Technical Specification for Soft Walls - Flight Control Systems to Limit the Flight Space of Commercial Aircraft

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VERY ROUGH DRAFT

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1 Introduction

See the companion document that discusses the issue.

2 Two-Dimensional Model

2.1 Two-Dimensional Aircraft Model

We consider first a two-dimensional model that addresses control of the heading of an aircraft. Let the position of the aircraft be a function

$$p\colon \mathit{Reals} \to \mathit{Reals}^2$$

where the domain is time (the reals) and the range is the position of the aircraft in two-dimensional space. Let $\dot{p}$ denote the derivative with respect to time (the velocity) and $\ddot{p}$ the second derivative with respect to time (the acceleration). Let p_x denote the position in the x direction (east-west, increasing to the east), and let p_y denote the position in the y direction (north-south, increasing to the north). Similarly, $\dot{p_x}$ and $\ddot{p_x}$ denote the speed and acceleration in the x direction (east).

Let the speed s of the aircraft be given by

$$\forall\, t \in \mathit{Reals}, \quad s(t) = |\dot{p}(t)|.$$

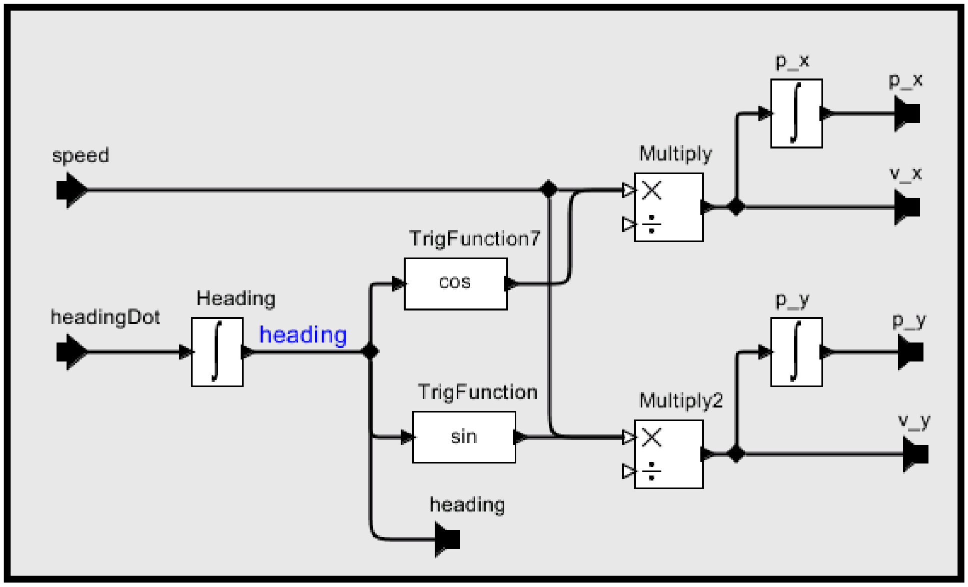

Figure 1: Two dimensional aircraft model.

Let

$$\theta\colon \textit{Reals} \to [-\pi, \pi)$$

be the heading of the aircraft, where 0 is due east, so that

$$\forall\, t \in \textit{Reals}, \quad \dot{p}(t) = (s(t)\cos(\theta(t)), s(t)\sin(\theta(t))).$$

Assume that under normal circumstances, the pilot controls the rate of change of heading, $\dot{\theta}$, through a combination of differential thrust on the engines and movement of the rudder and other control surfaces. Moreover, the pilot controls the speed via overall thrust (and vertical movement, which we are not yet considering). Thus, the inputs to the aircraft model are $\dot{\theta}$ and s.

The two dimensional model for the aircraft is shown in figure 1.

2.2 Turn Radius

Assume the speed is a constant s, so the pilot controls only heading. Then the turning radius is

$$r = s/\dot{\theta}.$$

To see this, assume the rate of change of heading is a constant, $\dot{\theta} = \alpha$. Assume that α is given in radians/second and s in meters per second. It takes $\tau = 2\pi/\alpha$ seconds to complete one circle. Upon completing the circle, the aircraft will have covered a distance of $s\tau = 2\pi s/\alpha$ meters. Since the radius of a circle times 2π is its circumference, the result follows.

