A simulation study to compare Pb dating data analyses

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The increasing interest to understand anthropogenic impacts on the environment have led to a considerable amount of studies that focus on sedimentary records of ~ 100 – 200 years. Dating this period is often complicated by the poor resolution and large errors associated with radiocarbon (14C) ages, which is the most popular dating technique. Instead, sediment dating with lead-210 (Pb) is widely used it provides absolute and continuous dates for ~ 100 – 150 years. The Pb dating method has traditionally relied on the Constant Flux (CF, also known as Constant Rate of Supply - CF) model which uses the radioactive decay equation resulting in a restrictive model to approximate dates. In this work, we compare the classical approach to Pb dating (CF) and its Bayesian alternative (*Plum*). For this, we created simulated 210Pb profiles following three different sedimentation processes, complying the assumptions imposed by the CF model, and analyzed them with both approaches. Results indicate that the CF model does not capture the true values even when the sediment is entirely dated, nor improves its accuracy as more information is available. On the other hand, the Bayesian alternative (*Plum*) provides consistently accurate results even with few samples, and its accuracy and precision constantly improves as more information is available.

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*Keywords:* Plum, Age-depth models, Chronology, Constant Rate of Supply, Comparison.

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# Introduction

Pb is a radioactive nuclide which naturally forms in the atmosphere (as well as in sediments) as result of the decay chain of U. This isotope, with a half-life if 22.23 yr, is commonly used to date recent recently accumulated sediments ( to yr). Unlike to other dating techniques such as C (radiocarbon dating), a single measurement of Pb is useless for dating and it is only when a suitable portion of the decay curve is measured (together with certain assumptions about the sedimentation process) that a chronology can be established. In recent decades, increasing amounts of palaeoecological and pollution studies have focused on recent sediments in order to measure the human impact in the environment. These studies strongly depend on the accuracy of the chronologies in order to correctly assign dates to chemical and biological changes. That is, unlike other dating techniques, an analysis of a series (data set) of Pb measurements must be carried out in order to obtain meaningful dates. In a lake sediment, or any other, sedimentation process, samples are taken along a core at different depths, from which Pb activity is measured. The whole series of Pb measurements need to be analyzed to attempt to produce a coherent chronology, see .

A range of traditional data analyses, or so called “models”, are available for dating recent sediment using Pb, most notably are the Constant Flux (CF, also known as the Constant Rate of Supply - CRS), Constant Flux : Constant sedimentation (CFCS) and Constant Activity (CA, also known as Constant Initial Concentration - CIC) models (Appleby and Oldfield, 1978; Robbins, 1978; Sanchez-Cabeza and Ruiz-Fernández, 2012). The CF model is by far the most popular (see Figure [1](#fig:210models)) and has the most flexible assumptions. It assumes a constant supply of Pb to the sediment from the atmosphere and allows for changes in the sedimentation rate. In order to estimate a chronology the CF model uses a ratio between the complete “inventory” (the complete estimate of the radioactivity in the column of the sediment between the surface and a certain depth where Pb from the atmosphere can no longer be found) and the remaining inventory from depth (, where is the complete inventory, and the decay constant of the Pb ).

Other, more restrictive, models such as CFCS and CA also require the assumption of a constant flux of Pb and other assumptions of the sedimentation process. The flexibility of the CF model comes at the cost of the need to measure a sufficient portion of the inventory or the use of interpolation in order to properly estimate the complete inventory of Pb in the sediment.

This model has received several revisions in order to improve its accuracy and applicability. There are two types of revisions to this model: revisions to its uncertainty, and to its applicability when extra information is available, such as extra dating sources (e.g. Cs profiles) or laminated sediments .

A recent inter-laboratory experiment presented a series of Pb data to different laboratories around the world. Each laboratory was ask to provide a chronology given the provided data. This experiment resulted in a whole range of different chronologies independently of the model used, and the authors strongly advised to the use of independent time markers (extra dating sources) to validate the chronologies. This work clearly shows the effect that user decisions have on the resulting chronologies, extremely important when trying to replicate the resulting chronologies. This requires not only access to the raw data but also to every user decision leading the resulting chronology, rarely reported.

![Frequency of ^{210}Pb dating models used in papers between 1964 and 2017. Data gathered by  from a literature review of 271 papers. The models include CF:CS model The Constant Flux - Constant Sedimentation;, CIC (Constant Initial Concentration)  and CRS - Constant Rate of Supply;. ](data:application/pdf;base64,)

Frequency of Pb dating models used in papers between 1964 and 2017. Data gathered by from a literature review of 271 papers. The models include CFCS model , CA (Constant Initial Concentration) and CF - .

Recently presented an alternative to these classical models, by introducing *Plum* (a Bayesian approach to Pb dating). This model treats every data point as originating from a system, which includes both the sedimentation process and the decay process. It also incorporates the levels of supported Pb, which naturally forms in the sediment and is normally threaded as a hindrance variable, and important component of the inferred processes. *Plum* assumes that there exists an (unknown) age-depth function that relates depth with a calendar age . Conditional on , the following model is assumed for the measured Pb for the sediment section form depths to

Here is the supported Pb in the sample and the supply of Pb to the sediment, see for details. The age-depth model is based on a piece-wise linear model constrained by prior information on the sediment’s accumulation rates .

