Functions and their Graphs

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Calculus I

Lecture 2

- Objective

  To be able to identify a function and to be able to graphically represent functions.

Relation

- Relation – pairs of quantities that are related to each other

- Example: The area *A* of a circle is related to its radius *r* by the formula

$$A = \pi r^2.$$

Function

- There are different kinds of relations.

- When a relation matches each item from one set with exactly one item from a different set the relation is called a ***function***.

Definition of a Function

- A ***function*** is a relationship between two variables such that each value of the first variable is paired with exactly one value of the second variable.

- The ***domain*** is the set of permitted x values.

- The ***range*** is the set of found values of y. These can be called ***images***.

Is it a Function?

- For each *x*, there is only one value of *y*.

$$y = x + 1$$

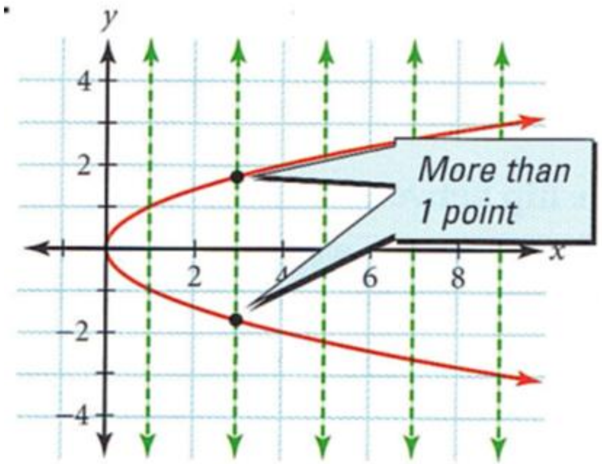
- Therefore, it <u>**IS**</u> a function.

Is it a function?

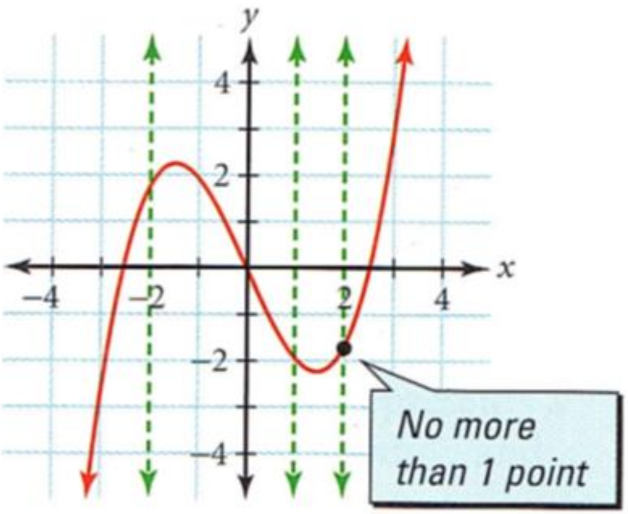
- $x^2 + y^2 = 4$
- it is **NOT** a function
- For each value of x we are getting two values of y.
- Why
- $y = \pm\sqrt{4 - x^2}$

Vertical Line Test

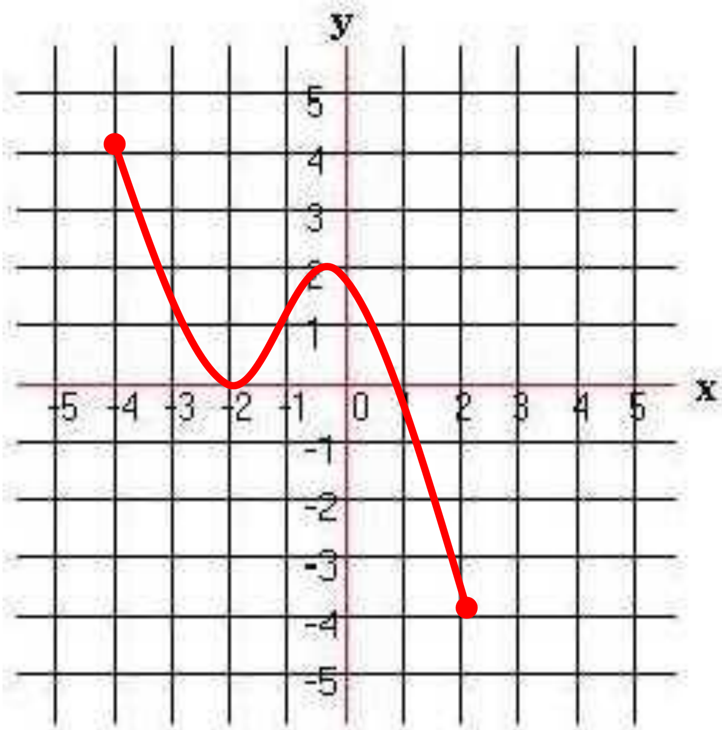
- Used to determine if a graph is a function.
- If a vertical line intersects the graph at more than one point, then the graph is **NOT** a function.