The rate of change of heading as a function of speed and steering angle is

$$\dot{\theta} = s/r.$$

If we assume that the minimum safe turning radius is r_{min}, then the control signal $\dot{\theta}$ must be kept in the range $[-s/r_{min}, s/r_{min}]$.

For example, if we assume an aircraft traveling at

$$s = 500\ kilometers/hour$$

or about 139 meters/second, and we assume the aircraft has a minimum safe turning radius of $r_{min} = 1000$ meters, then $\dot{\theta}$ must be constrained to lie in the range $[-0.139, 0.139]$ radians per second.

2.3 Blending Controller

Let the pilot's control be $\dot{\theta}_p$ and the control signal generated by the softwalls system be $\dot{\theta}_s$. We take the rate of change of heading of the aircraft to be

$$\dot{\theta} = \mathit{limit}_{[-s/r_{min}, s/r_{min}]}(\dot{\theta}_p - \dot{\theta}_s),$$

where $\mathit{limit}_{[a,b]}$ is a function

$$\mathit{limit}_{[a,b]}\colon \mathit{Reals} \to [a, b]$$

where

$$\forall\, u \in \mathit{Reals}, \quad \mathit{limit}_{[a,b]}(u) = \begin{cases} b & \text{if } u > b \\ a & \text{if } u < a \\ u & \text{otherwise} \end{cases}$$

This strategy for blending the softwalls controller and the pilot's control ensures that while the control parameter is within safe limits, the responsivity of the aircraft to the pilot commands remains normal. That is, the pilot's control signal is not attenuated.

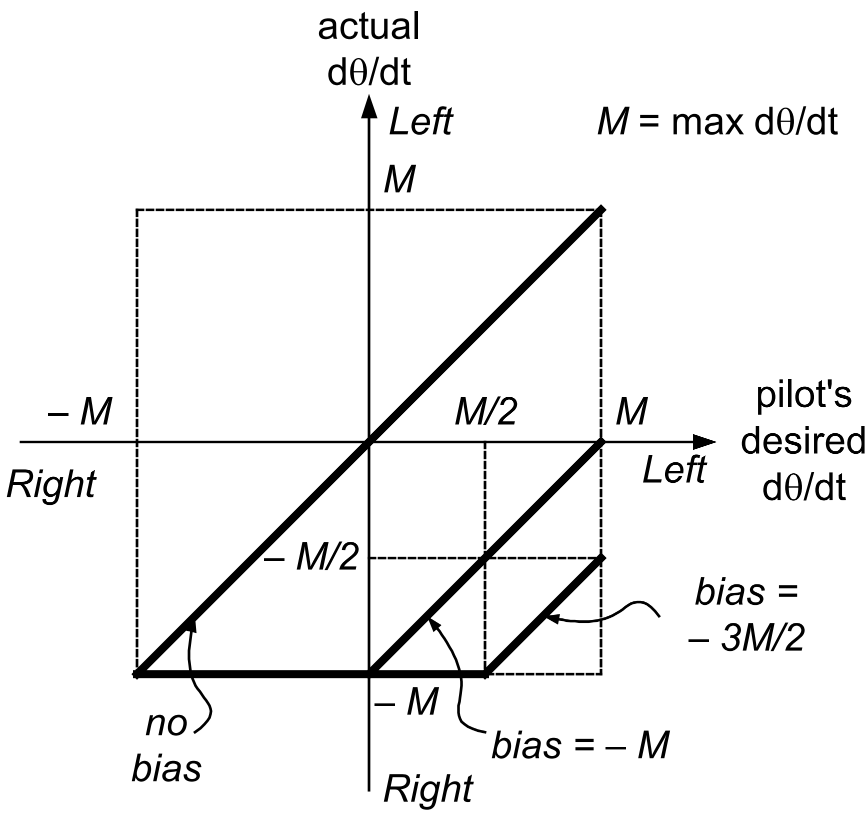

Figure 2: Rate of change of heading vs. pilot-specified rate of change of heading.

