Two-Dimensional Technical Specification for Soft Walls

J. Adam Cataldo, Edward A. Lee, and Xiaojun Liu
{acataldo, eal, liuxj}@eecs.berkeley.edu
Electrical Engineering & Computer Science
University of California, Berkeley

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1 Introduction

The terrorist attacks of September 11, 2001 proved that airplanes make deadly bombs. In response to this attack-by-aircraft threat, Edward Lee proposed *soft walls*, a flight control system that prevents aircrafts from entering *no-fly zones*, that is, restricted airspace, such as major cities, government centers, military installations, chemical factories, and nuclear-power plants [Lee 2001]. The major objective of this control scheme is to minimize the control imposed on pilots while protecting the no-fly zones. This control scheme uses a map from the aircraft's database together with position, velocity, and orientation information from onboard sensors to prevent no-fly zone entry.

This document describes our first approach at a softwalls control algorithm. We assume the aircraft travels in a horizontal plane at a constant velocity and can only turn as control. The no-fly zone is bounded by a line in the 2D plane. While these approximations are unrealistic, we chose a simple model, which we will later refine for accuracy.

2 Two-Dimensional Model

2.1 Two-Dimensional Aircraft Model

In our two-dimensional model we only control the aircraft's heading. Let the aircraft's position be a function

$$p\colon \mathit{Reals} \to \mathit{Reals} \times \mathit{Reals}$$

where the domain is time (the reals) and the range is the aircraft's two-dimensional position. Let $\dot{p}$ denote the time derivative (the velocity) and $\ddot{p}$ the second derivative with respect to time (the acceleration). Let p_x denote the x-direction position (east-west, increasing to the east) and p_y the y-direction position (north-south, increasing to the north). Similarly, $\dot{p_x}$ and $\ddot{p_x}$ denote the x-direction speed and acceleration.

Let the aircraft's speed s be given by

$$\forall\, t \in \mathit{Reals}, \quad s(t) = |\dot{p}(t)|.$$

Let

$$\theta\colon \mathit{Reals} \to [-\pi, \pi)$$

be the aircraft's heading, where 0 is due east, so that

$$\forall\, t \in \mathit{Reals}, \quad \dot{p}(t) = (s(t)\cos(\theta(t)), s(t)\sin(\theta(t))).$$

Assume that during flight, the pilot controls the heading's rate of change, $\dot{\theta}$, with differential thrust and movement of the rudder, ailerons, and elevator. Moreover, the pilot controls the speed via overall thrust and vertical movement. In this model, which we show in figure 1, the the aircraft-model's inputs are $\dot{\theta}$ and s.

2.2 Turn Radius

Assume the speed is a constant s, with s given in meters per second, so the pilot controls only heading. If the heading's rate of change is a constant, $\dot{\theta} = \alpha$, with α given in radians/second, it takes $\tau = 2\pi/\alpha$ seconds to complete one circle. Upon completing the circle, the aircraft has covered a $s\tau = 2\pi s/\alpha$ meter distance. Since the circle's radius times 2π gives its circumference, the turning radius is

