

Lense-Thirring-Effekt

Vortrag im Hauptseminar SoSe 2025

Marvin Henke - 10. Mai 2025

Betreuer: Dr. Nikodem Szpak

Lense-Thirring-Effekt

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- Metrik und Geodäten

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- Einsteinsche Feldgleichungen

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- Gravitoelektromagnetismus

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- Präzession

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$$\boldsymbol{\eta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Lense-Thirring-Effekt

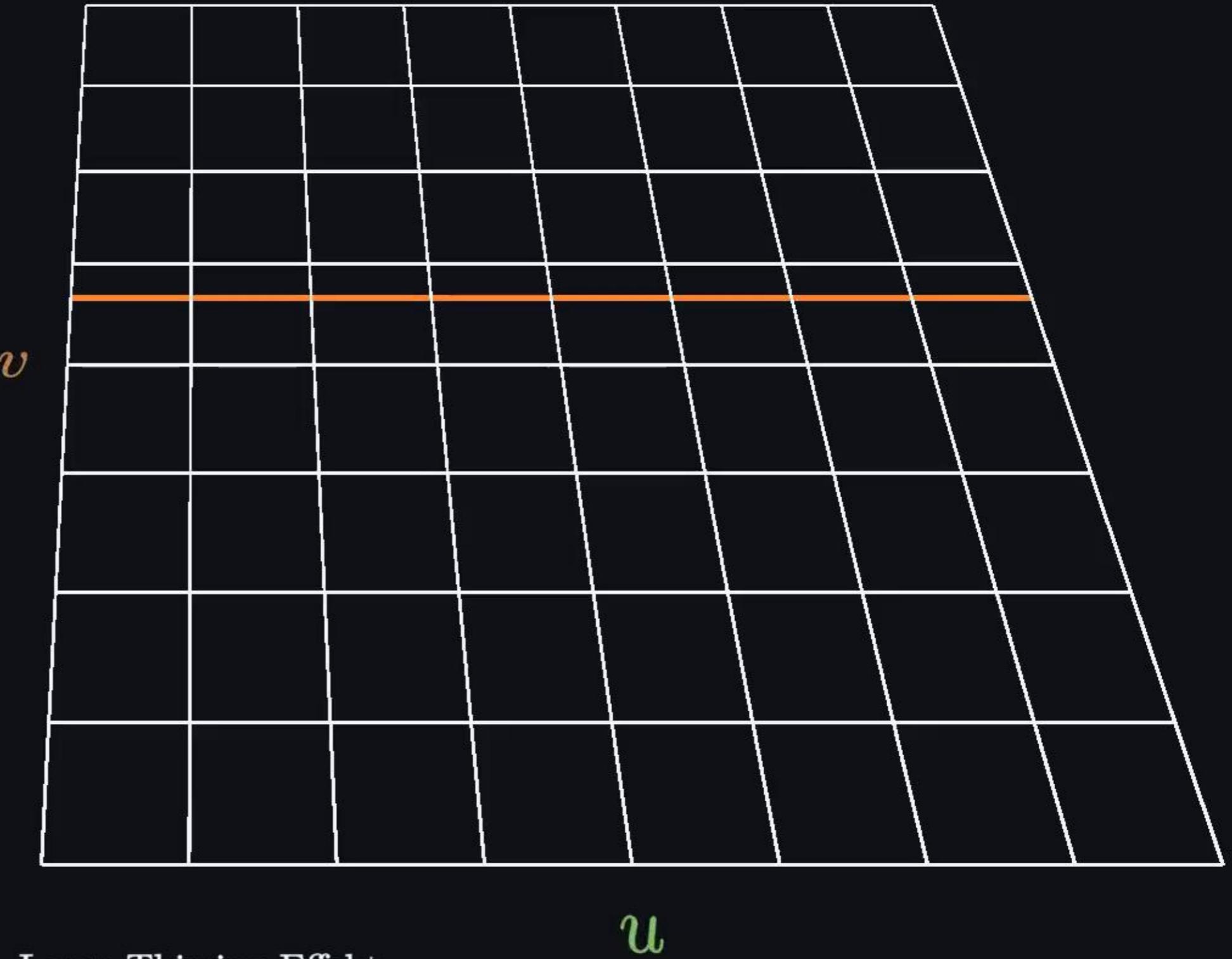
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Metrik und Geodäten