2.4 Maintaining Responsivity

Figure 2 illustrates how the blending controller maintains responsivity while biasing the pilot control. When there is no bias, the aircraft will turn as the pilot intends. That is, the actual $\dot{\theta}$ equals the desired $\dot{\theta}$. Suppose there is a bias of $-M = -s/r_{min}$, where M is the maximum rate of change in heading. The bias is to the right, and the pilot will not be able to make the aircraft turn left. If the pilot attempts to turn left at the maximum rate M, the aircraft will keep straight. When the bias increases to $-3M/2$, also to the right, the aircraft will turn right at a rate of at least $-M/2$, whatever the pilot tries to do.

Ideally, in most circumstances, only modest amounts of bias will be applied. A cooperative pilot will quickly realize that turning away from the soft wall will result in reduced biased. A maximally uncooperative pilot, however, will continue to attempt to turn towards the wall. When the bias exceeds $-M$, the pilot will no longer be able to overcome the bias, and with the controls saturated, the aircraft will turn anyway.

At all times, however, the responsivity of the controls when out of saturation remains normal. That is, when the slope of the response curve in figure 2 is not zero, it is one. A slope of one means that the aircraft responds exactly as the pilot expects.

3 Soft wall Controller – A First Design

Here we present our first attempt at designing a soft wall controller. Although later analysis and simulation showed that a malicious pilot can defeat the controller, the design illuminated the importance of criticality measures and inspired our criticality-based controller design.

3.1 Soft Wall Control Signal

Assume a single, flat soft wall with thickness d_s. The *inner boundary* delineates the no-fly zone. The *outer boundary* delineates the region where the soft wall controller starts to have some effect. The thickness d_s is the distance between these two boundaries. Let the distance of the aircraft from the inner boundary of the soft wall be given by

$$d \colon \mathit{Reals} \to \mathit{Reals}.$$

Define the *criticality* to be

$$c = \mathit{limit}_{[0,1]}(1 - \frac{d - r_{min}}{d_s - r_{min}}).$$

If the aircraft is further than d_s from the inner boundary of the soft wall, then $c = 0$, indicating that no corrective action is necessary. If the aircraft is closer than r_{min} to the inner boundary of the soft wall, then $c = 1$, indicating that maximum corrective action is necessary. If the aircraft is in between these two boundaries, then the criticality is between 0 and 1.

Let the angle of approach of the aircraft from the soft wall be given by

$$\theta\colon \mathit{Reals} \rightarrow [-\pi, \pi].$$

We set the corrective control signal to

$$\dot{\theta}_s = \begin{cases} 2\cos(\theta)cs/r_{min} & \text{if } -\pi/2 < \theta < \pi/2 \\ 0 & \text{otherwise} \end{cases}$$

This results in zero corrective control if the aircraft is moving away from the wall or parallel to the wall, and maximum corrective control if the aircraft is approaching head on ($\theta = 0$). The single point of maximum corrective control is when the aircraft is approaching the wall head on and has distance r_{min} from the inner boundary. At this point, the corrective control is twice that required to execute a minimum radius turn. It is twice that control so that when added to the pilot's control we are assured of having the maximum turning control, even if the pilot is maximally uncooperative in attempting to counteract the control.

3.2 Resistant Pilot

We were able to develop a malicious-pilot strategy that, even with the control scheme of the previous section, can fly the plane into the no-fly zone. This section explains why the strategy was successful.

The resistant pilot is initially heading towards the no-fly zone, so the initial $\dot{\theta}_p$ is 0. That is, the pilot intends to maintain forward motion and crash into the wall.

Since, before saturation, the input control is $\dot{\theta} = \dot{\theta}_p - \dot{\theta}_s$, the pilot will try to counteract the control signal by chosing $\dot{\theta}_p = \dot{\theta}_s$. In a cockpit, the pilot would be physically limited by a maximum $\dot{\theta}_p$, so the pilot uses

$$\dot{\theta}_p = \mathit{limit}_{[-s/r_{min}, s/r_{min}]}(\dot{\theta}_s)$$

Here the limit values are chosen for a pilot that would never move the plane in an unsafe position. In practice these limit points could be larger.

