EXERCISES 3.6

Derivatives of Rational Powers

Find dy/dx in Exercises 1-10.

1. $y = x^{9/4}$	**2.** $y = x^{-3/5}$
3. $y = \sqrt[3]{2x}$	**4.** $y = \sqrt[4]{5x}$
5. $y = 7\sqrt{x + 6}$	**6.** $y = -2\sqrt{x - 1}$
7. $y = (2x + 5)^{-1/2}$	**8.** $y = (1 - 6x)^{2/3}$
9. $y = x(x^2 + 1)^{1/2}$	**10.** $y = x(x^2 + 1)^{-1/2}$

Find the first derivatives of the functions in Exercises 11-18.

11. $s = \sqrt[7]{t^2}$	**12.** $r = \sqrt[4]{\theta^{-3}}$
13. $y = \sin[(2t + 5)^{-2/3}]$	**14.** $z = \cos[(1 - 6t)^{2/3}]$
15. $f(x) = \sqrt{1 - \sqrt{x}}$	**16.** $g(x) = 2(2x^{-1/2} + 1)^{-1/3}$
17. $h(\theta) = \sqrt[3]{1 + \cos(2\theta)}$	**18.** $k(\theta) = (\sin(\theta + 5))^{5/4}$

Differentiating Implicitly

Use implicit differentiation to find dy/dx in Exercises 19-32.

19. $x^2y + xy^2 = 6$	**20.** $x^3 + y^3 = 18xy$
21. $2xy + y^2 = x + y$	**22.** $x^3 - xy + y^3 = 1$
23. $x^2(x - y)^2 = x^2 - y^2$	**24.** $(3xy + 7)^2 = 6y$
25. $y^2 = \dfrac{x - 1}{x + 1}$	**26.** $x^2 = \dfrac{x - y}{x + y}$
27. $x = \tan y$	**28.** $xy = \cot(xy)$
29. $x + \tan(xy) = 0$	**30.** $x + \sin y = xy$
31. $y \sin\left(\dfrac{1}{y}\right) = 1 - xy$	**32.** $y^2 \cos\left(\dfrac{1}{y}\right) = 2x + 2y$

50. [illegible]

Find $dr/d\theta$ in Exercises 33-36.

33. $\theta^{1/2} + r^{1/2} = 1$

34. $r - 2\sqrt{\theta} = \frac{3}{2}\theta^{2/3} + \frac{4}{3}\theta^{3/4}$

35. $\sin(r\theta) = \frac{1}{2}$

36. $\cos r + \cot\theta = r\theta$

Second Derivatives

In Exercises 37-42, use implicit differentiation to find dy/dx and then d^2y/dx^2.

37. $x^2 + y^2 = 1$

38. $x^{2/3} + y^{2/3} = 1$

39. $y^2 = x^2 + 2x$

40. $y^2 - 2x = 1 - 2y$

41. $2\sqrt{y} = x - y$

42. $xy + y^2 = 1$

43. If $x^3 + y^3 = 16$, find the value of d^2y/dx^2 at the point $(2, 2)$.

44. If $xy + y^2 = 1$, find the value of d^2y/dx^2 at the point $(0, -1)$.

Slopes, Tangents, and Normals

In Exercises 45 and 46, find the slope of the curve at the given points.

45. $y^2 + x^2 = y^4 - 2x$ at $(-2, 1)$ and $(-2, -1)$

46. $(x^2 + y^2)^2 = (x - y)^2$ at $(1, 0)$ and $(1, -1)$

In Exercises 47-56, verify that the given point is on the curve and find the lines that are **(a)** tangent and **(b)** normal to the curve at the given point.

47. $x^2 + xy - y^2 = 1, \quad (2, 3)$

48. $x^2 + y^2 = 25, \quad (3, -4)$

49. $x^2y^2 = 9, \quad (-1, 3)$

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61. The devil's curve (Gabriel Cramer [the Cramer of Cramer's rule], 1750) Find the slopes of the devil's curve $y^4 - 4y^2 = x^4 - 9x^2$ at the four indicated points.

