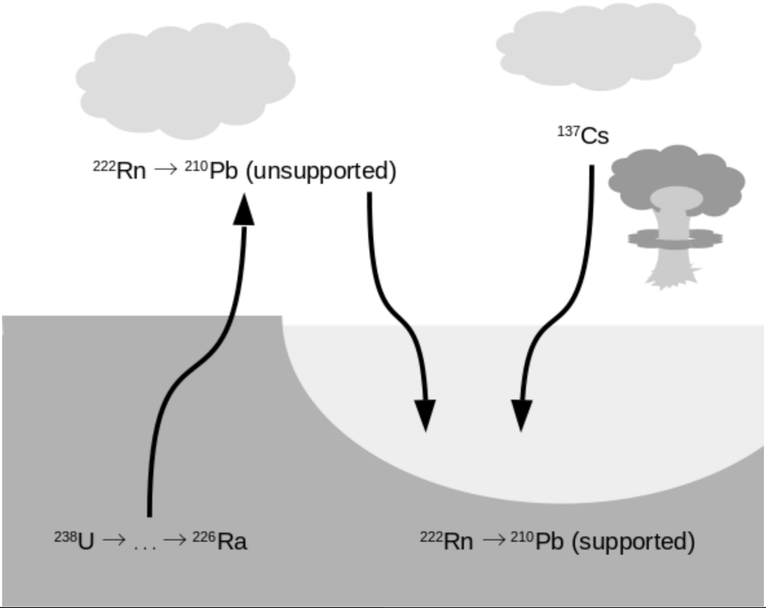
Bayesian methodologies for dating and interpreting palaeoecological data

Dr. Marco A. Aquino-López

October 31, 2022

Cycle ^{210}Pb



^{222}Rn → ^{210}Pb (unsupported)
^{137}Cs
^{238}U → . . . → ^{226}Ra
^{222}Rn → ^{210}Pb (supported)

Constant Rate of Supply assumption

By assuming a Constant Rate of Supply assumption the unsupported ^{210}Pb, for a given section (a,b),

$$C_0(t(x))r(t(x)) = \Phi,$$

This means that at higher rate of accumulation, less ^{210}Pb will be found a in a given section and vise versa. Given that ^{210}Pb is a radioactive isotope, we can use the decay equation,

$$C(t(x)) = C_0(t(x))e^{-\lambda t(x)}$$

Now considering the following equation,

$$r(t) = \rho(x)\left[\frac{dt(x)}{dx}\right]^{-1}$$

We get,

$$A(a,b) = \int_a^b C(z)\rho(z)dz = \int_a^b \Phi e^{-\lambda t(z)}dz$$

CRS Model

By integrating the previous equation over the whole section and solving for $t(x)$,

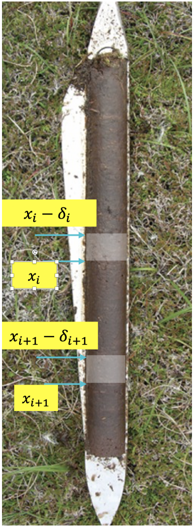
$$t(x) = \frac{1}{\lambda} \log \left(\frac{A_0}{A_x} \right)$$

Appleby & Oldfield (1979) ; Robbins (1979)

Plum Model

Plum uses a proper statistical framework to estimate the age of the sediments.
First we define a proper statistical distribution for each sample,

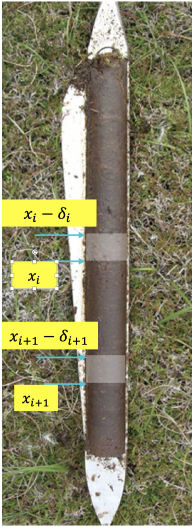
$$y_i \sim \mathcal{N}\left(\mu_i^s + \mu_i^u, \sigma_i\right)$$



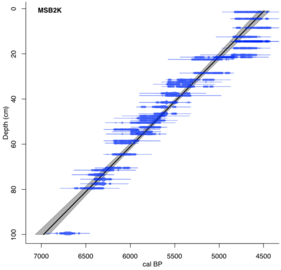
Plum model

Let μ_i^s be the "true" supported ^{210}Pb and μ_i^u the unsupported (excess) in sample y_i.
If we assume a constant flux of ^{210}Pb (Φ_i) in y_i,

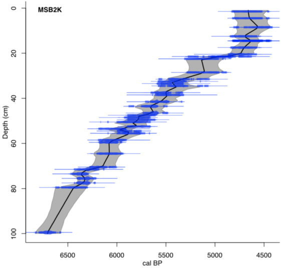
$$\mu_i^u = \frac{\Phi_i}{\lambda}\left(e^{-\lambda t(x_i-\delta)} - e^{-\lambda t(x_i)}\right)$$