This treatment of the data allows for a formal statistical inference and by using a Bayesian approach all the parameters of the model can be inferred. This differs from the CF model because the latter uses the decay equation to obtaine the age-depth function (which results in a more restrictive age-depth model), removes assumed values of supported Pb before modelling, and does not provide a formal statistical inference. *Plum* has shown to provide accurate results with a realistic precision depending on the different case scenarios - both in simulations as well as for real cores. Under optimal conditions *Plum* and the CF model have shown to provide similar results, with *Plum* providing more realistic uncertainties with minimal user interaction.

The objectives of this study were i) to compare the CF model and *Plum* by using simulated cores, i.e. sedimentation “scenarios”, and ii) to quantify the learning process of each of the models and how much information is needed to obtained a reasonable chronology for each model.

The paper is organized as follows: first section sets the tools for the comparison, it describes the simulations of the three different scenarios and we described a parameter which will facilitate the comparison called information percentage. Section 3 describes the comparison for both the overall chronologies and by single depths. Lastly section 4 presents the conclusions and discussion of the results obtained in section 3.

# Simulated data

In order to observe the accuracy and precision of a model, we need data where the true age-depth function is known. presented a methodology for simulating radiocarbon dates and their uncertainty, whereas presented an approach for simulating Pb data given an age-depth function . These simulations follow the equations presented by guaranteeing that the CF assumptions are met. By using the approach presented by for simulating Pb data and the structure of uncertainty estimation presented by , reliable Pb simulated data can be obtained.

## Simulation Construction

Three different scenarios (see Table [1](#tab:sim_param)) were chosen to simulate sedimentation processes, including their age-depth functions and parameters. This scenarios were selected as they provide three challenges for the models: the first scenario presents an age-depth function which is the result of increasing sedimentation and less compaction towards the present; the second (challenging) scenario is a drastic and quick change in sediment behaviour around depth 15 cm depth; the last scenario presents a cyclic and periodic change in accumulation. Using the age-depth functions and parameters in Table [1](#tab:sim_param), we may obtaine the Pb activity at any depth or interval. Although these concentrations may be interpreted as error-free measurements (Figure [2](#fig:true_210)), we need to replicate the Pb activity uncertainties, following a similar methodology to radiocarbon dates (Blaauw et al., 2008).

Simulated age-depth functions and parameters used in each scenario.

|  |  |  |  |
| --- | --- | --- | --- |
| Label | Age-depth |  | Supported Pb |
|  | function | () | () |
| Scenario 1 |  | 100 | 10 |
| Scenario 2 |  | 50 | 25 |
| Scenario 3 |  | 500 | 15 |

[tab:sim\_param]

![Simulated sedimentation scenarios with their corresponding ^{210}Pb profiles. Right: Age-depth function for the three different scenarios (Table 1). Left: Corresponding ^{210}Pb activity profiles in relation to depth.](data:application/pdf;base64,)

Simulated sedimentation scenarios with their corresponding Pb profiles. Right: Age-depth function for the three different scenarios (Table [1](#tab:sim_param)). Left: Corresponding Pb activity-depth profiles.