Absolute Extrema on Finite Closed Intervals

In Exercises 15–30, find the absolute maximum and minimum values of each function on the given interval. Then graph the function. Identify the points on the graph where the absolute extrema occur, and include their coordinates.

15. $f(x) = \frac{2}{3}x - 5, \quad -2 \le x \le 3$

16. $f(x) = -x - 4, \quad -4 \le x \le 1$

17. $f(x) = x^2 - 1, \quad -1 \le x \le 2$

18. $f(x) = 4 - x^2, \quad -3 \le x \le 1$

19. $F(x) = -\frac{1}{x^2}, \quad 0.5 \le x \le 2$

20. $F(x) = -\frac{1}{x}, \quad -2 \le x \le -1$

21. $h(x) = \sqrt[3]{x}, \quad -1 \le x \le 8$

22. $h(x) = -3x^{2/3}, \quad -1 \le x \le 1$

23. $g(x) = \sqrt{4 - x^2}, \quad -2 \le x \le 1$

24. $g(x) = -\sqrt{5 - x^2}, \quad -\sqrt{5} \le x \le 0$

25. $f(\theta) = \sin\theta, \quad -\frac{\pi}{2} \le \theta \le \frac{5\pi}{6}$

26. $f(\theta) = \tan\theta, \quad -\frac{\pi}{3} \le \theta \le \frac{\pi}{4}$

27. $g(x) = \csc x, \quad \frac{\pi}{3} \le x \le \frac{2\pi}{3}$

28. $g(x) = \sec x, \quad -\frac{\pi}{3} \le x \le \frac{\pi}{6}$

29. $f(t) = 2 - |t|, \quad -1 \le t \le 3$

30. $f(t) = |t - 5|, \quad 4 \le t \le 7$

In Exercises 31–34, find the function's absolute maximum and minimum values and say where they are assumed.

31. $f(x) = x^{4/3}, \quad -1 \le x \le 8$

32. $f(x) = x^{5/3}, \quad -1 \le x \le 8$

33. $g(\theta) = \theta^{3/5}, \quad -32 \le \theta \le 1$

34. $h(\theta) = 3\theta^{2/3}, \quad -27 \le \theta \le 8$

Finding Extreme Values

In Exercises 35–44, find the extreme values of the function and where

EXERCISES 4.2

Finding c in the Mean Value Theorem

Find the value or values of c that satisfy the equation

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

in the conclusion of the Mean Value Theorem for the functions and intervals in Exercises 1–4.

Exercises

1. $f(x) = x^2 + 2x - 1, \quad [0, 1]$
2. $f(x) = x^{2/3}, \quad [0, 1]$
3. $f(x) = x + \frac{1}{x}, \quad \left[\frac{1}{2}, 2\right]$
4. $f(x) = \sqrt{x - 1}, \quad [1, 3]$

Checking and Using Hypotheses

Which of the functions in Exercises 5–8 satisfy the hypotheses of the Mean Value Theorem on the given interval, and which do not? Give reasons for your answers.

5. $f(x) = x^{2/3}, \quad [-1, 8]$
6. $f(x) = x^{4/5}, \quad [0, 1]$
7. $f(x) = \sqrt{x(1 - x)}, \quad [0, 1]$
8. $f(x) = \begin{cases} \dfrac{\sin x}{x}, & -\pi \le x < 0 \\ 0, & x = 0 \end{cases}$

9. The functi

is zero at
rivative or
Theorem
Give reaso

10. For what v

satisfy the
$[0, 2]$?

Roots (Zer

11. a. Plot th
zeros o
i) $y =$
ii) $y =$
iii) $y =$
iv) $y =$

b. Use Rolle's Theorem to prove that between every two zeros of $x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$ there lies a zero of

$$nx^{n-1} + (n - 1)a_{n-1}x^{n-2} + \cdots + a_1.$$

12. Suppose that f'' is continuous on $[a, b]$ and that f has three zeros in the interval. Show that f'' has at least one zero in (a, b). Gener-

In Exercises 3
graph passes

33. $f'(x) =$

34. $g'(x) =$

EXERCISES 4.3

Analyzing f Given f'

Answer the following questions about the functions whose derivatives are given in Exercises 1–8:

a. What are the critical points of f?

b. On what intervals is f increasing or decreasing?

c. At what points, if any, does f assume local maximum and minimum values?