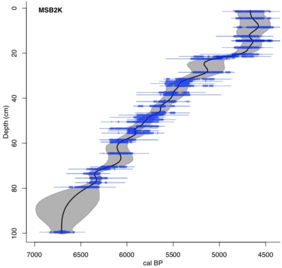
Age-depth model (Clam)



(A)



(B)



Age-depth model

Bacon model

$$
\begin{aligned}
G(d,m) &= \sum_{j=1}^{i} m_j \Delta c + m_{i+1}(d - c_i), \\
m_j &= \omega m_{j+1} + (1-\omega)\alpha_j
\end{aligned}
$$

Plum Model

Plum Likelihood

$$
\begin{aligned}
y_i \mid P_i^S, \Phi_i, \bar{t} \;&\sim\; \mathcal{N}\left(\mu_i^s + \frac{\Phi_i}{\lambda}\left(e^{-\lambda G(x_i-\delta_i)} - e^{-\lambda G(x_i)}\right), (\sigma_i \rho_i)^2\right) \\
\ell(Y \mid P_i^S, \Phi_i, \bar{t}) \;&\propto\; -\sum_{i=1}^{n} \frac{\left(y_i - \left(A_i^S + \frac{\Phi_i}{\lambda}\left(e^{-\lambda G(x_{i-1},m)} - e^{-\lambda G(x_i,m)}\right)\right)\right)^2}{2\sigma_i^2} \\
&\quad -\sum_{j=1}^{n_s} \frac{(y_j^S - P^S)}{2\sigma_j^2}
\end{aligned}
$$

Prior Distributions

- Supported ^{210}Pb:

$$\mu_i^s \propto Gamma(2, 1/10)$$

- Influx of ^{210}Pb:

$$\Phi \propto Gamma(2, 1/25)$$

- Accumulation rate:

$$\alpha_i \propto Gamma(1.5, 1.5/10)$$

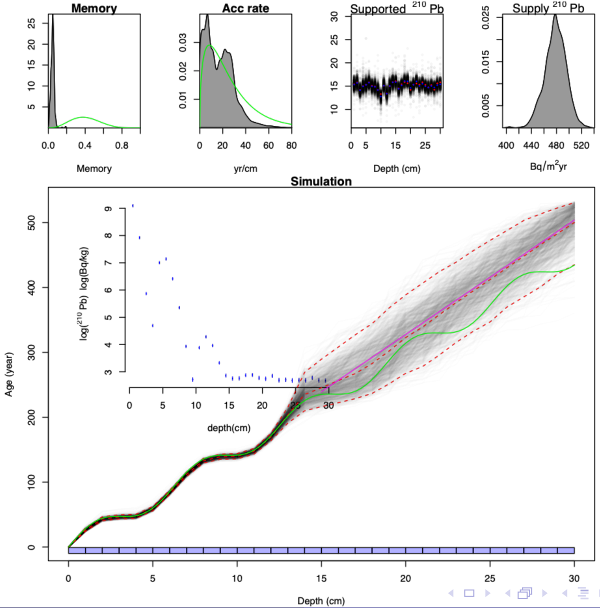
- Memory:

$$\omega_i \propto Beta(5, 5)$$

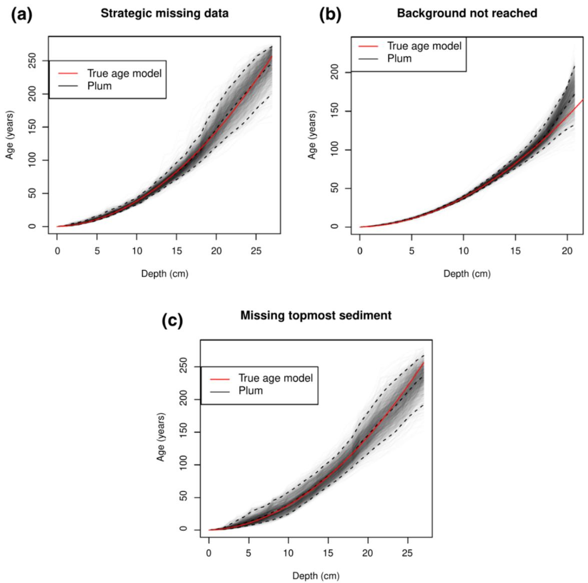
Note: In this case the influx of ^{210}Pb is assumed constant to facilite the computational calculations and to reduce parameters.

