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Dept:-

BSCS

Assignment#1

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⇒ Solve all questions.

Q:- Find rank of matrices.

i- $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & 0 \\ -2 & -1 & 3 \\ -1 & 4 & -2 \end{bmatrix}$

Solution:-

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & 0 \\ -2 & -1 & 3 \\ -1 & 4 & -2 \end{bmatrix} \Rightarrow \begin{matrix} x_1 + 2x_2 - 3x_3 \\ 2x_1 + x_2 \\ -2x_1 - x_2 + 3x_3 \\ -x_1 + 4x_2 - 2x_3 \end{matrix}$$

Rank A - ~~3~~ 4 > No of unknown
Solution is infinite

ii- $\begin{bmatrix} 1 & 3 & -2 & 5 & 4 \\ 1 & 4 & 1 & 3 & 5 \\ 1 & 4 & 2 & 4 & 3 \\ 1 & 7 & -3 & 6 & 13 \end{bmatrix}$

Solution:-

Rank A - 4 < No. of unknowns
Solution is infinite.

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Q2:- Use row reduction.

$$\Rightarrow \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} = (a-b)(b-c)(c-a)$$

Solution:-

$$\begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}$$

$R_2 - R_1,\ R_3 - R_1$

$$\begin{bmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{bmatrix}$$

Taking common $(b-a)(c-a)$

$$\begin{bmatrix} 1 & a & a^2 \\ 0 & 1 & b-a \\ 0 & 1 & c-a \end{bmatrix} (b-a)(c-a)$$

Now,

$$(b-a)(c-a)(c-a-b+a)$$
$$= -(a-b)(c-a)[-(b-c)$$
$$= +(a-b)(c-a)(b-c)$$

Ans:-

Q3:- Find the inverse of the matrix by Adjoint method.

$$\Rightarrow A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{bmatrix}$$

Solution:-

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$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{bmatrix}$$

$$3\begin{vmatrix} -1 & 8 \\ -2 & 7 \end{vmatrix} - 4\begin{vmatrix} 2 & 8 \\ 5 & 7 \end{vmatrix} + 5\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$

$3(-1\times7-(-2\times8)-4(2\times7-5\times8)+5(-4+5)$

$3(-7+16)-4(14-40)+5(1)$

$3(11)-4(-26)+5$

$33+104+5$

$|A| = 142$

Now,

$$AdjA \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$C_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 8 \\ -2 & 7 \end{vmatrix}$$

$= (1)(-7+16)$

$= 9$

$$C_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 8 \\ 5 & 7 \end{vmatrix}$$

$= -1(14-40)$

$= 26$

$$C_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$

$= (1)(-4+5)$

$= 1$

$$C_{21} = (-1)^{2+1}\begin{vmatrix} 4 & 5 \\ -2 & 7 \end{vmatrix}$$

$= -1(28+10)$

$= -38$

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$$C_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix}$$

$$= 1(21-25)$$

$$= -4$$

$$C_{23} = (-1)^{2+3} \begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix}$$

$$= -1(-6-20)$$

$$= 26$$

$$C_{31} = (-1)^{3+1} \begin{vmatrix} 4 & 5 \\ -1 & 8 \end{vmatrix}$$

$$= 1(32+5)$$

$$= 37$$

$$C_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix}$$

$$= -1(24-10)$$

$$= -14$$

$$C_{33} = (-1)^{3+3} \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix}$$

$$= (-1)^6 (-3-8)$$

$$= -11$$

$$\text{Adj}A = \begin{bmatrix} 9 & 26 & 1 \\ -38 & -4 & 26 \\ 37 & -14 & -11 \end{bmatrix}$$

For transpose,

$$\begin{bmatrix} 9 & -38 & 37 \\ 26 & -4 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$

Then,

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For AdjA

$$= \frac{1}{142}\begin{bmatrix} 9 & -32 & 37 \\ 26 & -4 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$

$$= \begin{bmatrix} 9/142 & -32/142 & 37/142 \\ 26/142 & -4/142 & -14/142 \\ 1/142 & 26/142 & -11/142 \end{bmatrix}$$

Ans:-

Q4- Solve system of linear equations, the field of scalar being R.

