Technical Specification for Soft Walls - Flight Control Systems to Limit the Flight Space of Commercial Aircraft

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1 Introduction

See the companion document that discusses the issue.

2 Two-Dimensional Model

2.1 Two-Dimensional Aircraft Model

We consider first a two-dimensional model that addresses control of the heading of an aircraft. Let the position of the aircraft be a function

$$p\colon Reals \to Reals^2$$

where the domain is time (the reals) and the range is the position of the aircraft in two-dimensional space. Let $\dot{p}$ denote the derivative with respect to time (the velocity) and $\ddot{p}$ the second derivative with respect to time (the acceleration). Let p_x denote the position in the x direction (east-west, increasing to the east), and let p_y denote the position in the y direction (north-south, increasing to the north). Similarly, $\dot{p}_x$ and $\ddot{p}_x$ denote the speed and acceleration in the x direction (east).

Let the speed s of the aircraft be given by

$$\forall\, t \in Reals, \quad s(t) = |\dot{p}(t)|.$$
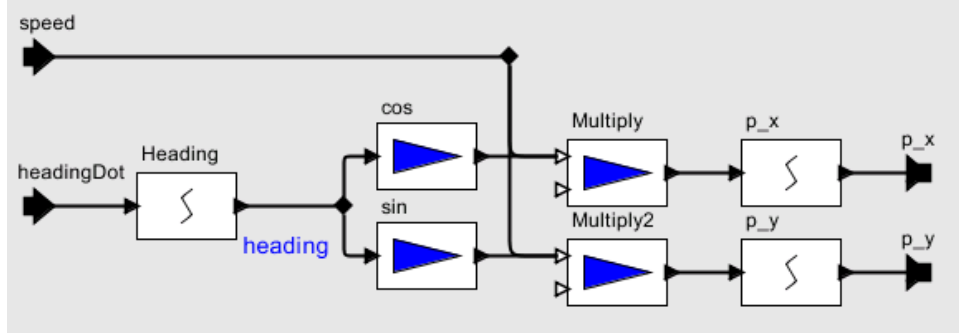
Let

$$\theta\colon Reals \to [-\pi, \pi)$$

Figure 1: Two dimensional aircraft model.

be the heading of the aircraft, where 0 is due east, so that

$$\forall\, t \in \text{\textit{Reals}}, \quad \dot{p}(t) = (s(t)\cos(\theta(t)), s(t)\sin(\theta(t))).$$

Assume that under normal circumstances, the pilot controls the rate of change of heading, $\dot{\theta}$, through a combination of differential thrust on the engines and movement of the rudder and other control surfaces. Moreover, the pilot controls the speed via overall thrust (and vertical movement, which we are not yet considering). Thus, the inputs to the aircraft model are $\dot{\theta}$ and s.

The two dimensional model for the aircraft is shown in figure 1.

2.2 Turn Radius

Assume the speed is a constant s, so the pilot controls only heading. Then the turning radius is

$$r = s/\dot{\theta}.$$

To see this, assume the rate of change of heading is a constant, $\dot{\theta} = \alpha$. Assume that α is given in radians/second and s in meters per second. It takes $\tau = 2\pi/\alpha$ seconds to complete one circle. Upon completing the circle, the aircraft will have covered a distance of $s\tau = 2\pi s/\alpha$ meters. Since the radius of a circle times 2π is its circumference, the result follows.

The rate of change of heading as a function of speed and steering angle is

$$\dot{\theta} = s/r.$$

If we assume that the minimum safe turning radius is r_{min}, then the control signal $\dot{\theta}$ must be kept in the range $[-s/r_{min}, s/r_{min}]$.

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For example, if we assume an aircraft traveling at

$$s = 500 \; kilometers/hour$$

or about 139 meters/second, and we assume the aircraft has a minimum safe turning radius of $r_{min} = 1000$ meters, then $\dot{\theta}$ must be constrained to lie in the range $[-0.139, 0.139]$ radians per second.

2.3 Blending Controller

Let the pilot's control be $\dot{\theta}_p$ and the control signal generated by the softwalls system be $\dot{\theta}_s$. We take the rate of change of heading of the aircraft to be

$$\dot{\theta} = limit_{[-s/r_{min}, s/r_{min}]}(\dot{\theta}_p - \dot{\theta}_s),$$

where $limit_{[a,b]}$ is a function

$$limit_{[a,b]} : Reals \rightarrow [a, b]$$

where

$$\forall u \in Reals, \quad limit_{[a,b]}(u) = \begin{cases} b & \text{if } u > b \\ a & \text{if } u < a \\ u & \text{otherwise} \end{cases}$$

This strategy for blending the softwalls controller and the pilot's control ensures that while the control parameter is within safe limits, the responsivity of the aircraft to the pilot commands remains normal. That is, the pilot's control signal is not attenuated.

2.4 Softwall Control Signal

Assume a single, flat soft wall with thickness d_s. The *inner boundary* delineates the no-fly zone. The *outer boundary* delineates the region where the softwall controller starts to have some effect. The thickness d_s is the distance between these two boundaries. Let the distance of the aircraft from the inner boundary of the soft wall be given by

$$d : Reals \rightarrow Reals.$$

Define the *criticality* to be

$$c = limit_{[0,1]}\Big(1 - \frac{d - r_{min}}{d_s - r_{min}}\Big).$$

If the aircraft is further than d_s from the inner boundary of the softwall, then $c = 0$, indicating that no corrective action is necessary. If the aircraft is closer than r_{min} to

the inner boundary of the softwall, then $c = 1$, indicating that maximum corrective action is necessary. If the aircraft is in between these two boundaries, then the criticality is between 0 and 1.