$$r = s/\dot{\theta}.$$

Thus, the heading's rate of change is

$$\dot{\theta} = s/r.$$

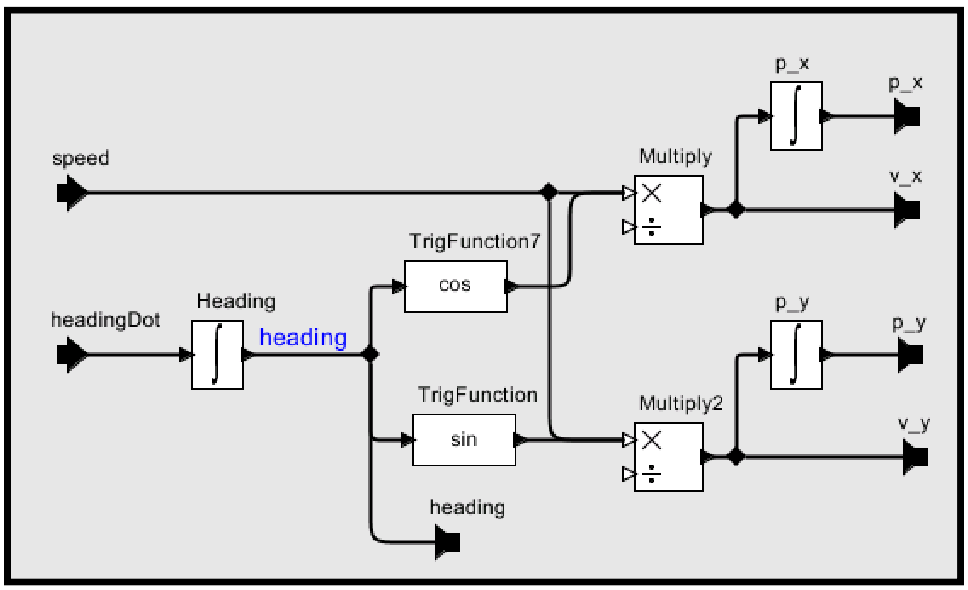

Figure 1: Two dimensional aircraft model.

If we know minimum-safe-turning radius is r_{min}, then the control signal $\dot{\theta}$ must be kept in the range $[-s/r_{min}, s/r_{min}]$.

For example, an aircraft traveling at

$$s = 500\ kilometers/hour$$

(139 meters/second) with a minimum safe turning radius $r_{min} = 1000$ meters constrains the pilot's safe $\dot{\theta}$ to the range $[-0.139, 0.139]$ radians per second.

2.3 Blending Controller

Let the pilot's control signal be $\dot{\theta}_p$ and the softwalls-generated control signal be $\dot{\theta}_s$. We take the aircraft heading's rate of change to be

$$\dot{\theta} = \mathit{limit}_{[-s/r_{min}, s/r_{min}]}(\dot{\theta}_p - \dot{\theta}_s),$$

where $\mathit{limit}_{[a,b]}$ is a function

$$\mathit{limit}_{[a,b]} \colon \mathit{Reals} \to [a, b]$$

where

$$\forall\, u \in \mathit{Reals}, \quad \mathit{limit}_{[a,b]}(u) = \begin{cases} b & \text{if } u > b, \\ a & \text{if } u < a, \\ u & \text{otherwise.} \end{cases}$$

This strategy blends the softwalls and pilot control signals ensuring that while the control parameter is within safe limits, the aircraft's response to the pilot's control signal remains unattenuated.

2.4 Maintaining Responsiveness

Figure 2 illustrates the blending controller maintaining responsivity while biasing the pilot control. When the softwalls controller adds no bias, the aircraft will turn as the pilot intends. That is, the actual $\dot{\theta}$ equals the pilot's $\dot{\theta}_p$. Suppose the softwalls bias is $-M = -s/r_{min}$, where M is the maximum rate of change in heading. The bias is rightward, and the pilot will be unable to turn the aircraft left, and if the pilot attempts to turn left at the maximum rate M, the aircraft will keep straight. When the bias increases to $-3M/2$, also rightward, the aircraft will turn right at a rate greater than or equal to $-M/2$ for any pilot control signal.

With this scheme, a cooperative pilot will turn away from the soft wall to reduce the bias. An uncooperative pilot, however, will attempt a turn towards the wall even with the bias applied. When the bias exceeds $-M$, this pilot will be unable to