3.3 Boundary Penetration

The resistant pilot succeeds in penetrating the no-fly zone boundray. Even though the plane's trajectory is changed, the $\cos(\theta)$ factor in the corrective control signal dampens the strength of the softwalls control signal, such that the pilot can fly into the soft wall at a $\pi/6$ angle from the wall.

If the $\cos(\theta)$ factor is removed from the control signal, this problem disapears. That is,

$$\dot{\theta}_s = \begin{cases} 2cs/r_{min} & \text{if } -\pi/2 < \theta < \pi/2 \\ 0 & \text{otherwise} \end{cases}$$

This control scheme is more restrictive however, because a plane cannot stay in the soft wall. Any aircraft between the inner and outer boundary will eventually be forced outside the outer boundary, so this motivated an alternative approach (described in the next section).

4 Criticality-Based Control

A measure of criticality assesses the level of threat that an aircraft poses to a no-fly zone. A general design pattern for soft wall controllers is to let their bias vary with respect to an appropriate criticality measure.

4.1 A Measure of Criticality

Here we propose a criticality measure based on how long it will take the plane to reach the no-fly zone in the worst case. Figure 3 illustrates this measure. In this figure, the black dots represent the position of an aircraft, and the arrows represent its heading. The worst case trajectories are shown as dotted lines.

Suppose the no-fly zone is defined by $\{(x, y)|\ x \geq b_x\}$. Then the criticality measure does not depend on the aircraft's y-position. The equations for calculating $c(x, \theta)$ are:

$$c(x,\theta) = \begin{cases} \frac{\theta}{M} + \frac{d - r_{min}\sin\theta}{s} & \text{if } d \geq r_{min}\sin\theta,\ 0 \leq \theta \leq \pi/2 \\ \frac{\theta - \arcsin\left(\frac{r_{min}\sin\theta - d}{r_{min}}\right)}{M} & \text{if } d < r_{min}\sin\theta,\ 0 \leq \theta \leq \pi/2 \\ \frac{2(\theta-\pi/2)}{M} + c(x, \pi - \theta) & \text{if } \pi/2 < \theta \leq \pi \\ c(x, |\theta|) & \text{if } -\pi \leq \theta < 0 \end{cases}$$

where $d = d_x - x$ is the distance between the aircraft and the no-fly zone. Note that s, r_{min}, and M are related by $M = s/r_{min}$.

This criticality measure is sensible in the following special cases. First, note that if the aircraft is at the wall and heading directly away from it, as shown at the top in figure 3, then the worst-case time that it will take the aircraft to hit the wall is the time it takes to traverse a semi-circle with the minumum turning radius. This is π/M. If the aircraft is at some distance d from the wall and heading

