Technical Specification for Soft Walls - Flight Control Systems to Limit the Flight Space of Commercial Aircraft

Edward A. Lee and Xiaojun Liu
eal@eecs.berkeley.edu, liuxj@eecs.berkeley.edu
Electrical Engineering & Computer Science
University of California, Berkeley

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1 Introduction

See the companion document that discusses the issue.

2 Two-Dimensional Model

2.1 Two-Dimensional Aircraft Model

We consider first a two-dimensional model that addresses control of the heading of an aircraft. Let the position of the aircraft be a function

$$p\colon \mathit{Reals} \to \mathit{Reals}^2$$

where the domain is time (the reals) and the range is the position of the aircraft in two-dimensional space. Let $\dot{p}$ denote the derivative with respect to time (the velocity) and $\ddot{p}$ the second derivative with respect to time (the acceleration). Let p_x denote the position in the x direction (east-west, increasing to the east), and let p_y denote the position in the y direction (north-south, increasing to the north). Similarly, $\dot{p_x}$ and $\ddot{p_x}$ denote the speed and acceleration in the x direction (east).

Let the speed s of the aircraft be given by

$$\forall\ t \in \mathit{Reals}, \quad s(t) = |\dot{p}(t)|.$$

Let

$$\theta\colon \mathit{Reals} \to [-\pi, \pi)$$

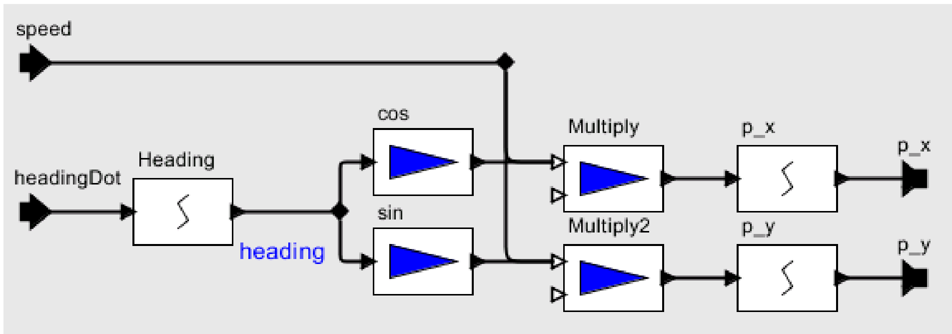

Figure 1: Two dimensional aircraft model.

be the heading of the aircraft, where 0 is due east, so that

$$\forall\, t \in \textit{Reals}, \quad \dot{p}(t) = (s(t)\cos(\theta(t)), s(t)\sin(\theta(t))).$$

Assume that under normal circumstances, the pilot controls the rate of change of heading, $\dot{\theta}$, through a combination of differential thrust on the engines and movement of the rudder and other control surfaces. Moreover, the pilot controls the speed via overall thrust (and vertical movement, which we are not yet considering). Thus, the inputs to the aircraft model are $\dot{\theta}$ and s.

The two dimensional model for the aircraft is shown in figure 1.

2.2 Turn Radius

Assume the speed is a constant s, so the pilot controls only heading. Then the turning radius is

$$r = s/\dot{\theta}.$$

To see this, assume the rate of change of heading is a constant, $\dot{\theta} = \alpha$. Assume that α is given in radians/second and s in meters per second. It takes $\tau = 2\pi/\alpha$ seconds to complete one circle. Upon completing the circle, the aircraft will have covered a distance of $s\tau = 2\pi s/\alpha$ meters. Since the radius of a circle times 2π is its circumference, the result follows.

The rate of change of heading as a function of speed and steering angle is

$$\dot{\theta} = s/r.$$

If we assume that the minimum safe turning radius is r_{min}, then the control signal $\dot{\theta}$ must be kept in the range $[-s/r_{min}, s/r_{min}]$.

For example, if we assume an aircraft traveling at

$$s = 500\ kilometers/hour$$

or about 139 meters/second, and we assume the aircraft has a minimum safe turning radius of $r_{min} = 1000$ meters, then $\dot{\theta}$ must be constrained to lie in the range $[-0.139, 0.139]$ radians per second.

2.3 Blending Controller

Let the pilot's control be $\dot{\theta}_p$ and the control signal generated by the softwalls system be $\dot{\theta}_s$. We take the rate of change of heading of the aircraft to be

$$\dot{\theta} = \mathit{limit}_{[-s/r_{min}, s/r_{min}]}(\dot{\theta}_p + \dot{\theta}_s),$$

where $\mathit{limit}_{[a,b]}$ is a function

$$\mathit{limit}_{[a,b]}\colon \mathit{Reals} \to [a, b]$$

where

$$\forall\ u \in \mathit{Reals}, \quad \mathit{limit}_{[a,b]}(u) = \begin{cases} b & \text{if } u > b \\ a & \text{if } u < a \\ u & \text{otherwise} \end{cases}$$

This strategy for blending the softwalls controller and the pilot's control ensures that while the control parameter is within safe limits, the responsivity of the aircraft to the pilot commands remains normal. That is, the pilot's control signal is not attenuated.

2.4 Softwall Control Signal

Assume a single, flat soft wall with thickness d_s. The *inner boundary* delineates the no-fly zone. The *outer boundary* delineates the region where the softwall controller starts to have some effect. The thickness d_s is the distance between these two boundaries. Let the distance of the aircraft from the inner boundary of the soft wall be given by

$$d\colon \mathit{Reals} \to \mathit{Reals}.$$

Define the *criticality* to be

$$c = \mathit{limit}_{[0,1]}(1 - \frac{d - r_{min}}{d_s - r_{min}}).$$

If the aircraft is further than d_s from the inner boundary of the softwall, then $c = 0$, indicating that no corrective action is necessary. If the aircraft is closer than r_{min} to the inner boundary of the softwall, then $c = 1$, indicating that maximum corrective action is necessary. If the aircraft is in between these two boundaries, then the criticality is between 0 and 1.

Let the angle of approach of the aircraft from the soft wall be given by

$$\phi\colon \mathit{Reals} \to [-\pi, \pi].$$

We set the corrective control signal to

$$\dot{\theta}_s = \begin{cases} 2\sin(\phi)cs/r_{min} & \text{if } 0 < \phi < \pi \\ 0 & \text{otherwise} \end{cases}$$

This results in zero corrective control if the aircraft is moving away from the wall or parallel to the wall, and maximum corrective control if the aircraft is approaching head on. The single point of maximum corrective control is when the aircraft is approaching the wall head on and has distance r_{min} from the inner boundary. At this point, the corrective control is twice that required to execute a minimum radius turn. It is twice that control so that when added to the pilot's control we are assured of having the maximum turning control, even if the pilot is maximally uncooperative in attempting to counteract the control.