Difference in Differences

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Outline

1. Causal Inference in Economics
2. Standard DID
3. Assumptions of DID
4. Extensions of DID

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Rubin Causal Model

Potential outcome framework (Rubin, 1974; Imbens and Rubin, 2015):

- Individual i has two potential outcomes $Y_i(1)$ and $Y_i(0)$ ($1 =$ treatment, $0 =$ control). The causal (treatment) effect for i is $Y_i(1) - Y_i(0)$.
- The fundamental problem for causal inference is a missing value problem, i.e., we cannot observe both $Y_i(1)$ and $Y_i(0)$.
- Let $W_i = 0, 1$ denote the the treatment status of i, we can observe $Y_i(W_i)$.
- Under reasonable assumptions, we can identify the average treatment effect $\tau_{ATE} = E(Y_i(1) - Y_i(0))$ in the population of interest.

Identification Strategies[1]

1. RCT (Randomized Controlled Trial)
2. PSM (Propensity Score Matching)
3. IV (Instrumental Variable)
4. DID (Difference in Differences)
5. SC (Synthetic Control)
6. RDD (Regression Discontinuity Design)

[1] See Angrist and Pischke (2008); Cunningham (2021); Abadie and Cattaneo (2018).

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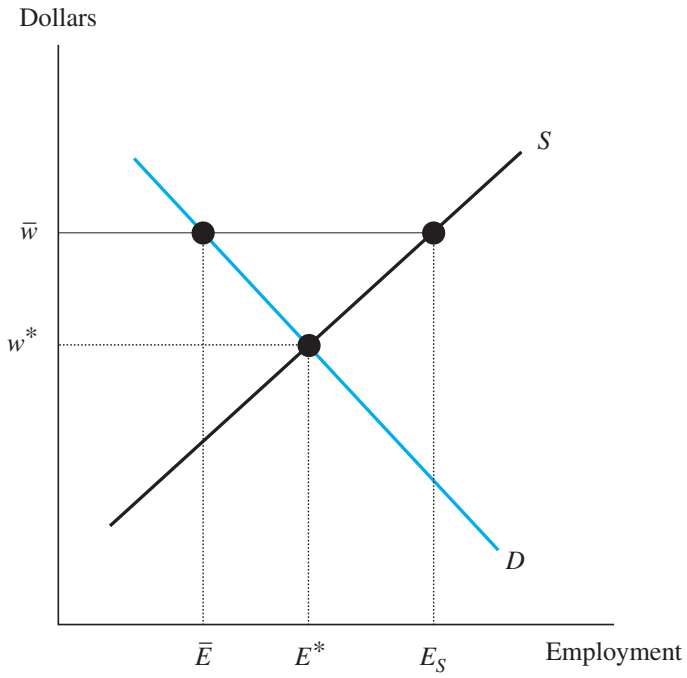
Motivation

- Card and Krueger (1994): minimum wage increase → employment.
  - *Policy*: In April 1992, New Jersey increased the minimum wage from \$4.25 to \$5.05. Pennsylvania's minimum wage stayed at \$4.25.
  - *Data*: They surveyed about 410 fast food stores in NJ and eastern PA before and after the minimum wage increase in NJ.



Motivation

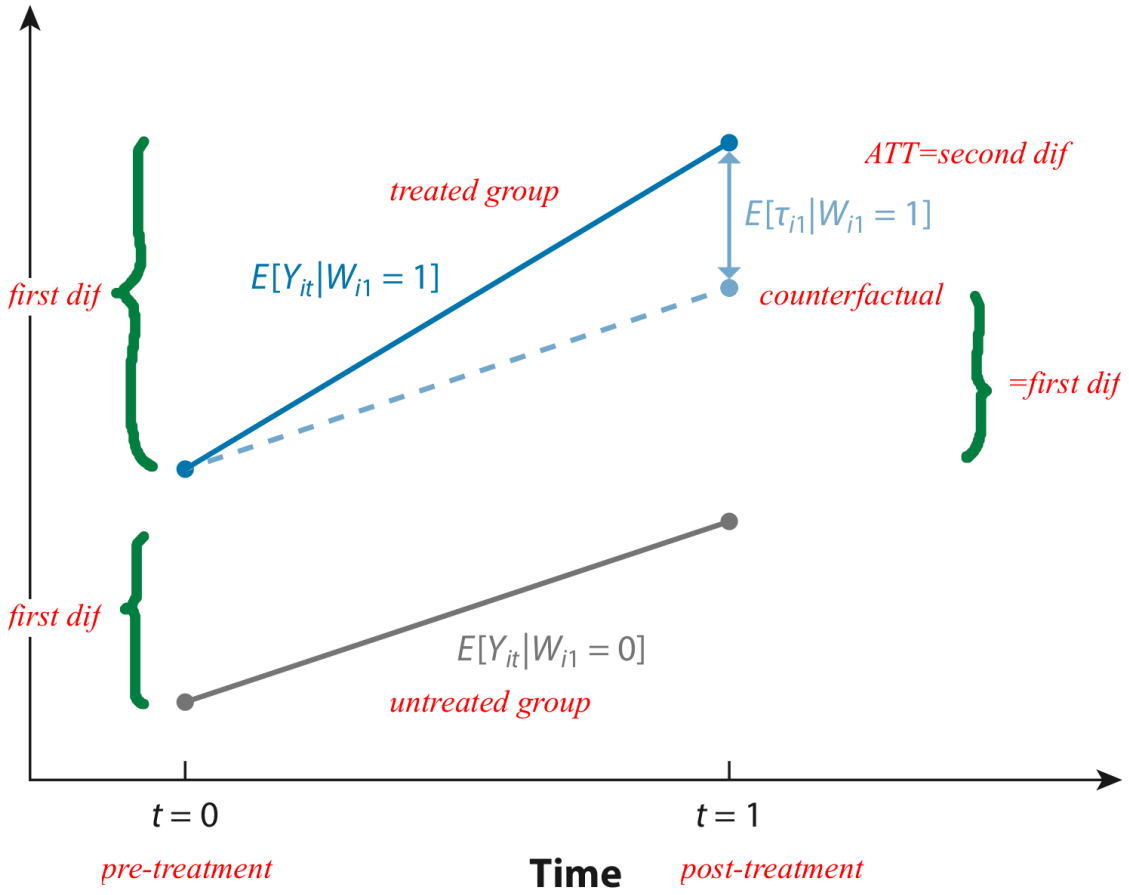
- Chen and Zhou (2007): famine → survivors' health/economic status.
  - *Policy*: A sharp drop in grain production in the 1959-1961 famine, but different regions had varied exposure.
  - *Data*: cross-sectional data of rural people born between 1954 and 1967, the excess death rates of regions in 1960, the worst year of the famine.