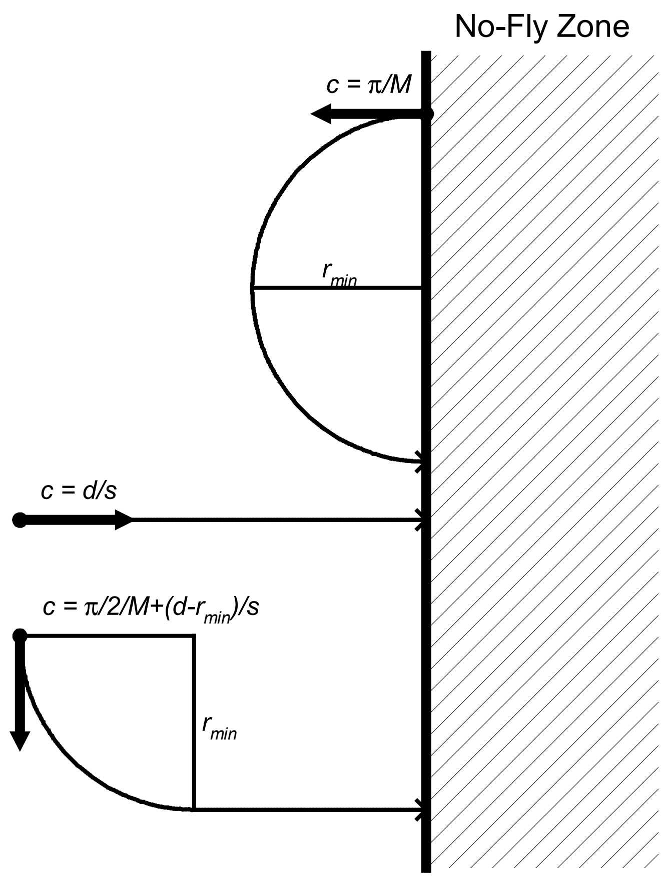

Figure 3: Rationale for criticality measure.

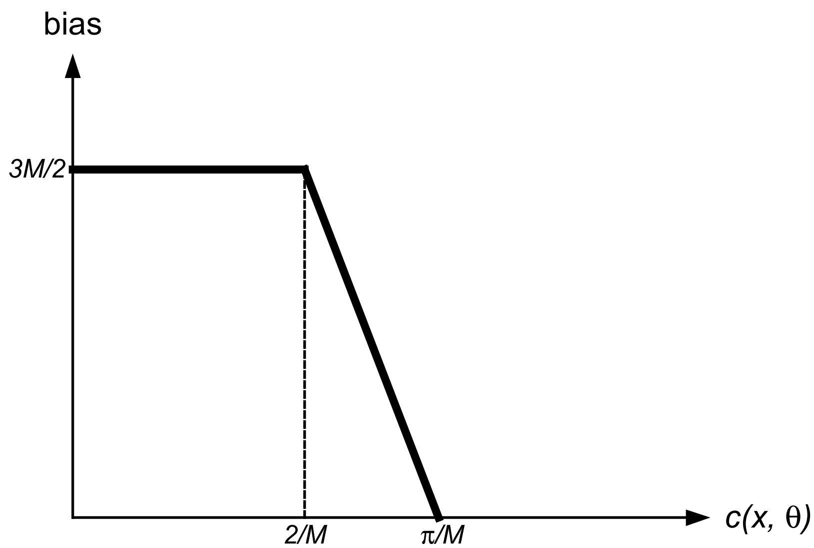

Figure 4: Bias as a function of criticality.

straight towards it, then the time it will take (in the worst case) to reach the wall is d/s, where s is the (fixed) speed. If the aircraft is at distance d from the wall but heading perpendicular to it, then the time it will take to reach the wall is

$$c = p/2/M + (d - r_{min})/s.$$

4.2 Criticality-Based Soft Wall Controller

The bias produced by a proposed criticality-based controller is shown in figure4. The threshold π/M is the value of $c(x, \theta)$ when $x = b_x$ (the aircraft is on the boundary of the no-fly zone), and $\theta = \pi$ (the aircraft is flying straight out of the no-fly zone). No bias is needed here, nor for larger criticality. The threshold $2/M$ is equal to $c(b_x - 2r_{min}, 0)$ – the aircraft is flying straight towards the no-fly zone, at a distance of $2r_{min}$ from the zone. The aircraft can still be safely turned away at half the maximum turning rate. Note that the maximum bias level is at $3M/2$, so