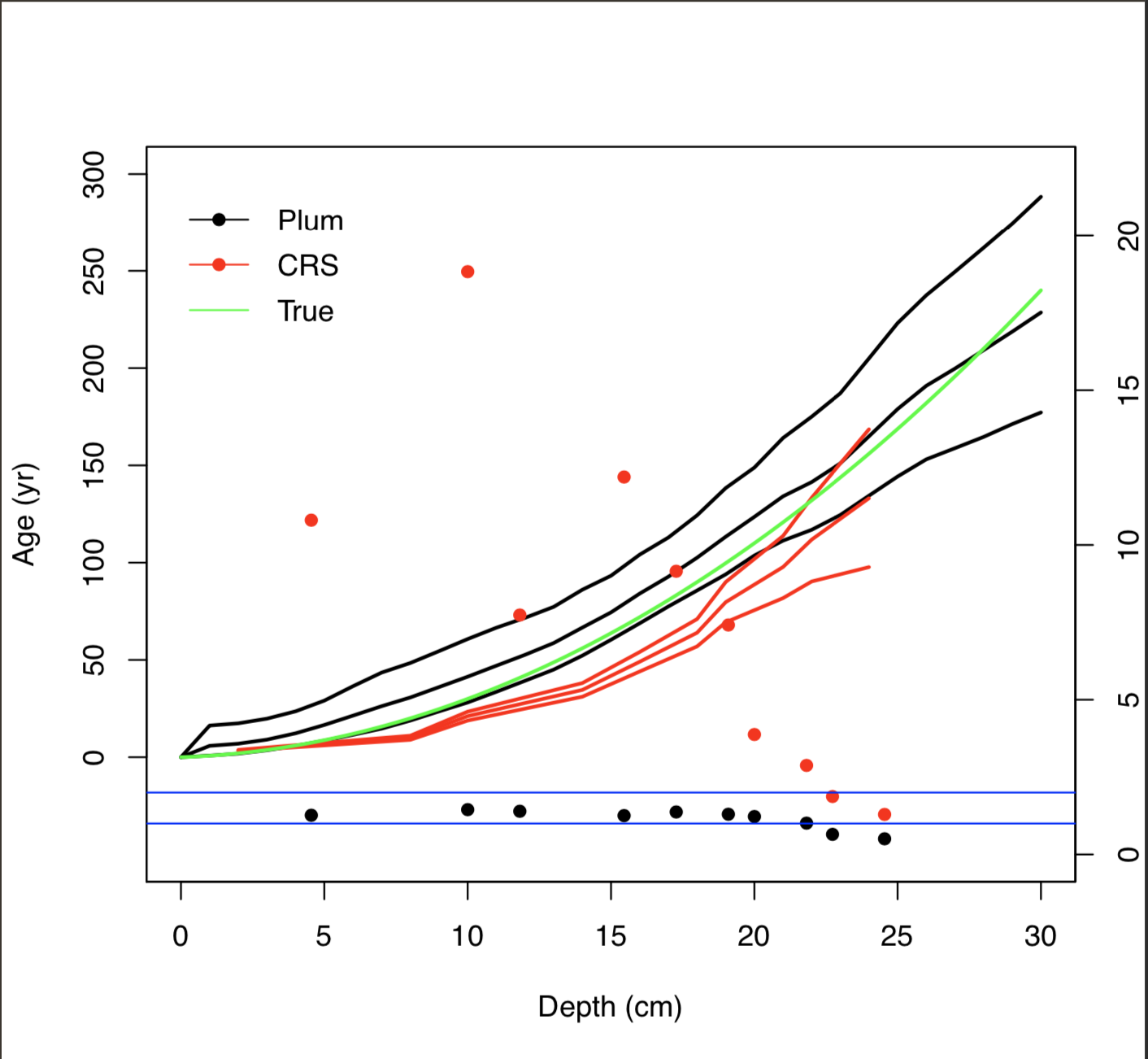
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Let be the true Pb concentration in the interval , given the age-depth function and parameters and in each scenario. To simulate disturbances in the material, we can introduce scatter centred around the true value, , where is the amount of scatter for this variable (in this case ). Now, to replicate outliers, a shift from the true value () is defined, which occurs with a probability . This results in a new variable which is defined as

Finally, to simulate the uncertainty provided by the laboratory, we can define the simulated measurements as , where is the standard deviation reported by the laboratory. is defined as , where is the minimum standard deviation assigned to a measurement. This variable differs between laboratories (we used a default value of ). Finally, is the analytical uncertainty (default .01) and an error multiplier (default 1.5). The default parameters were set in accordance with . For this this study we created a data set for each simulation by integrating in intervals of 1 cm, for depths from 0 to 30 cm where equilibrium was guaranteed . The complete simulated Pb data sets can be found in the Supplementary Material [5](#sec:supp_mat).

