**Using Simulations for model Comparison and analysing**

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To understand changes in peat accumulation in response to recent and rapid climate or anthropogenic change, accurate ages for the last 100-200 years are essential. Dating this period is often complicated by poor resolution and large errors associated with calibrating radiocarbon (14C) ages. The use of lead-210 () is a popular method as it allows for the measurement of absolute and continuous dates for the last  150 years of peat accumulation. In ombrotrophic peatlands, the lead-210 dating method has traditionally relied on the Constant Rate of Supply (CRS) model which uses the radioactive decay equation to provide a logarithmic model to approximate dates, resulting in a restrictive model. Key limitations of the CRS model are: (1) the accurate assessment of the supported lead which varies between sites and can be problematic if sampling of the total inventory is not continuous (e.g. interval measurements, lack of sample); (2) the inconsistent estimation of uncertainties. The Plum model was developed in a statistical framework with a Bayesian approach, notably resulting in longer chronologies, more realistic uncertainty estimations, and has the advantage of not double-modelling dates for final age-depth models, primarily radiocarbon and 210Pb chronologies. Here, we present two thorough tests of Plum. First, we created scenarios using simulated datasets with known age-depth functions in a range of shapes and with varying sampling resolution. These simulations are created using the physical behavior that most 210Pb dating models are based on. Plum and CRS model outputs are compared under each scenario. We also take this opportunity to demonstrate the new Plum’s R package, for use by non-statisticians in palaeoecological studies. We also compare the lead-210 dates derived from CRS models and from Plum using real peat cores with additional independent dating controls from Eastern Canada. These cores represent a thorough test for Plum, as permafrost thaw drastically changes stratigraphy and peat type (e.g. shift from ligneous peat to Sphagnum moss) which affects 210Pb retention within the peat. Recent decadal-scale changes are still poorly represented so accurate dating is now essential to quantify changes in carbon accumulation rates and predict future trends.

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# Introduction

# Simulations

In order to observe the accuracy and precision of any model we need data to which we know the true age-depth function. Blaauw et al. (2018) presented a methodology for simulating radiocarbon dates and as their uncertainty, on the other hand Aquino-López et al. (2018) presented an approach for simulating data given a age-depth function , it is important to note that this simulations follow the equations presented by Appleby and Oldfield (1978; Robbins 1978). By using the approach presented by Aquino-López et al. (2018) for obtaining simulated data from and the uncertainty estimations presented by Blaauw et al. (2018), we can obtained resalable simulated data.

For our simulations we constructed three different scenarios (see table [1](#Tab:sim_param)), each with their own age-depth functions.

Simulated age-depth function and parameters used in each simulation

|  |  |  |  |
| --- | --- | --- | --- |
| Label | Age-depth |  | Supported |
|  | function | () | () |
| Simulation 1 |  | 100 | 10 |
| Simulation 2 |  | 50 | 25 |
| Simulation 3 |  | 500 | 15 |

[Tab:sim\_param]

Using the age-depth functions, defined in table [1](#Tab:sim_param), we can obtained simulate the activity at any depth or interval (by integrating the curve in such interval).

Simulated concentration in relation to depth.

Simulated concentration in relation to depth.

These concentrations can be interpreted as error-free measurements. Because every equipment is subject to error, we need to replicate this measurement errors. (Blaauw et al. 2018) presents error structure for radiocarbon dates. We can use this structure to our measurements as both measurements are subject to similar measurement problems.

Let be the true concentration in the interval , given the age-depth function . To simulate disturbances in the material, we can introduce scatter centred around the true value, , where is the amount of scatter for this variable (in this case ). Now, to replicate outliers, we define a shift from the true value (), which occurs with a probability . This results in a new variable which is defined as

To simulate the uncertainty provided by the laboratory, we can define the simulated measurements as , where is the standard deviation reported by the laboratory. To simulate we use , where is the minimum standard deviation assigned to a measurement. This variable differs between laboratories (we will be using a default value of ). Finally, is the analytical uncertainty (default .01) and an error multiplier (default 1.5).