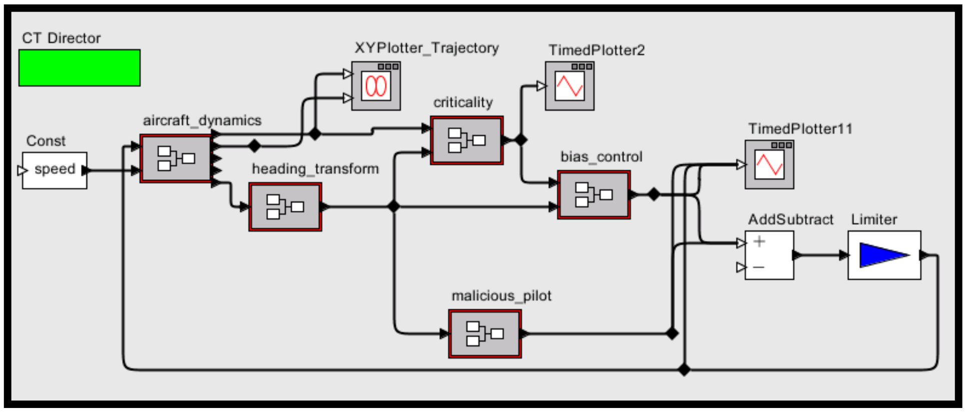

Figure 5: Top-level of a model with a maximally uncooperative pilot.

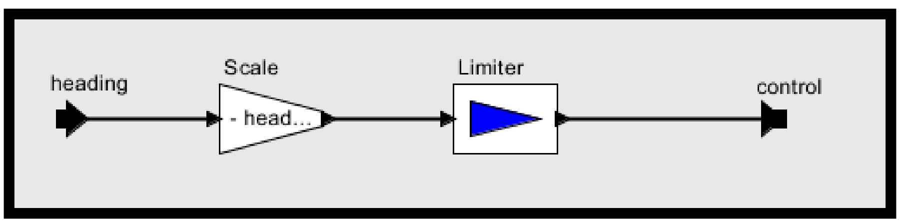

Figure 6: Maximally uncooperative pilot model. The scale factor is large.

the plane will be turned at a rate of at least half the maximum, whatever the pilot tries to do. The sign (plus – left, minus – right) of the bias is chosen to be the same as θ.

4.3 Simulation Results

The model shown in figure 5 is constructed in Ptolemy II to simulate the proposed controller. The *aircraft dynamics* component contains the aircraft model shown in figure 1. The *malicious pilot* component implements the control strategy shown in figure 6. The pilot tries to fly the aircraft into the no-fly zone by maintaining the heading θ at 0. This is accomplished by multiplying the heading by a large number and limiting the result to a number in the range of the range of the controls. Intuitively, the pilot will attempt to turn maximally towards the wall whenever

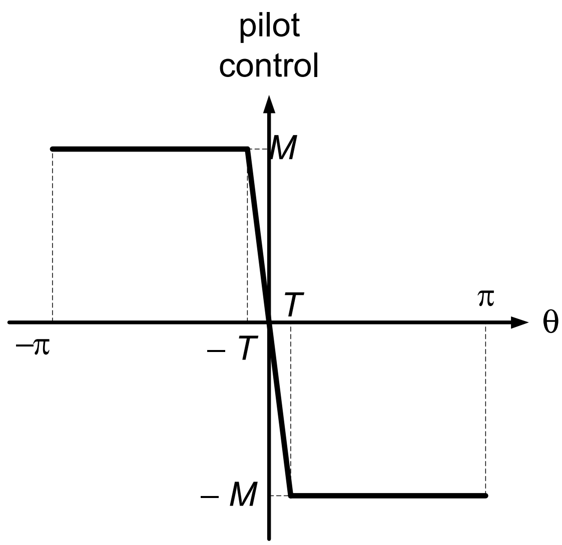

Figure 7: Pilot control as a function of heading for the maximally uncooperative pilot.

the current deviates significantly from one heading directly towards the wall, as shown in figure 7 The *criticality* component calculates $c(x, \theta)$. The *bias control* component implements the soft wall controller. The output from the pilot and the controller are combined and limited to the range $[-M, M]$ before fed back to the aircraft model.

A simulation run is shown in figure 8. The simulation is set up as follows. The aircraft is initially flying parallel to the no-fly zone ($\theta = \pi/2$), at a distance of 2 miles. The speed of the aircraft is a constant 360 miles per hour. The maximum turning rate is $2\pi/20$, so that the aircraft can complete a circle in a minimum of 20 seconds. (Note that these numbers are fictional. We have not tried to look up performance characteristics of any real aircraft yet.)

