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DEPT: BS (CS)

ASSIGNMENT #03"

Q No:- 1. Find the absolute maximum and minimum values of each function on the given interval.

(a) $h(x) = \sqrt[3]{x}$, $\quad -1 \leq x \leq 8$

Sol.

$$h'(x) = (x)^{1/3}$$

$$= \frac{1}{3}(x)^{\frac{1}{3}-1} \cdot (1)$$

$$= \frac{1}{3}(x^{-2/3})$$

$$= \frac{1}{3(x^{-2/3})}$$

critical point $x = 0$,

$h(-1) = (-1)^{1/3} = -1$

$h(0) = (0)^{1/3} = 0$

$h(8) = (8)^{1/3} = 2$

Absolute maximum at $x = 8$ and absolute minimum at $x = -1$.

b) $f(\theta) = \sin\theta, \quad -\pi/2 < \theta < 5\pi/6$

Soln.

$f'(\theta) = \cos\theta$

So, critical point $\theta = \pi/2$

$f(-\pi/2) = \sin(-\pi/2) = -1$

$f(\pi/2) = \sin(\pi/2) = 1$

$f(5\pi/6) = \sin(5\pi/6) = 1/2$

The absolute maximum at $\theta = \pi/2$
and absolute minimum at $\theta = -\pi/2$

Q No: 2:-

(a) What are the critical points of f'?
(b) local maximum and minimum values.
(c) increasing or decreasing?

(i) $f'(x) = (x-7)(x+1)(x+5)$

Sol.

$\Rightarrow$ put 0 in $f(x')$

$x-7=0,\quad x+1=0,\quad x+5=0$

$x=7,\quad x=-1,\quad x=-5$

Intervals $(-\infty,-5)\quad(-5,-1)\quad(-1,7)\quad(7,\infty)$

$f'(-6)=-65\quad f'(-2)=27\quad f'(0)=-35\quad f'(8)=117$

-ve $\quad$ +ve $\quad$ -ve $\quad$ +ve

decreasing $\quad$ increasing $\quad$ decreasing $\quad$ incrng

local maximum at $x=-1$ and local minimum at $x=-5$ & $x=7$.

(ii) $f'(x) = x^{-1/2}(x-3)$

$$f'(x) = \frac{x-3}{(x)^{1/2}}$$

$x-3=0$, $\quad x^{1/2}=0$

$x=3$, $\quad x=0$

Intervals

$(-\infty, 0)$	$(0,3)$	$(3,\infty)$
$f'(-1)=4$	$f'(1)=-2$	$f'(4)=\frac{1}{2}$
+ve	−ve	+ve
increasing	decreasing	increasing

Local maximum at $x=0$, and local minimum at $x=3$.

Q No 3. Use L'hopital's Rule to find the limits.

(i) $\lim_{x \to 1} \dfrac{2x^2 - (3x+1)\sqrt{x} + 2}{x - 1}$

Sol—

$\Rightarrow \dfrac{2x^2 - (3x^{1+1/2} + x^{1/2}) + 2}{x - 1}$

$\Rightarrow \dfrac{2x^2 - 3x^{3/2} + x^{1/2} + 2}{x - 1}$

$\Rightarrow \lim_{x \to 1} \dfrac{2x^2 - 3x^{3/2} + x^{1/2} + 2}{x - 1}, \dfrac{0}{0}$

$\Rightarrow$ Using L'hopital Rule

$\Rightarrow \dfrac{f(a)}{g(a)} = \dfrac{f'(a)}{g'(a)}$

$\Rightarrow \dfrac{4x - \frac{3}{2}(3x)^{3/2-1} \cdot (3) - \frac{1}{2}x^{1/2-1}}{1 - 0}$

$\Rightarrow 4x - \frac{3}{2}(3x)^{1/2} \cdot 3 - \frac{1}{2\sqrt{x}}$

$\Rightarrow 4x - \frac{9\sqrt{3x}}{2} - \frac{1}{2\sqrt{x}}$

put $\lim_{x \to 1} 4(1) - \frac{9}{2}\left(\sqrt{3(1)}\right] - \frac{1}{2\sqrt{(1)}}$