For this this study we created a data set for each simulation by integrating in intervals of 1 cm from depth 0 to 30 (where equilibrium was guaranteed).The complete data sets can be found in the Supplementary Material [4](#sec:supp_mat) and Figure [1](#fig:true_210) concentrations curves.

With these base data sets, we then define a new variable call percentage of information. This variable relates to how many much of the available information was measured. For this we assumed that background was reached at depth , information percentage is define as how much area of the core was measured, e.g. if background was reached at depth cm and the core was sampled very cm if 20 samples are measured, the percentage of information would be 20 %. This variable will help us to have a measuring tool for how many samples are needed for a good chronology without depending on the size of the samples.

# Model Comparison

In order to compare both the CRS and Plum under simular circumstances the previously described data sets will be randomly selected for samples given a information percentage, e.g. for a percentage information of 50% given our 30 sample data set, 14 samples will randomly selected from the samples at depths 1 to 29, and the samples at depth 30 will be always included. This was done to guaranty that background is reached, which is required for the CRS model. In the case of cores which have not reach background, *Plum (Aquino-López et al. 2018) has shown to provide accurate results without the need of user interference, the CRS can provide a chronology if inventory is completed, which means user innervation. To avoid this problem and to provide a more objective comparison every sampling set will have reach background.*

## Sampling Techniques

In order to observe how sampling affects the accuracy and precision of models, we used the three simulated data sets created for the previous section. This simulated cores were randomly selected given a percentage of information (e.g. for a 20% information sample, in the 30 cm cores, 6 random samples were selected). Becase the CRS model assumes that background has being reached, we decided to fix the last sample (30 cm depth) for every case, this will facilitate the CRS to provide more accurate results and also gives the model a single last depth to be removed as it is common practice when using this model. 100 individual samples were selected for information percentage from 10% to 90% in a 5% intervals (e.i 10%, 15%, 20%,...,85% and 90%).After a random sample was selected both the CRS model and Plum were performed and compared to the true age value to calculate its accuracy.

In order to observe the precision and accuracy we decided to calculate the mean of length of the 95% intervals (in yr), the offset (in yr) as well as the normalized accuracy (this variable will show us how far the model is from the true value given its uncertainty).

**Agregar gráficas aquí**

From this figure we can see that similar to the results presented by (Blaauw et al. 2018). The classical model (CRS) at first appears to provide a similar result (similar offset) with a more precise results (if we only look at the length of the 95% interval) but by looking at the normalized result we se on average the true age is almost always outside the 95% confidence intervals. This means that the CRS provides small uncertainties at the cost of its accuracy. It also appears that the length of the 95% interval and offset is not affected by how much information is provided to the model as well this could be a results of the implicit interpolation provided by the method.

On the other hand, *Plum*, which is a Bayesian method, shows more accurate results as more information is given to the model, with a consistently more accurate result compared to the CRS model, as well as the uncertainty. This result supports the clame that *Plum* provides more realistic uncertainties when compared to the CRS.

shows how its normalized accuracy remains similar at any number of samples used. On