The aircraft starts at the lower left, traveling upwards, and the no-fly zone is two miles to the right, with its boundary oriented vertically. Initially, the malicious pilot can freely turn the aircraft toward the no-fly zone. When the aircraft reaches within 1 mile from the zone, the controller starts to gradually bias the pilot control. Before the pilot control saturates, the pilot can mitigate the bias and keep the aircraft heading toward the no-fly zone. But the pilot control finally saturates and the soft wall controller turns the aircraft around at half the maximum rate. As

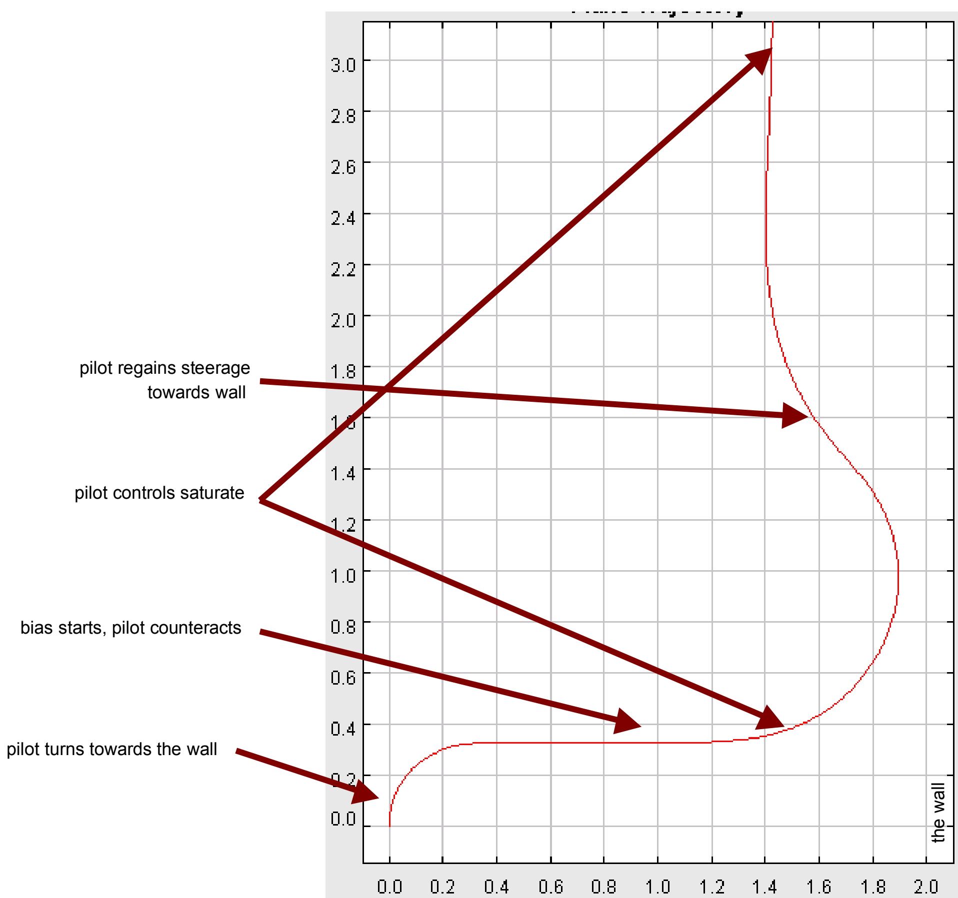

Figure 8: Simulation run with a maximally uncooperative pilot.

the criticality decreases, the bias from the controller becomes smaller. The pilot can regain steerage toward the no-fly zone for a while before the aircraft settles in flying parallel to the zone. At this time, the pilot is still trying in vain to fly the aircraft into the no-fly zone by placing the control at the right maximum.

Our work on this criticality-based control scheme is ongoing. We are working on simulating a variety of flight scenarios. We are investigating interactive simulation where the output of the pilot component is controlled by experimenters. We are also looking to prove the safety of our control scheme and the validity of the criticality measure.

5 Summary

We have described a very preliminary control algorithm for the two dimensional problem of keeping an aircraft out of a no-fly zone with a straight boundary while maintaining maximal responsivity to pilot controls subject to the constraint that the no-fly zone not be entered.