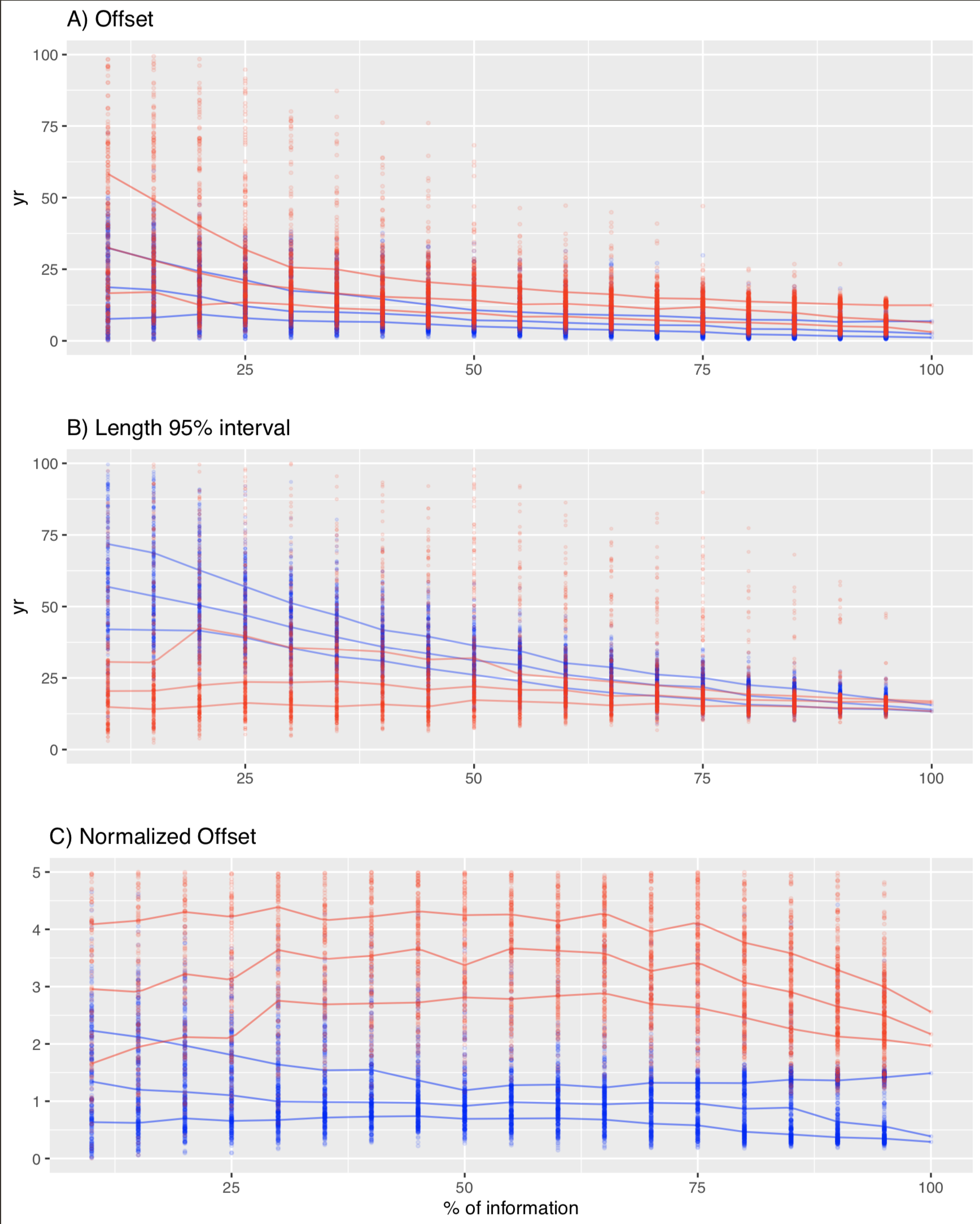
# Model Comparison

To allow for a reasonable comparison between models and to observe the effect that different amount of information have on the accuracy and precision of Pb models, we used the three simulated data sets(Table xxx). These simulated cores were randomly selected given a percentage of information (e.g. for a 20% information sample, in a 30 cm cores, 6 random 1-cm samples were selected). Because the CF model assumes that the background activity has being reached, we fixed the last sample (30 cm depth) for every case. This guaranties the proper use of the CF model and also gives the model a single last depth to be removed as it is common practice when using the CF model. 100 different samples were randomly selected for information percentages from 10% to 95% at 5% intervals (i.e., 10%, 15%, 20%,...,95%) and the complete sample was also used (i.e 100% percentage of information sample). After a random sample was selected, both the CF model and *Plum* were applied to the data set. To have an objective comparison, both models were run with their default configuration (*Plum* with default settings and CF estimates and uncertainties as described in ) and their outputs were compared against the true age value.



Comparison between Plum and the CF model against the true age-depth model using 50% of the sections (1-cm samples at depths 2, 8, 10, 14, 16, 18, 19, 21, 22, 24, 25, 27, 28, 29, 30). Lines show the age estimates with the 95% credible intervals (Plum) and the 95% confidence interval (CF). Dots show the normalized offset, the distance between the inferred age and the true age divided by the standard error (the standard deviation in the case of the CF model and the length of the confidence interval divided by 4 in the case of Plum).

Figure [3](#fig:comparison1r) shows an example of the comparison between the Pb models against the true value. Because we are dealing with 5333 simulations, in order to observe the precision and accuracy in general, we calculated the mean offset to the true age-depth model (in yr), the mean of the length of the 95% intervals (in yr), and the mean normalized accuracy, which indicates how far the model is from the true value given its own uncertainty at that depth.