$\Rightarrow$ $x_1 - 2x_2 - 7x_3 + 7x_4 = 5$

$-x_1 + 2x_2 + 8x_3 - 5x_4 = -7$

$3x_1 - 4x_2 - 17x_3 + 13x_4 = 14$

$2x_1 - 2x_2 - 11x_3 + 8x_4 = 7$

Solution:-

$$\begin{array}{c} \left[\begin{array}{cccc|c} 1 & -2 & -7 & 7 & 5 \\ -1 & 2 & 8 & -5 & -7 \\ 3 & -4 & -17 & 13 & 14 \\ 2 & -2 & -11 & 8 & 7 \end{array}\right] \end{array} \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$R_2 + R_1$, $R_3 - 3R_1$, $R_4 - 2R_1$

$$\left[\begin{array}{cccc|c} 1 & -2 & -7 & 7 & 5 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 2 & 4 & 8 & -1 \\ 0 & 2 & 3 & -6 & 3 \end{array}\right] \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$R_2 \longleftrightarrow R_3$

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$$\left[\begin{array}{cccc|c} 1 & -2 & -7 & 7 & 5 \\ 0 & 2 & 4 & 8 & -1 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 2 & 3 & -6 & 3 \end{array}\right] \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$1/2 R_2$

$$\left[\begin{array}{cccc|c} 1 & -2 & -7 & 7 & 5 \\ 0 & 1 & 2 & 4 & -1/2 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 2 & 3 & -6 & 3 \end{array}\right] \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$R_1 + 2R_2,\ R_4 - 2R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & -3 & 15 & 4 \\ 0 & 1 & 2 & 4 & -1/2 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & -1 & -14 & 2 \end{array}\right] \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

Now, $R_2 - 2R_3,\ R_1 + 3R_3,\ R_4 + R_3$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 21 & -2 \\ 0 & 1 & 0 & 0 & 7/2 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & -12 & 0 \end{array}\right] \begin{array}{c} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$\text{Rank}A \neq \text{Rank}AB$

Solution is infinite

Q5:- Find the value of λ .... solution.

$\Rightarrow (1-\lambda)x_1 + x_2 - x_3 = 0$

$x_1 - \lambda x_2 - 2x_3 = 0$

$x_1 + 2x_2 - \lambda x_3 = 0$

Solution:-

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For Augmented Matrix

$$\left[\begin{array}{ccc|c} 1-\lambda & 1 & -1 & 0 \\ 1 & -\lambda & -2 & 0 \\ 1 & 2 & -\lambda & 0 \end{array}\right]$$

Now, let, let $\lambda = 0$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & 0 & -2 & 0 \\ 1 & 2 & 0 & 0 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$R_2 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 1 & 2 & 0 & 0 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$R_3 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$R_3 + R_2$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

Here,

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

RankA = 2 < No of unknowns (3)

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So, system is non-trivial

$x_1 + x_2 - x_3 = 0$ ——— 1

$-x_2 - x_3 = 0$ ——— 2

Let $x_3 = t$

$-x_2 = t$

$x_2 = -t$

From 1

$x_1 = -x_2 + x_3$

$x_1 = t + t$

$x_1 = 2t$

Then,

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2t \\ -t \\ t \end{bmatrix} \Rightarrow t \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Here, $t \neq 0$

Ans:-

Q:- Use cramer's rule to solve a linear system.

$\Rightarrow x_1 + \quad + 2x_3 = 6$

$-3x_1 + 4x_2 + 6x_3 = 30$

$-x_1 - 2x_2 + 3x_3 = 8$

Solution:-

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 6 \\ -3 & 4 & 6 & 30 \\ -1 & -2 & 3 & 8 \end{array}\right]$$

Here, $\begin{bmatrix} 6 \\ 30 \\ 8 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

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Now,

$$\begin{bmatrix} 1 & 0 & 2 \\ -3 & 4 & 6 \\ -1 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow 1\begin{vmatrix} 4 & 6 \\ -2 & 3 \end{vmatrix} - 0 + 2\begin{vmatrix} -3 & 4 \\ -1 & -2 \end{vmatrix}$$

$1(12+12) - 0 + 2(6+4)$

$24 + 20 \Rightarrow 44$

$|A| = 44$

Then,

$$\begin{bmatrix} 6 & 0 & 2 \\ 30 & 4 & 6 \\ 8 & -2 & 3 \end{bmatrix}$$

$$6\begin{vmatrix} 4 & 6 \\ -2 & 3 \end{vmatrix} - 0 + 2\begin{vmatrix} 30 & 4 \\ 8 & -2 \end{vmatrix}$$

$= 6(12+12) - 0 + 2(-60-32)$

$= 6(24) + 2(92)$

$= 328$

Now,

$$\begin{bmatrix} 1 & 6 & 2 \\ -3 & 30 & 6 \\ -1 & 8 & 3 \end{bmatrix}$$

$$= 1\begin{vmatrix} 30 & 6 \\ 8 & 3 \end{vmatrix} - 6\begin{vmatrix} -3 & 6 \\ -1 & 3 \end{vmatrix} + 2\begin{vmatrix} -3 & 30 \\ -1 & 8 \end{vmatrix}$$