1. $f'(x) = x(x-1)$

2. $f'(x) = (x-1)(x+2)$

3. $f'(x) = (x-1)^2(x+2)$

4. $f'(x) = (x-1)^2(x+2)^2$

5. $f'(x) = (x-1)(x+2)(x-3)$

6. $f'(x) = (x-7)(x+1)(x+5)$

7. $f'(x) = x^{-1/3}(x+2)$

8. $f'(x) = x^{-1/2}(x-3)$

Extremes of Given Functions

In Exercises 9–28:

a. Find the intervals on which the function is increasing and decreasing.

b. Then identify the function's local extreme values, if any, saying where they are taken on.

c. Which, if any, of the extreme values are absolute?

T **d.** Support your findings with a graphing calculator or computer grapher.

9. $g(t) = -t^2 - 3t + 3$

10. $g(t) = -3t^2 + 9t + 5$

11. $h(x) = -x^3 + 2x^2$

12. $h(x) = 2x^3 - 18x$

13. $f(\theta) = 3\theta^2 - 4\theta^3$

14. $f(\theta) = 6\theta - \theta^3$

15. $f(r) = 3r^3 + 16r$

16. $h(r) = (r+7)^3$

Solve the initial value problems in Exercises 67–86.

xercises

67. $\frac{dy}{dx} = 2x - 7, \quad y(2) = 0$

68. $\frac{dy}{dx} = 10 - x, \quad y(0) = -1$

69. $\frac{dy}{dx} = \frac{1}{x^2} + x, \quad x > 0; \quad y(2) = 1$

70. $\frac{dy}{dx} = 9x^2 - 4x + 5, \quad y(-1) = 0$

71. $\frac{dy}{dx} = 3x^{-2/3}, \quad y(-1) = -5$

72. $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}, \quad y(4) = 0$

73. $\frac{ds}{dt} = 1 + \cos t, \quad s(0) = 4$

74. $\frac{ds}{dt} = \cos t + \sin t, \quad s(\pi) = 1$

75. $\frac{dr}{d\theta} = -\pi \sin \pi\theta, \quad r(0) = 0$

76. $\frac{dr}{d\theta} = \cos \pi\theta, \quad r(0) = 1$

77. $\frac{dv}{dt} = \frac{1}{2} \sec t \tan t, \quad v(0) = 1$

78. $\frac{dv}{dt} = 8t + \csc^2 t, \quad v\left(\frac{\pi}{2}\right) = -7$

79. $\frac{d^2y}{dx^2} = 2 - 6x; \quad y'(0) = 4, \quad y(0) = 1$

80. $\frac{d^2y}{dx^2} = 0; \quad y'(0) = 2, \quad y(0) = 0$

81. $\frac{d^2r}{dt^2} = \frac{2}{t^3}; \quad \left.\frac{dr}{dt}\right|_{t=1} = 1, \quad r(1) = 1$

82. $\frac{d^2s}{dt^2} = \frac{3t}{8}; \quad \left.\frac{ds}{dt}\right|_{t=4} = 3, \quad s(4) = 4$

83. $\frac{d^3y}{dx^3} = 6; \quad y''(0) = -8, \quad y'(0) = 0, \quad y(0) = 5$

84. $\frac{d^3\theta}{dt^3} = 0; \quad \theta''(0) = -2, \quad \theta'(0) = -\frac{1}{2}, \quad \theta(0) = \sqrt{2}$

85. $y^{(4)} = -\sin t + \cos t;$
$y'''(0) = 7, \quad y''(0) = y'(0) = -1, \quad y(0) = 0$

86. $y^{(4)} = -\cos x + 8 \sin 2x;$
$y'''(0) = 0, \quad y''(0) = y'(0) = 1, \quad y(0) = 3$

Finding Curves

87. Find the curve $y = f(x)$ in the xy-plane that passes through the point $(9, 4)$ and whose slope at each point is $3\sqrt{x}$.