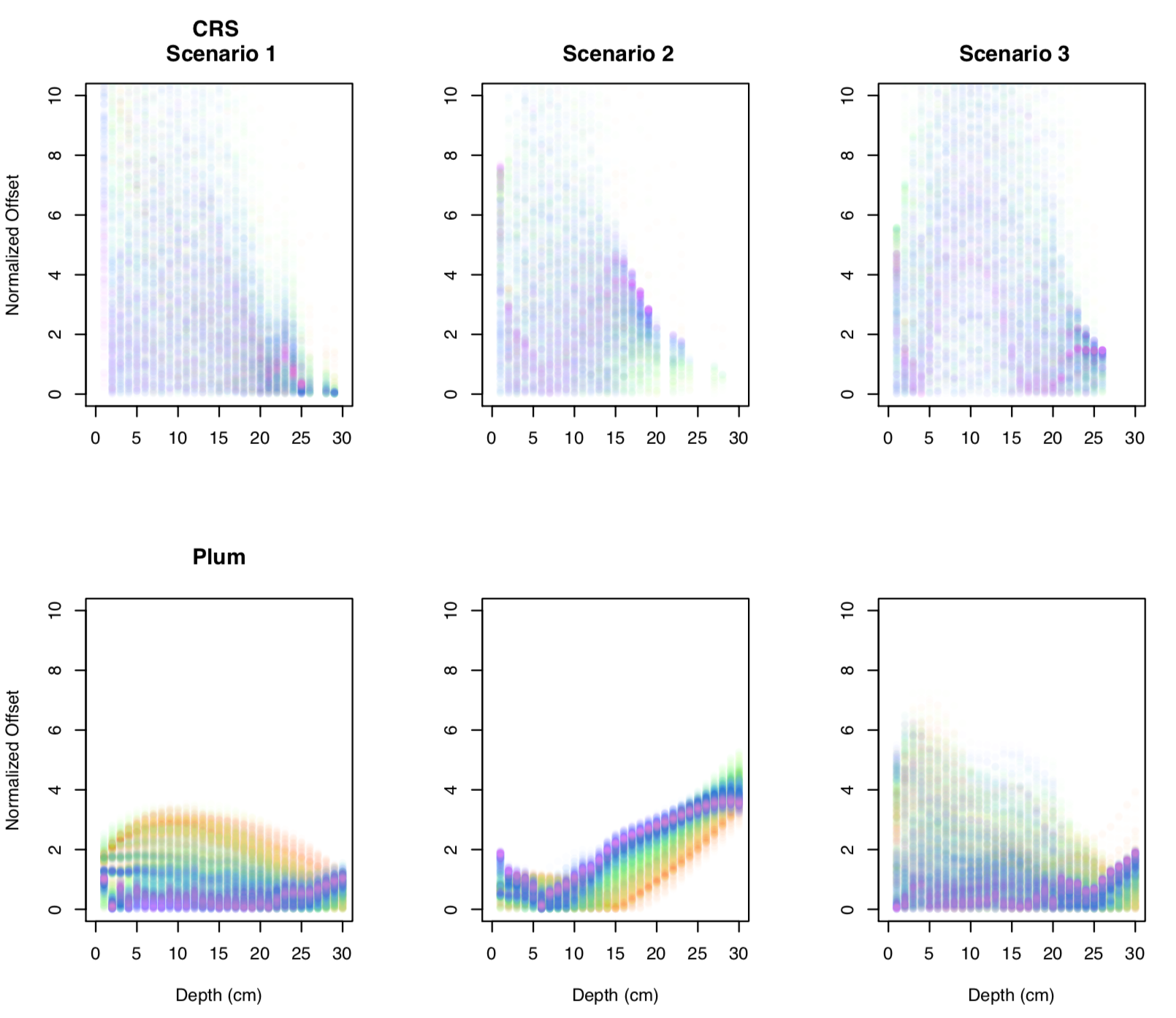


Top panel A) shows the offset between the modelled and true age of the CF (red) and Plum (blue). This panel shows how Plum provides a small offset in almost every scenario with both models improving their offset as more information is available. Middle panel B) shows the 95% confidence intervals. It is clear, from this panel, than the uncertainty provided by Plum is a lot bigger for low percentage of information and it constantly improves as more data is available, whereas the length of the intervals provided by the CF appear to stay constant regardless of the available information. Bottom panel C) shows the normalized offsets, presenting the distance between the modelled age and the true age normalized divided by the standard deviation (in the case of Plum, the length of the 95% interval divided by 4). This panel presents a worrying situation where the CF model’s calculated standard deviation (on average) is incapable of of capturing the true age. On the other hand, Plum’s credible intervals almost always capture the true age even when little information is available.

Figure [4](#fig:accpre) show similar results those presented by . The classical model (CF) at first appears to provide a similar results (similar offsets) to the Bayesian alternative (*Plum*), but at higher estimated precision (if we only look at the length of the 95% interval). These results can be misleading if we do not analyze the effects of both the offset and length of the interval together. To have a more realistic representation of how the models capture the true age-depth models we can observe the normalized offset. This variable shows to which degree the average models contain the truth within their uncertainty intervals (normalized to one standard deviation). Any model with normalized offset larger than two (two standard deviations) is incapable of capturing the true ages within their uncertainty intervals. This means that the CF estimates smaller uncertainties, yet at the cost of its accuracy. It also appears that the length of the 95% interval and offset are not affected by how much information is provided to the CF model.

On the other hand, *Plum* shows more accurate results as more information is given to the model. This again coincides with the results found by . When we observe the regular offset (not normalized), we observed that textitPlum provides a smaller offset in comparison to the CF model; this in combination with slightly larger modelled uncertainties results in consistently accurate age-depth models which are capable of capturing the true values within their uncertainty intervals. This result supports the claim that *Plum* provides more realistic uncertainties than those of the CF.

Another important statistic to take into account is that 87.86% (4686/5333) of *Plum*’s runs remain under the 2 standard deviations, whereas the CF model only has 7.48% (399/5333) under the 2 standard deviations, and only 0.54% (29/5333) lies under the 1 standard deviation, which is the most commonly reported interval when reporting CF results. We can also observe a clear structure in the way *Plum* increases its accuracy and precision to obtaine a better chronology as more information is available, whereas the CF model does not appears to learn from more data. These results are presented for the overall chronology (the mean offset, interval and normalized offset of the overall chronology). In order to observe if certain models are better predicting at certain section of the sediment, we have to look at the normalized offset of every depth.