$= 1(30\times3 - 8\times6) - 6(-3\times3 + 6\times1) + 2(-24+30)$

$= 1(90-48) - 6(-9+6) + 2(6)$

$= 42 + 18 + 12$

$= 72$

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$$= \begin{bmatrix} -1 & 0 & 6 \\ -3 & 4 & 30 \\ -1 & -2 & 8 \end{bmatrix}$$

$$-1\begin{vmatrix} 4 & 30 \\ -2 & 8 \end{vmatrix} - 0\begin{bmatrix} 0 \end{bmatrix} + 6\begin{vmatrix} -3 & 4 \\ -1 & -2 \end{vmatrix}$$

$= -1(32+60) + 6(+6+4)$

$= -92 + 60$

$= -32$

Now,

$x_1 = \frac{328}{44}$, $x_2 = \frac{72}{44}$, $x_3 = \frac{-32}{44}$

$x_1 = \frac{164}{22} = \frac{82}{11}$, $x_2 = \frac{36}{22} = \frac{18}{11}$

$x_3 = \frac{16}{22} \Rightarrow \frac{8}{11}$

Here,

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 82/11 \\ 18/11 \\ 8/11 \end{bmatrix}$$

Ans:-

Q7:- Show that det(A)=0, without directly evaluating the determinant.

$$A = \begin{bmatrix} -2 & 8 & 1 & 4 \\ 3 & 2 & 5 & 1 \\ 1 & 10 & 6 & 5 \\ 4 & -6 & 4 & -3 \end{bmatrix}$$

Solution:-

Here

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$$\begin{array}{cccc} C_1 & C_2 & C_3 & C_4 \end{array}$$
$$\begin{bmatrix} -2 & 8 & 1 & 4 \\ 3 & 2 & 5 & 1 \\ 1 & 10 & 6 & 5 \\ 4 & -6 & 4 & -3 \end{bmatrix}$$

$\| C_2 - 2C_4 \|$

$C_2 = 2C_4$

$$\begin{bmatrix} 8 \\ 2 \\ 10 \\ -6 \end{bmatrix} = 2 \begin{bmatrix} 4 \\ 1 \\ 5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} 8 \\ 2 \\ 10 \\ -6 \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ 10 \\ -6 \end{bmatrix}$$

Hence, second column vector is a scalar multiple of the forth column vector. Ans.

Q8:- Find minor and cofactor.
Find M_{24} and C_{24}
Find M_{41} and C_{41}

$\Rightarrow$ $A = \begin{bmatrix} 2 & 3 & -1 & 1 \\ -3 & 2 & 0 & 3 \\ 3 & -2 & 1 & 0 \\ 3 & -2 & 1 & 4 \end{bmatrix}$

Solution:-

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$$\begin{bmatrix} 2 & 3 & -1 & 1 \\ -3 & 2 & 0 & 3 \\ 3 & -2 & 1 & 0 \\ 3 & 2 & 1 & 4 \end{bmatrix}$$

for M_{24} and C_{24}.

$$M_{24} = \begin{bmatrix} 2 & 2 & -1 \\ 3 & -2 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

$$2\begin{vmatrix} -2 & 1 \\ 2 & 1 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ 3 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 3 & -2 \\ 3 & 2 \end{vmatrix}$$

$= 2(-2-2) - 2(3-3) + (-1)(6+6)$

$= 2(-4) - 0 - (12)$

$= -8 - 12 \rightarrow -20$

Now,

$C_{ij} = (-1)^{2+4} - 20$

$= -20$

for M_{41} and C_{41}.

$$M_{41} = \begin{bmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ -2 & 1 & 0 \end{bmatrix}$$

$$= 3\begin{vmatrix} 0 & 3 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} - 1\begin{vmatrix} 2 & 0 \\ 2 & 1 \end{vmatrix}$$

$= 3(0) - 1(0+6) - 1(2+0)$

$= 0 - 6 - 2$

$= -8$

Then,

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$C_{ij} = (-1)^{4+1} - 8$

$= (-1) - 8 \Rightarrow 8$

Ans:-

Q1:- Solve the linear - - - - matrix.

a- $-x_1 + 4x_2 + x = b_1$

$x_1 + 9x_2 - 2x_3 = b_2$

$6x_1 + 4x_2 - 8x_3 = b_3$

Here $b_1 = 0$, $b_2 = 1$, $b_3 = 0$

Solution:-

$-x_1 + 4x_2 + x = 0$

$x_1 + 9x_2 - 2x_3 = 1$

$6x_1 + 4x_2 - 8x_3 = 0$

$$\left[\begin{array}{ccc|c} -1 & 4 & 1 & 0 \\ 1 & 9 & -2 & 1 \\ 6 & 4 & -8 & 0 \end{array}\right]$$

