CA HW #2

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$$T(n) = aT(n/b) + \Theta(n^k \log^p n)$$

$\underline{a \geq 1 \quad b > 1 \quad k \geq 0,}$ $\quad p \in \mathbb{R}$

Part 1

a) $T(n) = 2T(n/2) + n \log n$

$a = 2 \quad b = 2 \quad k = 1 \quad p = 1$

$a = b^k \quad 2 = 2^1$ CASE 2 SUBCASE A as $p > -1$

$T(n) = \Theta(n \log_2^2 \log^2 n) = \Theta(n \log^2 n)$

b) $T(n) = 2T(n/2) + n/\log n$

$a = 2 \quad b = 2 \quad k = 1 \quad p = -1$ $\quad | \quad \frac{1}{\log n} = \log^{-1} n$

$a = b^k \quad 2 = 2^1$ CASE 2

SUBCASE B as $p = -1$

$T(n) = \Theta(n^{\log_2 2} \log\log n) = \Theta(n \log\log n)$

c) $T(n) = 2T(n/4) + n^{0.51}$

$a = 2 \quad b = 4 \quad k = 0.51 \quad p = 0$

$\boxed{a < b^k} \quad 2 < 4^{0.51}$

$4^{0.51} \approx 2.03$

CASE 3

SUBCASE A as $p = 0$

$T(n) = \Theta(n^{0.51} \log^0 n) \simeq \Theta(n^{0.51})$ $\quad | \quad \log^0 n = 1$

d) $T(n) = \sqrt{2}\,T(n/2) + n^2 \log n$

$a = \sqrt{2} \quad b = 2 \quad k = 2 \quad p = 1$

$\boxed{a < b^k} \quad \sqrt{2} < 2^2$

CASE 3

SUBCASE A $p = 1$ & $p \geq 0$

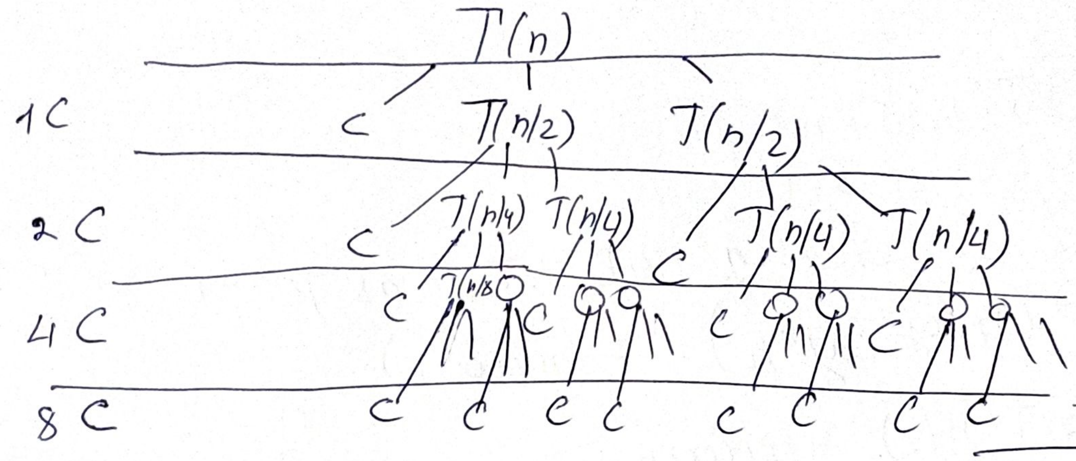
$T(n) = \Theta(n^2 \log n)$

Part 2

$T(n) = 2T(n/2) + c \qquad n > 1$

$\quad\;\; = 1 \qquad\qquad\qquad\quad n = 1$

1/2



$T(n) = c + 2c + 4c + \dots\ n \cdot c =$

$= 2^0 c + 2^1 c + 2^2 c + \dots + 2^{\log n - 1} =$

$= \sum_{h=0}^{\log n - 1} 2^h c = 2n - 1 = O(n)$ because there should be $n-1$ internal node

so, total number is $2n-1$.

$$\boxed{\begin{array}{l} \text{height} = \log_2 n \\ 2^h = 2^{\log_2 n} = n \end{array}}$$

$O(n)$

FINAL ANSWER

Part 3

$$T(n) = T(n-1) + \log n \qquad n > 0$$
$$= 1 \qquad n = 0$$

$$T(n-1) = T(n-2) + \log(n-1)$$
$$T(n-2) = T(n-3) + \log(n-2)$$
$$\vdots$$
$$T(n) = \log n + \log(n-1) + \log(n-2) + \ldots \log(n-k) + T(0)$$

One more try!

$$T(n) = T(n-1) + \log n$$
$$T(n) = [T(n-2) + \log(n-1)] + \log n$$
$$T(n) = [T(n-3) + \log(n-2) + \log(n-1)] + \log n$$
$$\vdots$$
$$T(n) = T(n-k) + (\log 1 + \log 2 + \ldots \log(n-1) + \log n)$$

$\Downarrow$
$n = k$
because until $T(n) = 1$
$n = 0$ so $n = k$

$\log[n(n-1)(n-2)\ldots]$
$\sum^{n} \log i = \log(n!)$

$$T(n) = T(0) + \log(n!) =$$
$$= 1 + \log n! =$$
$$= O(n \log n)$$