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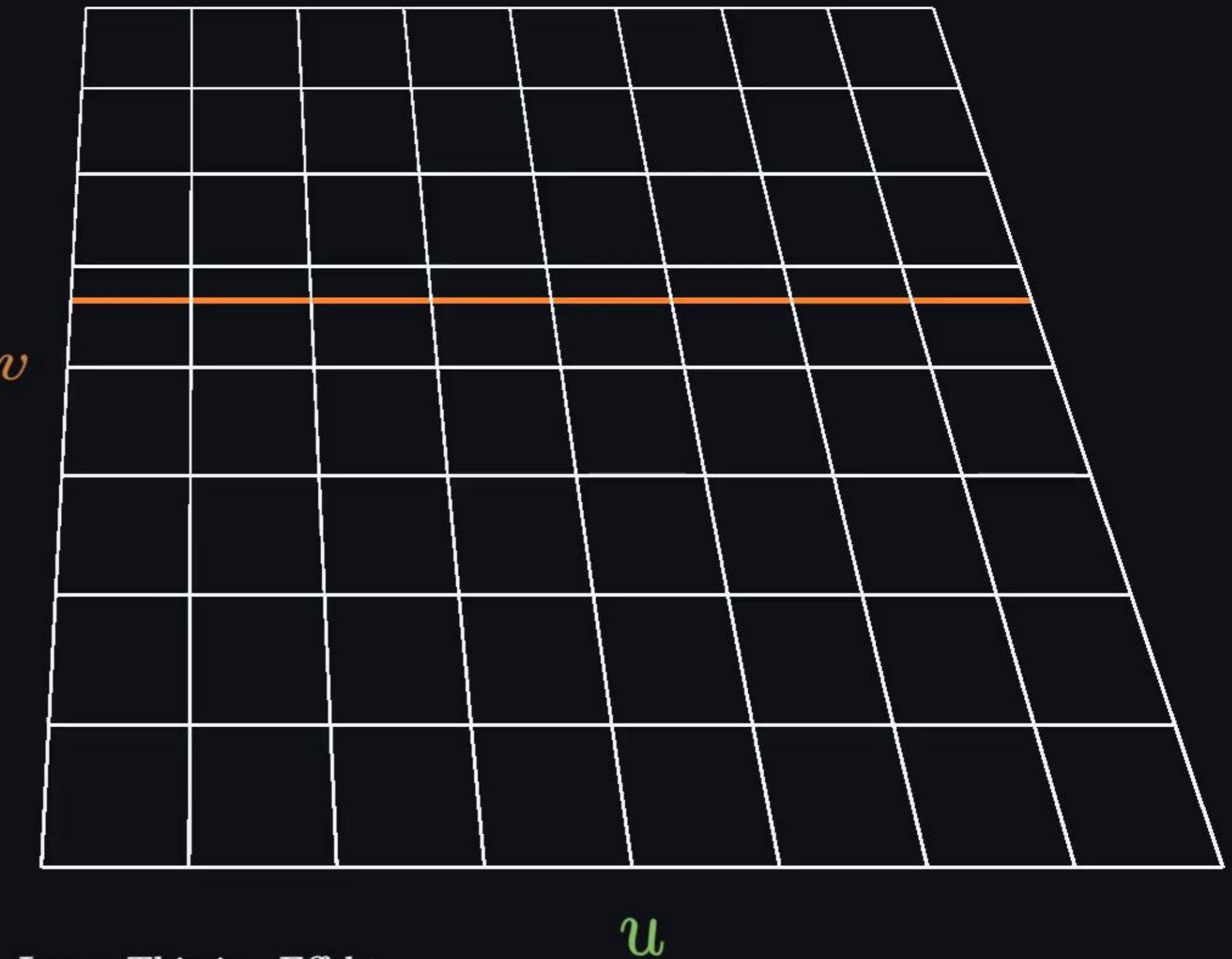