$R_1 \longleftrightarrow R_2$

$$\left[\begin{array}{ccc|c} 1 & 9 & -2 & 1 \\ -1 & 4 & 1 & 0 \\ 6 & 4 & -8 & 0 \end{array}\right]$$

$R_2 + R_1$, $R_3 - 6R_1$

$$\begin{array}{c|c} \left[\begin{array}{ccc|c} 1 & 9 & -2 & 1 \\ 0 & 13 & -1 & 1 \\ 0 & -50 & 4 & -6 \end{array}\right] & \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \end{array}$$

$1/13 R_2$

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$$\left[\begin{array}{ccc|c} 1 & 9 & -2 & 1 \\ 0 & 1 & -1/13 & 1/13 \\ 0 & -50 & 4 & -6 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$R_1 - 9R_2, \quad R_3 + 50R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3/5 & 4/13 \\ 0 & 1 & -1/13 & 1/13 \\ 0 & 0 & 2/13 & 128/13 \end{array}\right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$x_1 + (-3/5\,x_3) = 4/13$ —— 1

$x_2 - 1/13\,x_3 = 1/13$ —— 2

$2/13\,x_3 = 128/13$ —— 3

$$x_3 = \frac{122}{3} \times \frac{2}{13}$$

$$x_3 = \frac{1664}{6} = \frac{832}{3}$$

From (2)

$$x_2 - \frac{1}{13}\left(\frac{832}{3}\right) = \frac{1}{13}$$

$$\cancel{x_2 = \frac{832}{39} = \frac{1}{13}} \qquad x_2 - \frac{64}{3} = \frac{1}{13}$$

$$\cancel{x_2 = \frac{39}{\ }} \qquad x_2 = \frac{1}{13} + \frac{64}{3}$$

$$x_2 = \frac{835}{39}$$

From eq-(1)

$$x_1 - \frac{\cancel{3}}{5}\left(\frac{832}{\cancel{3}}\right) = \frac{4}{13}$$

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$$x_1 = \frac{4}{13} + \frac{232}{5}$$

$$x_1 = 236$$

Here,

$x_1 = 236$, $x_2 = \frac{235}{39}$, $x_3 = \frac{232}{3}$

b. $-x_1 + 4x_2 + x_3 = -3$

$x_1 + 9x_2 - 2x_3 = 4$

$6x_1 + 4x_2 - 8x_3 = -5$

Solution:-

$$\begin{array}{c} \left[\begin{array}{ccc:c} -1 & 4 & 1 & -3 \\ 1 & 9 & -2 & 4 \\ 6 & 4 & -8 & -5 \end{array}\right] \end{array} \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

$R_1 \longleftrightarrow R_2$

$$\left[\begin{array}{ccc:c} 1 & 9 & -2 & 4 \\ -1 & 4 & 1 & -3 \\ 6 & 4 & -8 & -5 \end{array}\right]$$

$R_2 + R_1$, $R_3 - 6R_1$

$$\left[\begin{array}{ccc:c} 1 & 9 & -2 & 4 \\ 0 & 13 & -1 & 1 \\ 0 & -50 & 4 & -19 \end{array}\right] \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

$1/13\, R_2$

$$\left[\begin{array}{ccc:c} 1 & 9 & -2 & 4 \\ 0 & 1 & -1/13 & 1/13 \\ 0 & -50 & 4 & -19 \end{array}\right] \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

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$R_1 - 9R_2$, $R_3 + 50R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3/5 & 43/13 \\ 0 & 1 & -1/13 & 1/13 \\ 0 & 0 & 2/13 & -197/13 \end{array}\right]$$

$$x_1 - 3/5 x_3 = \frac{43}{13} \quad \text{--- } 1$$

$$x_2 - 1/13 x_3 = \frac{1}{13} \quad \text{--- } 2$$

$$\frac{2}{\cancel{13}} x_3 = \frac{-197}{\cancel{13}} \quad \text{--- } 3$$

$$x_3 = \frac{-197}{2}$$

From 2

$$x_2 - \frac{1}{13}\left(\frac{-197}{2}\right) - \frac{1}{13}$$

$$x_2 = \frac{1}{13} + \frac{197}{26}$$

$$x_2 = \frac{199}{26}$$

From 1

$$x_1 - \frac{3}{5}\left(\frac{-197}{2}\right) = \frac{43}{13}$$

$$x_1 + \frac{591}{10} = \frac{43}{13}$$

$$x_1 = \frac{43}{13} - \frac{591}{10}$$

$$x_1 = \frac{7253}{130}$$

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$x_1 = \frac{7253}{130}$, $x_2 = \frac{199}{26}$, $x_3 = -\frac{197}{2}$

Q20:- Show that matrix is idempotent.

$$A = \begin{bmatrix} 2 & -2 & 4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$

Solution:-