Let the angle of approach of the aircraft from the soft wall be given by

$$\phi : Reals \to [-\pi, \pi].$$

We set the corrective control signal to

$$\dot{\theta}_s = \begin{cases} 2\sin(\phi)cs/r_{min} & \text{if } 0 < \phi < \pi \\ 0 & \text{otherwise} \end{cases}$$

This results in zero corrective control if the aircraft is moving away from the wall or parallel to the wall, and maximum corrective control if the aircraft is approaching head on. The single point of maximum corrective control is when the aircraft is approaching the wall head on and has distance r_{min} from the inner boundary. At this point, the corrective control is twice that required to execute a minimum radius turn. It is twice that control so that when added to the pilot's control we are assured of having the maximum turning control, even if the pilot is maximally uncooperative in attempting to counteract the control.

3 Control Model Failure

We were able to develop a malicious pilot strategy that, even with the control scheme of Section 2, can fly the plane into the no-fly zone. This section explains this strategy.

3.1 Resistant Pilot

The resistant pilot is initially heading towards the no fly zone, so the iinitial $\dot{\theta}_p$ is 0. That is, the pilot intends to maintain forward motion and crash into the wall.

Since, before saturation, the input control is $\dot{\theta} = \dot{\theta}_p - \dot{\theta}_s$, the pilot will try to counteract the control signal by chosing $\dot{\theta}_p = \dot{\theta}_s$. In a cockpit, the pilot would be physically limited by a maximum $\dot{\theta}_p$, so the pilot uses

$$\dot{theta}_p = limit_{[-s/r_{min}, s/r_{min}]}(\dot{\theta}_s)$$

Here the limit values are chosen for a pilot that would never move the plane in an unsafe position. In practice the limit points could be larger.

3.2 Boundary Penetration

The resistant pilot succeeds in penetrating the no-fly zone boundray. Even though the plane's trajectory is changed, the $\sin\phi$ factor in the corrective control signal dampens the strength of the softwalls control signal, such that the pilot can fly into the softwall at a $pi/6$ angle from the wall.

If the $\sin\phi$ factor is removed from the control signal, this problem disapears. That is,

$$\dot{\theta}_s = \begin{cases} 2cs/r_{min} & \text{if } 0 < \phi < \pi \\ 0 & \text{otherwise} \end{cases}$$

This control scheme is more restrictive however, because a plane cannot stay in the softwall. Any pilot between the inner and outer boundary will eventually be forced outside the outer boundary.

4 Criticality-Based Control

A measure of criticality assesses the level of threat that an aircraft poses to a no-fly zone. The softwall controller can adjust its bias according to an appropriate criticality measure.

4.1 Maintaining Responsivity

Figure A illustrates how the softwall controller maintains responsivity while biasing the pilot control. When there is no bias, the aircraft will turn as the pilot intends. When there is a bias of M (the maximum rate of change in heading) to the right, the pilot will not be able to make the aircraft turn left – the aircraft will keep straight as the pilot tries to turn the rudder to the left extreme position. When the bias increases to $3M/2$ to the right, the aircraft will turn right at a rate of at least $M/2$, whatever the pilot tries to do.

4.2 A Measure of Criticality

Here we propose a criticality measure based on how long it will take the plane to reach the no-fly zone in the worst case. Figure B illustrates this measure. The worst case trajectories are shown as dotted lines.

Suppose the no-fly zone is defined by $\{(x, y) \mid x \geq b_x\}$. The criticality measure does not depend on the aircraft's y-position. The equations for calculating $c(x, \theta)$

are:

$$c(x, \theta) = \begin{cases} \frac{\theta}{M} + \frac{b_x - x - r_{min} \sin \theta}{v} & \text{if } x \leq b_x - r_{min} \sin \theta, \ 0 \leq \theta \leq \pi/2 \\ \frac{\theta - \arcsin \left(\frac{r_{min} \sin \theta - (b_x - x)}{r_{min}} \right)}{M} & \text{if } x > b_x - r_{min} \sin \theta, \ 0 \leq \theta \leq \pi/2 \\ \frac{2(\theta - \pi/2)}{M} + c(x, \pi - \theta) & \text{if } \pi/2 < \theta \leq \pi \\ c(x, |\theta|) & \text{if } -\pi \leq \theta < 0 \end{cases}$$

Note that v, r_{min}, and M are related by $M = v/r_{min}$.

4.3 A Malicious Pilot

Figure C illustrates the control strategy of a malicious pilot, who tries to fly the aircraft into the no-fly zone (maintaining $\theta = 0$).

4.4 Criticality-Based Softwall Controller

The bias produced by a proposed criticality-based controller is shown in figure D. The threshold π/M is the value of $c(x, \theta)$ when $x = b_x$ (the aircraft is on the boundary of the no-fly zone), and $\theta = \pi$ (the aircraft is flying straight out of the no-fly zone). No bias is needed here. The threshold $2/M$ is equal to $c(b_x - 2r_{min}, 0)$ – the aircraft is flying head on to the no-fly zone, but still can be safely turned away at half the maximum turning rate. Note that the bias level is at $3M/2$, so the plane will be turned at a rate of at least half the maximum, whatever the pilot tries to do. The sign (plus – left, minus – right) of the bias is chosen to be the same as θ.