Normalized offset of every random sampling at every depth for the three simulated scenarios - CF age estimates at samples depths and Plum’s age estimates at 1 cm intervals. Dots go from lowest information percentage samples (few dated depths; red) to high percentage samples (nearly completely dated cores; purple). The CF’s normalized offset shows no structure at any particular depth regardless of the available information. This means that the model can provide a reasonable chronology with low levels of information or a very inaccurate age estimate with high levels of information at any given depth resulting in a distrustful age-depth model. On the other hand, Plum demostrares a systematic improvement in its age estimates as more data is available. This results assures that a Bayesian approach would consistently provide more reliable results.

Figure [5](#fig:depths) shows the normalized accuracy of every simulation by depth for both models. *Plum* shows a clear learning structure which depends on the information available to the model. The information percentage appears to be irrelevant to the accuracy of the CF model, contrary to the results obtained by *Plum*. It is important to note that the inaccuracies of the CF model are not exclusive to any particular sections of the chronology; this is most likely caused by the small uncertainties estimated by the CF model.

# Conclusions and Discussion

These results clearly show the bias of the CF model. discussed this point and states that the bias is the product of the use of a logarithmic function for the age-depth function. It is also evident from these results that the CF model’s uncertainty estimates are not sufficient to capture the true age-depth function. This is an important point given the fact that it is common practice among the Pb dating community to report credible intervals to one single deviation (instead of the 95% confidence intervals, which have become common practice in most other chronology reconstructions). Other confidence intervals can be calculated for this model but the fact that these intervals are even smaller than the ones obtained by error propagation is of concern.

Previous work on model comparison has shown the problems with the variability of Pb results even when different users apply the same or similar models to a single set of data. In this work, user input was reduced to the minimum in an effort to show the potential effects of different information percentages have on the resulting chronology. The results of this experiment showed that the CF model can provide extremely different results even when the data originates from the same data set. Figure [4](#fig:accpre) showed that the CF model appears not to learn from using more data. This explains why over the years, many authors have insisted in the use of other dating techniques to validate the chronology provided by the CF model. These results highly encourage the need of validating the CF chronology before it is use.

On the other hand, *Plum* shows a consistently accurate result by capturing the true values within the 95% credible intervals in by most of the simulated sampling strategies. It is important to note that one of the big advantages of *Plum* is its increasing accuracy and precision reduction as more data becomes available. In the case of proposal 2, we observe that *Plum* appears to behave worse as more data is available, which would be of concern if we do not take into consideration that this sedimentation simulation was extremely unusual in the real world and also if users would not double check the resulting chronologies. If more information is available about the core or the sediment, this information can easily be implemented as prior information in *Plum*, and this will result in a much better chronology. It is also important to note that even when *Plum* performs badly in this specific case, it is providing a much better chronology to upper first 10-15 cm of the core than that of the chaotic CF age-depth model model.

In conclusion, the use of CF can only be accommodated if users may validate their chronology with external dates as the Cs time markers or other time markers. If not extra information is available for this validation, the use of *Plum* is still valid, specially if provide it with as as many Pb measurements as possible (at least 60% of the available information of the core). Even so, additional dating information may also be formally included in *Plum* to improve the resulting chronology .

# Supplementary Material

Data for each simulation and code used is hosted at: https://github.com/maquinolopez/Paper\_Simulations

