CSC 101 Applied Physics

Spring 2023

Today's topic:
Moment of a Couple

Chapter-4 Force System Resultants

4.6 Moment of a Couple

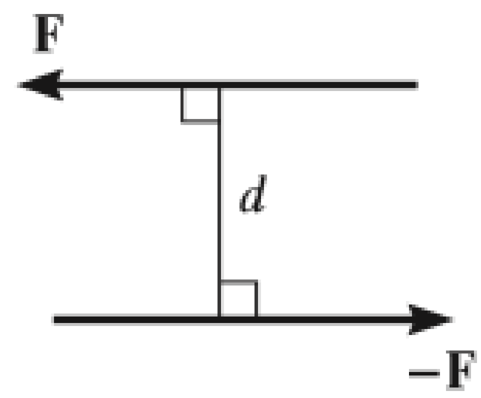


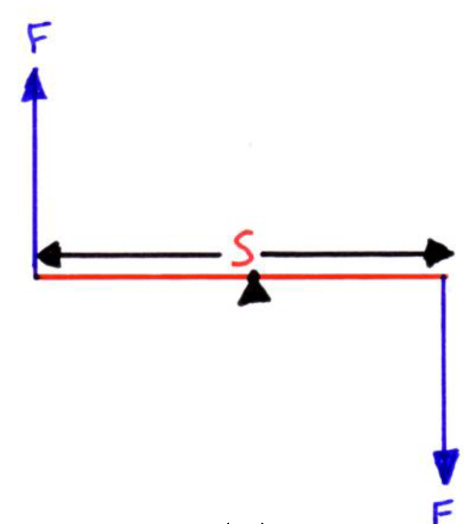
Fig. 4–25 (a)

- *Couple:* Two equal and opposite elements. is a special form of Moment

- *A couple in vectors is defined as two parall forces that have the same magnitude, but opposite directions, and are separated by a perpendicular distance d, Fig. 4−25 (a)*

- *Two Forces Fs are acting as parallel but they are opposite to each other Fig (b)*

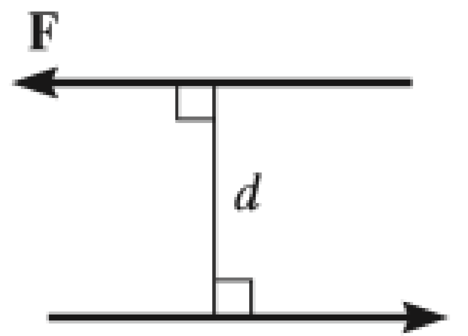
- *Since the resultant force is zero, because forces are equal and opposite, therefore only effect of a couple is to produce an actual rotation*



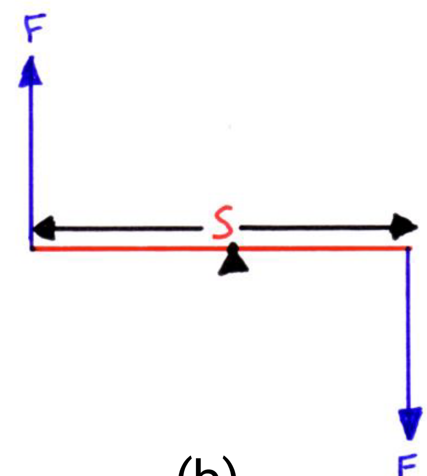
(b)

4.6 Moment of a Couple

- when any movement is possible, there is a tendency of rotation in a specified direction in diagram (a)
- In diagram (b), the two forces are parallel, and a rotation is possible
- Two forces that are equal in magnitude, opposite in sense and do not share a line of action (It means their point of acting is at different positions/locations)



(a)



(b)

