

Exercise# 1-2

"Gaussian Elimination" Q15 to Q17

=> Solve the ---- method.

$$15 - 2u_1 + u_2 + 3u_3 = 0$$

$$u_1 + 2u_2 = 0$$

$$u_2 + u_3 = 0$$

Solution:-

$$2u_1 + u_2 + 3u_3 = 0 \quad \text{--- 1}$$

$$u_1 + 2u_2 + 0 = 0 \quad \text{--- 2}$$

$$0 + u_2 + u_3 = 0 \quad \text{--- 3}$$

$$= \left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$R_1 - 2R_2$

$$\begin{array}{ccc} -2 & (1 & 2 & 0) \\ -2 & -4 & 0 \\ \hline 2 & 1 & 3 \\ \hline 0 & -3 & 3 \end{array}$$

Then,

$$\left[\begin{array}{ccc|c} 0 & -3 & 3 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

add 3 in (R_1)

$$\left[\begin{array}{ccc|c} 0 & 0 & 6 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$6u_3 = 0 \rightarrow (A)$$

Then, From eq (3)

$$u_2 + u_3 = 0$$

$$\text{Let } u_3 = 0$$

$$u_2 + 0 = 0$$

$$u_2 = 0$$

From eq (2)

$$2u_1 + u_2 + 3u_3 = 0$$

$$\text{put } u_2 = 0 \text{ and } u_3 = 0$$

$$2u_1 + 0 + 3(0) = 0$$

$$2u_1 = 0$$

$$u_1 = 0$$

Here, $u_1 = 0, u_2 = 0, u_3 = 0$ Ans:-

$$16. \quad 2x - y - 3z = 0$$

$$-x + 2y - 3z = 0$$

$$x + y + 4z = 0$$

Solution:-

$$\left[\begin{array}{ccc|c} 2 & -1 & -3 & 0 \\ -1 & 2 & -3 & 0 \\ 1 & 1 & 4 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$= R_1/2$$

$$\begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -0.5 & -1.5 & 0 \\ -1 & 2 & -3 & 0 \\ 1 & 1 & 4 & 0 \end{array} \right]$$

$$R_2 + R_1$$

$$\left[\begin{array}{ccc|c} 1 & -0.5 & -1.5 & 0 \\ 0 & 1.5 & -4.5 & 0 \\ 1 & 1 & 4 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$= R_3 - R_1$$

$$\left[\begin{array}{ccc|c} 1 & -0.5 & -1.5 & 0 \\ 0 & 1.5 & -4.5 & 0 \\ 0 & -1.5 & -5.5 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$= R_2 / 1.5$$

$$\left[\begin{array}{ccc|c} 1 & -0.5 & -1.5 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & -1.5 & -5.5 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$R_3 + (1.5)R_2$$

$$\left[\begin{array}{ccc|c} 1 & -0.5 & -1.5 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & -10 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$R_1 + (0.5)R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & -10 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$$R_3 / -10$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

Now, $R_1 + 3R_3$

$$\begin{array}{ccc|c} 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 1 & 0 & -3 & 0 \\ \hline 1 & 0 & 0 & 0 \end{array}$$

Then,

$R_2 + 3R_3$

$$\begin{array}{ccc|c} 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 1 & -3 & 0 \\ \hline 0 & 1 & 0 & 0 \end{array}$$

Here,

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & R_1 \\ 0 & 1 & 0 & R_2 \\ 0 & 0 & 1 & R_3 \end{array} \right]$$

Here, $x=0, y=0, z=0$

Ans.

$$17 - 3x_1 + x_2 + x_3 + x_4 = 0$$

$$5x_1 - x_2 + x_3 - x_4 = 0$$

Solution:-

$$= \begin{bmatrix} 3 & 1 & 1 & 1 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \end{matrix}$$

$R_1 \div 3$

$$= \begin{bmatrix} 1 & 1/3 & 1/3 & 1/3 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{bmatrix}$$