# Supplementary Material

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Label | Depth | Density | 210Pb | sd(210Pb) | Thickness | 226Ra | sd(226Ra) |
|  | (cm) | () | (Bq/kg) |  | (cm) | (Bq/kg) |  |
| Sim01-01 | 1 | 0.10009 | 63.50103 | 2.85755 | 1 | 23.8045 | 1.125 |
| Sim01-02 | 2 | 0.10064 | 80.08738 | 3.60393 | 1 | 23.2924 | 1.125 |
| Sim01-03 | 3 | 0.10173 | 98.32806 | 4.42476 | 1 | 23.434 | 1.125 |
| Sim01-04 | 4 | 0.10334 | 125.45705 | 5.64557 | 1 | 26.0873 | 1.125 |
| Sim01-05 | 5 | 0.10547 | 141.27971 | 6.35759 | 1 | 22.8041 | 1.125 |
| Sim01-06 | 6 | 0.10809 | 130.27571 | 5.86241 | 1 | 23.4333 | 1.125 |
| Sim01-07 | 7 | 0.11116 | 134.04051 | 6.03182 | 1 | 25.6156 | 1.125 |
| Sim01-08 | 8 | 0.11466 | 129.69245 | 5.83616 | 1 | 26.1371 | 1.125 |
| Sim01-09 | 9 | 0.11855 | 134.93655 | 6.07214 | 1 | 25.4813 | 1.125 |
| Sim01-10 | 10 | 0.12278 | 109.39886 | 4.92295 | 1 | 25.8877 | 1.125 |
| Sim01-11 | 11 | 0.12731 | 110.68133 | 4.98066 | 1 | 24.4414 | 1.125 |
| Sim01-12 | 12 | 0.13209 | 102.38094 | 4.60714 | 1 | 24.9053 | 1.125 |
| Sim01-13 | 13 | 0.13706 | 75.80895 | 3.4114 | 1 | 22.9151 | 1.125 |
| Sim01-14 | 14 | 0.14218 | 77.60406 | 3.49218 | 1 | 24.4808 | 1.125 |
| Sim01-15 | 15 | 0.14738 | 68.4401 | 3.0798 | 1 | 24.9343 | 1.125 |
| Sim01-16 | 16 | 0.15262 | 60.72037 | 2.73242 | 1 | 25.2659 | 1.125 |
| Sim01-17 | 17 | 0.15782 | 50.28147 | 2.26267 | 1 | 22.961 | 1.125 |
| Sim01-18 | 18 | 0.16294 | 44.24641 | 1.99109 | 1 | 22.9139 | 1.125 |
| Sim01-19 | 19 | 0.16791 | 39.85997 | 1.7937 | 1 | 28.3774 | 1.125 |
| Sim01-20 | 20 | 0.17269 | 38.40823 | 1.72837 | 1 | 23.5379 | 1.125 |
| Sim01-21 | 21 | 0.17722 | 32.75922 | 1.47416 | 1 | 25.4363 | 1.125 |
| Sim01-22 | 22 | 0.18145 | 28.02545 | 1.26115 | 1 | 24.8995 | 1.125 |
| Sim01-23 | 23 | 0.18534 | 27.8749 | 1.25437 | 1 | 22.6783 | 1.125 |
| Sim01-24 | 24 | 0.18884 | 30.74797 | 1.38366 | 1 | 24.8575 | 1.125 |
| Sim01-25 | 25 | 0.19191 | 28.36187 | 1.27628 | 1 | 24.8724 | 1.125 |
| Sim01-26 | 26 | 0.19453 | 27.24535 | 1.22604 | 1 | 24.3778 | 1.125 |
| Sim01-27 | 27 | 0.19666 | 23.59236 | 1.06166 | 1 | 24.7209 | 1.125 |
| Sim01-28 | 28 | 0.19827 | 25.74855 | 1.15868 | 1 | 24.6615 | 1.125 |
| Sim01-29 | 29 | 0.19936 | 25.05368 | 1.12742 | 1 | 24.7199 | 1.125 |
| Sim01-30 | 30 | 0.19991 | 25.0065 | 1.12529 | 1 | 24.4937 | 1.125 |