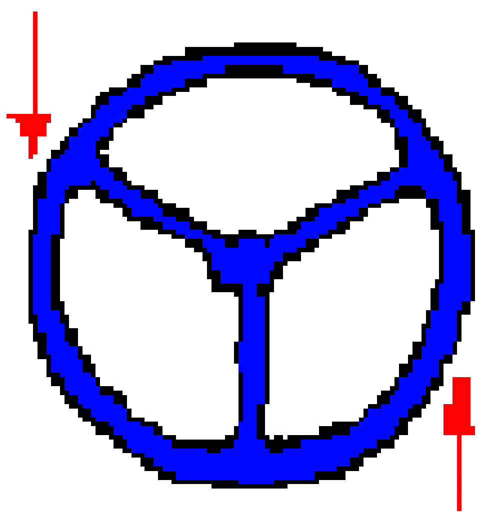
4.6 Moment of a Couple

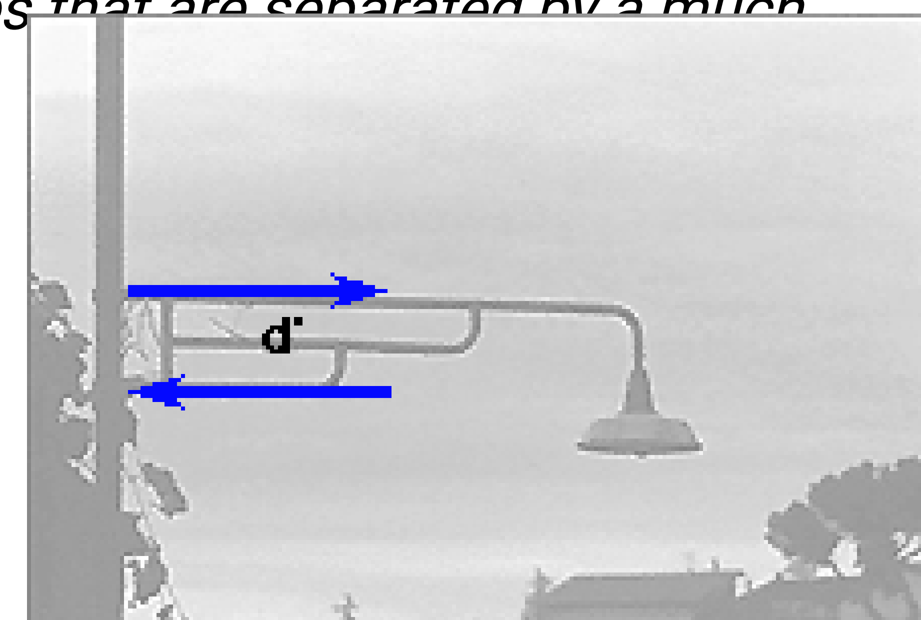
For example:

imagine that you are driving a car with both hands on the steering wheel and you are making a turn. One hand will push up on the wheel while the other hand pulls down, which causes the steering wheel to rotate

For example:

A common street lamp creates a moment. The two forces of the couple can be seen and tension forces that are separated by a much small





4.6 Moment of a Couple

Couple-Moment

- The moment produced by a couple is called a **couple moment**
- We can determine its value by finding the sum of the moments of both couple forces about any arbitrary point.
- This couple moment will have both **Magnitude** and **Direction**

4.6 Moment of a Couple

Couple-Moment

- *Two forces are acting as shown in f ig 4-26; F and -F*

- *The perpendicular distance between these two forces is r*

- *The Force F is acting at a line of action passes through point B*

- *The Force –F is acting at a line of action that passes through point A*

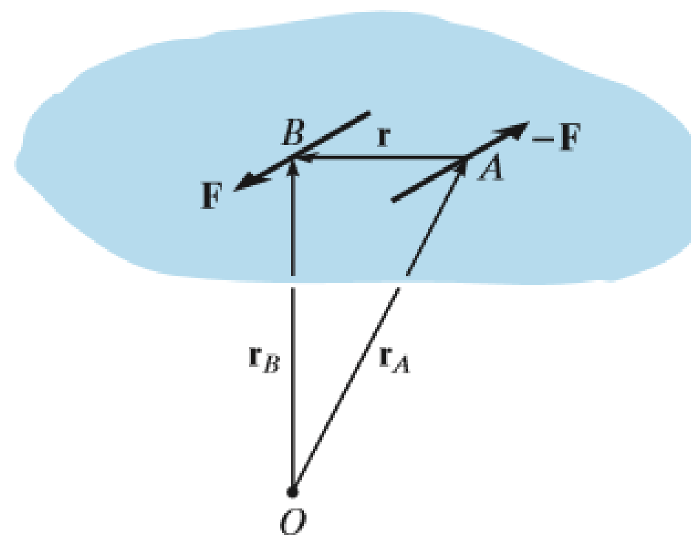


Fig. 4–26

4.6 Moment of a Couple

- *In Fig. 4–26, position vectors r_A and r_B are directed from point O to points A and B lying on the line of action of -F and F.*