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Metrik und Geodäten

Fläche

$$\vec{x}(u, v) = (u, v, 0)$$



Lense-Thirring-Effekt

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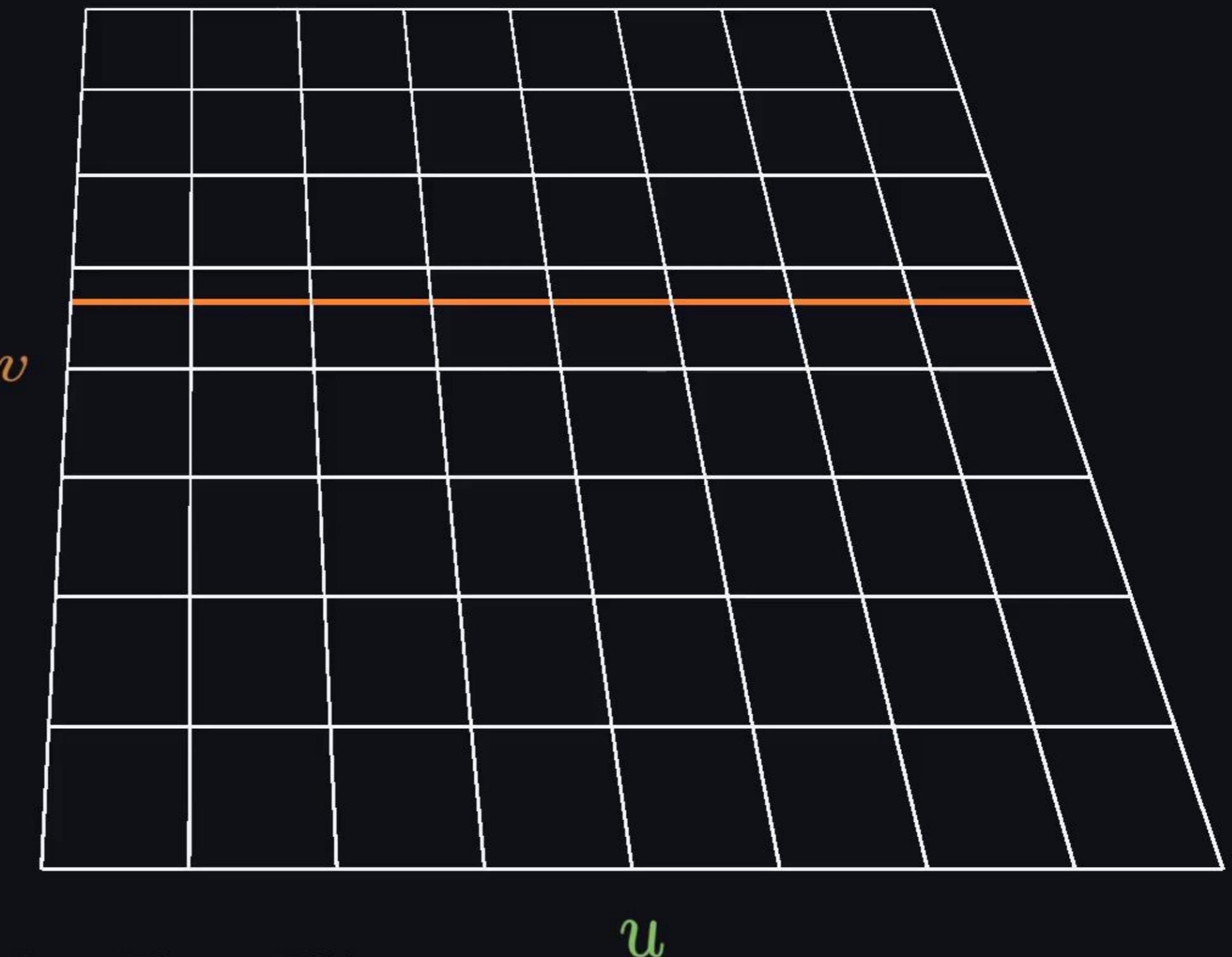
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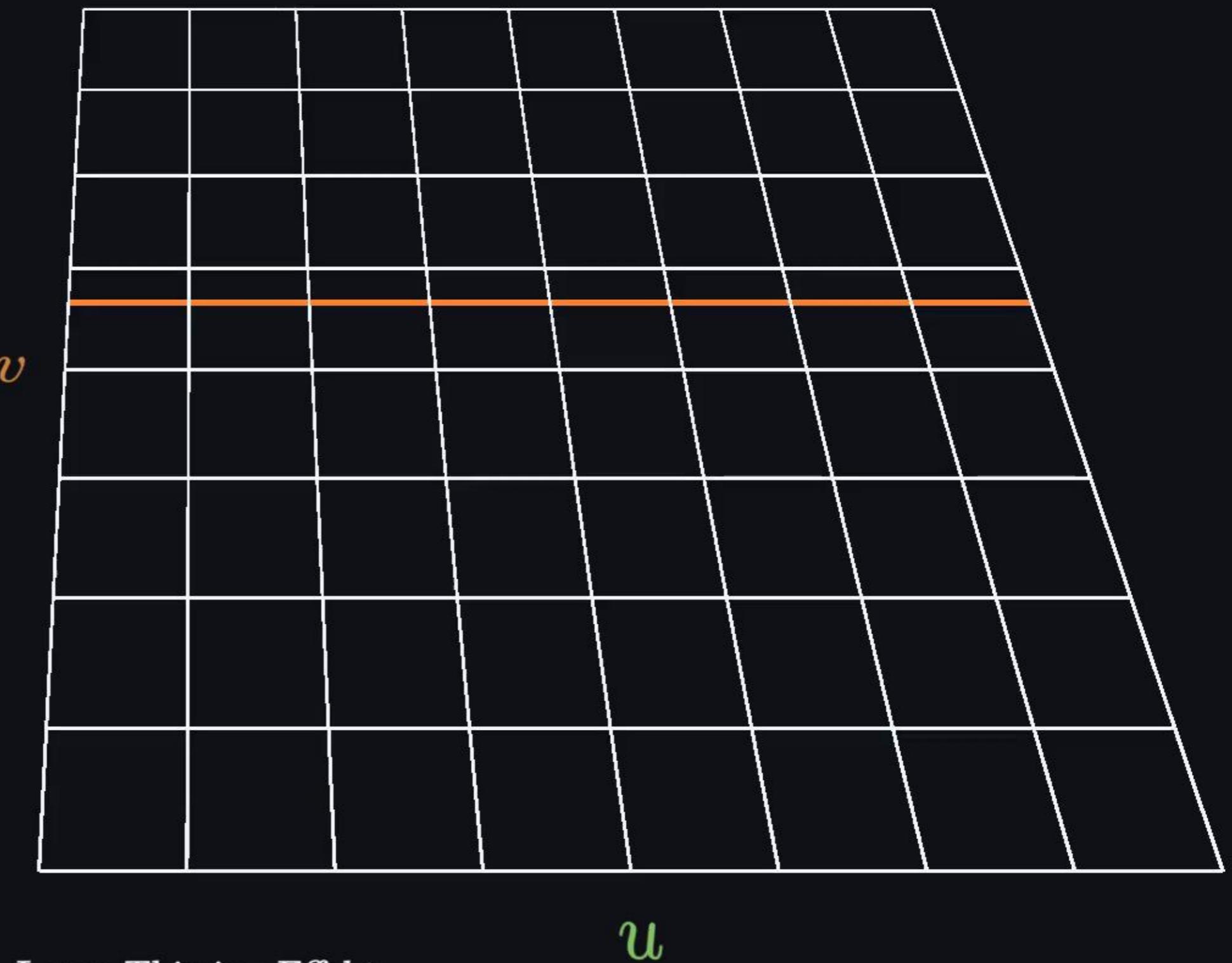
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$$\frac{d^2 \vec{x}^\lambda}{d\tau^2} = 0$$