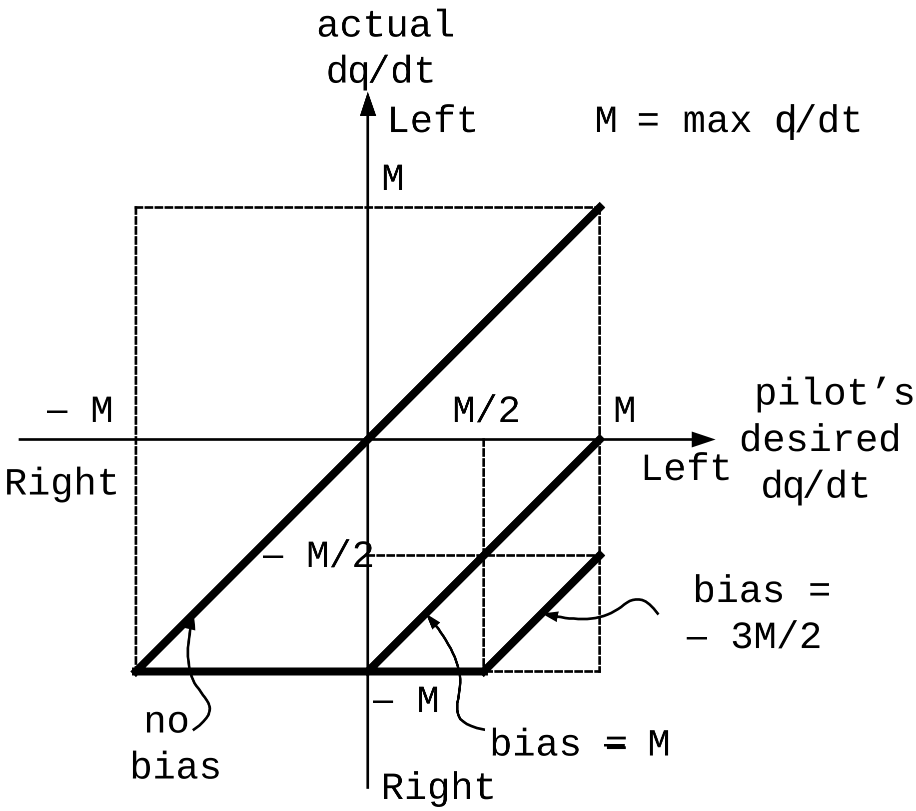

Figure 2: Rate of change of heading vs. pilot-specified rate of change of heading. Here a left turn is on the right side of the graph because a positive $d\theta/dt$ will cause the plane to turn left.

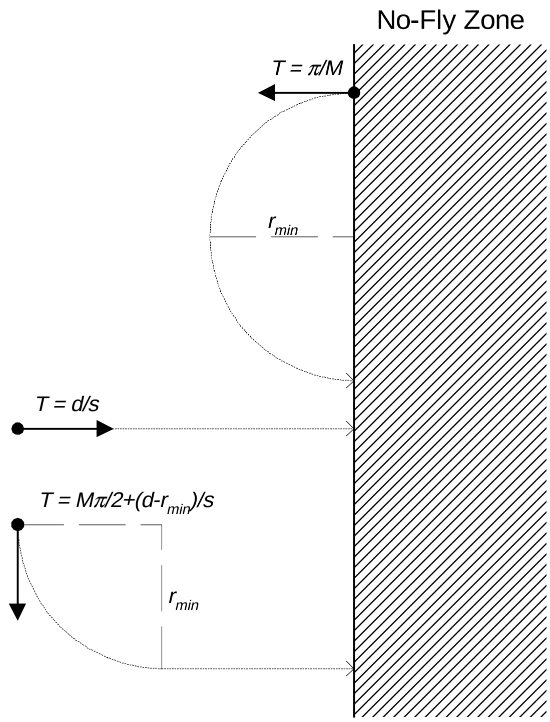

Figure 3: Calculating $T(x, \theta)$ for criticality measure.

overcome the bias, and with the controls saturated, the aircraft will turn away from the soft wall.

Until the actual $\dot{\theta}$ saturates, the aircraft responds exactly as the pilot expects. That is, when the response curve's slope (figure 2) is not zero, it is one.

3 Criticality-Based Control

To asses an aircraft's threat to a no-fly zone, we created a criticallity measurement. From this we compute the bias, θ_s, if any.

3.1 A Measure of Criticality

Our criticality measurement is inversely proportional to the minimum time it takes the aircraft to enter the no-fly zone. Figure 3 illustrates this measure. In this figure, the black dots represent the aircraft's position, and the arrows represent its heading. For each position and heading, we plot the worst-case trajectory, i.e., the path that takes the aircraft into the no-fly zone faster than all other paths, as dotted lines. In this sense, we are calculating an optimal path for the plane to collide with the no-fly zone.

