MODULE *GalsHeat*

EXTENDS *FiniteSets*, *Integers*

CONSTANTS
- *Nodes*, Total number of nodes
- *NEIGHBOURS*, A tuple of the set of neighbours with the index being the node number
- *MAXTIME* Maximum time for the systems

A variable that stores the state (t, cs, ns) of each node where "t" is the current time of the node, "cs" is the count of messages received for the current time and "ns" is the count of messages recieved for $t+1$

VARIABLE *state*

Initialize the state of each node so the $t = 0$, all the messages for $t = 0$ (cs) have been received (the nodes are ready to progress to $t = 1$) and no messages for $t = 1$ (ns) have been received

$$
\begin{aligned}
&GSInit \triangleq\ state = \\
&\quad [n \in 1 \,..\, Nodes \mapsto \\
&\qquad [t \mapsto 0,\ cs \mapsto Cardinality(NEIGHBOURS[n]),\ ns \mapsto 0]]
\end{aligned}
$$

$$
\begin{aligned}
&Next(n) \triangleq\ \land\ state[n].t < MAXTIME \\
&\quad \text{Check if messages for the current time have been received from all the neighbours} \\
&\quad \land\ state[n].cs = Cardinality(NEIGHBOURS[n]) \\
&\quad \land\ state' = [i \in 1 \,..\, Nodes \mapsto \\
&\qquad \text{IF } i = n \text{ THEN} \\
&\qquad\quad [\ t\ \mapsto state[i].t + 1, \\
&\qquad\quad\ \ cs\ \mapsto state[i].ns, \\
&\qquad\quad\ \ ns\ \mapsto 0] \\
&\qquad \text{ELSE IF } i \in NEIGHBOURS[n] \text{ THEN} \\
&\qquad\quad \text{IF } state[i].t = state[n].t + 1 \text{ THEN} \\
&\qquad\qquad \text{Message received for the current time} \\
&\qquad\qquad [state[i] \text{ EXCEPT } !.cs = 1 + @] \\
&\qquad\quad \text{ELSE IF } state[i].t = state[n].t \text{ THEN} \\
&\qquad\qquad \text{Message received for time } t+1 \\
&\qquad\qquad [state[i] \text{ EXCEPT } !.ns = 1 + @] \\
&\qquad\quad \text{ELSE FALSE} \\
&\qquad \text{ELSE} \\
&\qquad\quad state[i]]
\end{aligned}
$$

$$GSNext \triangleq\ \exists\, n \in 1 \,..\, Nodes : Next(n)$$

$$GSSpec \triangleq\ GSInit \land \Box[GSNext]_{\langle state \rangle}$$

Check that the neighbours have a two way link

$$NeighbourOK \triangleq$$
$$\forall\, n \in 1\,..\,Nodes :$$
$$\quad \forall\, a \in NEIGHBOURS[n] : n \in NEIGHBOURS[a]$$

Check that the time difference between neighbours is never greater than 1

$$TimeDiffOK \triangleq$$
$$\forall\, n \in 1\,..\,Nodes :$$
$$\quad \forall\, a \in NEIGHBOURS[n] :$$
$$\qquad \land\ state[n].t - state[a].t < 2$$
$$\qquad \land\ state[n].t - state[a].t > -2$$

Check that the values of the state variables are correct

$$StateVariablesOK \triangleq$$
$$\forall\, n \in 1\,..\,Nodes :$$
$$\quad \land\ state[n].t \leq MAXTIME$$
$$\quad \land\ state[n].cs \leq Cardinality(NEIGHBOURS[n])$$
$$\quad \land\ state[n].ns \leq Cardinality(NEIGHBOURS[n])$$