Then, $R_2 - 5R_1$

$$= \begin{bmatrix} 1 & 1/3 & 1/3 & 1/3 & 0 \\ 0 & -8/3 & -2/3 & -8/3 & 0 \end{bmatrix}$$

Multiply R_2 with $-3/8$

$$= \begin{bmatrix} 1 & 1/3 & 1/3 & 1/3 & 0 \\ 0 & 1 & 1/4 & 1 & 0 \end{bmatrix}$$

$$n_1 + \frac{1}{3}n_2 + \frac{1}{3}n_3 + \frac{1}{3}n_4 = 0 \quad -1$$

$$n_2 + \frac{1}{4}n_3 + n_4 = 0 \quad -2$$

Let $n_3 = s$, $n_4 = t$

$$n_2 + \frac{1}{4}s + t = 0$$

$$n_2 = -\frac{1}{4}s - t$$

put x_2, x_3 and x_4 in eq. (1)

$$x_1 + \frac{-15-t}{3} + \frac{15+t}{3} + \frac{1+t}{3} = 0$$

$$x_1 - \frac{15}{12} - \frac{1}{3}t + \frac{15}{3} + \frac{1}{3}t = 0$$

$$x_1 - \frac{15 + 45}{12} = 0$$

$$x_1 + \frac{15}{4} = 0$$

$$x_1 = -\frac{15}{4} \quad \text{Ans: -}$$

18- $v + 3w - 2u = 0$
 $2u + v - 4w + 3u = 0$
 $2u + 3v + 2w - u = 0$
 $-4u - 3v + 5w - 4u = 0$

Solution:-

$$\left[\begin{array}{cccc|c} 0 & 1 & 3 & -2 & 0 \\ 2 & 1 & -4 & 3 & 0 \\ 2 & 3 & 2 & -1 & 0 \\ -4 & -3 & 5 & -4 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$R_1 \leftrightarrow R_2$

$$\left[\begin{array}{cccc|c} 2 & 1 & -4 & 3 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 2 & 3 & 2 & -1 & 0 \\ -4 & -3 & 5 & -4 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$\Rightarrow \frac{1}{2} R_1$

$$\left[\begin{array}{cccc|c} 1 & 1/2 & -2 & 3/2 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 2 & 3 & 2 & -1 & 0 \\ -4 & -3 & 5 & -4 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$R_3 - 2R_1$

$$\left[\begin{array}{cccc|c} -2 & 1 & 1/2 & -2 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ -4 & -3 & 5 & -4 & 0 \end{array} \right]$$

$$\begin{array}{rrrr} -2 & -1 & 4 & -3 \\ 2 & 3 & 2 & -1 \\ \hline 0 & 2 & 6 & -4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 1/2 & -2 & 3/2 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 2 & 6 & -4 & 0 \\ -4 & -3 & 5 & -4 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$$R_4 + 4R_1$$

$$\begin{array}{rrrr} +4(1 & 1/2 & -2 & 3/2) \\ +4 & +2 & -8 & +6 \\ \hline -4 & -3 & 5 & -4 \\ \hline 0 & -1 & -3 & 2 \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 1/2 & -2 & 3/2 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 2 & 6 & -4 & 0 \\ 0 & -1 & -3 & 2 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$$= R_1 - \frac{R_2}{2}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & -7/2 & 5/2 & 0 \\ 0 & 1 & 3 & -2 & 0 \\ 0 & 2 & 6 & -4 & 0 \\ 0 & -1 & -3 & 2 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$$= R_3 - 2R_2$$

$$= \begin{bmatrix} 1 & 0 & -7/2 & 5/2 & 1 & 0 \\ 0 & 1 & 3 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & -3 & 2 & 1 & 0 \end{bmatrix}$$