Suppose we define the no-fly zone as the regioin $\{(x, y)|\ x \geq b_x\}$. Then the criticality measurement, c, and the aircraft's y-position are independent. We let $c(x, \theta) = 1/T(x, \theta)$ where $T(x, \theta)$, the minimum time a plane needs to contact the no-fly zone, comes from

$$T(x,\theta) = \begin{cases} \frac{\theta}{M} + \frac{d - r_{min}\sin\theta}{s} & \text{if } d \geq r_{min}\sin\theta,\ 0 \leq \theta \leq \pi/2 \\ \frac{\theta - \arcsin\left(\frac{r_{min}\sin\theta - d}{r_{min}}\right)}{M} & \text{if } d < r_{min}\sin\theta,\ 0 \leq \theta \leq \pi/2 \\ \frac{2(\theta-\pi/2)}{M} + T(x, \pi-\theta) & \text{if } \pi/2 < \theta \leq \pi \\ T(x, |\theta|) & \text{if } -\pi \leq \theta < 0 \end{cases} \tag{1}$$

where $d = b_x - x$ is the distance between the aircraft and the no-fly zone. Note that s, r_{min}, and M are related by $M = s/r_{min}$.

Note that if the aircraft is at the wall and heading directly away from it, as in the top diagram of figure 3, then the minimum time for aircraft/no-fly zone collision is the time required to traverse a semi-circle with radius r_{min}. This is π/M. If the aircraft is at distance d from the wall and heading straight towards it, then the minimum time to contact is d/s, where s is the (fixed) speed. If the aircraft is at distance d from the wall (greater than r_{min}) but heading parallel to it, then the time it will take to reach the wall is

$$T = \pi M/2 + (d - r_{min})/s.$$

3.2 Criticality-Based Soft Wall Controller

The proposed criticality-based controller produces the bias shown in figure 4. The threshold M/π is the value of $c(x, \theta)$ when $x = b_x$ (the aircraft is on the boundary of the no-fly zone), and $\theta = \pi$ (the aircraft is flying straight out of the no-fly zone). No bias is needed here, nor for smaller criticality. The threshold $M/2$ is equal to $c(b_x - 2r_{min}, 0)$ – the aircraft is flying straight towards the no-fly zone at a distance of $2r_{min}$ from the zone. The aircraft can still safely turn away at half the maximum turning rate. Note that the maximum bias level is at $3M/2$, so the plane will be turned at a rate of at least half the maximum, irrespective of pilot input. The bias sign convention is the same as θ (positive – left, negative – right).

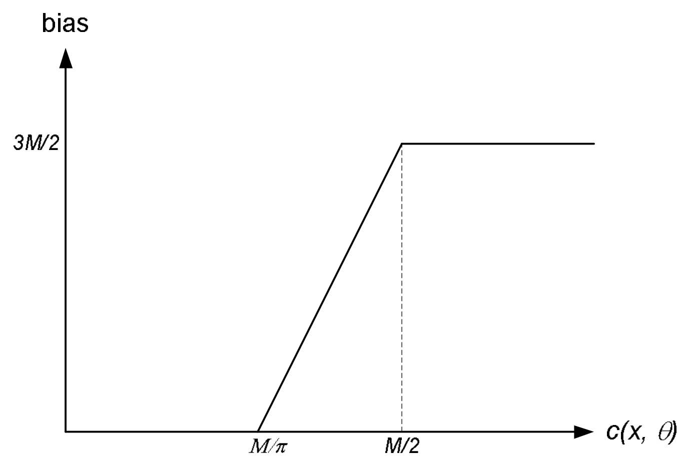

Figure 4: Bias as a function of criticality.