Death rates in the sampled provinces: unit 0.1%

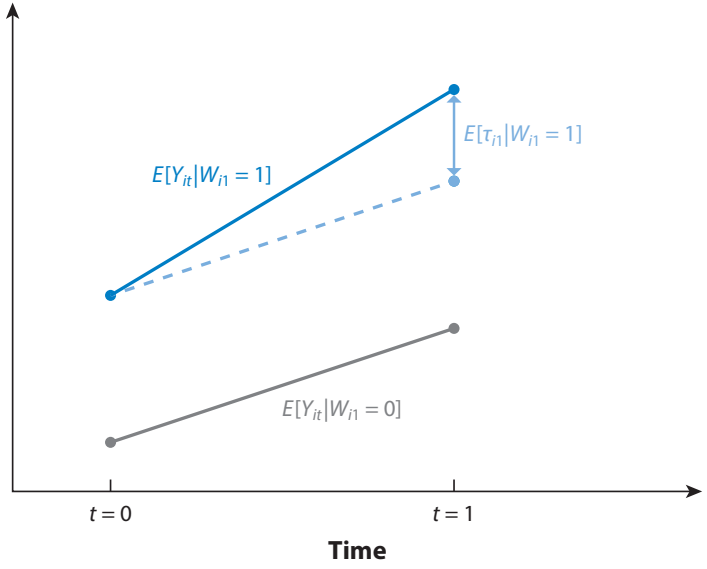
Provinces	1956	1957	1958	1959	1960	1961	1962	1963	1964
Panel A: Death rates in the sampled provinces: 1956–1964									
Liaoning	6.6	9.4	6.6	11.8	11.5	17.5	8.5	7.9	9.3
Jiangsu	13	10.3	9.4	14.6	18.4	13.4	10.4	9	10.1
Shandong	12.1	12.1	12.8	18.2	23.6	18.4	12.4	11.8	12
Henan	14	11.8	12.7	14.1	39.6	10.2	8	9.4	10.6
Hubei	10.8	9.6	9.6	14.5	21.2	9.1	8.8	9.8	10.9
Hunan	11.5	10.4	11.7	13	29.4	17.5	10.2	10.3	12.9
Guangxi	12.5	12.4	11.7	17.5	29.5	19.5	10.3	10.1	10.6
Guizhou	7.5	8.8	13.7	16.2	45.4	17.7	10.4	9.4	10.5
Nation	11.4	10.8	12.0	14.6	25.4	14.2	10.0	10.0	11.5
Panel B: Excess death rates in the: sampled provinces: 1959–1961									
Liaoning				4.3	4.0	9.97			
Jiangsu				3.7	7.5	2.5			
Shandong				5.9	11.3	6.07			
Henan				1.3	26.8	−2.63			
Hubei				4.5	11.2	−0.9			
Hunan				1.8	18.2	6.3			
Guangxi				5.3	17.3	7.3			
Guizhou				6.2	35.4	7.7			
Average for sampled provinces				4.1	16.4	4.5			
Nationwide average				3.7	14.4	3.2			

Source: Panel A is adapted from Lin and Yang (2000), Table 3, P147. Excess death rates in Panel B are calculated as the difference between death rates in the famine year and the average of death rates in 1956–1958.

Idea


ATT=second dif
treated group
$E[\tau_{i1}|W_{i1}=1]$
first dif
$E[Y_{it}|W_{i1}=1]$
counterfactual
=first dif
first dif
$E[Y_{it}|W_{i1}=0]$
untreated group
$t=0$
$t=1$
pre-treatment
Time
post-treatment

Identification



- Time $t = 0, 1$ (pre-treatment/post-treatment period), treatment group $W_{it} = 0, 1$ (treated/untreated group), treatment ω_{it}:

$$\omega_{i0}(W_{it} = 0) = \omega_{i0}(W_{it} = 1) = \omega_{i1}(W_{it} = 0) = 0, \omega_{i1}(W_{it} = 1) = 1. \quad (1)$$

- We are interested in the average treatment effect on the treated or ATT:

$$E(Y_{i1}(\omega_{i1} = 1) - Y_{i1}(\omega_{i1} = 0)|W_{it} = 1) = E(\tau_{i1}|W_{it} = 1), \quad (2)$$

where *on the treated* means $t = 1, W = 1$.

Identification

- The ATT can be written as

$$
\begin{aligned}
&E(\tau_{i1}|W_{it}=1)=E(Y_{i1}(1)-Y_{i1}(0)|W_{it}=1)\\
&=E(Y_{i1}(1)|W_{it}=1)-E(Y_{i1}(0)|W_{it}=1)\\
&=E(Y_{i1}(1)|W_{it}=1)-E(Y_{i0}(0)|W_{it}=1)\\
&+E(Y_{i0}(0)|W_{it}=1)-E(Y_{i1}(0)|W_{it}=1)\\
&=E(Y_{i1}(1)-Y_{i0}(0)|W_{it}=1)-E(Y_{i1}(0)-Y_{i0}(0)|W_{it}=1)\\
&=E(Y_{i1}(1)-Y_{i0}(0)|W_{it}=1)-E(Y_{i1}(0)-Y_{i0}(0)|W_{it}=0).
\end{aligned}
\tag{3}
$$

by the common trend assumption,

$$E(Y_{i1}(0)-Y_{i0}(0)|W_{it}=1)=E(Y_{i1}(0)-Y_{i0}(0)|W_{it}=0).$$

Estimation

- DID estimator: $\tau_{DID} = \bar{Y}_{1,1} - \bar{Y}_{1,0} - \left(\bar{Y}_{0,1} - \bar{Y}_{0,0}\right)$, where $\bar{Y}_{w,t}$ is the sample average of Y for group w and time t.
- For example, in Card and Krueger (1994)'s case study, Y is employment, group is New Jersey or Pennsylvania, time is before or after New Jersey increased the minimum wage.

