

Assignment #1

Subject: Linear Algebra and Differential Equations.

Solve All Questions.

Q1. Find the rank of the following matrices

$$i) \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & 0 \\ -2 & -1 & 3 \\ -1 & 4 & -2 \end{bmatrix}$$

$$ii) \begin{bmatrix} 1 & 3 & -2 & 5 & 4 \\ 1 & 4 & 1 & 3 & 5 \\ 1 & 4 & 2 & 4 & 3 \\ 2 & 7 & -3 & 6 & 13 \end{bmatrix}$$

Q2. Use row reduction to show that

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

Q3. Find the inverse of matrix by Adjoint method.

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 7 \end{bmatrix}$$

Q4. Solve system of linear equations, the field of scalars being \mathbb{R} .

$$\begin{aligned} x_1 - 2x_2 - 7x_3 + 7x_4 &= 5 \\ -x_1 + 2x_2 + 8x_3 - 5x_4 &= -7 \\ 3x_1 - 4x_2 - 17x_3 + 13x_4 &= 14 \\ 2x_1 - 2x_2 - 11x_3 + 8x_4 &= 7 \end{aligned}$$

Q5. For what value of λ , the homogenous equations has nontrivial solution.

$$(1-\lambda)x_1 + x_2 - x_3 = 0$$

$$x_1 - \lambda x_2 - 2x_3 = 0$$

$$x_1 + 2x_2 - \lambda x_3 = 0$$

Q6. Use Cramer's Rule to solve a linear system.

$$\begin{aligned}x_1 + 2x_3 &= 6 \\ -3x_1 + 4x_2 + 6x_3 &= 30 \\ -x_1 - 2x_2 + 3x_3 &= 8\end{aligned}$$

Q7. Show that $\det(A) = 0$, without directly evaluating the determinant.

$$A = \begin{bmatrix} -2 & 8 & 1 & 4 \\ 3 & 2 & 5 & 1 \\ 1 & 10 & 6 & 5 \\ 4 & -6 & 4 & -3 \end{bmatrix}$$

Q8. Let $A = \begin{bmatrix} 2 & 3 & -1 & 1 \\ -3 & 2 & 0 & 3 \\ 3 & -2 & 1 & 0 \\ 3 & -2 & 1 & 4 \end{bmatrix}$

i) Find M_{24} and C_{24}

ii) M_{41} and C_{41}

Q9. Solve the linear system together by reducing appropriate augmented matrix.

$$-x_1 + 4x_2 + x_3 = b_1$$

$$x_1 + 9x_2 - 2x_3 = b_2$$

$$6x_1 + 4x_2 - 8x_3 = b_3$$

i) $b_1 = 0$, $b_2 = 1$, $b_3 = 0$

ii) $b_1 = -3$, $b_2 = 4$, $b_3 = -5$

Q10. Show that the given matrix is idempotent

$$A = \begin{bmatrix} 2 & -2 & 4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$$