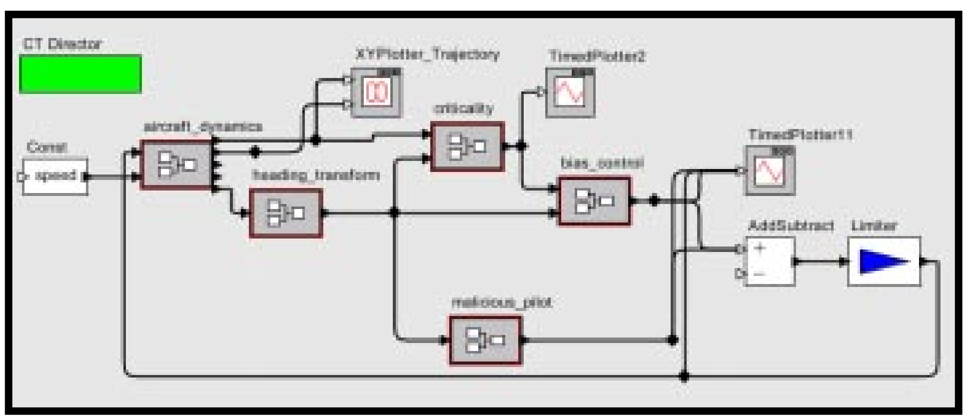

Figure 5: Top-level of a model with a maximally uncooperative pilot.

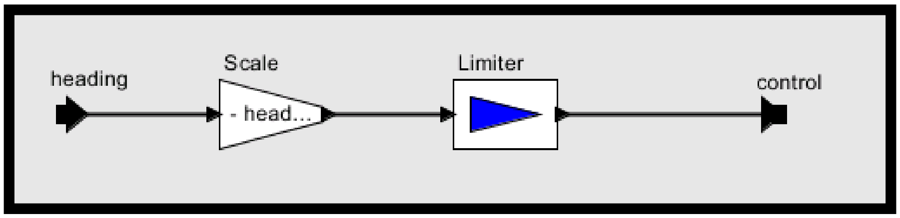

Figure 6: Maximally uncooperative pilot model. The scale factor is large.

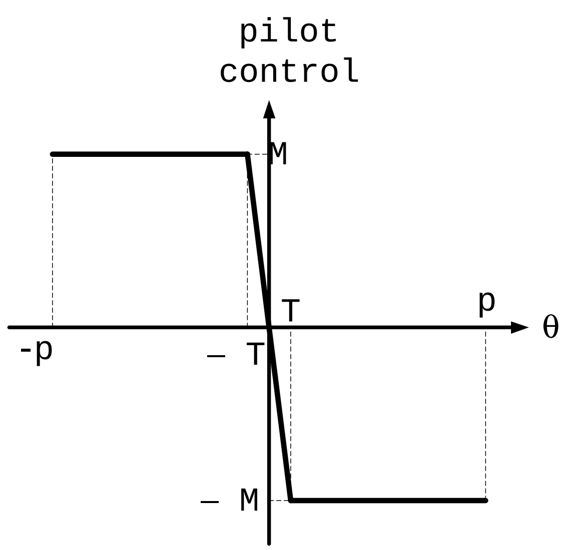

Figure 7: Pilot control as a function of heading for the maximally uncooperative pilot.

3.3 Simulation Results

We constructed the model of figure 5 with Ptolemy II to simulate the proposed controller. The *aircraft dynamics* component contains the aircraft model of figure 1. The *malicious pilot* component implements the control strategy of figure 6. Here, the pilot tries to fly the aircraft into the no-fly zone by maintaining the heading θ at 0. This is accomplished by multiplying the heading by a large number and limiting the result to a number in the safe-control range. Intuitively, the pilot will attempt to turn maximally towards the wall whenever the current heading deviates from the heading directly towards the wall, as shown in figure 7. When the plane is approaching the wall, this pilot does not try to turn. The nonzero slope near $\theta = 0$ prevents the system from exhibiting Zeno behavior. The *criticality* component calculates $c(x, \theta)$. The *bias control* component implements the soft wall controller. The output from the pilot and the controller are combined and limited to the range $[-M, M]$ before fed back to the aircraft model.

Figure 8 shows a simulation run. In our simulation the aircraft initially flies parallel to the no-fly zone ($\theta = \pi/2$), at a distance of 2 miles. The speed of the aircraft is a constant 360 miles per hour. The maximum turning rate is $2\pi/20$, so that the aircraft can complete a circle in 20 seconds. (Note that these numbers are fictional. Later simulations will use real aircraft-performance characteristics.)