NOT a Function



Is it a function? Give the domain and range.



FUNCTION

$$Domain: [-4,2]$$

$$Range: [-4,4]$$

Functional Notation

- We have seen an equation written in the form y = some expression in x.
- Another way of writing this is to use ***functional notation.***
- For Example, you could write $y = x^2$ as $f(x) = x^2$.

Functional Notation: Find the following

$$f(x) = 3x^2 - x + 2$$

$$f(-3)$$

$$3(-3)^2 - (-3) + 2$$
$$27 + 3 + 2$$
$$30 + 2$$
$$32$$

$$f(x) = x^2 - x + 2$$

$$f(m+3)$$

$$(m+3)^2 - (m+3) + 2$$
$$(m+3)(m+3) - m - 3 + 2$$
$$m^2 + 3m + 3m + 9 - m - 3 + 2$$
$$m^2 + 5m + 8$$

Piecewise-Defined Function

- A piecewise-defined function is a function that is defined by two or more equations over a specified domain.

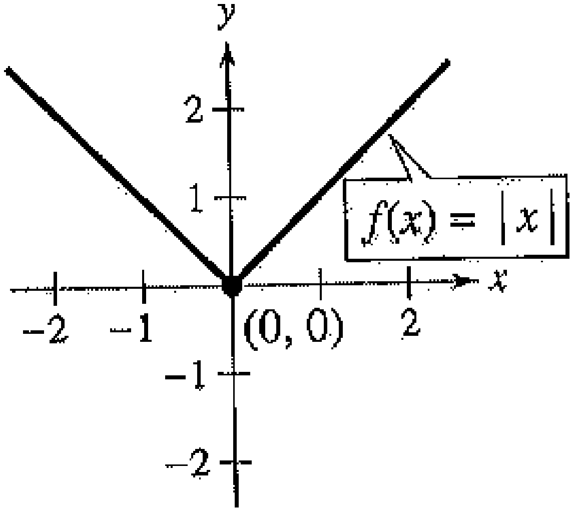
- The absolute value function $f(x) = |x|$ can be written as a piecewise-defined function.

- The basic characteristics of the absolute value function are summarized on the next page.

Absolute Value Function is a Piecewise Function

Graph of $f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$



Example

- Evaluate the function when x = -1 and 0.

$$f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$$

Solution

Because $x = -1$ is less than 0, use $f(x) = x^2 + 1$ to obtain

$$f(-1) = (-1)^2 + 1 = 2.$$

For $x = 0$, use $f(x) = x - 1$ to obtain

$$f(0) = (0) - 1 = -1.$$

Domain of a Function

- The domain of a function can be **implied** by the expression used to define the function

- The implied domain is the set of all real numbers for which the expression is defined.

- For example,

- The function $f(x) = \frac{1}{x^2 - 4}$ has an implied domain that consists of all real x other than x = ±2