	Employment in Typical Fast-Food Restaurant (in full-time equivalents)	
	New Jersey	Pennsylvania
Before New Jersey increased the minimum wage	20.4	23.3
After New Jersey increased the minimum wage	21.0	21.2
Difference	0.6	−2.1
Difference-in-differences	**2.7**	

Estimation

- Linear model with interactions:

$$Y_i = \beta_0 + \beta_1 W_i + \beta_2 T_i + \beta_3 W_i T_i + u_i. \tag{4}$$

$E(Y_i \mid T_i, W_i)$	$T_i = 0$	$T_i = 1$	Δ
$W_i = 0$	β_0	$\beta_0 + \beta_2$	β_2
$W_i = 1$	$\beta_0 + \beta_1$	$\beta_0 + \beta_1 + \beta_2 + \beta_3$	$\beta_2 + \beta_3$
Δ	β_1	$\beta_1 + \beta_3$	β_3

The OLS estimator $\hat{\beta}_3$ is a 2×2 DID estimator with panel or repeated cross-sectional data.

Estimation

- The effect of the one-child policy on the probability of being a boy (sex ratio imbalance) in China (Li et al., 2011):

$$S_i = \alpha_0 + \alpha_1 H_i + \alpha_2 T_i + \alpha_3 H_i \times T_i + \varepsilon_i. \tag{5}$$

  - $S_i = 1$ if child i is a boy, $S_i = 0$ if child i is a girl.
  - $H_i = 1$ for Han, $H_i = 0$ for minorities.
  - $T_i = 1$ if child i was born in the post-policy change period (after 1979).

	Han	Minority	Difference
Before 1979	$\alpha_0 + \alpha_1$	α_0	α_1
After 1979	$\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3$	$\alpha_0 + \alpha_2$	$\alpha_1 + \alpha_3$
Difference	$\alpha_2 + \alpha_3$	α_2	α_3

	Han	Minority
Before 1979	$(\alpha_0 + \alpha_1) / [1 - (\alpha_0 + \alpha_1)]$	$\alpha_0 / (1 - \alpha_0)$
After 1979	$(\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3) / [1 - (\alpha_0 + \alpha_1 + \alpha_2 + \alpha_3)]$	$(\alpha_0 + \alpha_2) / [1 - (\alpha_0 + \alpha_2)]$

Estimation

- Panel data model with two-way fixed effects (TWFE):

$$Y_{it} = \alpha + \beta D_{it} + \mu_i + \delta_t + \varepsilon_{it}. \tag{6}$$

  - $i = 1, \cdots, N, t = 0, 1$ (can be easily extended to multiple groups and periods).
  - D_{it} is a treatment indicator, $D_{it} = 1$ if $W_{it} = 1$ and $t = 1$.
  - μ_i individual (group) fixed effects, δ_t time (period) fixed effects.
- The OLS estimator $\hat{\beta}$ is a DID estimator with panel data.

$$E(Y_{i0}|W_{it} = 0) = \alpha + E(\mu_i|W_{it} = 0) + \delta_0. \tag{7}$$

$$E(Y_{i1}|W_{it} = 0) = \alpha + E(\mu_i|W_{it} = 0) + \delta_1. \tag{8}$$

$$E(Y_{i0}|W_{it} = 1) = \alpha + E(\mu_i|W_{it} = 1) + \delta_0. \tag{9}$$

$$E(Y_{i1}|W_{it} = 1) = \alpha + \beta + E(\mu_i|W_{it} = 1) + \delta_1. \tag{10}$$

By the common trend assumption, $ATT = ((10) - (9)) - ((8) - (7))$.

Inference

- The outcome variables of interest in DID are often serially correlated, and conventional standard errors often severely understate the standard deviation of the estimators (Bertrand et al., 2004).
- The most common solution is adjusting standard errors by clustering at the group level with software packages, e.g., in Stata.
- However, simple solutions like clustering the standard errors no longer suffice when the number of clusters is small.

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Common Trend Assumption

- Common Trend Assumption

$$E(Y_{i1}(0) - Y_{i0}(0)|W_{it} = 1) = E(Y_{i1}(0) - Y_{i0}(0)|W_{it} = 0)$$

is actually untestable. But as a robustness check we often examine the parallel of pre-treatment trends.

- Event study approach is a systematic way to check the presence of pre-trend and also the long-term effects.

$$Y_{it} = \sum_s \tau_s W_i \times T_s + \mu_i + \delta_t + \varepsilon_{it}, \tag{11}$$

where $T_s = 1(t = s)$ and we expect that $\tau_s = 0, s \leq 0, \tau_s \neq 0, s > 0$.

Common Trend Assumption

- In DID settings, we may need to include covariates to satisfy the common trend assumption,

$$Y_{it} = \tau D_{it} + \mu_i + \delta_t + \beta' X_{it} + \varepsilon_{it}. \tag{12}$$

- Common trend assumption with covariates:

$$E(Y(0,1) - Y(0,0) \mid X, W = 1) = E(Y(0,1) - Y(0,0) \mid X, W = 0), \tag{13}$$

where X should be predetermined variables to avoid bad controls.

SUTVA[2]

Assumption (The Stable Unit Treatment Value Assumption or SUTVA)

(1) ***No Interference****: the potential outcomes for any unit do not vary with the treatments assigned to other units;*
(2) ***No Hidden Variations of Treatments****: for each unit, there are no different forms or versions of each treatment level, which lead to different potential outcomes.*

[2] See Imbens and Rubin (2015).