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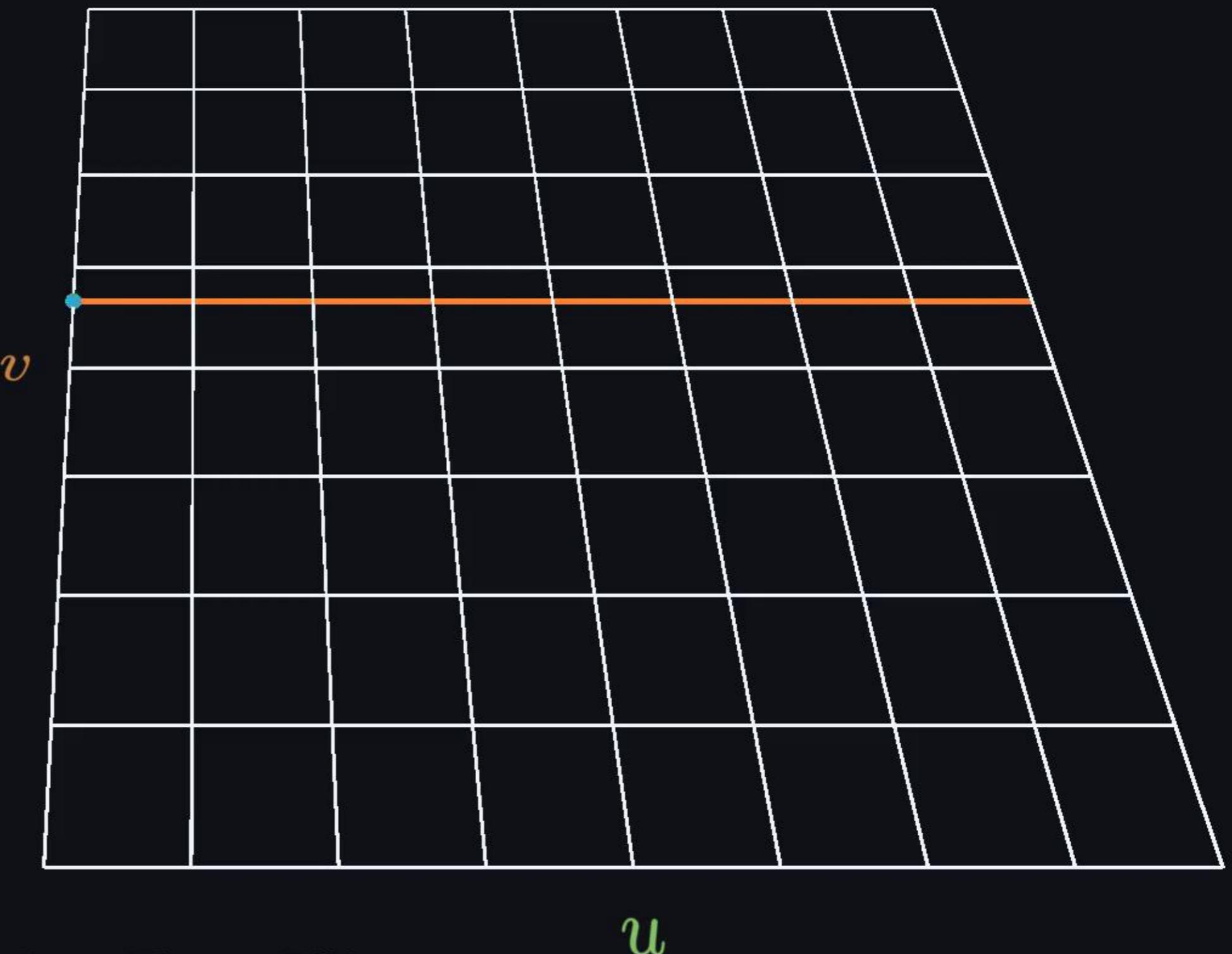
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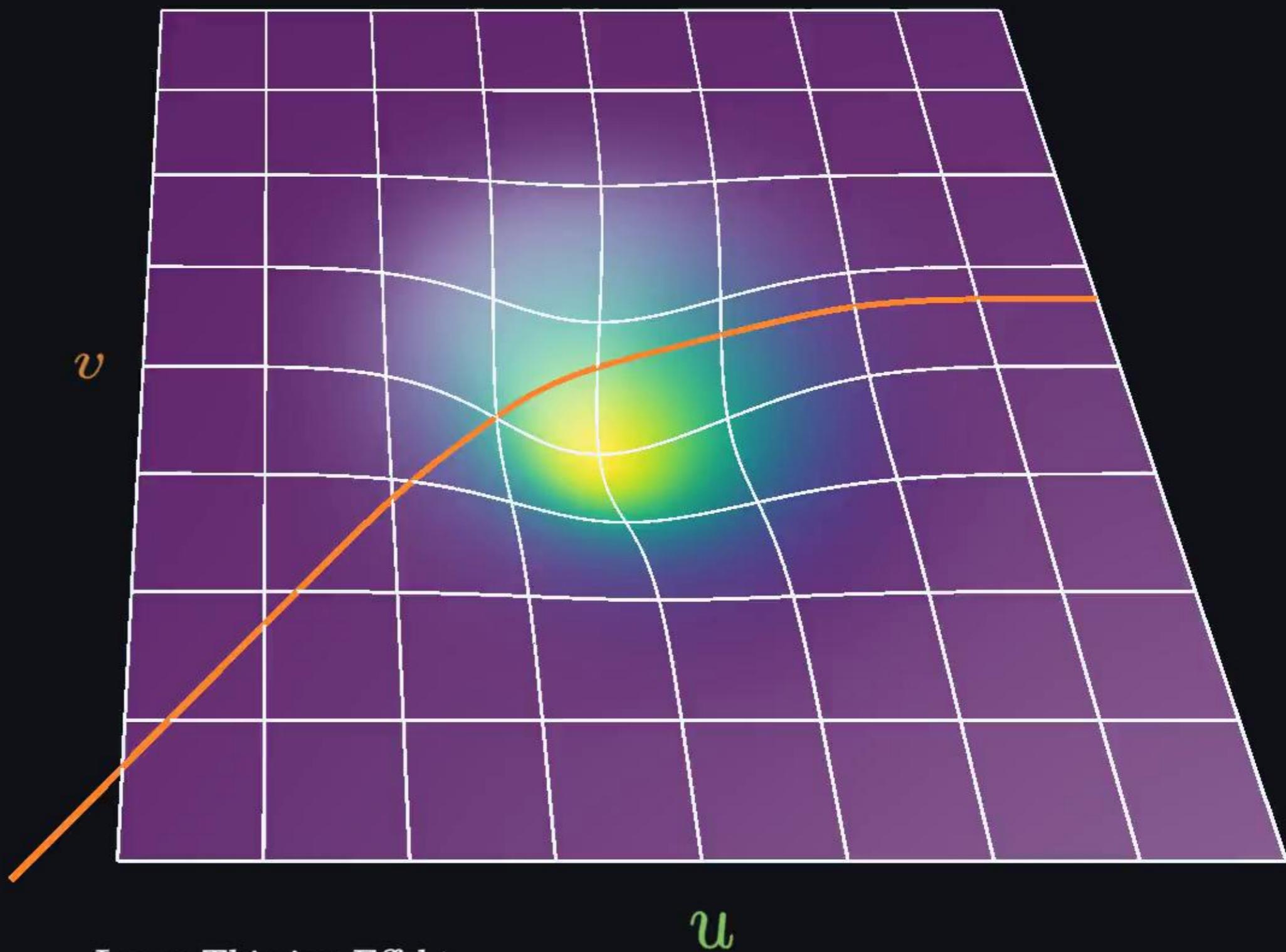