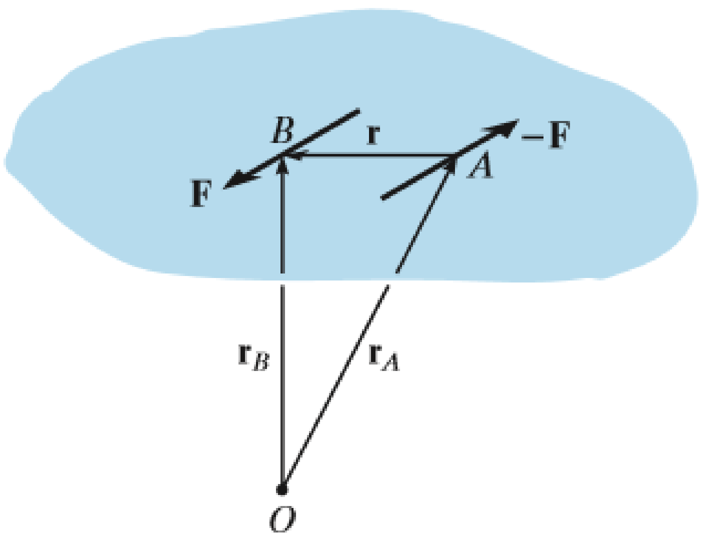


Fig. 4–26

- *The couple moment determined about O would be therefore given by:*

$$\mathbf{M} = \mathbf{r}_B \times \mathbf{F} + \mathbf{r}_A \times -\mathbf{F} = (\mathbf{r}_B - \mathbf{r}_A) \times \mathbf{F}$$

However $\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}$ or $\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A$, so that

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4.6 Moment of a Couple

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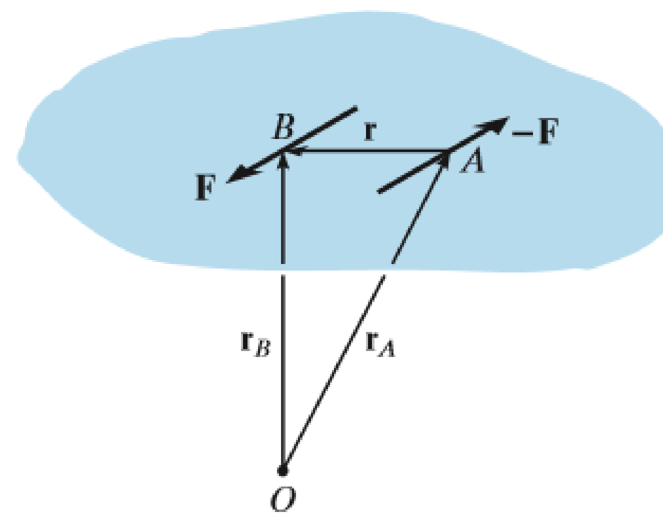


Fig. 4–26

- *This result indicates that a couple moment is a free vector, it means that*
- *Moment of a couple M depends only upon the position vector r directed between the forces while*
-
- *M does not depend on the position vectors r_A and r_B, directed from the arbitrary point O to the forces.*

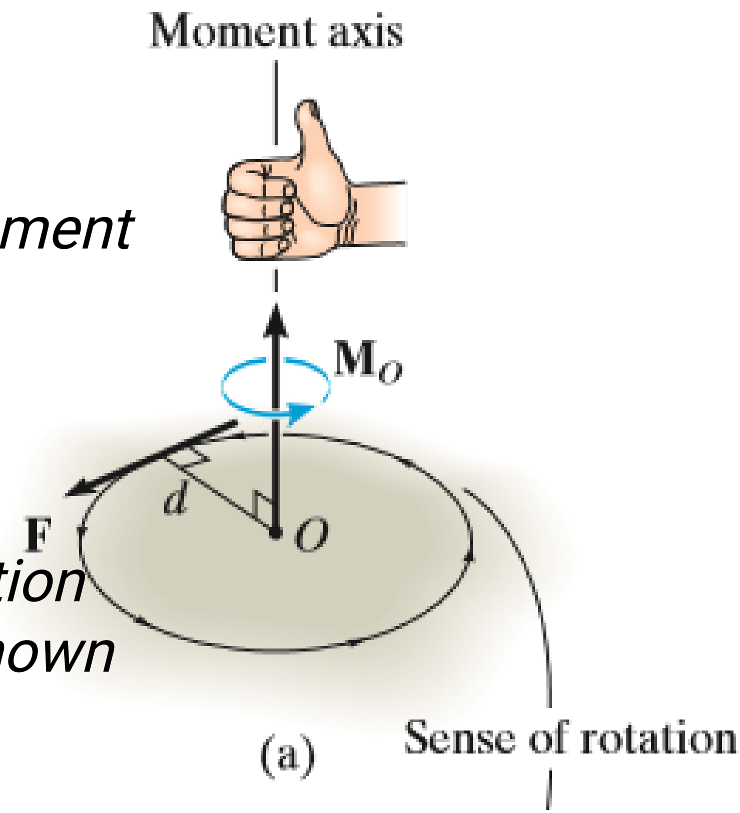
4.6 Moment of a Couple

Scaler Formation of Moment of a couple:

There would be a magnitude of the couple moment

- It means that the Magnitude of Moment of couple can be calculated

- As we had calculated magnitude and direction in our previous lecture about Moment as shown in fig (a) here at right hand side



(a)