SUTVA

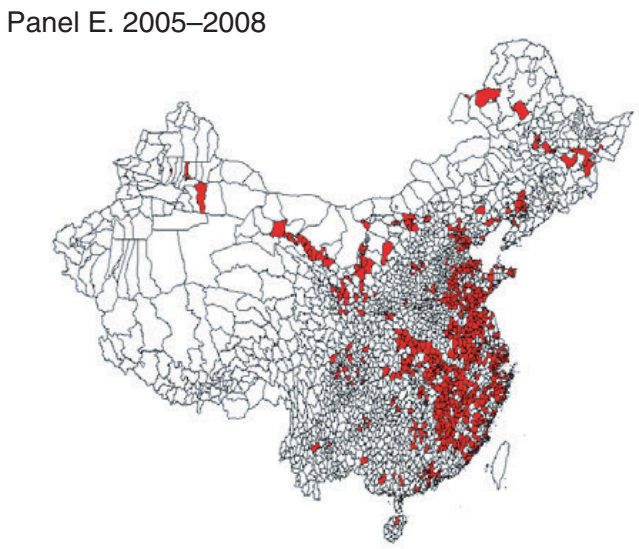

FIGURE 1. SPECIAL ECONOMIC ZONES

- Lu et al. (2019) estimates the spillover effect of China's Economic Zone Program.
  - Direct estimation of the spillover effect to surrounding non-SEZ villages.
  - Concentric Ring Analysis: a series of estimates $\hat{\gamma}^j, j \in \mathbb{N}^+$ corresponding to the exclusion of non-SEZ villages located within $2j$ km of an SEZ village.

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Staggered DID

Panel A. First round, November 2009

Panel B. Second round, July–October 2010

Panel C. Third round, July–September 2011

Panel D. Fourth round, July–October 2012

FIGURE 1. COUNTY-BY-COUNTY ROLLOUT OF THE NRPS OVER TIME

Staggered DID

- Huang and Zhang (2021) uses a two-way FE regression to investigate behavioral responses to the New Rural Pension Scheme in China:

$$Y_{ict} = \beta_0 + \beta_1 NRPS_{ct} + \delta_c + \delta_t + X_{ict} + \varepsilon_{ict}. \tag{14}$$

  - Individual i, county c, year t.
  - The dependent variable Y_{ict} is the potential outcome: household incomes, expenditures, private transfers, and so forth.
  - $NRPS_{ct}$ indicates whether county c implemented the NRPS in year t.
  - County fixed effects δ_c, year fixed effects δ_t.

DID Decomposition Theorem

- Two-way fixed-effects regression (TWFE):

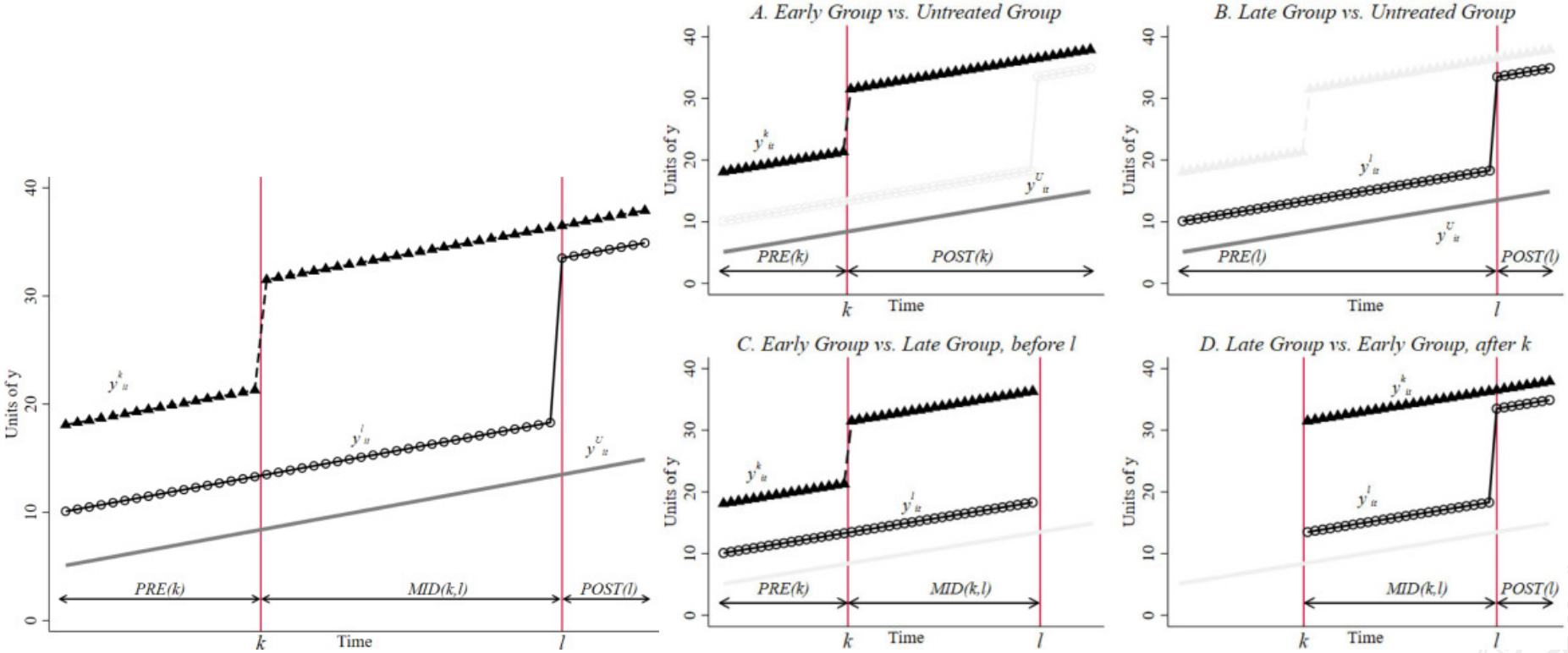
$$Y_{it} = \alpha + \beta D_{it} + \mu_i + \delta_t + \varepsilon_{it}. \tag{15}$$