88.

Sa

Ex

eac

89.

91.

Ap

93.

Express the sums in Exercises 11–16 in sigma notation. The form of your answer will depend on your choice of the lower limit of summation.

11. $1 + 2 + 3 + 4 + 5 + 6$

12. $1 + 4 + 9 + 16$

13. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$

14. $2 + 4 + 6 + 8 + 10$

15. $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}$

16. $-\frac{1}{5} + \frac{2}{5} - \frac{3}{5} + \frac{4}{5} - \frac{5}{5}$

Values of Finite Sums

17. Suppose that $\sum_{k=1}^{n} a_k = -5$ and $\sum_{k=1}^{n} b_k = 6$. Find the values of

a. $\sum_{k=1}^{n} 3a_k$ **b.** $\sum_{k=1}^{n} \frac{b_k}{6}$ **c.** $\sum_{k=1}^{n} (a_k + b_k)$

d. $\sum_{k=1}^{n} (a_k - b_k)$ **e.** $\sum_{k=1}^{n} (b_k - 2a_k)$

18. Suppose that $\sum_{k=1}^{n} a_k = 0$ and $\sum_{k=1}^{n} b_k = 1$. Find the values of

a. $\sum_{k=1}^{n} 8a_k$ **b.** $\sum_{k=1}^{n} 250b_k$

c. $\sum_{k=1}^{n} (a_k + 1)$ **d.** $\sum_{k=1}^{n} (b_k - 1)$

Evaluate the sums in Exercises 19–28.

19. a. $\sum_{k=1}^{10} k$ **b.** $\sum_{k=1}^{10} k^2$ **c.** $\sum_{k=1}^{10} k^3$

20. a. $\sum_{k=1}^{13} k$ **b.** $\sum_{k=1}^{13} k^2$ **c.** $\sum_{k=1}^{13} k^3$

21. $\sum_{k=1}^{7} (-2k)$

22. $\sum_{k=1}^{5} \frac{\pi k}{15}$

23. $\sum_{k=1}^{6} (3 - k^2)$

24. $\sum_{k=1}^{6} (k^2 - 5)$

25. $\displaystyle\sum_{k=1}^{5} k(3k+5)$

26. $\displaystyle\sum_{k=1}^{7} k(2k+1)$

27. $\displaystyle\sum_{k=1}^{5} \frac{k^3}{225} + \left(\sum_{k=1}^{5} k\right)^3$

28. $\displaystyle\left(\sum_{k=1}^{7} k\right)^2 - \sum_{k=1}^{7} \frac{k^3}{4}$

Rectangles for Riemann Sums

In Exercises 29–32, graph each function $f(x)$ over the given interval. Partition the interval into four subintervals of equal length. Then add to your sketch the rectangles associated with the Riemann sum $\sum_{k=1}^{4} f(c_k)\,\Delta x_k$, given that c_k is the (a) left-hand endpoint, (b) right-hand endpoint, (c) midpoint of the kth subinterval. (Make a separate sketch for each set of rectangles.)

29. $f(x) = x^2 - 1, \quad [0, 2]$

30. $f(x) = -x^2, \quad [0, 1]$

31. $f(x) = \sin x, \quad [-\pi, \pi]$

32. $f(x) = \sin x + 1, \quad [-\pi, \pi]$

33. Find the norm of the partition $P = \{0, 1.2, 1.5, 2.3, 2.6, 3\}$.

34. Find the norm of the partition $P = \{-2, -1.6, -0.5, 0, 0.8, 1\}$.

Limits of Upper Sums

For the functions in Exercises 35–40 find a formula for the upper sum obtained by dividing the interval $[a, b]$ into n equal subintervals. Then take a limit of these sums as $n \to \infty$ to calculate the area under the curve over $[a, b]$.

35. $f(x) = 1 - x^2$ over the interval $[0, 1]$.

36. $f(x) = 2x$ over the interval $[0, 3]$.