$\Rightarrow \frac{4}{1} - \frac{9}{2} - \frac{1}{2}$

$\Rightarrow \frac{8 - 10}{2}$

$\Rightarrow \frac{-\cancel{2}}{\cancel{2}} = -1$

(ii) $\lim_{r \to 1} \frac{a(r^n - 1)}{r - 1}$

$\Rightarrow \frac{a(r^n - 1)}{r - 1} = \frac{0}{0}$ (

$\Rightarrow$ Using L'hopital Rule

$\Rightarrow \frac{a(nar^{n-1})}{1}$

$\Rightarrow$ put $\lim_{r \to 1} an(1)^{n-1}$

$\Rightarrow an$

(iii) $\lim_{x \to \infty} x \tan \frac{1}{x}$

Sol.

Divide by $1/x$

$\Rightarrow \dfrac{\tan 1/x}{1/x}$

$\Rightarrow$ Using L'hospital Rule

$\Rightarrow \dfrac{\cancel{-\frac{1}{x^2}} \sec^2 \frac{1}{x}}{\cancel{-1/x^2}}$

$\Rightarrow$ put $\lim_{x \to \infty} \sec^2 (0)$

$\Rightarrow 1$ Ans.

Q No. 41. Find the most general indefinite integral.

(i) $\displaystyle\int \frac{4+\sqrt{t}}{t^3}\,dt$

Sol.

$$\Rightarrow \int \frac{4}{t^3} + \frac{t^{1/2}}{t^3}\,dt$$

$$\Rightarrow \int \left(4t^{-3} + t^{-5/2}\right) dt$$

$$\Rightarrow 4\int \frac{t^{-3+1}}{-3+1} + \int \frac{t^{-5/2+1}}{-5/2+1}\,dt$$

$$\Rightarrow \frac{4t^{-2}}{-2} + \frac{t^{-3/2}}{-3/2} + c$$

$$\Rightarrow -\frac{2}{t^2} - \frac{2}{3t^{3/2}} + C$$

(ii) $\int (2 + \tan^2\theta)\, d\theta$

Sol.

$\Rightarrow \int 2 + (1 - \sec^2\theta)\, d\theta$

$\Rightarrow 2\int d\theta \pm 1 \int d\theta + \int \sec^2\theta \int d\theta$

$\Rightarrow 2\theta \pm \theta + \tan^2\theta + c$

$\Rightarrow \theta + \tan^2\theta + c$

(iii) $\int \cot^2\theta\, d\theta$

Sol.

$\Rightarrow \int (\csc^2\theta - 1)\, d\theta$

$\Rightarrow \int -\cot\theta - \int 1\, d\theta \int d\theta$

$\Rightarrow -\cot\theta - \theta + c$

Q No. 5. Solve the initial Value Problems

$$\frac{d^3\theta}{dt^3} = 0; \quad \theta''(0) = -2, \quad \theta'(0) = \frac{-1}{2}, \quad \theta(0) = \sqrt{2}$$

Sol..

$\Rightarrow \quad t''' = 0,$

$\Rightarrow \quad t'' = C_1$

$\Rightarrow \quad \theta''(0) = -2$

$\Rightarrow \quad 0 = -2 \quad , C_1 = -2$

$\Rightarrow \quad t'' = -2$

$\Rightarrow \quad t' = -2t\theta$

$\Rightarrow \quad t' = -2\theta + C_2$

$\Rightarrow \quad \theta'(0) = -\frac{1}{2}$

$\Rightarrow \quad 0 = -2(0) - \frac{1}{2} + C_2$

$\Rightarrow \quad t' = -\frac{1}{2} + C_2 \quad , C_2 = -\frac{1}{2}$

$\Rightarrow \quad t' = -2\theta - \frac{1}{2} \cancel{+C_2}$

$\Rightarrow \quad t = -2\theta^2 - \frac{\theta}{2} + C_3$

$\Rightarrow \quad t(0) = \sqrt{2}$

$0 = -2(0)^2 - \frac{0}{2} + \sqrt{2} + C_3$

So, we have $C_3 = \sqrt{2}$

$\Rightarrow \quad t = -2\theta^2 - \frac{1}{2}\theta + \sqrt{2}$