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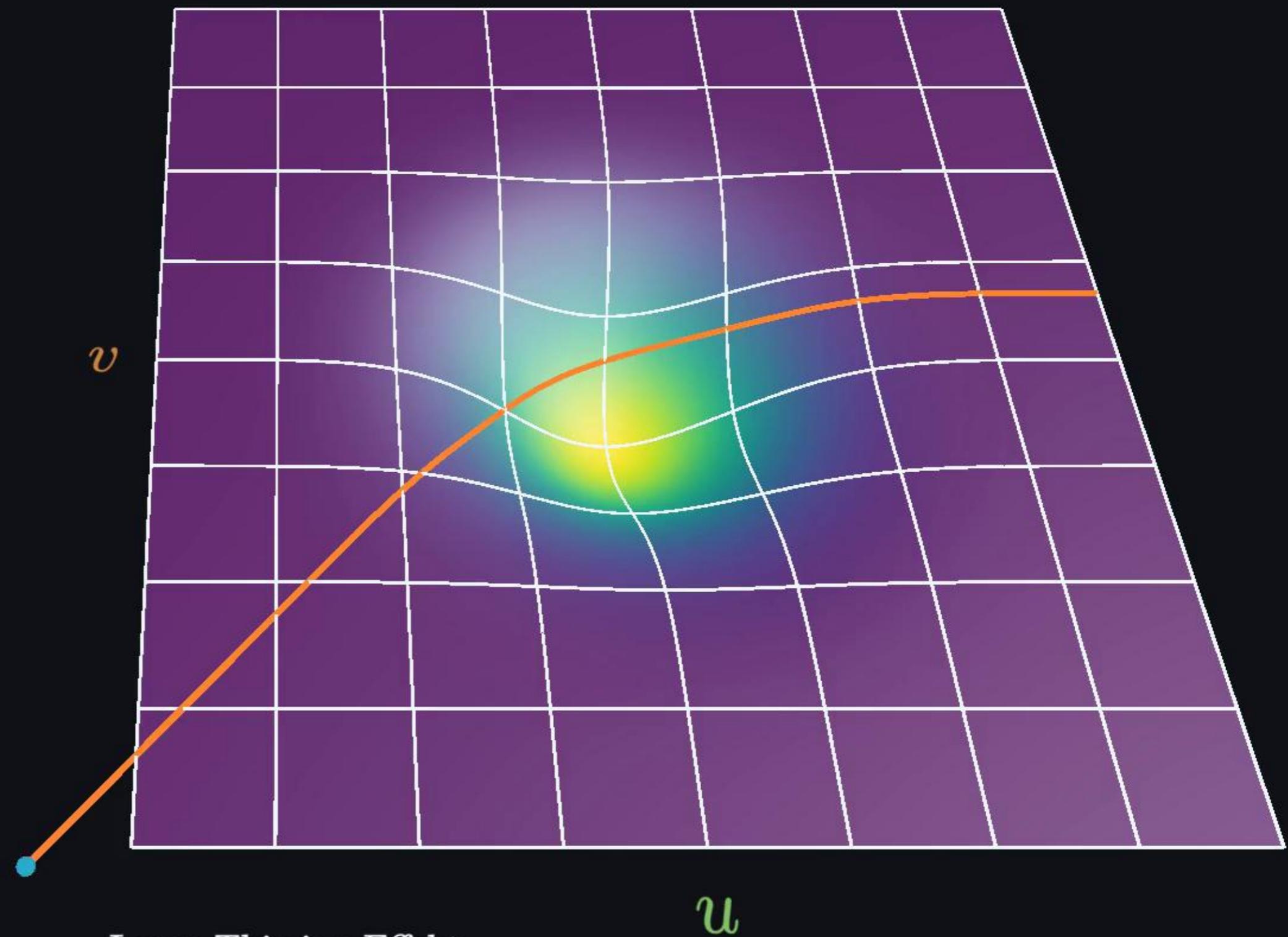
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Einsteinsche Feldgleichungen