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Label | Depth | Density | 210Pb | sd(210Pb) | Thickness | 226Ra | sd(226Ra) |
|  | (cm) | () | (Bq/kg) |  | (cm) | (Bq/kg) |  |
| Sim01-01 | 1 | 0.10009 | 63.50103 | 2.85755 | 1 | 23.8045 | 1.125 |
| Sim01-02 | 2 | 0.10064 | 80.08738 | 3.60393 | 1 | 23.2924 | 1.125 |
| Sim01-03 | 3 | 0.10173 | 98.32806 | 4.42476 | 1 | 23.434 | 1.125 |
| Sim01-04 | 4 | 0.10334 | 125.45705 | 5.64557 | 1 | 26.0873 | 1.125 |
| Sim01-05 | 5 | 0.10547 | 141.27971 | 6.35759 | 1 | 22.8041 | 1.125 |
| Sim01-06 | 6 | 0.10809 | 130.27571 | 5.86241 | 1 | 23.4333 | 1.125 |
| Sim01-07 | 7 | 0.11116 | 134.04051 | 6.03182 | 1 | 25.6156 | 1.125 |
| Sim01-08 | 8 | 0.11466 | 129.69245 | 5.83616 | 1 | 26.1371 | 1.125 |
| Sim01-09 | 9 | 0.11855 | 134.93655 | 6.07214 | 1 | 25.4813 | 1.125 |
| Sim01-10 | 10 | 0.12278 | 109.39886 | 4.92295 | 1 | 25.8877 | 1.125 |
| Sim01-11 | 11 | 0.12731 | 110.68133 | 4.98066 | 1 | 24.4414 | 1.125 |
| Sim01-12 | 12 | 0.13209 | 102.38094 | 4.60714 | 1 | 24.9053 | 1.125 |
| Sim01-13 | 13 | 0.13706 | 75.80895 | 3.4114 | 1 | 22.9151 | 1.125 |
| Sim01-14 | 14 | 0.14218 | 77.60406 | 3.49218 | 1 | 24.4808 | 1.125 |
| Sim01-15 | 15 | 0.14738 | 68.4401 | 3.0798 | 1 | 24.9343 | 1.125 |
| Sim01-16 | 16 | 0.15262 | 60.72037 | 2.73242 | 1 | 25.2659 | 1.125 |
| Sim01-17 | 17 | 0.15782 | 50.28147 | 2.26267 | 1 | 22.961 | 1.125 |
| Sim01-18 | 18 | 0.16294 | 44.24641 | 1.99109 | 1 | 22.9139 | 1.125 |
| Sim01-19 | 19 | 0.16791 | 39.85997 | 1.7937 | 1 | 28.3774 | 1.125 |
| Sim01-20 | 20 | 0.17269 | 38.40823 | 1.72837 | 1 | 23.5379 | 1.125 |
| Sim01-21 | 21 | 0.17722 | 32.75922 | 1.47416 | 1 | 25.4363 | 1.125 |
| Sim01-22 | 22 | 0.18145 | 28.02545 | 1.26115 | 1 | 24.8995 | 1.125 |
| Sim01-23 | 23 | 0.18534 | 27.8749 | 1.25437 | 1 | 22.6783 | 1.125 |
| Sim01-24 | 24 | 0.18884 | 30.74797 | 1.38366 | 1 | 24.8575 | 1.125 |
| Sim01-25 | 25 | 0.19191 | 28.36187 | 1.27628 | 1 | 24.8724 | 1.125 |
| Sim01-26 | 26 | 0.19453 | 27.24535 | 1.22604 | 1 | 24.3778 | 1.125 |
| Sim01-27 | 27 | 0.19666 | 23.59236 | 1.06166 | 1 | 24.7209 | 1.125 |
| Sim01-28 | 28 | 0.19827 | 25.74855 | 1.15868 | 1 | 24.6615 | 1.125 |
| Sim01-29 | 29 | 0.19936 | 25.05368 | 1.12742 | 1 | 24.7199 | 1.125 |
| Sim01-30 | 30 | 0.19991 | 25.0065 | 1.12529 | 1 | 24.4937 | 1.125 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Label | Depth | Density | 210Pb | sd(210Pb) | Thickness | 226Ra | sd(226Ra) |
|  | (cm) | () | (Bq/kg) |  | (cm) | (Bq/kg) |  |
| Sim02-01 | 1 | 0.1001 | 909.3928 | 40.9227 | 1 | 8.9761 | 0.45 |
| Sim02-02 | 2 | 0.1006 | 683.9989 | 30.7799 | 1 | 10.0607 | 0.45 |
| Sim02-03 | 3 | 0.1017 | 453.0503 | 20.3873 | 1 | 9.8701 | 0.45 |
| Sim02-04 | 4 | 0.1033 | 310.7897 | 13.9855 | 1 | 10.37 | 0.45 |
| Sim02-05 | 5 | 0.1055 | 218.0058 | 9.8103 | 1 | 10.0418 | 0.45 |
| Sim02-06 | 6 | 0.1081 | 158.6974 | 7.1414 | 1 | 10.104 | 0.45 |
| Sim02-07 | 7 | 0.1112 | 113.9062 | 5.1258 | 1 | 10.2049 | 0.45 |
| Sim02-08 | 8 | 0.1147 | 75.5493 | 3.3997 | 1 | 9.334 | 0.45 |
| Sim02-09 | 9 | 0.1185 | 56.6252 | 2.5481 | 1 | 10.5145 | 0.45 |
| Sim02-10 | 10 | 0.1228 | 44.1595 | 1.9872 | 1 | 9.8677 | 0.45 |
| Sim02-11 | 11 | 0.1273 | 34.7448 | 1.5635 | 1 | 9.7694 | 0.45 |
| Sim02-12 | 12 | 0.1321 | 25.384 | 1.1423 | 1 | 10.5134 | 0.45 |