4.6 Moment of a Couple

Scaler Formation of Moment of a couple: Magnitude and Direction

The moment of a couple, M, Fig. 4–27, is defined as having a magnitude of M = Fd

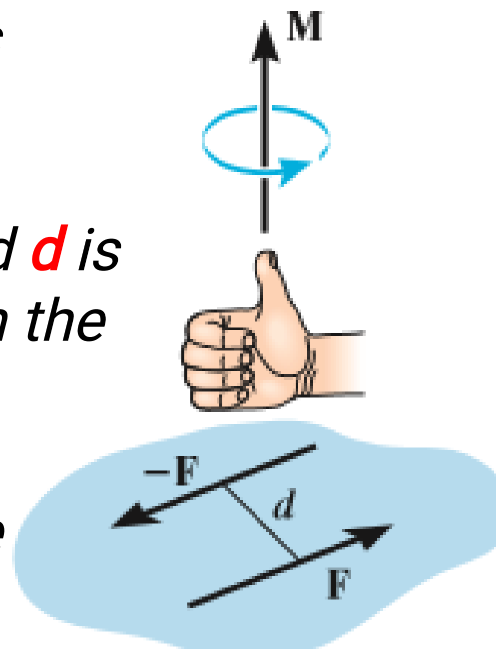


Fig. 4–27

- *where **F** is the magnitude of one of the forces and **d** is the perpendicular distance or moment arm between the forces.*
- *The direction and sense of the couple moment are determined by the right-hand rule.*
- *The thumb indicates this direction when the fingers are curled with the sense of rotation caused by the couple forces.*
- *In all cases, M will act perpendicular to the plane containing these forces.*

4.6 Moment of a Couple

Vector Formation: Magnitude and Direction by Using Cross Product of F and d

The moment of a couple can also be expressed by the vector cross product using Eq. 4–13, i.e.,
$M = r \times F$ equation 4-15

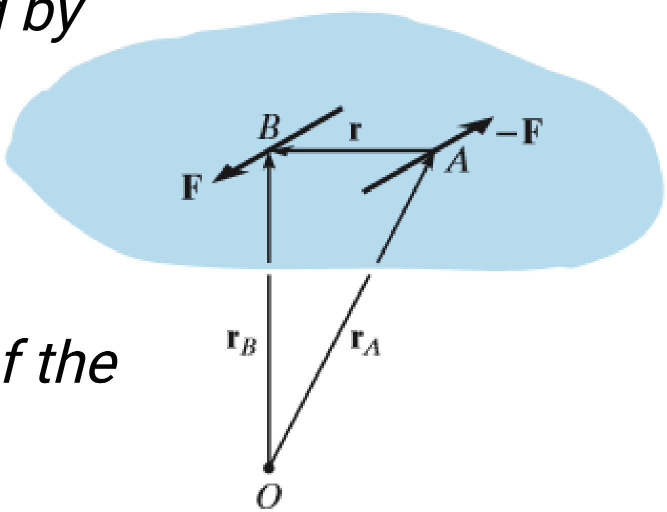


Fig. 4–26

- Taking the moments of both forces F and −F about a point lying on the line of action of one of the forces.

For example, if moments are taken about point A in Fig. 4–26,

- The moment of -F is zero about this point, and the moment of F is defined from Eq. 4–15 above

4.6 Moment of a Couple

*Therefore, in the vector formulation **r is crossed with the force F** to which it is directed.*