Simulación de *Plum*

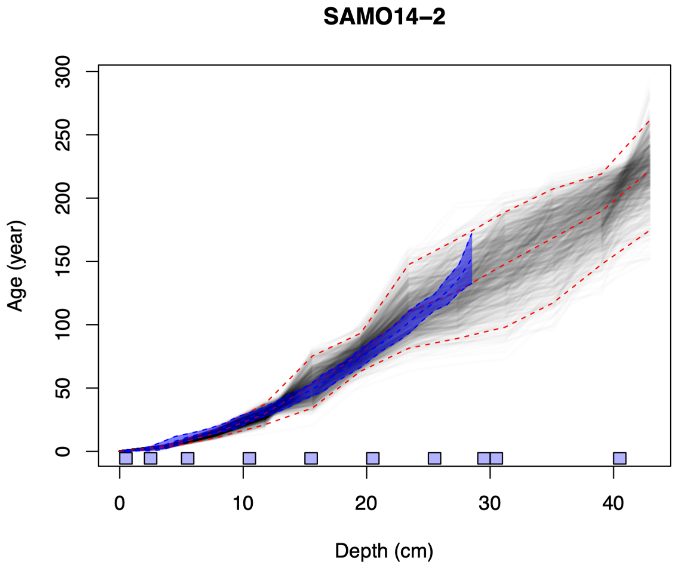
Learning process



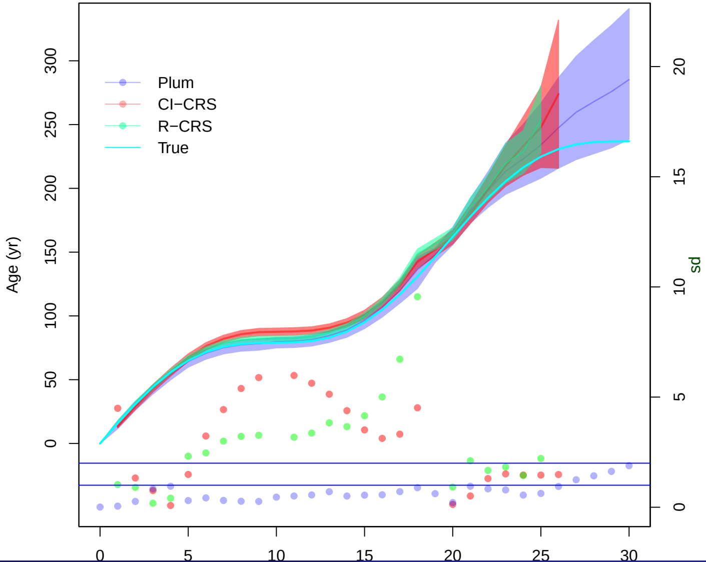
Learning process



Precision with realistic uncertainties



Precision with realistic uncertainties

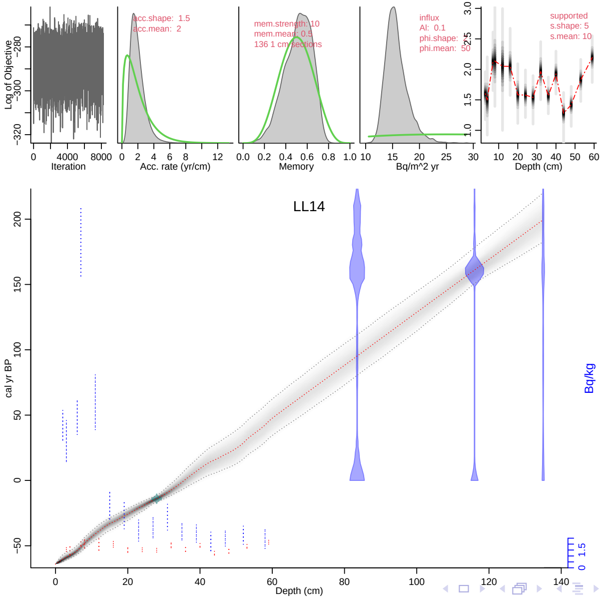


Other Dating sources

137Cs and ^{14}C

$$\mathcal{L}\left(\Theta\right) = \mathcal{L}_{^{210}Pb}\left(\Theta\right)\mathcal{L}_{^{14}C}\left(\Theta\right)\mathcal{L}_{^{137}Cs}\left(\Theta\right). \tag{1}$$