| Sim02-13 | 13 | 0.1371 | 24.0007 | 1.08 | 1 | 10.4589 | 0.45 |
| Sim02-14 | 14 | 0.1422 | 21.3643 | 1 | 1 | 9.9504 | 0.45 |
| Sim02-15 | 15 | 0.1474 | 17.7932 | 1 | 1 | 10.5135 | 0.45 |
| Sim02-16 | 16 | 0.1526 | 15.0416 | 1 | 1 | 10.3362 | 0.45 |
| Sim02-17 | 17 | 0.1578 | 14.2937 | 1 | 1 | 10.5131 | 0.45 |
| Sim02-18 | 18 | 0.1629 | 12.3844 | 1 | 1 | 10.368 | 0.45 |
| Sim02-19 | 19 | 0.1679 | 12.6023 | 1 | 1 | 10.5297 | 0.45 |
| Sim02-20 | 20 | 0.1727 | 11.9329 | 1 | 1 | 10.0924 | 0.45 |
| Sim02-21 | 21 | 0.1772 | 9.301 | 1 | 1 | 10.118 | 0.45 |
| Sim02-22 | 22 | 0.1815 | 10.7777 | 1 | 1 | 10.249 | 0.45 |
| Sim02-23 | 23 | 0.1853 | 12.9491 | 1 | 1 | 10.134 | 0.45 |
| Sim02-24 | 24 | 0.1888 | 10.6571 | 1 | 1 | 10.1151 | 0.45 |
| Sim02-25 | 25 | 0.1919 | 9.6297 | 1 | 1 | 9.6608 | 0.45 |
| Sim02-26 | 26 | 0.1945 | 8.4331 | 1 | 1 | 8.7821 | 0.45 |
| Sim02-27 | 27 | 0.1967 | 10.4921 | 1 | 1 | 9.8995 | 0.45 |
| Sim02-28 | 28 | 0.1983 | 11.135 | 1 | 1 | 9.2481 | 0.45 |
| Sim02-29 | 29 | 0.1994 | 10.109 | 1 | 1 | 10.4398 | 0.45 |
| Sim02-30 | 30 | 0.1999 | 9.5404 | 1 | 1 | 10.1114 | 0.45 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Label | Depth | Density | 210Pb | sd(210Pb) | Thickness | 226Ra | sd(226Ra) |
|  | (cm) | () | (Bq/kg) |  | (cm) | (Bq/kg) |  |
| Sim03-01 | 1 | 0.1001 | 6384.1354 | 287.2861 | 1 | 15.8007 | 0.675 |
| Sim03-02 | 2 | 0.1006 | 3550.0809 | 159.7536 | 1 | 14.5245 | 0.675 |
| Sim03-03 | 3 | 0.1017 | 1954.5702 | 87.9557 | 1 | 15.6527 | 0.675 |
| Sim03-04 | 4 | 0.1033 | 1183.8917 | 53.2751 | 1 | 14.5175 | 0.675 |
| Sim03-05 | 5 | 0.1055 | 760.2132 | 34.2096 | 1 | 14.9242 | 0.675 |
| Sim03-06 | 6 | 0.1081 | 360.2553 | 16.2115 | 1 | 14.801 | 0.675 |
| Sim03-07 | 7 | 0.1112 | 212.9402 | 9.5823 | 1 | 14.8738 | 0.675 |
| Sim03-08 | 8 | 0.1147 | 104.2684 | 4.6921 | 1 | 14.9028 | 0.675 |
| Sim03-09 | 9 | 0.1185 | 44.3849 | 1.9973 | 1 | 15.0768 | 0.675 |
| Sim03-10 | 10 | 0.1228 | 18.6447 | 1 | 1 | 15.3764 | 0.675 |
| Sim03-11 | 11 | 0.1273 | 23.2778 | 1.0475 | 1 | 14.6231 | 0.675 |
| Sim03-12 | 12 | 0.1321 | 53.1587 | 2.3921 | 1 | 15.1629 | 0.675 |
| Sim03-13 | 13 | 0.1371 | 97.363 | 4.3813 | 1 | 14.3047 | 0.675 |
| Sim03-14 | 14 | 0.1422 | 116.9788 | 5.264 | 1 | 14.0261 | 0.675 |
| Sim03-15 | 15 | 0.1474 | 153.2901 | 6.8981 | 1 | 15.9723 | 0.675 |
| Sim03-16 | 16 | 0.1526 | 151.8496 | 6.8332 | 1 | 14.7579 | 0.675 |
| Sim03-17 | 17 | 0.1578 | 136.3609 | 6.1362 | 1 | 16.114 | 0.675 |
| Sim03-18 | 18 | 0.1629 | 107.2736 | 4.8273 | 1 | 15.4595 | 0.675 |
| Sim03-19 | 19 | 0.1679 | 76.8966 | 3.4603 | 1 | 15.9439 | 0.675 |
| Sim03-20 | 20 | 0.1727 | 48.9213 | 2.2015 | 1 | 14.6235 | 0.675 |
| Sim03-21 | 21 | 0.1772 | 40.4439 | 1.82 | 1 | 14.6716 | 0.675 |
| Sim03-22 | 22 | 0.1815 | 26.5638 | 1.1954 | 1 | 16.2541 | 0.675 |
| Sim03-23 | 23 | 0.1853 | 21.714 | 1 | 1 | 14.4826 | 0.675 |
| Sim03-24 | 24 | 0.1888 | 17.6428 | 1 | 1 | 15.5109 | 0.675 |
| Sim03-25 | 25 | 0.1919 | 17.3533 | 1 | 1 | 13.6898 | 0.675 |
| Sim03-26 | 26 | 0.1945 | 17.4211 | 1 | 1 | 14.4684 | 0.675 |
| Sim03-27 | 27 | 0.1967 | 16.4246 | 1 | 1 | 15.3889 | 0.675 |
| Sim03-28 | 28 | 0.1983 | 12.4828 | 1 | 1 | 15.0698 | 0.675 |
| Sim03-29 | 29 | 0.1994 | 13.5514 | 1 | 1 | 15.2346 | 0.675 |
| Sim03-30 | 30 | 0.1999 | 14.3145 | 1 | 1 | 14.7846 | 0.675 |

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