Theorem (DID Decomposition (Goodman-Bacon, 2021))

Assume that the data contain $k = 1, \cdots, K$ timing groups of units ordered by the time when they receive a binary treatment, $k \in (1, T]$. There may be one timing group U that includes units that never receive treatment. The OLS estimate in a two-way fixed-effects regression is a weighted average of all possible two-by-two DID estimators:

$$\hat{\beta} = \sum_{k \neq U} s_{kU} \hat{\beta}_{kU}^{2\times 2} + \sum_{k \neq U} \sum_{\ell > k} \left[s_{k\ell}^{k} \hat{\beta}_{k\ell}^{2\times 2,k} + s_{k\ell}^{\ell} \hat{\beta}_{k\ell}^{2\times 2,\ell} \right]. \tag{16}$$

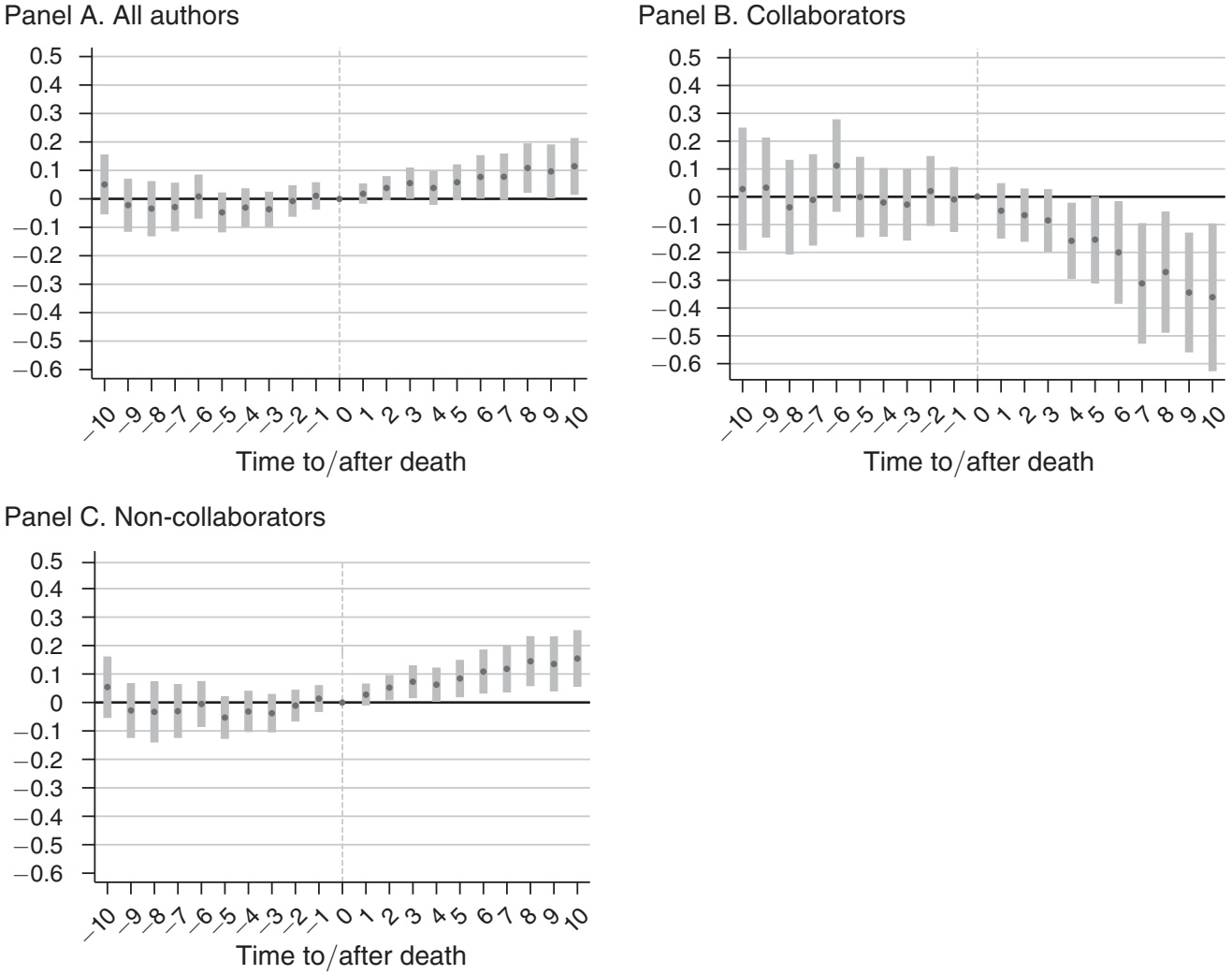
DID Decomposition Theorem



DID Decomposition Theorem

- The two-way fixed effects estimator equals a weighted average of all possible two-group/two-period DD estimators in the data.
- A causal interpretation of two-way fixed effects DID estimates requires both a parallel trends assumption and treatment effects that are constant over time.
- Implications:
  1. When treatment effects are homogeneous, DID estimator is ATT.
  2. When treatment effects are heterogeneous across units, DID estimator is a variance-weighted treatment effect that is not ATT.
  3. When treatment effects change over time, DID estimator is biased and stacked DID/event study may be more appropriate.
- But it's still unclear whether researchers should systematically abandon TWFE estimators.

Stacked DID/Event Study/Dynamic DID[3]



[3]See Azoulay et al. (2019). For recent developments in TWFE and DID with heterogeneous treatment effects, see de Chaisemartin and D'Haultfoeuille (2022).