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| --- | --- | --- | --- | --- | --- | --- | --- |
| Label | Depth | Density | 210Pb | sd(210Pb) | Thickness | 226Ra | sd(226Ra) |
|  | (cm) | () | (Bq/kg) |  | (cm) | (Bq/kg) |  |
| Sim02-01 | 1 | 0.1001 | 909.3928 | 40.9227 | 1 | 8.9761 | 0.45 |
| Sim02-02 | 2 | 0.1006 | 683.9989 | 30.7799 | 1 | 10.0607 | 0.45 |
| Sim02-03 | 3 | 0.1017 | 453.0503 | 20.3873 | 1 | 9.8701 | 0.45 |
| Sim02-04 | 4 | 0.1033 | 310.7897 | 13.9855 | 1 | 10.37 | 0.45 |
| Sim02-05 | 5 | 0.1055 | 218.0058 | 9.8103 | 1 | 10.0418 | 0.45 |
| Sim02-06 | 6 | 0.1081 | 158.6974 | 7.1414 | 1 | 10.104 | 0.45 |
| Sim02-07 | 7 | 0.1112 | 113.9062 | 5.1258 | 1 | 10.2049 | 0.45 |
| Sim02-08 | 8 | 0.1147 | 75.5493 | 3.3997 | 1 | 9.334 | 0.45 |
| Sim02-09 | 9 | 0.1185 | 56.6252 | 2.5481 | 1 | 10.5145 | 0.45 |
| Sim02-10 | 10 | 0.1228 | 44.1595 | 1.9872 | 1 | 9.8677 | 0.45 |
| Sim02-11 | 11 | 0.1273 | 34.7448 | 1.5635 | 1 | 9.7694 | 0.45 |
| Sim02-12 | 12 | 0.1321 | 25.384 | 1.1423 | 1 | 10.5134 | 0.45 |
| Sim02-13 | 13 | 0.1371 | 24.0007 | 1.08 | 1 | 10.4589 | 0.45 |
| Sim02-14 | 14 | 0.1422 | 21.3643 | 1 | 1 | 9.9504 | 0.45 |
| Sim02-15 | 15 | 0.1474 | 17.7932 | 1 | 1 | 10.5135 | 0.45 |
| Sim02-16 | 16 | 0.1526 | 15.0416 | 1 | 1 | 10.3362 | 0.45 |
| Sim02-17 | 17 | 0.1578 | 14.2937 | 1 | 1 | 10.5131 | 0.45 |
| Sim02-18 | 18 | 0.1629 | 12.3844 | 1 | 1 | 10.368 | 0.45 |
| Sim02-19 | 19 | 0.1679 | 12.6023 | 1 | 1 | 10.5297 | 0.45 |
| Sim02-20 | 20 | 0.1727 | 11.9329 | 1 | 1 | 10.0924 | 0.45 |
| Sim02-21 | 21 | 0.1772 | 9.301 | 1 | 1 | 10.118 | 0.45 |
| Sim02-22 | 22 | 0.1815 | 10.7777 | 1 | 1 | 10.249 | 0.45 |
| Sim02-23 | 23 | 0.1853 | 12.9491 | 1 | 1 | 10.134 | 0.45 |
| Sim02-24 | 24 | 0.1888 | 10.6571 | 1 | 1 | 10.1151 | 0.45 |
| Sim02-25 | 25 | 0.1919 | 9.6297 | 1 | 1 | 9.6608 | 0.45 |
| Sim02-26 | 26 | 0.1945 | 8.4331 | 1 | 1 | 8.7821 | 0.45 |
| Sim02-27 | 27 | 0.1967 | 10.4921 | 1 | 1 | 9.8995 | 0.45 |
| Sim02-28 | 28 | 0.1983 | 11.135 | 1 | 1 | 9.2481 | 0.45 |
| Sim02-29 | 29 | 0.1994 | 10.109 | 1 | 1 | 10.4398 | 0.45 |
| Sim02-30 | 30 | 0.1999 | 9.5404 | 1 | 1 | 10.1114 | 0.45 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Label | Depth | Density | 210Pb | sd(210Pb) | Thickness | 226Ra | sd(226Ra) |
