
MODULE *GalsIzhikevichOriginal*

EXTENDS *FiniteSets, Integers*

CONSTANTS *Neurons*, Total number of neurons
 A tuple representing the set of in neighbours of each neuron
 InNeighbours,
 A tuple representing the set of out neighbours of each neuron
 OutNeighbours,
 MaxTime

The state variable is a function with domain the set of neurons and range a record with fields: *t* (current time of neuron), *p* (number of pending fires), *c* (count of recieved messages)

VARIABLES *state*

Initialize the state of each neuron with $t = 0$, having 1 pending fire ($p = 1$) and not having received any messages ($c = 0$)

$GIInit \triangleq state = [n \in 1 \dots Neurons \mapsto$
 [$t \mapsto 0, p \mapsto 1, c \mapsto 0$]
]

$Next(n) \triangleq \wedge state[n].t < MaxTime$
 $\wedge state[n].p > 0$
 $\wedge state' =$
 [$a \in 1 \dots Neurons \mapsto$
 IF $a = n$ THEN
 [$t \mapsto state[a].t + 1,$
 $p \mapsto state[a].p - 1,$
 $c \mapsto 0$
]
 ELSE IF $a \in OutNeighbours[n]$ THEN
 Increment pending fires if count equal to the number of in neighbours
 IF $Cardinality(InNeighbours[a]) = state[a].c + 1$ THEN
 [$state[a]$ EXCEPT
 $!.p = 1 + @,$
 $!.c = 0$
]
 ELSE
 [$state[a]$ EXCEPT $!.c = 1 + @$]
 ELSE
 $state[a]$
]

$$GINext \triangleq \exists n \in 1 \dots Neurons : Next(n)$$

$$GISpec \triangleq GIInit \wedge \Box[GINext]_{\langle state \rangle}$$

Check that the connections are correct

$$\begin{aligned} NeighbourOK &\triangleq \forall n \in 1 \dots Neurons : \\ &\quad \wedge \forall i \in InNeighbours[n] : n \in OutNeighbours[i] \\ &\quad \wedge \forall o \in OutNeighbours[n] : n \in InNeighbours[o] \end{aligned}$$

Check that the out neighbour is not more then 1 timestep ahead, need to receive the message of current time step before jumping the next one

$$\begin{aligned} TimeDiffOK &\triangleq \forall n \in 1 \dots Neurons : \\ &\quad \wedge \forall i \in InNeighbours[n] : \\ &\quad \quad \wedge state[n].t - state[i].t < 2 \\ &\quad \wedge \forall o \in OutNeighbours[n] : \\ &\quad \quad \wedge state[n].t - state[o].t > -2 \end{aligned}$$

Check that the values of the state variables are correct

$$\begin{aligned} TypeOK &\triangleq \wedge \forall n \in 1 \dots Neurons : \\ &\quad \wedge state[n].t \leq MaxTime \\ &\quad \wedge state[n].c \leq Cardinality(InNeighbours[n]) \end{aligned}$$