- **The domain excludes *x*-values that result in division by zero.**

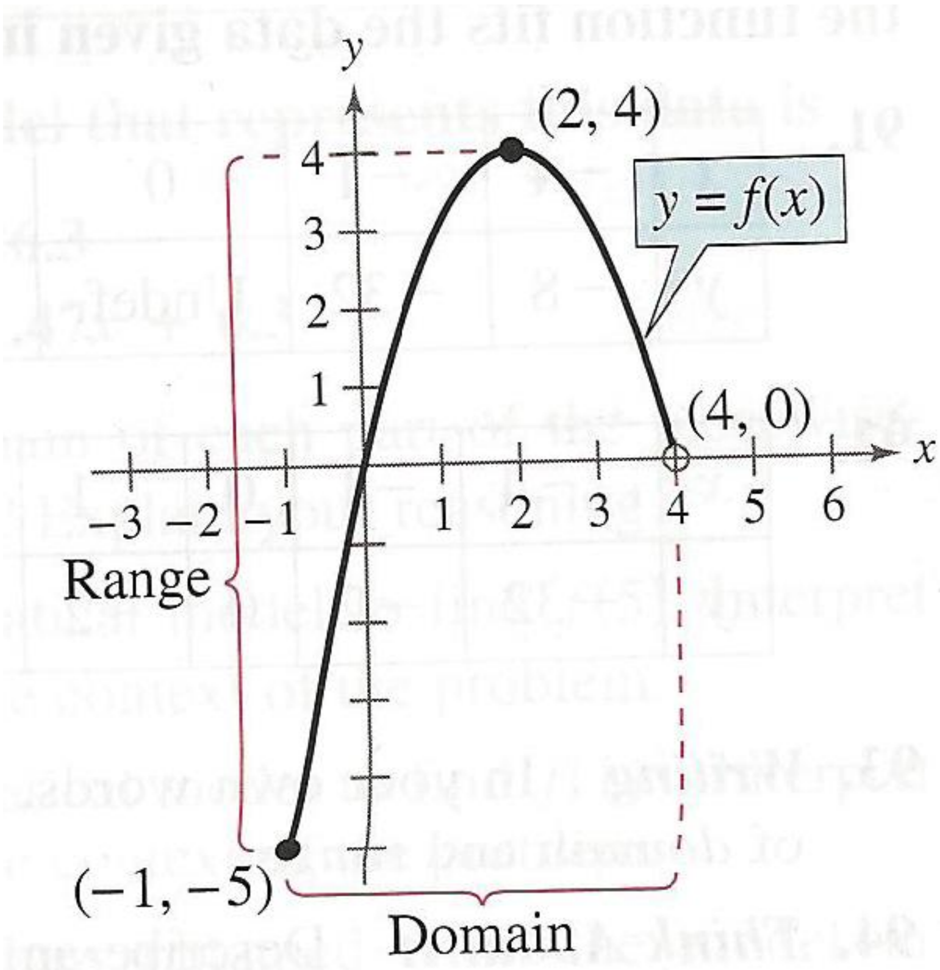
- Another common type of implied domain is that used to avoid even roots of negative numbers.
- EX: $$f(x) = \sqrt{x}$$

is defined only for $x \geq 0$.

The domain excludes *x*-values that result in even roots of negative numbers.