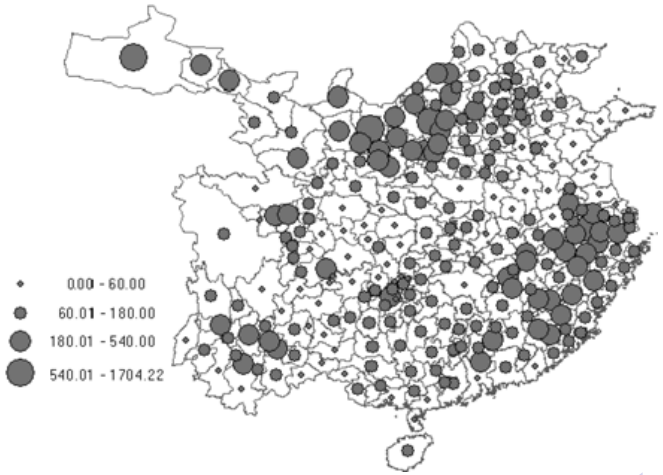
Generalized DID

- Bai and Jia (2016) examines the link between quotas per capita and revolution participation before and after the abolition of *kejv*:

$$R_{p,t} = \beta \ln \text{Quota}_p \times \text{Post}_t + \nu \ln \text{Pop}_p \times \text{Post}_t + \theta \text{X}_p \times \text{Post}_t + \lambda_p + \gamma_t + \delta_{\text{prov}} \times \gamma_t + \varepsilon_{p,t}. \tag{17}$$

  - $R_{p,t}$ indicates whether there is any revolutionary in prefecture p and year t.
  - $Quota_p$ is the quotas for the entry-level exam.
  - $Post_t$ is 0 for the years before 1905 and 1906.

(b) Spatial Distribution of Quotas per Million People



Cohort DID

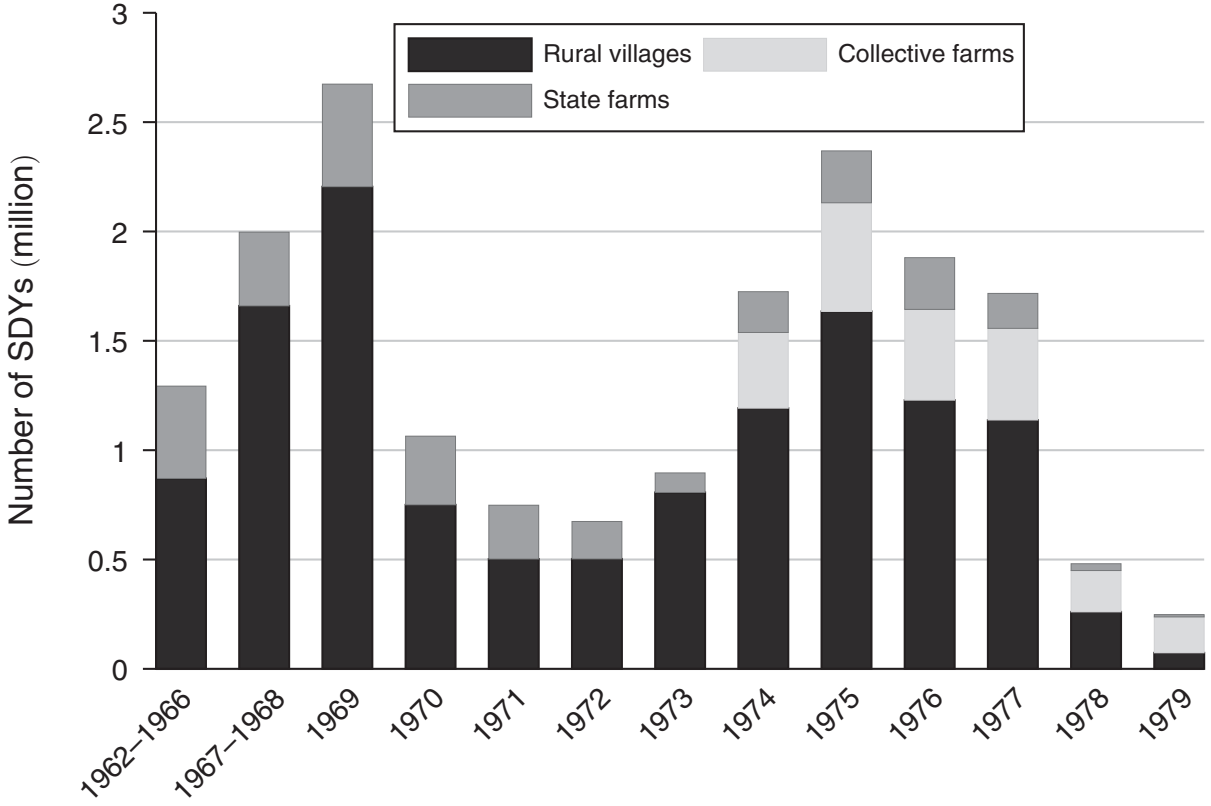


FIGURE 1. NUMBER OF SDYS BY RESETTLEMENT, 1962–1979

- Chen et al. (2020) estimates the effects on rural education of the send-down movement during the Cultural Revolution.

Cohort DID

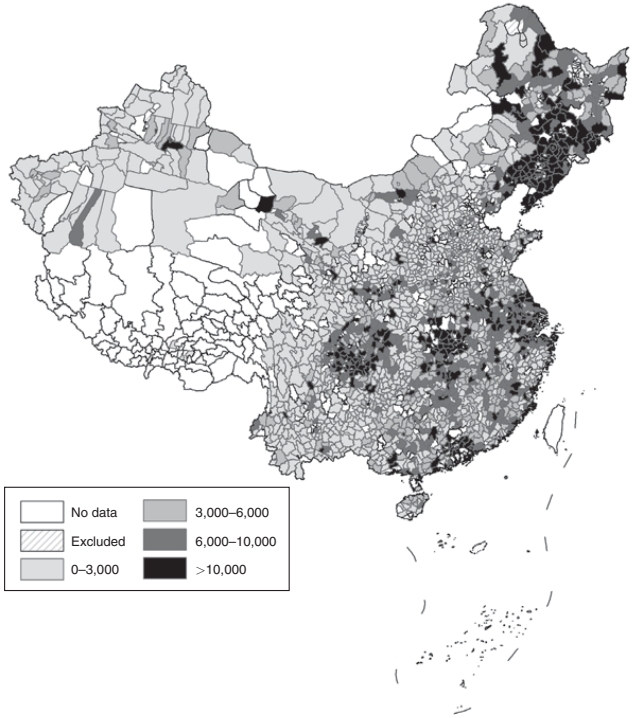


FIGURE 2. NUMBER OF RECEIVED SDYS IN EACH COUNTY

1. Counties received different numbers of SDYs (sent-down youths).
2. Within the same county, children from different cohorts were exposed differently depending on schooling years overlapped with the movement.