The aircraft starts at the lower left, traveling upwards, and the no-fly zone is two miles to the right, with its boundary oriented vertically. Initially, the malicious pilot freely turns the aircraft toward the no-fly zone. When the aircraft is within 1 mile from the zone, the controller starts biasing the pilot control. Before the bias control reaches M, the pilot mitigates the bias and keeps the aircraft heading toward the no-fly zone, but the pilot control finally saturates when $\theta_s = M$, and the soft wall controller turns the aircraft around at half the maximum rate. As the criticality decreases, the bias from the controller becomes smaller. The pilot regains steerage towards the no-fly zone, but the aircraft settles in flying parallel to the zone. At this time, the pilot is still trying in vain to fly the aircraft into the no-fly zone by placing the control at the right maximum.

We are still improving this criticality-based control scheme. At present, we are simulating a variety of flight scenarios, and investigating interactive simulation where experimenters control the pilot's output.

3.4 Criticality-Based Control Verification

3.4.1 Validity of the Criticality Measure

The trajectories we use to calculate $T(x,\ \theta)$ are illustrated in figure 3. Such trajectories are achieved by first turning the aircraft toward flying straight on to the no-fly

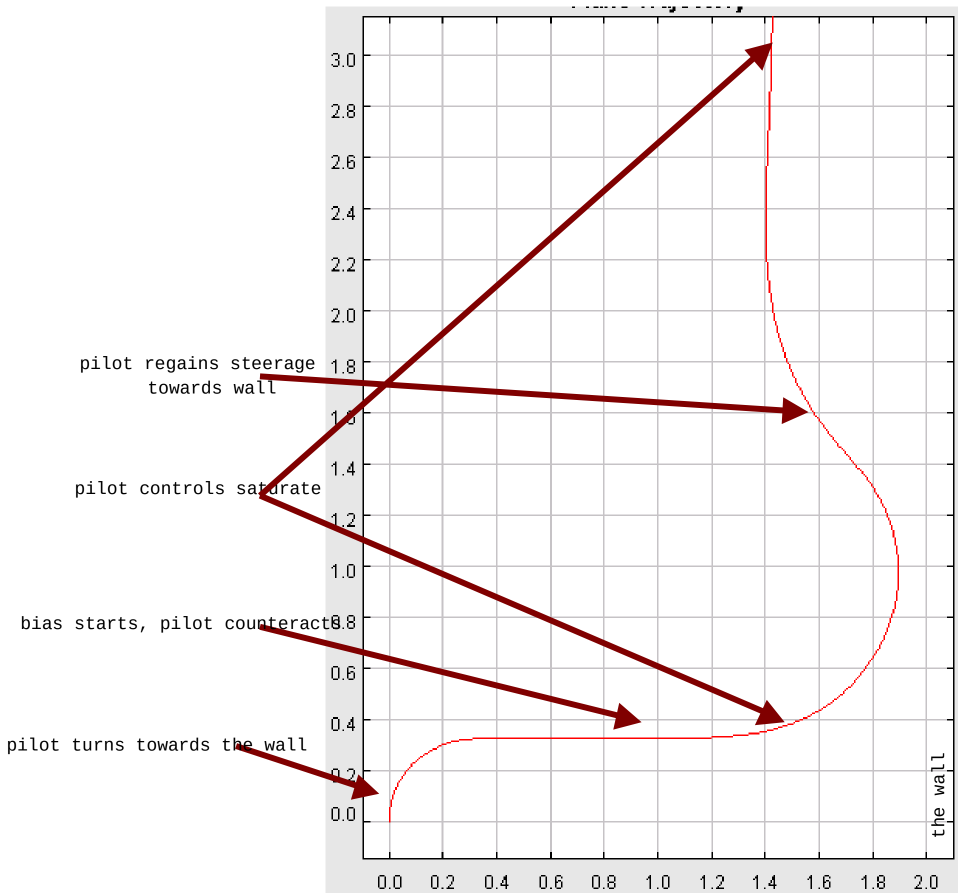

Figure 8: Simulation run with a maximally uncooperative pilot.

zone at the maximum rate M, then maintaining that direction. In the following we argue that such trajectories indeed yield the shortest time to reach the no-fly zone.