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Einstein'sche Feldgleichungen

2D Fläche \rightarrow 4D Mannigfaltigkeit

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Einsteinsche Feldgleichungen

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Einsteinsche Feldgleichungen

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Energie-Impuls-Tensor: $T_{\mu\nu}$

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Linearisierung

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Gravitoelektromagnetismus

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Rotierende Kugelmasse

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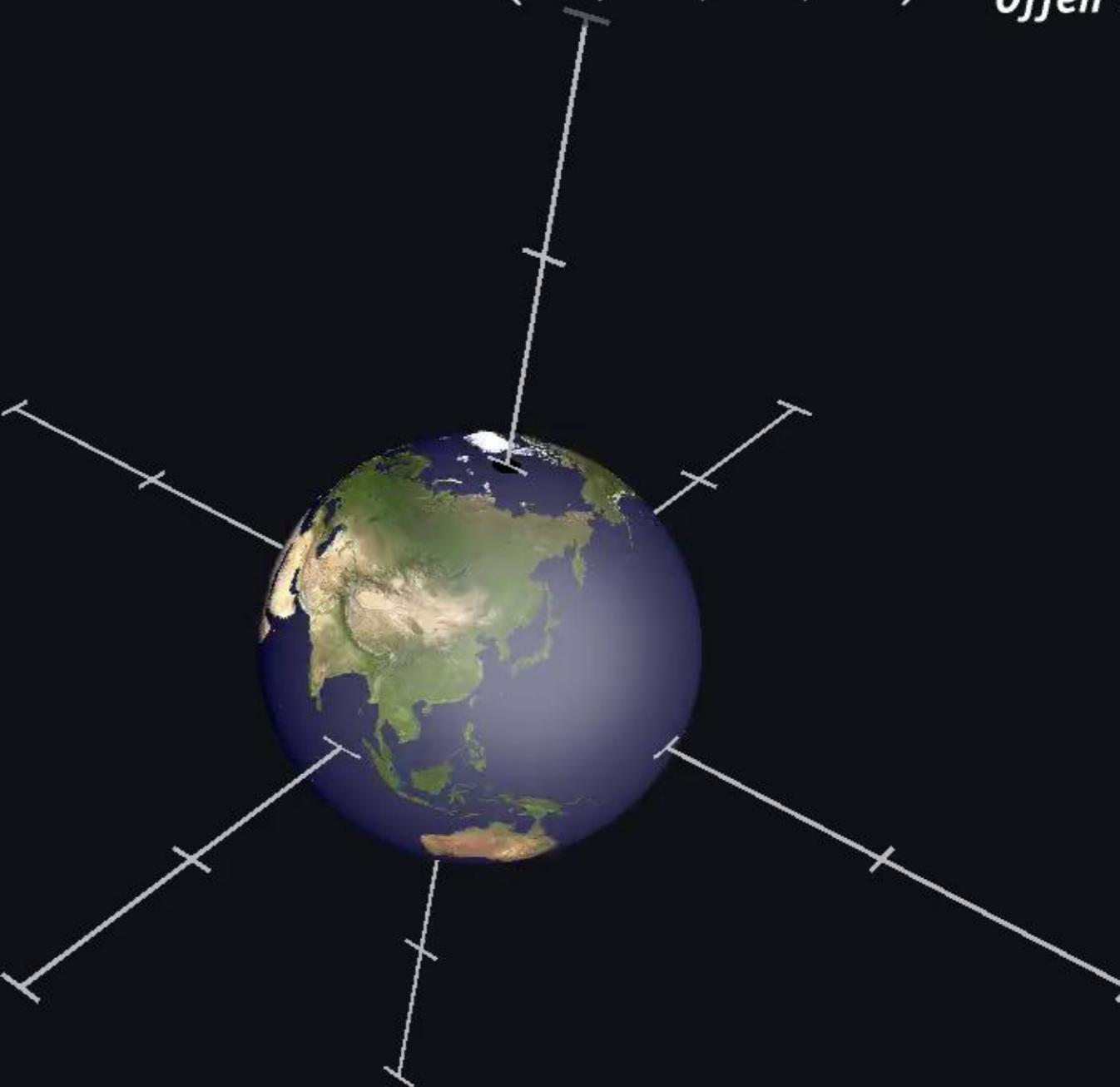
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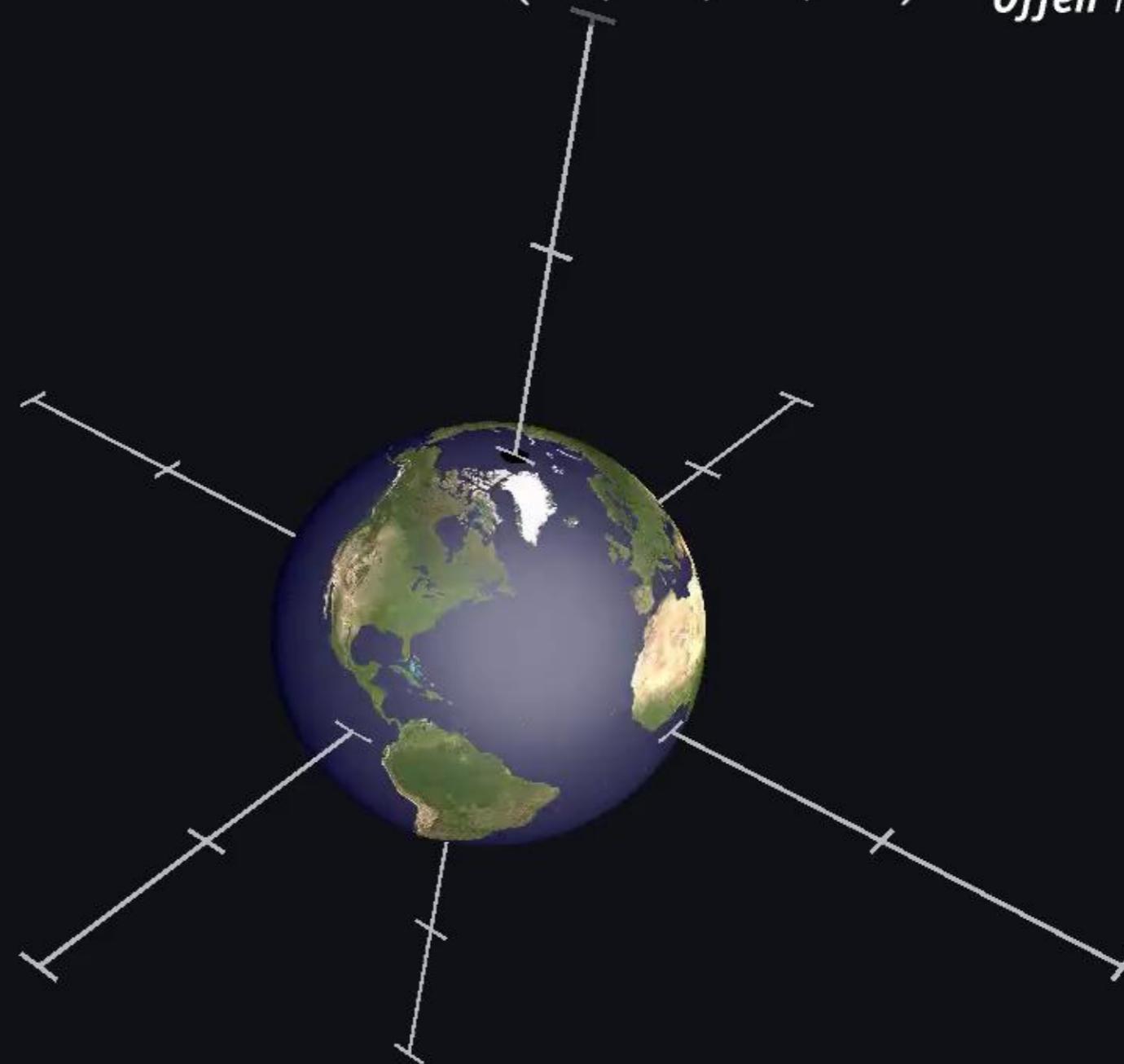
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$$\vec{S} = I \vec{\omega}$$