Cohort DID

- A Standard cohort DID (control group are cohorts 1946-1955):

$$\begin{aligned} Y_Edu_{i,g,c,p} =& \beta_0 + \beta_1 \%SDY_{c,p} \times I(1956 \leq g \leq 1969) + \beta_2 \mathbf{X}_{i,g,c,p} \\ &+ \lambda_c + \mu_{g,p} + \Lambda_c \times \mu_g + \varepsilon_{i,g,c,p}. \end{aligned} \tag{18}$$

- Separately estimating the effects of SDYs on each cohort (the 1941-1945 cohorts serve as the baseline):

$$\begin{aligned} Y_Edu_{i,g,c,p} = \beta_0 + \sum_{\gamma=1946}^{1969} \beta_{1,\gamma} \%SDY_{c,p} \times I(g = \gamma) + \beta_2 \mathbf{X}_{i,g,c,p} \\ + \lambda_c + \mu_{g,p} + \Lambda_c \times \mu_g + \varepsilon_{i,g,c,p}. \end{aligned} \tag{19}$$

  - $Y_Edu_{i,g,c,p}$ is the years of education of individual i of cohort g in county c of province p.
  - $\%SDY_{c,p}$ is the density of received SDYs in county c.
  - λ_c are county FEs, $\mu_{g,p}$ are province-cohort FEs, Λ_c is county base education before the SDYs' arrival.

Cohort DID

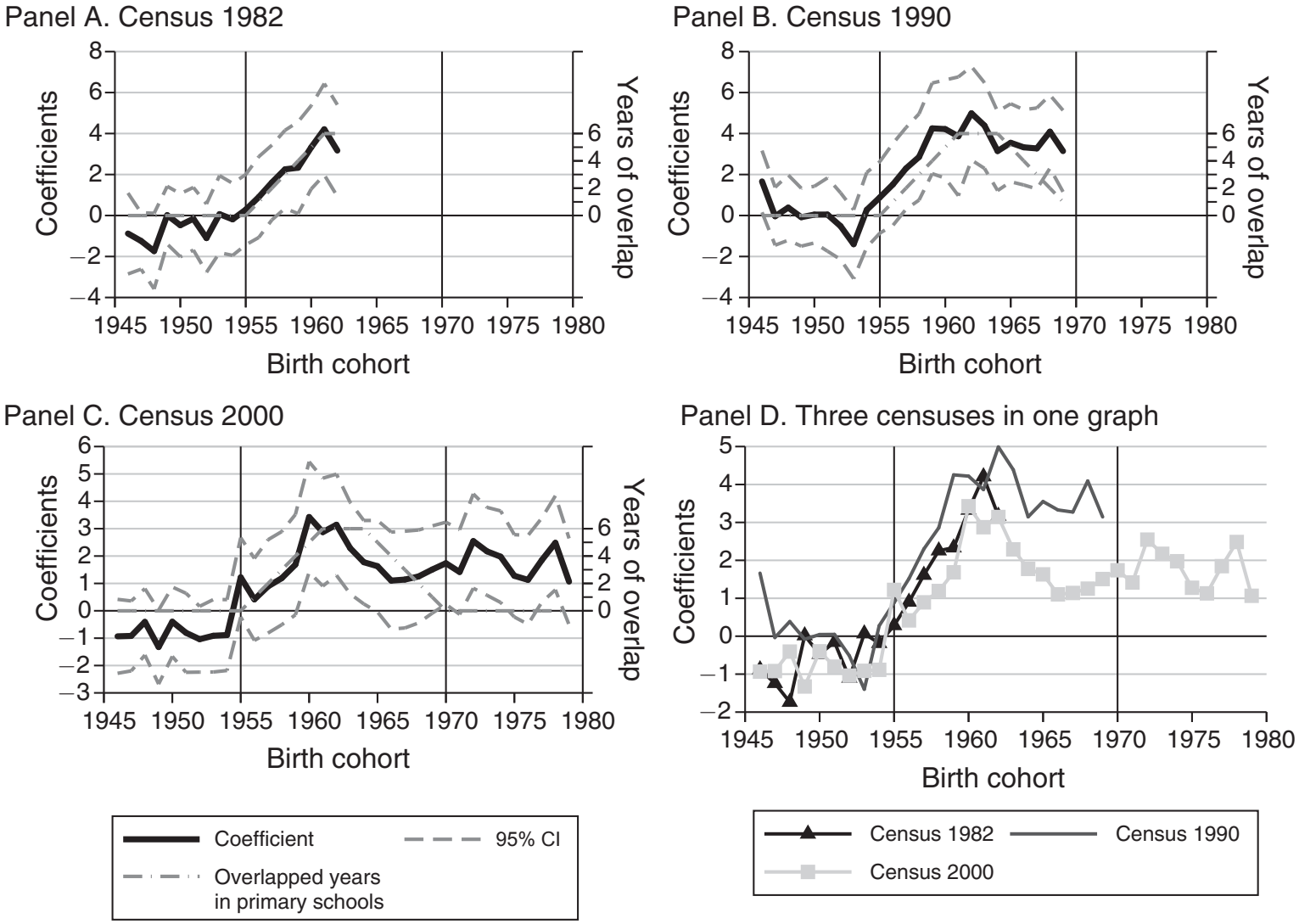


FIGURE 3. EFFECT OF SDYS ON THE EDUCATIONAL ATTAINMENT OF DIFFERENT COHORTS

DDD[4]