Let $\dot{x}(t)$ denote the plane's velocity in the direction perpendicular to the wall, and let $\theta(t)$ denote the plane's heading. From our aircraft model, where s is the plane's constant speed,

$$\dot{x}(t) = s\cos\left(\int_0^t \dot{\theta}(\hat{t})d\hat{t}\right)$$

Here $\dot{\theta}(t)$ is the heading's rate of change. When the softwalls system applies no bias, this signal equals the pilot input. θ is always in the range $[-s/r_{min}, s/r_{min}]$.

When $x(t) < b_x$, i.e., the plane is left of the no-fly zone, the plane approaches the no-fly zone faster as $\dot{x}(t)$ increases. The maximum value of $\dot{x}(t)$ is s. If $\theta(t) = 0$, then $\dot{x}(t) = s$, so an input of $\dot{\theta}(t) = 0$ will cause the plane to move to the wall the fastest. When $\theta(t) \neq 0$, as $\theta(t) \to 0, \dot{x}(t) \to s$. For $\theta(t) \in (0, \pi]$, the fastest way to make $x(t) \to s$ is to set $\dot{\theta}(t) = -M$, where M is the maximum turning rate. In this range of angles, $\dot{x}(t)$ will be strictly increasing at the maximum rate, so this is the fastest approach to the wall. Similarly, when $\theta(t) \in (-\pi, 0)$, the fastest approach to the wall uses $\dot{\theta}(t) = M$ until $\theta(t) = 0$. The criticality measure uses this strategy to calculate the minimum time for aircraft/no-fly zone collision, so it is a valid minimum-time calculation.

3.4.2 Safety of Criticality-Based Control

We assume that the initial position of the aircraft is at least $2r_{min}$ from the no-fly zone. With the criticality-based control strategy discussed earlier in this section, we show that the aircraft cannot be flown into the no-fly zone, no matter what the pilot does.

At the initial position, the criticality $c \leq M/2$. Because c is a continuous function of x and θ, along any potential trajectory from the initial position to the no-fly zone, there must be a point where $c(x,\ \theta) = M/2$. The pair $x,\ \theta$ satisfying this equation is related by

$$x = \begin{cases} b_x - r_{min}(2 + \sin|\theta| - |\theta|) & |\theta| \leq 2 \\ b_x - r_{min}(\sin|\theta| - \sin(|\theta| - 2)) & 2 < |\theta| \leq \pi/2 + 1 \end{cases}$$

$c(x,\ \theta)$ is always less than $M/2$ when $|\theta| > \pi/2 + 1$.

If a malicious pilot wants to fly the aircraft from a point where $c(x,\ \theta) = M/2$ into the no-fly zone, the pilot has to prevent the criticality from decreasing to a value less than $M/2$. Given the bias added by our control strategy, the aircraft will be turned away from the no-fly zone at a minimum rate of $M/2$ when $c(x,\ \theta) \geq$

$M/2$. Starting at a point where $c(x,\ \theta) = M/2$, the maximum x-coordinate that the aircraft can reach is given by

$$x_{max} = \begin{cases} d_x - r_{min}(3\sin|\theta| - |\theta|) & |\theta| \leq \pi/2 \\ d_x - r_{min}(2 + \sin|\theta| - |\theta|) & \pi/2 < |\theta| \leq 2 \\ d_x - r_{min}(\sin|\theta| - \sin(|\theta| - 2)) & 2 < |\theta| \leq \pi/2 + 1 \end{cases}$$

x_{max} is never greater than b_x, so the aircraft is never inside the no-fly zone.

4 Summary

We have described a simple control algorithm to keep an aircraft out of a no-fly zone with a straight-line boundary in two dimensions. Our strategy maintains maximal responsiveness to pilot controls subject to the constraint that we forbid no-fly zone entry.

References

[Lee 2001] Edward A. Lee. “Soft Walls - Modifying Flight Control Systems to Limit the Flight Space of Commercial Aircraft”. *Technical Memorandum UCB/ERL M01/31*. 15 September 2001.