37. $f(x) = x^2 + 1$ over the interval $[0, 3]$.

38. $f(x) = 3x^2$ over the interval $[0, 1]$.

39. $f(x) = x + x^2$ over the interval $[0, 1]$.

40. $f(x) = 3x + 2x^2$ over the interval $[0, 1]$.

a. $\int_{1} f(u)\,du$ b. $\int_{1} \sqrt{3} f(z)\,dz$

c. $\int_{2}^{1} f(t)\,dt$ d. $\int_{1}^{2} [-f(x)]\,dx$

12. Suppose that $\int_{-3}^{0} g(t)\,dt = \sqrt{2}$. Find

a. $\int_{0}^{-3} g(t)\,dt$ b. $\int_{-3}^{0} g(u)\,du$

c. $\int_{-3}^{0} [-g(x)]\,dx$ d. $\int_{-3}^{0} \frac{g(r)}{\sqrt{2}}\,dr$

13. Suppose that f is integrable and that $\int_{0}^{3} f(z)\,dz = 3$ and $\int_{0}^{4} f(z)\,dz = 7$. Find

a. $\int_{3}^{4} f(z)\,dz$ b. $\int_{4}^{3} f(t)\,dt$

14. Suppose that h is integrable and that $\int_{-1}^{1} h(r)\,dr = 0$ and $\int_{-1}^{3} h(r)\,dr = 6$. Find

a. $\int_{1}^{3} h(r)\,dr$ b. $-\int_{3}^{1} h(u)\,du$

Using Area to Evaluate Definite Integrals

In Exercises 15–22, graph the integrands and use areas to evaluate the integrals.

15. $\int_{-2}^{4} \left(\frac{x}{2} + 3\right) dx$ **16.** $\int_{1/2}^{3/2} (-2x + 4)\,dx$

Average Value

In Exercises 55–62, graph the function and find its average value over the given interval.

55. $f(x) = x^2 - 1$ on $[0, \sqrt{3}]$

56. $f(x) = -\frac{x^2}{2}$ on $[0, 3]$ **57.** $f(x) = -3x^2 - 1$ on $[0, 1]$

58. $f(x) = 3x^2 - 3$ on $[0, 1]$

59. $f(t) = (t - 1)^2$ on $[0, 3]$

60. $f(t) = t^2 - t$ on $[-2, 1]$

61. $g(x) = |x| - 1$ on **a.** $[-1, 1]$, **b.** $[1, 3]$, and **c.** $[-1, 3]$

62. $h(x) = -|x|$ on **a.** $[-1, 0]$, **b.** $[0, 1]$, and **c.** $[-1, 1]$

Theory and Examples

17. $\int_{-3}^{3} \sqrt{9 - x^2}\, dx$

18. $\int_{-4}^{0} \sqrt{16 - x^2}\, dx$

19. $\int_{-2}^{1} |x|\, dx$

20. $\int_{-1}^{1} (1 - |x|)\, dx$

21. $\int_{-1}^{1} (2 - |x|)\, dx$

22. $\int_{-1}^{1} \left(1 + \sqrt{1 - x^2}\right) dx$

Use areas to evaluate the integrals in Exercises 23–26.

23. $\int_{0}^{b} \frac{x}{2}\, dx, \quad b > 0$

24. $\int_{0}^{b} 4x\, dx, \quad b > 0$

25. $\int_{a}^{b} 2s\, ds, \quad 0 < a < b$

26. $\int_{a}^{b} 3t\, dt, \quad 0 < a < b$

Evaluations

Use the results of Equations (1) and (3) to evaluate the integrals in Exercises 27–38.

27. $\int_{1}^{\sqrt{2}} x\, dx$

28. $\int_{0.5}^{2.5} x\, dx$

29. $\int_{\pi}^{2\pi} \theta\, d\theta$

30. $\int_{\sqrt{2}}^{5\sqrt{2}} r\, dr$

31. $\int_{0}^{\sqrt[3]{7}} x^2\, dx$

32. $\int_{0}^{0.3} s^2\, ds$

33. $\int_{0}^{1/2} t^2\, dt$

34. $\int_{0}^{\pi/2} \theta^2\, d\theta$

35. $\int_{a}^{2a} x\, dx$

36. $\int_{a}^{\sqrt{3a}} x\, dx$

37. $\int_{0}^{\sqrt[3]{b}} x^2\, dx$

38. $\int_{0}^{3b} x^2\, dx$

Use the rules in Table 5.3 and Equations (1)–(3) to evaluate the integrals in Exercises 39–50.

