③ "Linear Independence"

1- $\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}7\\9\\-2\end{bmatrix}, \begin{bmatrix}4\\9\\5\end{bmatrix}$

Solution:-

$$C_1\begin{bmatrix}1\\2\\3\end{bmatrix} + C_2\begin{bmatrix}7\\9\\-2\end{bmatrix} + C_3\begin{bmatrix}4\\9\\5\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$C_1 + 7C_2 + 4C_3 = 0$ ——— 1

$2C_1 + 9C_2 + 9C_3 = 0$ ——— 2

$3C_1 + (-2C_2) + 5C_3 = 0$ ——— 3

Now,

× eq ① with "2"

$2C_1 + 7C_2 + 4C_3$

Now, sub with eq ②

$2C_1 + 14C_2 + 8C_3 = 0$

$2C_1 + 9C_2 + 9C_3 = 0$

$5C_2 - C_3 = 0$

$C_2 = \frac{C_3}{5}$ ——— ④

Then,

× eq ① with "3"

$3C_1 + 21C_2 + 12C_3 = 0$

Sub with eq ③

$3C_1 + 21C_2 + 12C_3 = 0$

$3C_1 - 2C_2 + 5C_3 = 0$

$23C_2 + 7C_3 = 0$ ——— ⑤

From eq (5) put C_2

$$23C_2 + 7C_3 = 0$$

$$23\left(\frac{C_3}{5}\right) + 7C_3 = 0$$

$$\frac{23C_3 + 35C_3}{5} = 0$$

$$\frac{58C_3}{5} = 0$$

$$C_3 = 0$$

put value of C_3 in eq (4)

$$C_2 = \frac{0}{5}$$

$$C_2 = 5$$

put C_2 and C_3 in eq (1)

$$C_1 + 0 + 0 = 0$$

$$C_1 = 0$$

So, this function linear independent.

③ $\begin{bmatrix} 1 \\ -2 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} 1 \\ -6 \\ 1 \\ 4 \end{bmatrix}$

Solution:-

$C_1 + 2C_2 + C_3 = 0$ ——— 1

$-2C_1 + C_2 - 6C_3 = 0$ ——— 2

$4C_1 + 0 + C_3 = 0$ ——— 3

$C_1 - 3C_2 + 4C_3 = 0$ ——— 4

From eq ① x with "2"

$2(C_1 + 2C_2 + C_3) = 0$

$2C_1 + 4C_2 + 2C_3$

sub with eq ②

$2C_1 + 4C_2 + 2C_3 = 0$

$-2C_1 + C_2 - 6C_3 = 0$

$5C_2 - 4C_3 = 0$ ——— ⑤

From eq ① x with "4" and sub with ③

$4C_1 + 8C_2 + 4C_3 = 0$

$\pm 4C_1 + 0 \pm C_3 = 0$

$8C_2 + 3C_3 = 0$ ——— ⑥

$C_2 = \frac{-3C_3}{8}$

put in eq ⑤

$5\left(\frac{-3C_3}{8}\right) - 4C_3 = 0$

$\frac{-47C_3}{8} = 0 \Rightarrow C_3 = 0$

Put $c_3 = 0$ in eq (5)

$$5c_2 - 4(0) = 0$$
$$5c_2 = 0$$
$$c_2 = 0$$

Put $c_2 = 0$ and $c_3 = 0$ in eq (1)

$$c_1 + 0 + 0 = 0$$
$$c_1 = 0$$ Ans:-

3- $\begin{bmatrix}1\\2\\3\end{bmatrix}, \begin{bmatrix}2\\5\\7\end{bmatrix}, \begin{bmatrix}4\\3\\5\end{bmatrix}$

Solution:-

$$c_1 + 2c_2 + c_3 = 0 \quad \text{---- 1}$$
$$2c_1 + 5c_2 + 3c_3 = 0 \quad \text{---- 2}$$
$$3c_1 + 7c_2 + 5c_3 = 0 \quad \text{---- 3}$$

× eq (1) with (2) and sub with eq (2)

$$2c_1 + 4c_2 + 2c_3$$
$$-2c_1 + 5c_2 + 3c_3$$
$$-c_2 - c_3 = 0$$
$$-c_2 = c_3 \quad \text{---- (4)}$$

× eq 1 with (3) and sub with eq (3)

$$3c_1 + 6c_2 + 3c_3$$
$$+3c_1 + 7c_2 + 5c_3$$
$$-c_2 - 2c_3 = 0 \quad \text{---- (5)}$$

$-C_2 = -2C_3$

put eq (4) in eq (5)

$C_3 - 2C_3 = 0$

$-C_3 = 0$

$C_3 = 0$

put in eq (4)

$-C_2 = 0$

$C_2 = 0$

put $C_2 = 0$ and $C_3 = 0$ in eq (1)

$C_1 + 0 + 0 = 0$

$C_1 = 0$ Ans:-

Linear dependent

4- (3, 0, -3) (-1, 1, 2), (4, 2, -2) and (2, 1, 1)

Solution:-

$C_1(3, 0, -3), C_2(-1, 1, 2), C_3(4, 2, -2)$ $C_4(2, 1, 1)$

$3C_1 + (-C_2) + 4C_3 + 2C_4 = 0$

$0 + C_2 + 2C_3 + C_4 = 0$

$-3C_1 + 2C_2 - 2C_3 + C_4 = 0$

$3C_1 - C_2 + 4C_3 + 2C_4 = 0$ — 1

$C_2 + 2C_3 + C_4 = 0$ — 2

$-3C_1 + 2C_2 - 2C_3 + C_4 = 0$ — 3

sub [redacted] ② and ③

[redacted]

$$-3C_1 + 2C_2 - 2C_3 + C_4 = 0$$
$$+C_2 + 2C_3 + C_4 = 0$$
$$-3C_1 + C_2 - 4C_3 = 0 \quad \text{---} (4)$$

Now add ① and ④

$$3C_1 - C_2 + 4C_3 + 2C_4 = 0$$
$$-3C_1 + C_2 - 4C_3 = 0$$
$$2C_4 = 0$$
$$C_4 = 0$$

From eq ②

$$C_2 + 2C_3 + C_4 = 0$$
$$C_2 + 2C_3 + 0 = 0$$
$$C_2 + 2C_3 = 0$$
$$C_2 = -2C_3$$

Now from eq ④

$$-3C_1 - 2C_3 - 4C_3 = 0$$
$$-3C_1 - 6C_3 = 0$$
$$-3C_1 = +6C_3$$
$$C_1 = -2C_3$$

Here,

$$C_1 = C_2 = -2C_3, \; d = 0$$

So,

$= -2(3, 0, -3) - 2(-1, 1, 2) + 1(4, 2, -2)$
$+ 0(2, 1, 1)$

$= (-6 - 0 + 6) + (+2 - 2 - 2) + (4 + 2 - 2) + 0$
$(0, 0, 0, 0)$

Linear dependent.