$R_4 - R_2$

$$= \begin{bmatrix} 1 & 0 & -7/2 & 5/2 & 1 & 0 \\ 0 & 1 & 3 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$u - 7/2w + 5/2v = 0 \quad \text{--- 1}$$

$$v + 3w - 2u = 0 \quad \text{--- 2}$$

Let $u = t$, $w = s$

From (2)

$$v + 3s - 2t = 0$$

$$v = -3s + 2t$$

From (1)

$$u - 7/2s + 5/2t = 0$$

$$u = 7/2s - 5/2t$$

Ans:-

$$19- \quad 2x + 2y + 4z = 0$$

$$w \quad -y - 3z = 0$$

$$2w + 3x + y + 2 = 0$$

$$-2w + x + 3y - 2z = 0$$

Solution:-

$$\left[\begin{array}{cccc|c} 0 & 2 & 2 & 4 & 1 & 0 \\ 1 & 0 & -1 & -3 & 1 & 0 \\ 2 & 3 & 1 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 1 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$$R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 1 & 0 \\ 0 & 2 & 2 & 4 & 1 & 0 \\ 2 & 3 & 1 & 1 & 1 & 0 \\ -2 & 1 & 3 & -2 & 1 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$$R_2 - 2R_1$$

$$-2 \left(\begin{array}{cccc} 1 & 0 & -2 & -3 \end{array} \right)$$

$$\begin{array}{cccc} -2 & 0 & 4 & 6 \end{array}$$

$$\begin{array}{cccc} 2 & 3 & 1 & 1 \end{array}$$

$$\begin{array}{cccc} 0 & 3 & 5 & 7 \end{array}$$

$$\text{Then, } R_3 + 2R_1$$

$$2 \left(\begin{array}{cccc} 1 & 0 & -2 & -3 \end{array} \right)$$

$$\begin{array}{cccc} 2 & 0 & -2 & -6 \end{array}$$

$$\begin{array}{cccc} -2 & 1 & 3 & -2 \end{array}$$

$$\begin{array}{cccc} 0 & 1 & 1 & -8 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 2 & 2 & 4 & 0 \\ 0 & 3 & 5 & 7 & 0 \\ 0 & 1 & 1 & -8 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$$R_2 - R_4$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & -4 & 0 \\ 0 & 3 & 5 & 7 & 0 \\ 0 & 1 & 1 & -8 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$$R_3 - 3R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & -4 & 0 \\ 0 & 0 & -2 & 19 & 0 \\ 0 & 1 & 1 & -8 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$$R_4 - R_2$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & -4 & 0 \\ 0 & 0 & -2 & 19 & 0 \\ 0 & 0 & 0 & -4 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$$-1/2 R_3$$

$$\left[\begin{array}{cccc|c} 1 & 0 & -1 & -3 & 0 \\ 0 & 1 & 1 & -4 & 0 \\ 0 & 0 & 1 & 19/2 & 0 \\ 0 & 0 & 0 & -4 & 0 \end{array} \right] \begin{array}{l} R_1 \\ R_2 \\ R_3 \\ R_4 \end{array}$$

$$R_2 - R_3$$

$$\begin{bmatrix} 1 & 0 & -1 & -3 & 1 & 0 \\ 0 & 1 & 0 & -27/2 & 1 & 0 \\ 0 & 0 & 1 & 19/2 & 1 & 0 \\ 0 & 0 & 0 & -4 & 1 & 0 \end{bmatrix}$$

$$= R_1 + R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & 13/2 & 1 & 0 \\ 0 & 1 & 0 & -27/2 & 1 & 0 \\ 0 & 0 & 1 & 19/2 & 1 & 0 \\ 0 & 0 & 0 & -4 & 1 & 0 \end{bmatrix}$$

Then,

$$w + 13/2z = 0 \quad \text{--- 1}$$

$$x - 27/2z = 0 \quad \text{--- 2}$$

$$y + 19/2z = 0 \quad \text{--- 3}$$

$$-4z = 0$$

$$z = 0$$

put $z = 0$ in eq ①, ②, ③

$$w = 0$$

$$x = 0$$

$$y = 0$$

$$z = 0$$

Ans: -