39. $\int_{3}^{1} 7\, dx$

40. $\int_{0}^{-2} \sqrt{2}\, dx$

41. $\int_{0}^{2} 5x\, dx$

42. $\int_{3}^{5} \frac{x}{8}\, dx$

43. $\int_{0}^{2} (2t - 3)\, dt$

44. $\int_{0}^{\sqrt{2}} \left(t - \sqrt{2}\right) dt$

45. $\int_{2}^{1} \left(1 + \frac{z}{2}\right) dz$

46. $\int_{3}^{0} (2z - 3)\, dz$

47. $\int_{1}^{2} 3u^2\, du$

48. $\int_{1/2}^{1} 24u^2\, du$

49. $\int_{0}^{2} (3x^2 + x - 5)\, dx$

50. $\int_{1}^{0} (3x^2 + x - 5)\, dx$

Finding Area

In Exercises 51–54 use a definite integral to find the area of the region between the given curve and the x-axis on the interval $[0, b]$.

51. $y = 3x^2$

52. $y = \pi x^2$

53. $y = 2x$

54. $y = \frac{x}{2} + 1$

EXERCISES 5.4

Evaluating Integrals

Evaluate the integrals in Exercises 1–26.

1. $\int_{-2}^{0}(2x+5)\,dx$
2. $\int_{-3}^{4}\left(5-\frac{x}{2}\right)dx$
3. $\int_{0}^{4}\left(3x-\frac{x^3}{4}\right)dx$
4. $\int_{-2}^{2}(x^3-2x+3)\,dx$
5. $\int_{0}^{1}(x^2+\sqrt{x})\,dx$
6. $\int_{0}^{5}x^{3/2}\,dx$
7. $\int_{1}^{32}x^{-6/5}\,dx$
8. $\int_{-2}^{-1}\frac{2}{x^2}\,dx$
9. $\int_{0}^{\pi}\sin x\,dx$
10. $\int_{0}^{\pi}(1+\cos x)\,dx$
11. $\int_{0}^{\pi/3}2\sec^2 x\,dx$
12. $\int_{\pi/6}^{5\pi/6}\csc^2 x\,dx$
13. $\int_{\pi/4}^{3\pi/4}\csc\theta\cot\theta\,d\theta$
14. $\int_{0}^{\pi/3}4\sec u\tan u\,du$
15. $\int_{\pi/2}^{0}\frac{1+\cos 2t}{2}\,dt$
16. $\int_{-\pi/3}^{\pi/3}\frac{1-\cos 2t}{2}\,dt$
17. $\int_{-\pi/2}^{\pi/2}(8y^2+\sin y)\,dy$
18. $\int_{-\pi/3}^{-\pi/4}\left(4\sec^2 t+\frac{\pi}{t^2}\right)dt$
19. $\int_{1}^{-1}(r+1)^2\,dr$
20. $\int_{-\sqrt{3}}^{\sqrt{3}}(t+1)(t^2+4)\,dt$
21. $\int_{\sqrt{2}}^{1}\left(\frac{u^7}{2}-\frac{1}{u^5}\right)du$
22. $\int_{1/2}^{1}\left(\frac{1}{v^3}-\frac{1}{v^4}\right)dv$
23. $\int_{1}^{\sqrt{2}}\frac{s^2+\sqrt{s}}{s^2}\,ds$
24. $\int_{9}^{4}\frac{1-\sqrt{u}}{\sqrt{u}}\,du$
25. $\int_{-4}^{4}|x|\,dx$
26. $\int_{0}^{\pi}\frac{1}{2}(\cos x+|\cos x|)\,dx$