Graphs of Functions

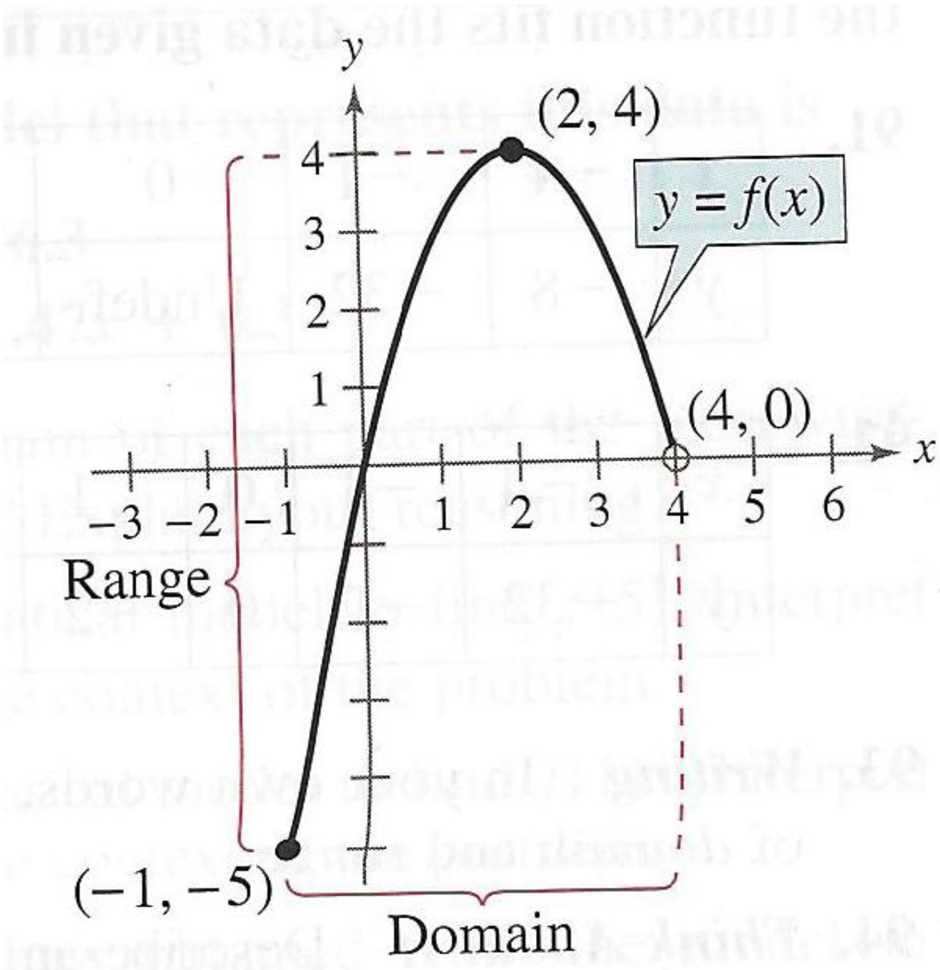
Domain & Range of a Function



What is the domain of the graph of the function f?

$$A: [-1,4)$$

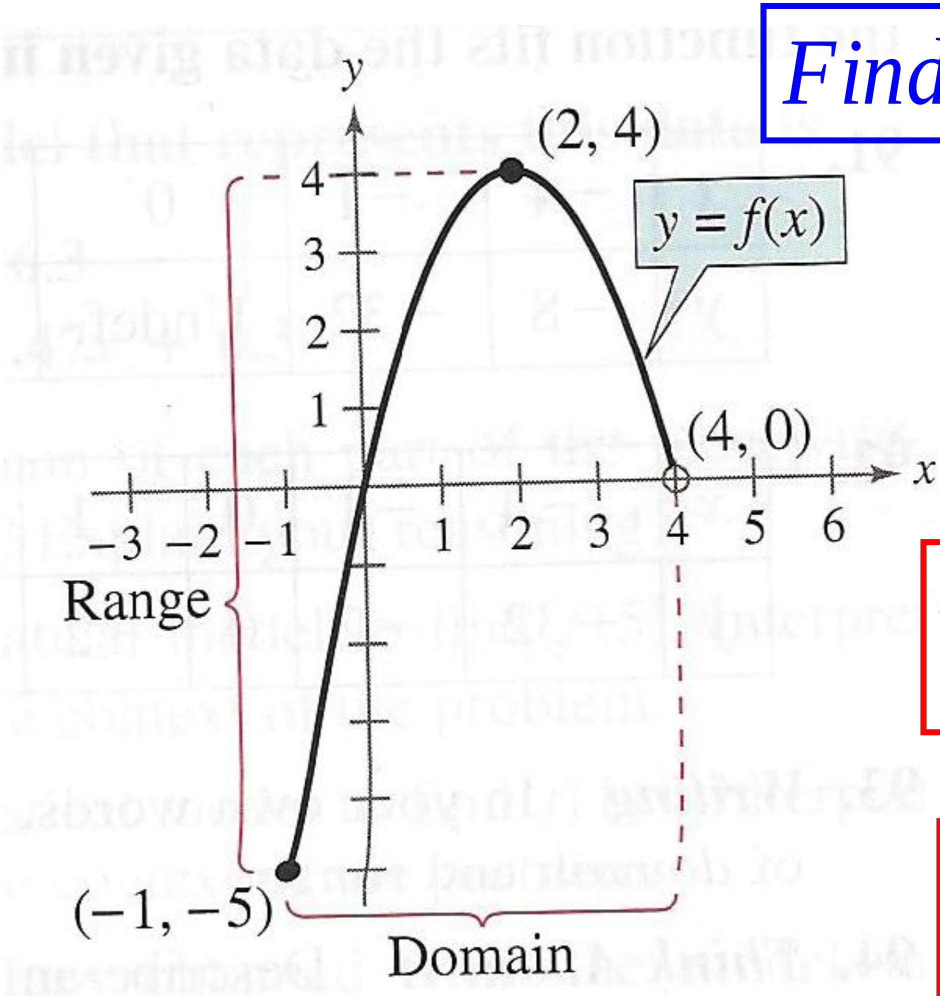
Domain & Range of a Function



What is the range of the graph of the function f?

