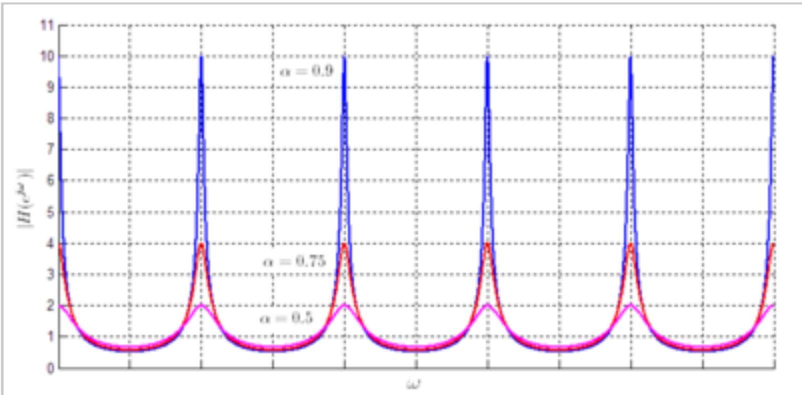
$$(1 - \alpha z^{-K})Y(z) = X(z)$$

The transfer function is therefore:

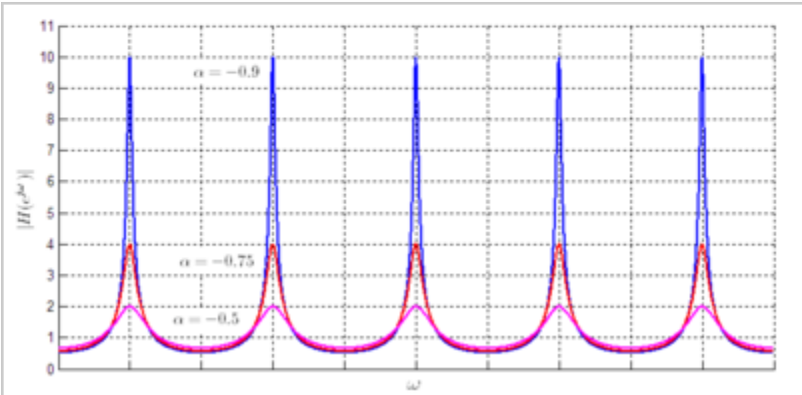
$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-\alpha z^{-K}} = \frac{z^K}{z^K - \alpha}$$

Frequency response

If we make the substitution $z = e^{j\omega}$ into the Z-domain expression for the feedback comb filter, we get:



Feedback magnitude response for various positive values of α



Feedback magnitude response for various negative values of α

$$H(e^{j\omega}) = \frac{1}{1-\alpha e^{-j\omega K}}$$

The magnitude response is as follows:

$$|H(e^{j\omega})| = \frac{1}{\sqrt{(1+\alpha^2) - 2\alpha\cos(\omega K)}}$$

Again, the response is periodic, as the graphs to the right demonstrate. The feedback comb filter has some properties in common with the feedforward form:

- The response periodically drops to a local minimum and rises to a local maximum.
- The maxima for positive values of α coincide with the minima for negative values of α, and vice versa.

However, there are also some important differences because the magnitude response has a term in the denominator:

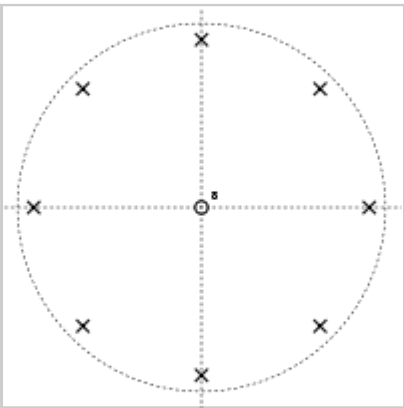
- The levels of the maxima and minima are no longer equidistant from 1.
- The filter is only stable if $|\alpha|$ is strictly less than 1. As can be seen from the graphs, as $|\alpha|$ increases, the amplitude of the maxima rises increasingly rapidly.

Pole-zero interpretation

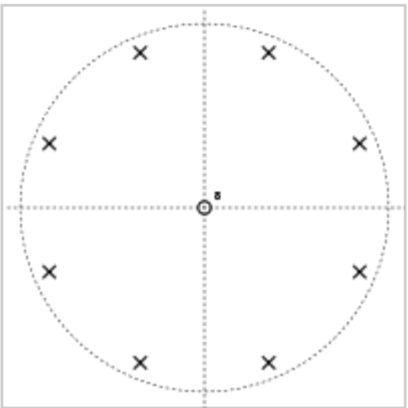
Looking again at the Z-domain transfer function of the feedback comb filter:

$$H(z) = \frac{z^K}{z^K - \alpha}$$

This time, the numerator is zero at $z^K = 0$, giving K zeros at $z = 0$. The denominator is equal to zero whenever $z^K = \alpha$. This has K solutions, equally spaced around a circle in the complex plane; these are the poles of the transfer function. This leads to a pole-zero plot like the ones shown below.



Pole-zero plot of feedback comb filter with $K = 8$ and $\alpha = 0.5$



Pole-zero plot of feedback comb filter with $K = 8$ and $\alpha = -0.5$

Continuous-time comb filters

Comb filters may also be implemented in continuous time. The feedforward form may be described by the following equation:

$$y(t) = x(t) + \alpha x(t - \tau)$$

and the feedback form by:

$$y(t) = x(t) + \alpha y(t - \tau)$$

where τ is the delay (measured in seconds).

They have the following frequency responses, respectively:

$$H(\omega) = 1 + \alpha e^{-j\omega\tau}$$
$$H(\omega) = \frac{1}{1 - \alpha e^{-j\omega\tau}}$$

Continuous-time implementations share all the properties of the respective discrete-time implementations.

See also

- Filter (signal processing)
- Digital filter
- Finite impulse response
- Infinite impulse response

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