Derivatives of Integrals

EXERCISES 5.5

Evaluating Integrals

Evaluate the indefinite integrals in Exercises 1–12 by using the given substitutions to reduce the integrals to standard form.

ses

1. $\int \sin 3x\, dx, \quad u = 3x$

2. $\int x \sin(2x^2)\, dx, \quad u = 2x^2$

ses

5. $\int 28(7x-2)^{-5}\, dx, \quad u = 7x - 2$

6. $\int x^3(x^4-1)^2\, dx, \quad u = x^4 - 1$

7. $\int \dfrac{9r^2\, dr}{\sqrt{1-r^3}}, \quad u = 1 - r^3$

8. $\int 12(y^4 + 4y^2 + 1)^2(y^3 + 2y)\, dy, \quad u = y^4 + 4y^2 + 1$

9. $\int \sqrt{x} \sin^2(x^{3/2} - 1)\, dx, \quad u = x^{3/2} - 1$

10. $\int \dfrac{1}{x^2} \cos^2\left(\dfrac{1}{x}\right) dx, \quad u = -\dfrac{1}{x}$

11. $\int \csc^2 2\theta \cot 2\theta\, d\theta$

    a. Using $u = \cot 2\theta$

    b. Using $u = \csc 2\theta$

12. $\int \dfrac{dx}{\sqrt{5x+8}}$

    a. Using $u = 5x + 8$

    b. Using $u = \sqrt{5x+8}$

3. $\int \sec 2t \tan 2t \, dt, \quad u = 2t$

4. $\int \left(1 - \cos \frac{t}{2}\right)^2 \sin \frac{t}{2} \, dt, \quad u = 1 - \cos \frac{t}{2}$

EXERCISES 6.3

Lengths of Parametrized Curves

Find the lengths of the curves in Exercises 1–6.

1. $x = 1 - t, \quad y = 2 + 3t, \quad -2/3 \le t \le 1$
2. $x = \cos t, \quad y = t + \sin t, \quad 0 \le t \le \pi$
3. $x = t^3, \quad y = 3t^2/2, \quad 0 \le t \le \sqrt{3}$
4. $x = t^2/2, \quad y = (2t + 1)^{3/2}/3, \quad 0 \le t \le 4$
5. $x = (2t + 3)^{3/2}/3, \quad y = t + t^2/2, \quad 0 \le t \le 3$
6. $x = 8 \cos t + 8t \sin t, \quad y = 8 \sin t - 8t \cos t, \quad 0 \le t \le \pi/2$

Finding Lengths of Curves

Find the lengths of the curves in Exercises 7–16. If you have a grapher, you may want to graph these curves to see what they look like.

7. $y = (1/3)(x^2 + 2)^{3/2}$ from $x = 0$ to $x = 3$
8. $y = x^{3/2}$ from $x = 0$ to $x = 4$
9. $x = (y^3/3) + 1/(4y)$ from $y = 1$ to $y = 3$
   (*Hint:* $1 + (dx/dy)^2$ is a perfect square.)
10. $x = (y^{3/2}/3) - y^{1/2}$ from $y = 1$ to $y = 9$
    (*Hint:* $1 + (dx/dy)^2$ is a perfect square.)
11. $x = (y^4/4) + 1/(8y^2)$ from $y = 1$ to $y = 2$
    (*Hint:* $1 + (dx/dy)^2$ is a perfect square.)
12. $x = (y^3/6) + 1/(2y)$ from $y = 2$ to $y = 3$
    (*Hint:* $1 + (dx/dy)^2$ is a perfect square.)
13. $y = (3/4)x^{4/3} - (3/8)x^{2/3} + 5, \quad 1 \le x \le 8$
14. $y = (x^3/3) + x^2 + x + 1/(4x + 4), \quad 0 \le x \le 2$
15. $x = \displaystyle\int_0^y \sqrt{\sec^4 t - 1}\, dt, \quad -\pi/4 \le y \le \pi/4$
16. $y = \displaystyle\int_{-2}^x \sqrt{3t^4 - 1}\, dt, \quad -2 \le x \le -1$