EXERCISE 4

Solve the following systems of linear equations, the field of scalars being R:

1.
$$\begin{aligned} 2x_1 + x_3 &= 1 \\ 2x_1 + 4x_2 - x_3 &= -2 \\ x_1 - 8x_2 - 3x_3 &= 2 \end{aligned}$$

2.
$$\begin{aligned} x_1 + x_2 + x_3 &= a \\ x_1 + (1+a)x_2 + x_3 &= 2a \\ x_1 + x_2 + (1+a)x_3 &= 3a \end{aligned}$$

3.
$$\begin{aligned} x_1 - x_2 + x_3 - x_4 + x_5 &= 1 \\ 2x_1 + x_2 + 3x_3 + 4x_5 &= 3 \\ 3x_1 - 2x_2 + 2x_3 + x_4 + x_5 &= 1 \\ x_2 + x_4 + x_5 &= 0 \end{aligned}$$

4.
$$\begin{aligned} x_1 + x_2 - x_3 &= 1 \\ x_2 + x_3 - x_4 &= 1 \\ x_3 + x_4 - x_5 &= 1 \\ -x_3 + x_4 + x_5 &= 1 \\ -x_2 + x_3 + x_4 &= 1 \end{aligned}$$

5.
$$\begin{aligned} x_1 - 2x_2 - 7x_3 + 7x_4 &= 5 \\ -x_1 + 2x_2 + 8x_3 - 5x_4 &= -7 \\ 3x_1 - 4x_2 - 17x_3 + 13x_4 &= 14 \\ 2x_1 - 2x_2 - 11x_3 + 8x_4 &= 7 \end{aligned}$$

6.
$$\begin{aligned} x_1 + 2x_2 + x_3 &= -1 \\ 6x_1 + x_2 + x_3 &= -4 \\ 2x_1 - 3x_2 - x_3 &= 0 \\ x_1 - x_2 &= 1. \end{aligned}$$

7.
$$\begin{aligned} 2x_1 + x_2 + 5x_3 &= 4 \\ 3x_1 - 2x_2 + 2x_3 &= 2 \\ 5x_1 - 8x_2 - 4x_3 &= 1. \end{aligned}$$

8. Solve the system of equations having the given matrices as their augmented matrices:

(i) $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 1 & 1 & 1 \end{array}\right]$

(ii) $\left[\begin{array}{ccc|c} 1 & 2 & 4 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 1 & 3 & 3 & 0 \end{array}\right]$

(iii) $\left[\begin{array}{ccc|c} 4 & 2 & -1 & 0 \\ 3 & 3 & 6 & 3 \\ 5 & 1 & -8 & -1 \end{array}\right]$

(iv) $\left[\begin{array}{ccc|c} 2 & -1 & 3 & 4 \\ 0 & 4 & 1 & 8 \\ 0 & 0 & 2 & 0 \end{array}\right]$

For what values of λ do the following homogeneous equations have nontrivial solutions? Find these solutions: (Problems 9 – 11):

9. $$\begin{aligned}(1-\lambda)x_1 + x_2 &= 0\\ x_1 + (1-\lambda)x_2 &= 0\end{aligned}$$

10. $$\begin{aligned}(3-\lambda)x_1 - x_2 + x_3 &= 0\\ x_1 - (1-\lambda)x_2 + x_3 &= 0\\ x_1 - x_2 + (1-\lambda)x_3 &= 0\end{aligned}$$

11. $$\begin{aligned}(1-\lambda)x_1 + x_2 - x_3 &= 0\\ x_1 - \lambda x_2 - 2x_3 &= 0\\ x_1 + 2x_2 - \lambda x_3 &= 0\end{aligned}$$

In each of the following use Gauss-Jordan method to reduce the given system to reduced echelon form, indicating the operations performed and determine the solution if any: (Problems 12 – 19):

12. $$\begin{aligned}6x_1 - 6x_2 + 6x_3 &= 6\\ 2x_1 - 4x_2 - 6x &= 12\\ 10x_1 - 5x_2 + 5x_3 &= 30\end{aligned}$$

13. $$\begin{aligned}5x_1 + 5x_2 - x_3 &= 0\\ 10x_1 + 5x_2 + 2x_3 &= 0\\ 5x_1 + 15x_2 - 9x_3 &= 0\end{aligned}$$

14. $$\begin{aligned}5x_1 - 2x_2 + x_3 &= 3\\ 3x_1 + 2x_2 + 7x_3 &= 5\\ x_1 + x_2 + 3x_3 &= 2\end{aligned}$$