$$[-5,4]$$

Domain & Range of a Function



$Find\ \ f(-1) and\ \ f(2).$

$$f(-1) = -5$$

$$f(2) = 4$$

- Let’s look at domain and range of a function using an algebraic approach.

- Then, let’s check it with a graphical approach.

Find the domain and range of $f(x) = \sqrt{x-4}$.

- **Algebraic Approach**

The expression under the radical can not be negative.
Therefore, $x - 4 \geq 0.$ → **Domain**

$A: \; x \geq 4$

or

$[4, \infty)$

Since the domain is never negative the range is the set of all nonnegative real numbers.

$A: \; y \geq 0$

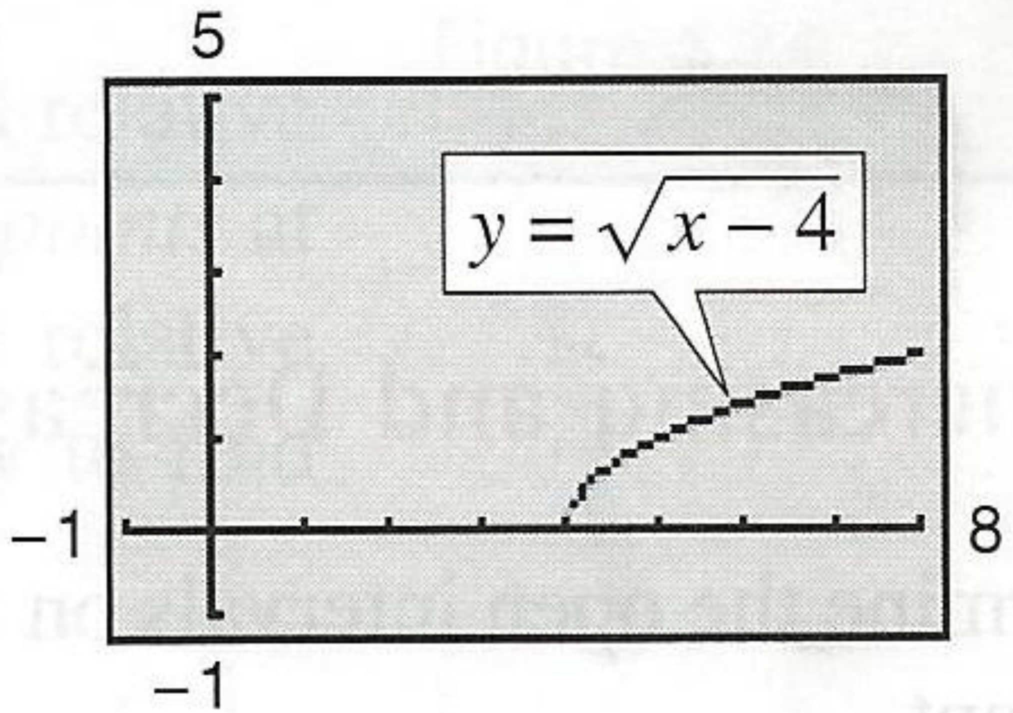
or

$[0, \infty)$

→ $\boxed{Range}$

Find the domain and range of $f(x) = \sqrt{x-4}$.