- Difference-in-difference-in-differences (DDD) ot triple difference (TD) is a way to relax common trend assumption, whose estimator can be computed as the difference between two DID estimators.
- The difference between two biased DID estimators will be unbiased as long as the bias is the same in both estimators, which is the common difference-in-trends assumption.
- Model with interactions:

$$\begin{aligned} Y_i = & \beta_0 + \beta_1 W_i + \beta_2 T_i + \beta_3 G_i \\ & + \beta_4 W_i T_i + \beta_5 W_i G_i + \beta_6 T_i G_i \\ & + \beta_7 W_i T_i G_i + u_i, \end{aligned} \tag{20}$$

where $\hat{\beta}_7$ is DDD estimator and ATE on treated ($T = 1, W = 1, G = g$) is identified.

[4] See Olden and Møen (2020).

DDD

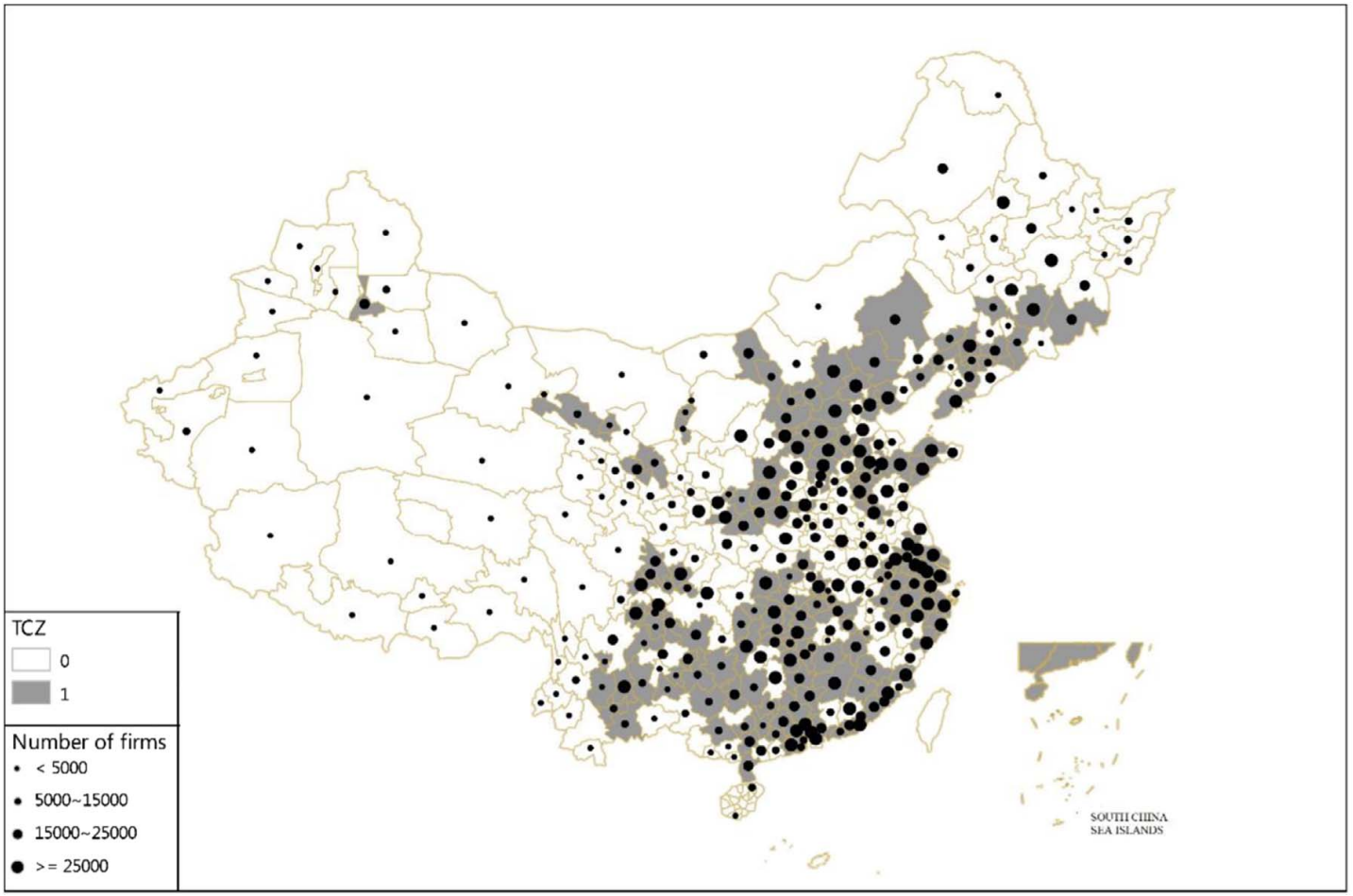


Fig. 1. Geographic distribution of TCZ cities. *Data source*: The national Environmental Protection Bureau, "The Proposal of Designation for Acid Rain Control Areas and SO_2 Pollution Control Areas"; Chinese First National Census of Basic Units.

DDD

- Cai et al. (2016) uses Two Control Zones policy (1998) to investigate the effects of environmental regulation on inbound foreign direct investment:

$$Y_{ict} = \gamma \cdot TCZ_c \times Post_t \times \mathrm{SO}_{2i} + \eta_{ct} + \lambda_{ic} + \varphi_{it} + \varepsilon_{ict}. \tag{21}$$

  - Y_{ict} is the measurement of FDI flows in industry i of city c at year t.
  - $TCZ_c = 1$ if city c is a TCZ city.
  - $Post_t = 1$ if $t \geq 1998$.
  - SO_{2i} is the degree of SO_2 emission by industry i.
  - $\eta_{ct}, \lambda_{ic}, \varphi_{it}$ are industry-year fixed effects, industry-city fixed effects, and city-year fixed effects.

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