Other Dating sources



Reservoir Effect

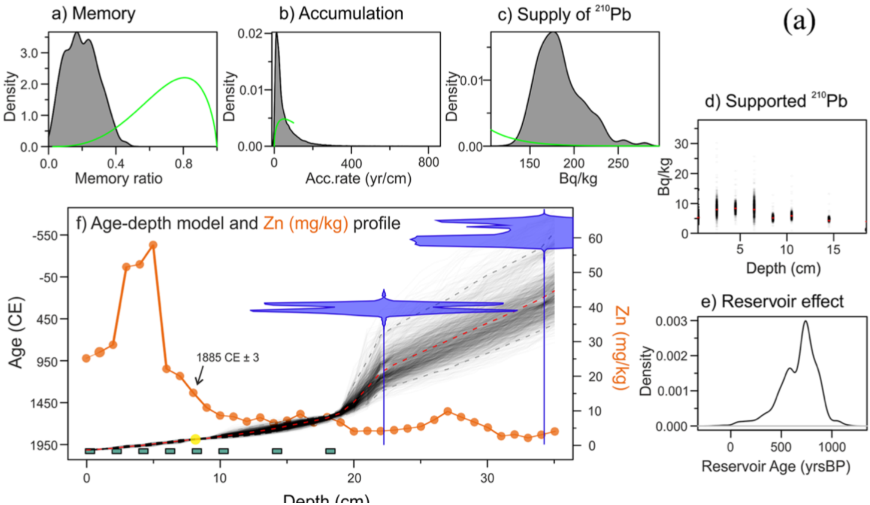
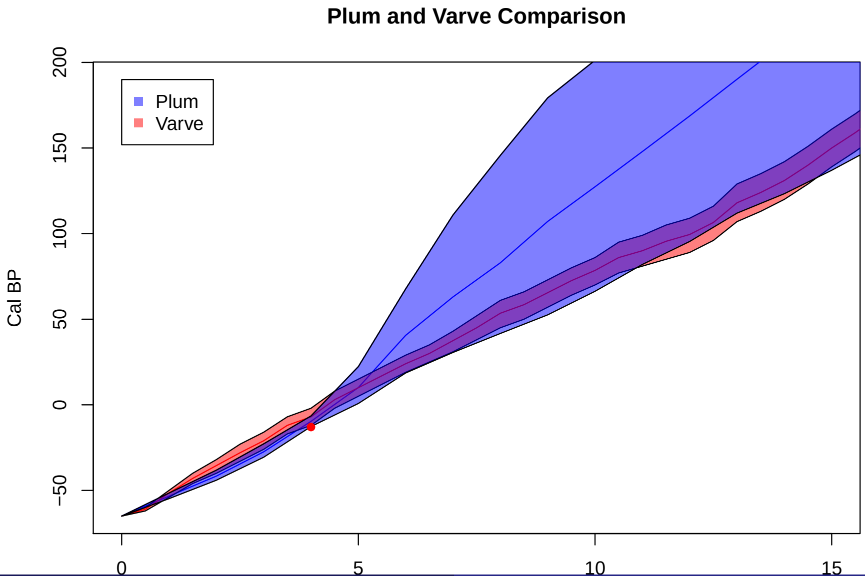
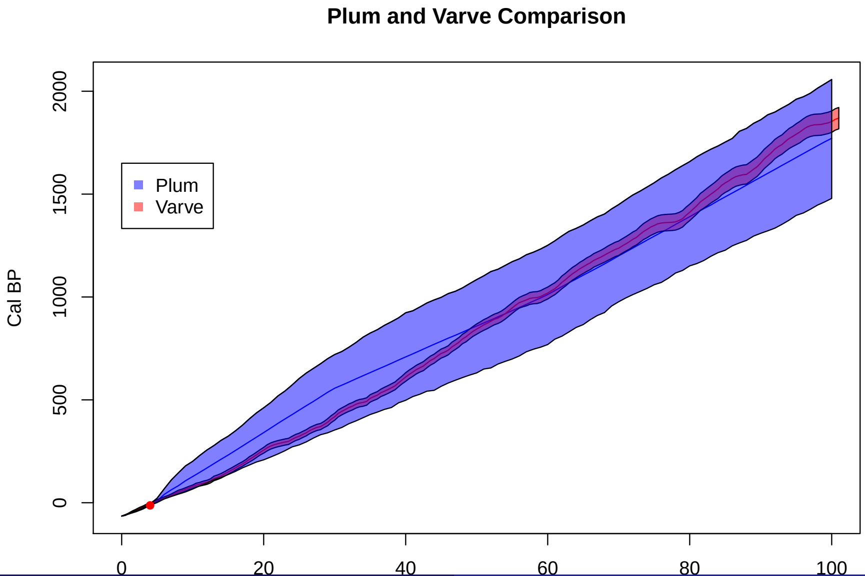


Figure:

Reservoir Effect



Reservoir Effect



Gracias

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