That means F is directed towards r, as in fig-4-26

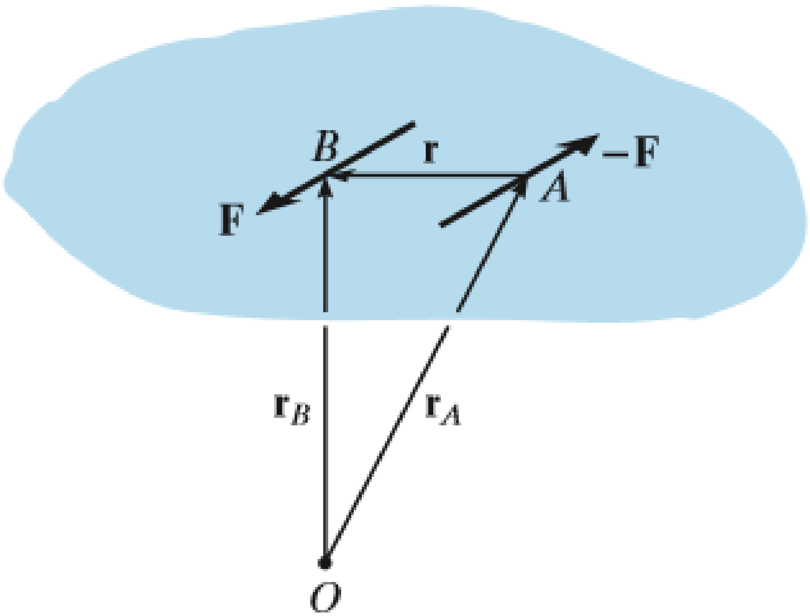
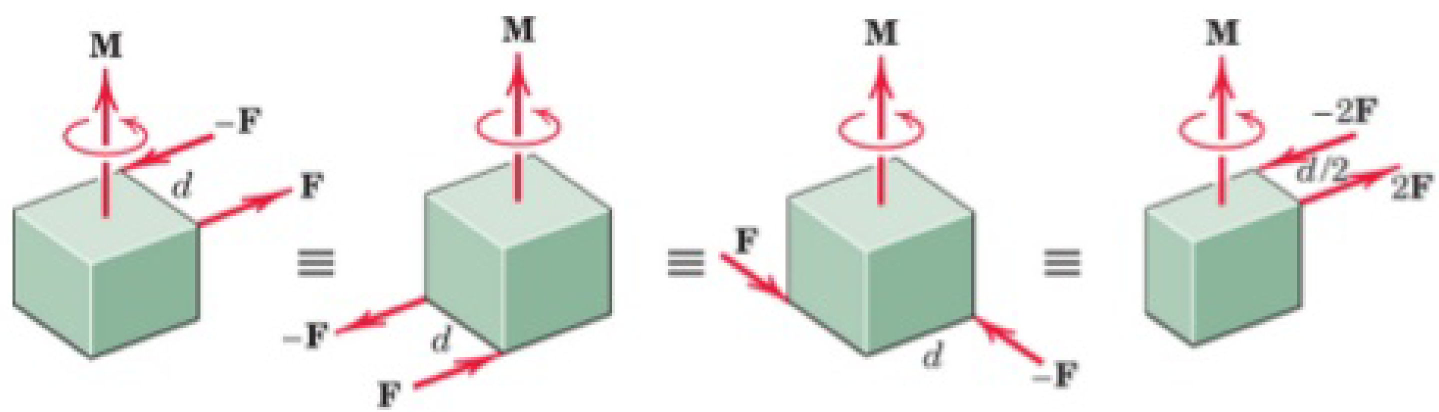


Fig. 4–26

4.6 Moment of a Couple: Examples

The two forces with different magnitudes are applied on different objects as shown in figure here below:

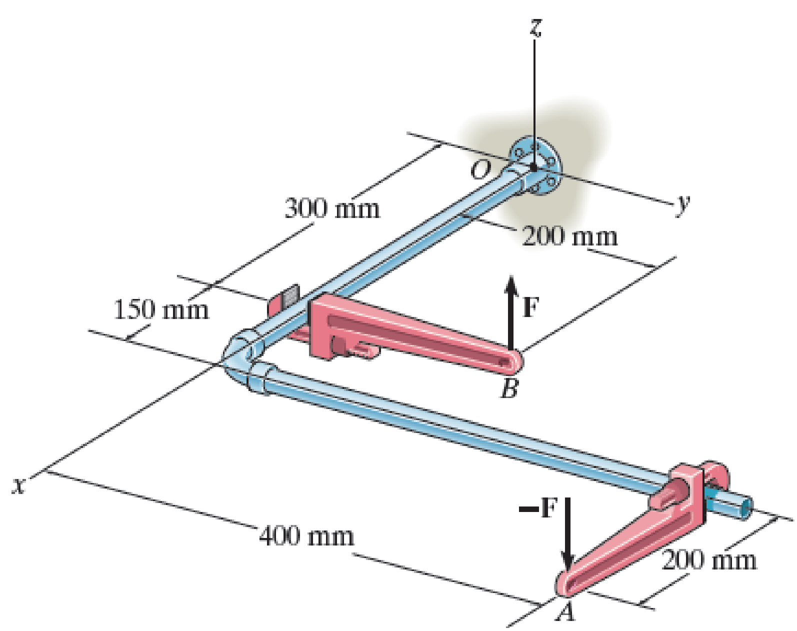
Examples:

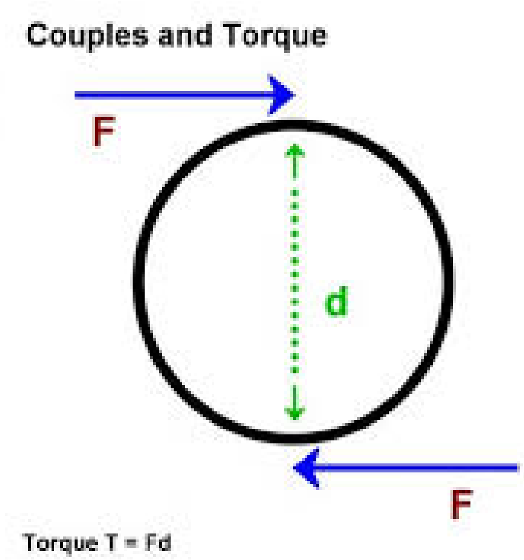


Internet resources

4.6 Moment of a Couple: Examples

The two forces with different magnitudes are applied on different objects as shown in figure here below:





Internet resources

Chapter-5

Equilibrium of a Rigid Body
and
Conditions for Rigid-Body Equilibrium

5.1 Conditions for Rigid Body Equilibrium

Rigid Body: A rigid body means, an object/body which is tough and difficult to deform/change

- *It means, that when we apply force on that object, there is hardly any change in the formation (molecules) of that body*

- *We can also say, that there is very small deformation/change in that object*

- *We can further say, that distance between two atoms/or two points do ot change when force is applied; so the object is RIGID (HARD)*

Chapter-5 Equilibrium of a Rigid Body

5.1 Conditions for Rigid Body Equilibrium

Rigid Body: A rigid body means, an object/body which is tough and difficult to deform/change

- *A rigid body is considered as a Continuous-distribution of mass*
- *Actually, in a very hard object, almost no mechanical energy is lost if you tap it*

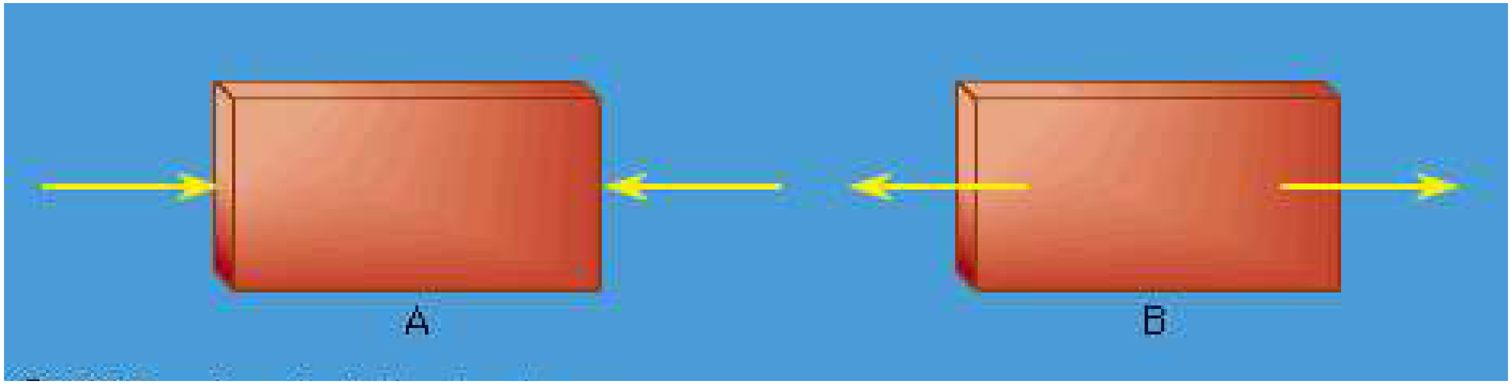
Examples:
A ball-bearing made of hardened steel
A perfect spring

Conditions of Equilibrium of Rigid Body:
We need to define some conditions for the equilibrium of a rigid-body, so that when forces are applied on that body, there is no change/movement

5.1 Conditions for Rigid Body Equilibrium

A body is formally regarded as rigid if the distance between any set of two points in it is always constant.

Equal and opposite forces acting on a rigid body may act so as to compress the body (Figure A) or to stretch it (Figure B).



5.1 Conditions for Rigid Body Equilibrium

Rigid Body:

Shown in Fig 5-1, a rigid body and there are different forces applied on it

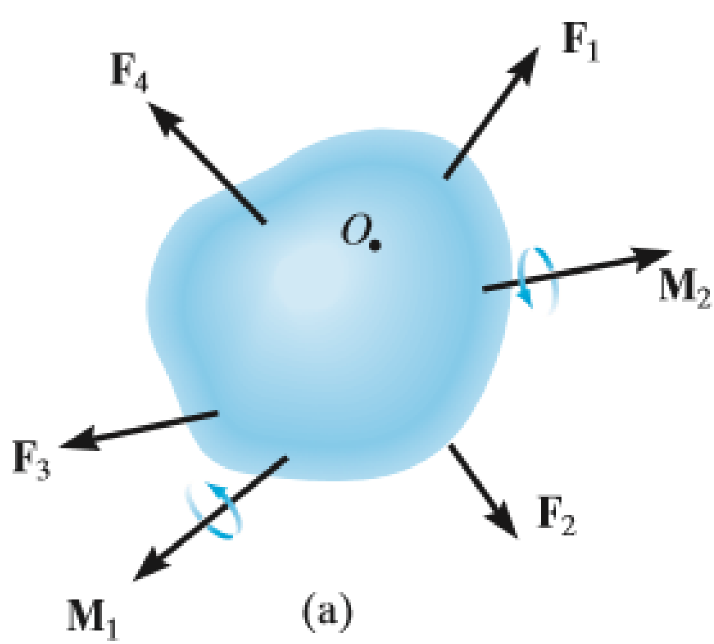


Fig. 5–1

5.1 Conditions for Rigid Body Equilibrium

Rigid Body:

