Lesson 4

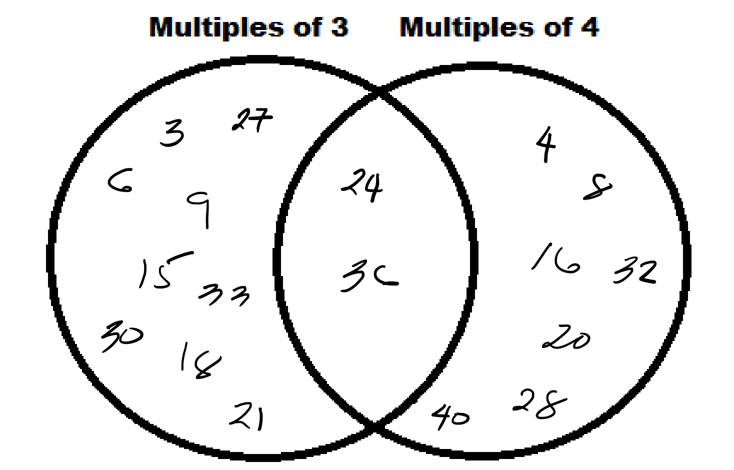
Common Divisors, Common Multiples and Division Theorem

Learning Objectives:

A. Differentiate between common divisors and common multiples.
B. Give the greatest common divisors and least common multiples
C. Find the remainder and quotient on division algorithm.

Preliminary Activity:

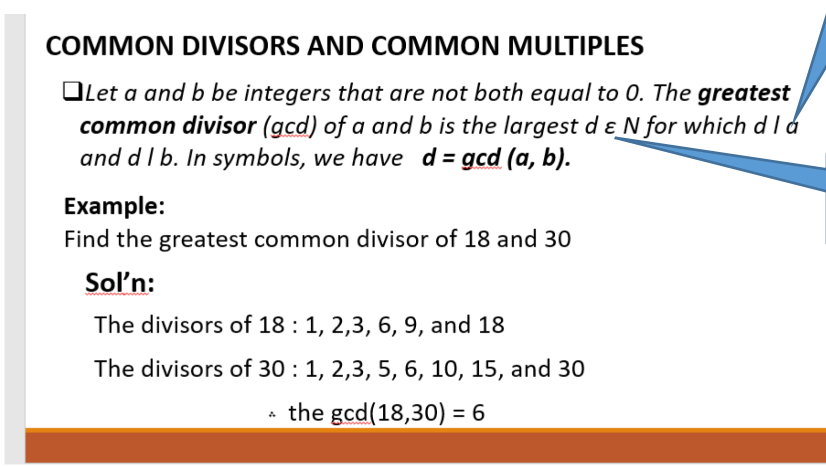
Consider the Venn Diagram below.

Arrange the integers in the region from the Venn diagram at the right. The first set is multiples of 3 and the other is multiples of 4. The integers are : 3, 4, 6,8,9,15, 16, 18, 20, 21, 24, 27, 28, 30, 32, 33, 36, 40.	

Questions:
What can you say about the integers which are multiples of 3? of 4?
What can you say about the integers common to both sets?

Discussion

Please refer to the presentations below:



Example 2: Find

a) gcd (234, 540), b). gcd (19, -57), c). gcd(77, 250)

Answers:

a. 18 b. 19 c. 1

Using the tree factorization in lesson 2, the divisors of
234 : 2 x 3 x 3 x 13 and
540 : 2 x 2 x 3 x 3 x 3 x 15
Hence, the gcd(234,540) = 2 x 3 x 3 = 18

Relatively Prime

- Let *a*, *b* ε *Z*. We say *a* and *b* are relatively prime if gcd(*a*, *b*) = 1
- Let *p* be prime, and let *n* be an integer. If *p* does not divide *n*, then gcd(*p*, *n*) = 1.

Examples:

✓77 and 250 ✓12 and 13

✓21 and 22

Common Multiple

- If two integers each divide a number m, then m is called a **common multiple** of these integers.
- Let a and b be nonzero integers. The **least common multiple** (LCM) of a and b is the smallest m ε N for which a l m and b l m. In symbols,

m = lcm (a,b).

Examples

Find the least common multiple of each pair of integers.

a. lcm(1960, 1100) ➡ $1960 = 2^3 * 5^1 * 7^2$, $1100 = 2^2 * 5^2 * 11^1$ } $2^3*5^2*7^2*11$ = 107, 800

b. lcm(27, 85) ➡ $27 = 3^3$, $85 = 5 * 17$ } $3^3*5*17=2295$

c. lcm(11, -132) ➡ 11 = 11, -132 = -3*4*11 } 3*4*11=132

Lcm (27,85) = 3^3 x 5 x 17 = 2295
Divisors of 27: 3 x 3 x 3
Divisors of 85: 5 x 17
Lcm (27, 85) : 3 x 3 x 3 x 5 x 15 = 3^3 x 5 x 17 = 2295

Alignment of same factors

LCM (11, -132) = 132
Divisors of 11: 11
Divisors of -132: 11 * 4 * -3
LCM (11, -132): 11 * 4 * -3

Alignment of same factors

The Division Theorem

- Let $a \in Z$ and $d \in N$. Then there exist unique integers q and r such that $a = dq + r$ and $0 \leq r < d$.

Examples:

1. Divide 329 by 67

   329 = 4*67 + 61

2. Divide -120 by 50

   -120 = -3*50 + 30

r : remainder
d : divisor
q : quotient
d & q: factors of a
a : dividend

3. What are the quotient and remainder when 101 is divided by 11?

   Sol'n: 101 = 11*9 + 2

   ∴ Hence, q = 9 and r = 2

4. What are the quotient and remainder when -11 is divided by 3?

   Sol'n: -11 = 3 (-4) + 1

   ∴ Hence, q = -4 and r = 1

Additional Readings/Video Clips:

1. https://www.youtube.com/watch?v=PQMCbNKUB-E
2. https://math.libretexts.org/Courses/Mount_Royal_University/MATH_2150%3A_Higher_Arithmetic/4%3A_Greatest_Common_Divisor%2C_least_common_multiple_and_Euclidean_Algorithm/4.1%3A_Greatest_Common_Divisor
3. https://math.libretexts.org/Courses/Mount_Royal_University/MATH_2150%3A_Higher_Arithmetic/4%3A_Greatest_Common_Divisor%2C_least_common_multiple_and_Euclidean_Algorithm/4.3%3A_Least_Common_Multiple

Please accomplish SAE W1L4 to be posted in the MS Teams