|  | (cm) | () | (Bq/kg) |  | (cm) | (Bq/kg) |  |
| Sim03-01 | 1 | 0.1001 | 6384.1354 | 287.2861 | 1 | 15.8007 | 0.675 |
| Sim03-02 | 2 | 0.1006 | 3550.0809 | 159.7536 | 1 | 14.5245 | 0.675 |
| Sim03-03 | 3 | 0.1017 | 1954.5702 | 87.9557 | 1 | 15.6527 | 0.675 |
| Sim03-04 | 4 | 0.1033 | 1183.8917 | 53.2751 | 1 | 14.5175 | 0.675 |
| Sim03-05 | 5 | 0.1055 | 760.2132 | 34.2096 | 1 | 14.9242 | 0.675 |
| Sim03-06 | 6 | 0.1081 | 360.2553 | 16.2115 | 1 | 14.801 | 0.675 |
| Sim03-07 | 7 | 0.1112 | 212.9402 | 9.5823 | 1 | 14.8738 | 0.675 |
| Sim03-08 | 8 | 0.1147 | 104.2684 | 4.6921 | 1 | 14.9028 | 0.675 |
| Sim03-09 | 9 | 0.1185 | 44.3849 | 1.9973 | 1 | 15.0768 | 0.675 |
| Sim03-10 | 10 | 0.1228 | 18.6447 | 1 | 1 | 15.3764 | 0.675 |
| Sim03-11 | 11 | 0.1273 | 23.2778 | 1.0475 | 1 | 14.6231 | 0.675 |
| Sim03-12 | 12 | 0.1321 | 53.1587 | 2.3921 | 1 | 15.1629 | 0.675 |
| Sim03-13 | 13 | 0.1371 | 97.363 | 4.3813 | 1 | 14.3047 | 0.675 |
| Sim03-14 | 14 | 0.1422 | 116.9788 | 5.264 | 1 | 14.0261 | 0.675 |
| Sim03-15 | 15 | 0.1474 | 153.2901 | 6.8981 | 1 | 15.9723 | 0.675 |
| Sim03-16 | 16 | 0.1526 | 151.8496 | 6.8332 | 1 | 14.7579 | 0.675 |
| Sim03-17 | 17 | 0.1578 | 136.3609 | 6.1362 | 1 | 16.114 | 0.675 |
| Sim03-18 | 18 | 0.1629 | 107.2736 | 4.8273 | 1 | 15.4595 | 0.675 |
| Sim03-19 | 19 | 0.1679 | 76.8966 | 3.4603 | 1 | 15.9439 | 0.675 |
| Sim03-20 | 20 | 0.1727 | 48.9213 | 2.2015 | 1 | 14.6235 | 0.675 |
| Sim03-21 | 21 | 0.1772 | 40.4439 | 1.82 | 1 | 14.6716 | 0.675 |
| Sim03-22 | 22 | 0.1815 | 26.5638 | 1.1954 | 1 | 16.2541 | 0.675 |
| Sim03-23 | 23 | 0.1853 | 21.714 | 1 | 1 | 14.4826 | 0.675 |
| Sim03-24 | 24 | 0.1888 | 17.6428 | 1 | 1 | 15.5109 | 0.675 |
| Sim03-25 | 25 | 0.1919 | 17.3533 | 1 | 1 | 13.6898 | 0.675 |
| Sim03-26 | 26 | 0.1945 | 17.4211 | 1 | 1 | 14.4684 | 0.675 |
| Sim03-27 | 27 | 0.1967 | 16.4246 | 1 | 1 | 15.3889 | 0.675 |
| Sim03-28 | 28 | 0.1983 | 12.4828 | 1 | 1 | 15.0698 | 0.675 |
| Sim03-29 | 29 | 0.1994 | 13.5514 | 1 | 1 | 15.2346 | 0.675 |
| Sim03-30 | 30 | 0.1999 | 14.3145 | 1 | 1 | 14.7846 | 0.675 |

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