We can also see in the Fig 5-1, some moments M1 and M2 are shown.

These moments are due to the forces, that are applied on the rigid body/object

Moreover, there is also some rotational effects on that object as can be seen by moment M1 and moment M2

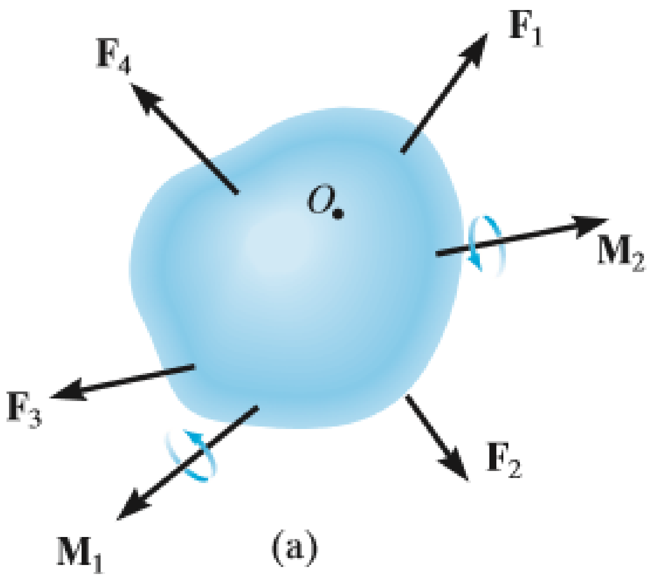


Fig. 5–1

5.1 Conditions for Rigid Body Equilibrium

External Forces and Internal Forces:

External Forces:

As shown in Fig 5-1, some forces are applied on the object, it is clear that these forces are externally applied

It means that these forces are applied on the surface of the rigid body/object; examples of such forces are:

- *Gravitational force*
- *Magnetic force*
- *Electrical-field (which is also a force E=F/Q)*

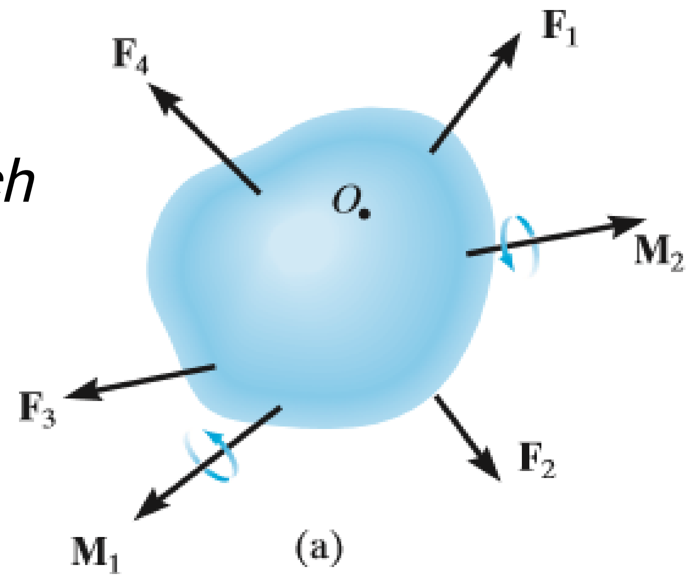


Fig. 5–1

5.1 Conditions for Rigid Body Equilibrium

External Forces and Internal Forces:

Internal Forces:

Some forces are applied internally on the object

It means that these forces are applied on the inner surfaces due to the interaction of particles/molecules/atoms, for example