- **Graphical Approach**



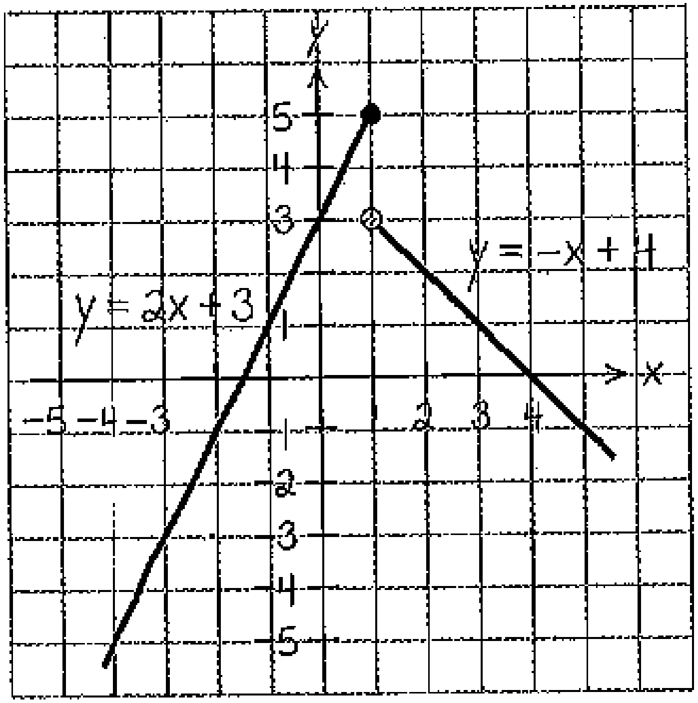
Step Functions and Piecewise-Defined Functions

Let's graph a Piecewise-Defined Function

- Sketch the graph of

$$f(x)=\begin{cases} 2x+3, & x\le 1 \\ -x+4, & x>1 \end{cases}$$

Notice when open dots and closed dots are used. Why?



The Greatest Integer Function

- The function whose value at any number x is the greatest integer less than or equal to x is called **the greatest integer function** or **the integer floor function**.
- It is denoted as $\lfloor x \rfloor$
- Observe that
- $\lfloor 2.4 \rfloor = 2, \lfloor 1.9 \rfloor = 1, \quad \lfloor 0 \rfloor = 0,$
- $\lfloor 0.2 \rfloor = 0, \lfloor -1.2 \rfloor = -2, \lfloor -2 \rfloor = -2,$
- $\lfloor -0.3 \rfloor = -1, \lfloor 2 \rfloor = 2$

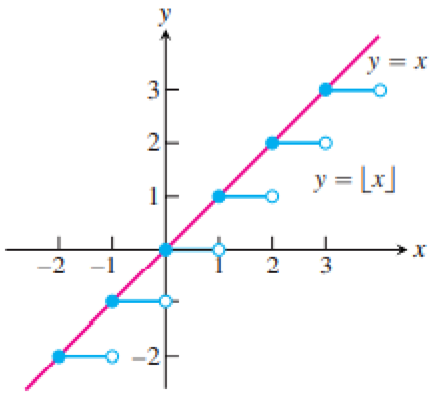


FIGURE 1.31 The graph of the greatest integer function $y = \lfloor x \rfloor$ lies on or below the line $y = x$, so it provides an integer floor for x

The Least Integer Function

- The function whose value at any number x is the smallest integer greater than or equal to x is called the **least integer function** or the **integer ceiling function**.
- It is denoted as $\lceil x \rceil$
- Observe that
- $\lceil 2.4 \rceil = 3, \lceil 1.9 \rceil = 2, \lceil 0 \rceil = 0,$
- $\lceil 0.2 \rceil = 1, \lceil -1.2 \rceil = -1, \lceil -2 \rceil = -2,$
- $\lceil -0.3 \rceil = 0, \lceil 2 \rceil = 2$

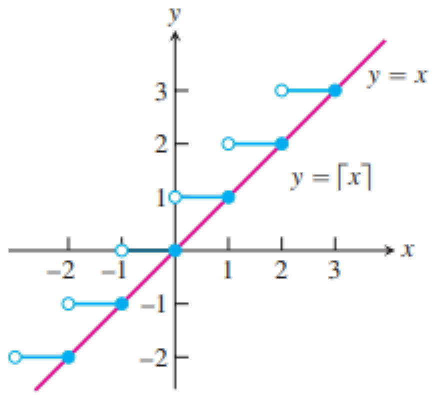


FIGURE 1.32 The graph of the least integer function $y = \lceil x \rceil$ lies on or above the line $y = x$, so it provides an integer ceiling for x

Practice Questions

Exercise no 1.3
Q no1 – Q no 8
Q no 23 – Q no 26