15. $$\begin{aligned}5x_1 - 2x_2 + x_3 &= 2\\ 3x_1 + 2x_2 + 7x_3 &= 3\\ x_1 + x_2 + 3x_3 &= 2\end{aligned}$$

16. $$\begin{aligned}2x_1 - x_2 + 3x_3 &= 3\\ 3x_1 + x_2 - 5x_3 &= 0\\ 4x_1 - x_2 + x_3 &= 3\end{aligned}$$

17. $$\begin{aligned}x_1 + 3x_2 + 5x_3 - 4x_4 &= 1\\ x_1 + 2x_2 + x_3 - x_4 + x_5 &= -1\\ x_1 - 2x_2 + 3x_3 + 2x_4 - x_5 &= 3\\ x_1 + 5x_2 + 3x_3 + x_4 + x_5 &= -11\\ x_1 + 3x_2 - x_3 + x_4 + 2x_5 &= -3\end{aligned}$$

18. $$\begin{aligned}3x_1 + 2x_2 + 4x_3 &= 7\\ 2x_1 + x_2 + x_3 &= 4\\ x_1 + 3x_2 + 5x_3 &= 3\end{aligned}$$

19. $$\begin{aligned}5x_1 + 4x_3 + 2x_4 &= 3\\ x_1 - x_2 + 2x_3 + x_4 &= 1\\ 4x_1 + x_2 + 2x_3 &= 1\\ x_1 + x_2 + x_3 + x_4 &= 0\end{aligned}$$

20. Show that the system

$$\begin{aligned}2x_1 - x_2 + 3x_3 &= a\\ 3x_1 + x_2 - 5x_3 &= b\\ -5x_1 - 5x_2 + 21x_3 &= c\end{aligned}$$

is inconsistent if $c \neq 2a - 3b$.

EXERCISE 4 (Page 174)

1. $x_1 = 1/5, x_2 = -9/20, x_3 = 3/5$ **2.** $x_1 = a - 3, x_2 = 1, x_3 = 2$

3. $x_1 = 1/2 - 5x_5, x_2 = 1/2 - 3x_5, x_3 = 1/2 + 3x_5, x_4 = -1/2 + 2x_5$
and value of x_5 in arbitrary.

4. $x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 1, x_5 = 1$ **5.** $x_1 = 3, x_2 = -9/2, x_3 = 0, x_4 = -1$

6. $x_1 = -1, x_2 = -2, x_3 = 4$. The system has unique solution.

7. The system has no solution.

8. **(i)** System inconsistent **(ii)** No nontrivial solution

(iii) System inconsistent **(iv)** $x_1 = 3, x_2 = 2, x_3 = 0$

9. $\lambda = 0, x_2 = -x_1$; $\lambda = 2, x_2 = x_1$

10. $\lambda = 0, x_1 = 0, x_2 = x_3$, $\lambda = 1, x_1 = x_2 = -x_3$

$\lambda = 4, x_1 = 2x_3, x_3 = -x_2$

11. $\lambda = 0, x_1 = -2x_2, x_2 = -x_3$ **12.** $x_1 = 5, x_2 = 11/5, x_1 = -9/5$

13. $x_1 = -\frac{3}{5}x_3, \quad x_2 = \frac{4}{5}x_3 \quad x_3$ arbitrary.

14. The sytem has infinite number of solutions. $x_1 = 1 - x_3, x_2 = 1 - 2x_3, x_3$ arbitrary.

15. The system has no solution.

16. $x_1 = 1, x_2 = 2, x_3 = 1$

17. $x_1 = -19, x_2 = -10, x_3 = 10, x_4 = 0, x_5 = 28$

18. $x_1 = 2, x_2 = -1/2, x_3 = 1/2$ **19.** $x_1 = 1, x_2 = -1, x_3 = -1, x_4 = 1$

21. $\begin{bmatrix} 2-s & -1-t \\ -1-s & 1-t \\ s & t \end{bmatrix}$, $s, t \in \boldsymbol{R}$. No.

22. The amounts spent on radio, magazine and T.V. are Rs. 100,000; Rs. 200,000 and Rs 300,000 respectively.

23. **(iii)** 850

24. **(ii)** Along $AB = 100, BC = 100, CE = 400 - s, s - 200, CF = s, 200 \le s \le 400$

(iii) Along $EA = 100, AB = 200, BC = 200$

$CE = 300 - s, 0 \le s \le 300$

$CF = s - 200, 0 \le s \le 200$