- *Attractive force due to opposite atoms*
- *Repulsive force due to same atoms*

However, these internal forces are not shown in Fig-5-1

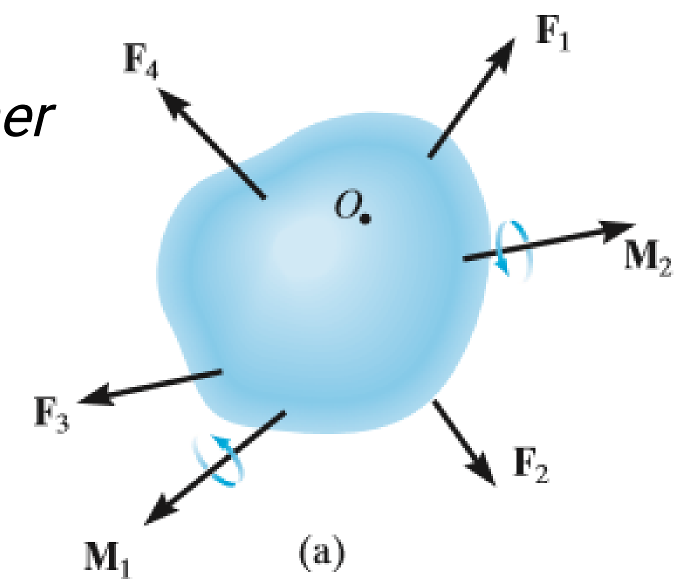


Fig. 5–1

Chapter-5 Equilibrium of a Rigid Body

5.1 Conditions for Rigid Body Equilibrium

5.1 Conditions for Rigid-Body Equilibrium

In this section, we will develop both the necessary and sufficient conditions for the equilibrium of the rigid body in Fig. 5–1*a*. As shown, this body is subjected to an external force and couple moment system that is the result of the effects of gravitational, electrical, magnetic, or contact forces caused by adjacent bodies. The internal forces caused by interactions between particles within the body are not shown in this figure because these forces occur in equal but opposite collinear pairs and hence will cancel out, a consequence of Newton's third law.

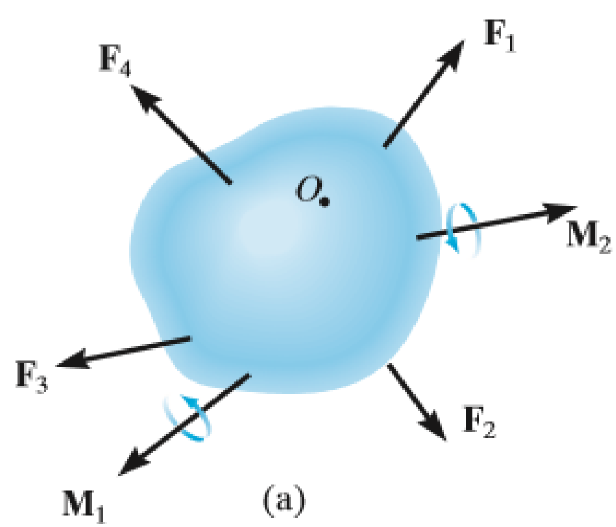


Fig. 5–1

5.1 Conditions for Rigid Body Equilibrium

Conditions for Equilibrium:

Similarly, based on our previous knowledge, we can say that, sum of all the moments that are observed on the object is also zero

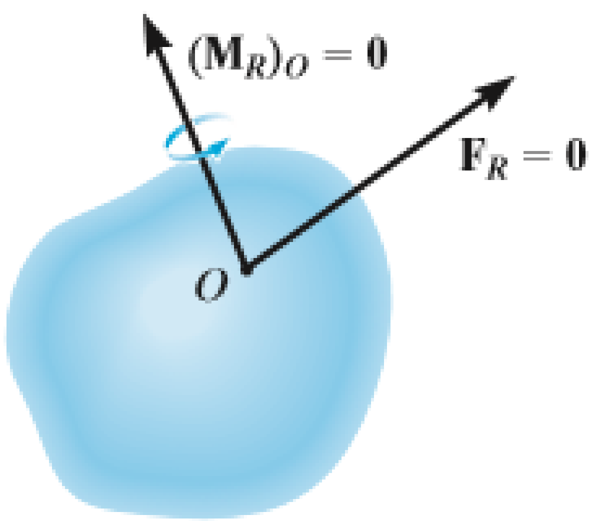
Hence, we can say that the object is in equilibrium since::

- *sum of all the forces that are acting externally is zero*
- *sum of all forces acting internally is also zero*
- *sum of all the moments is also zero*

Chapter-5 Equilibrium of a Rigid Body

5.1 Conditions for Rigid Body Equilibrium

Using the methods of the previous chapter, the force and couple moment system acting on a body can be reduced to an equivalent resultant force and resultant couple moment at any arbitrary point O on or off the body, Fig. 5–1b. If this resultant force and couple moment are both equal to zero, then the body is said to be in ***equilibrium***. Mathematically, the equilibrium of a body is expressed as



(b)

$$\mathbf{F}_R = \Sigma \mathbf{F} = \mathbf{0}$$
$$(\mathbf{M}_R)_O = \Sigma \mathbf{M}_O = \mathbf{0} \qquad (5\text{–}1)$$