EM-Felder

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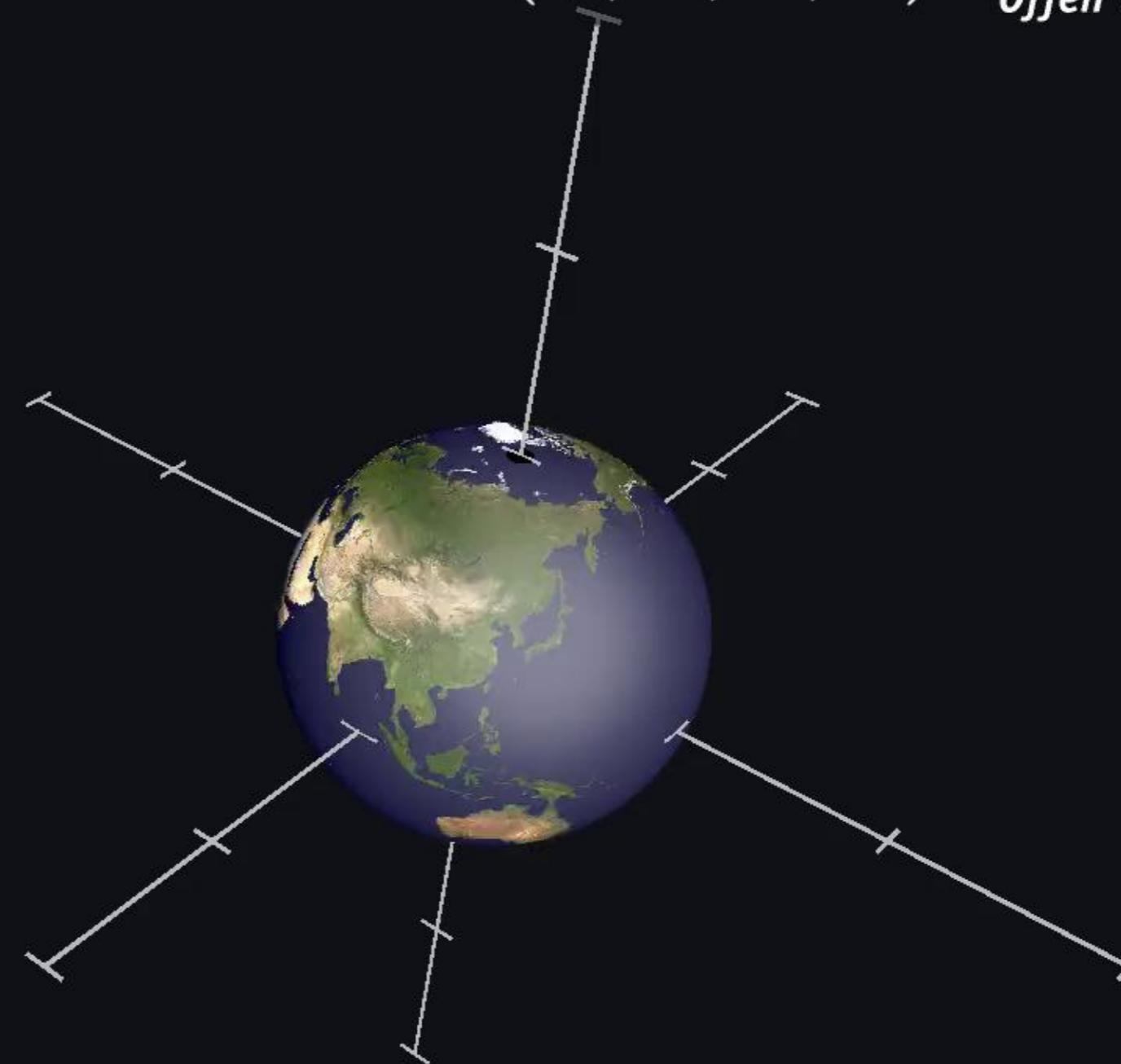
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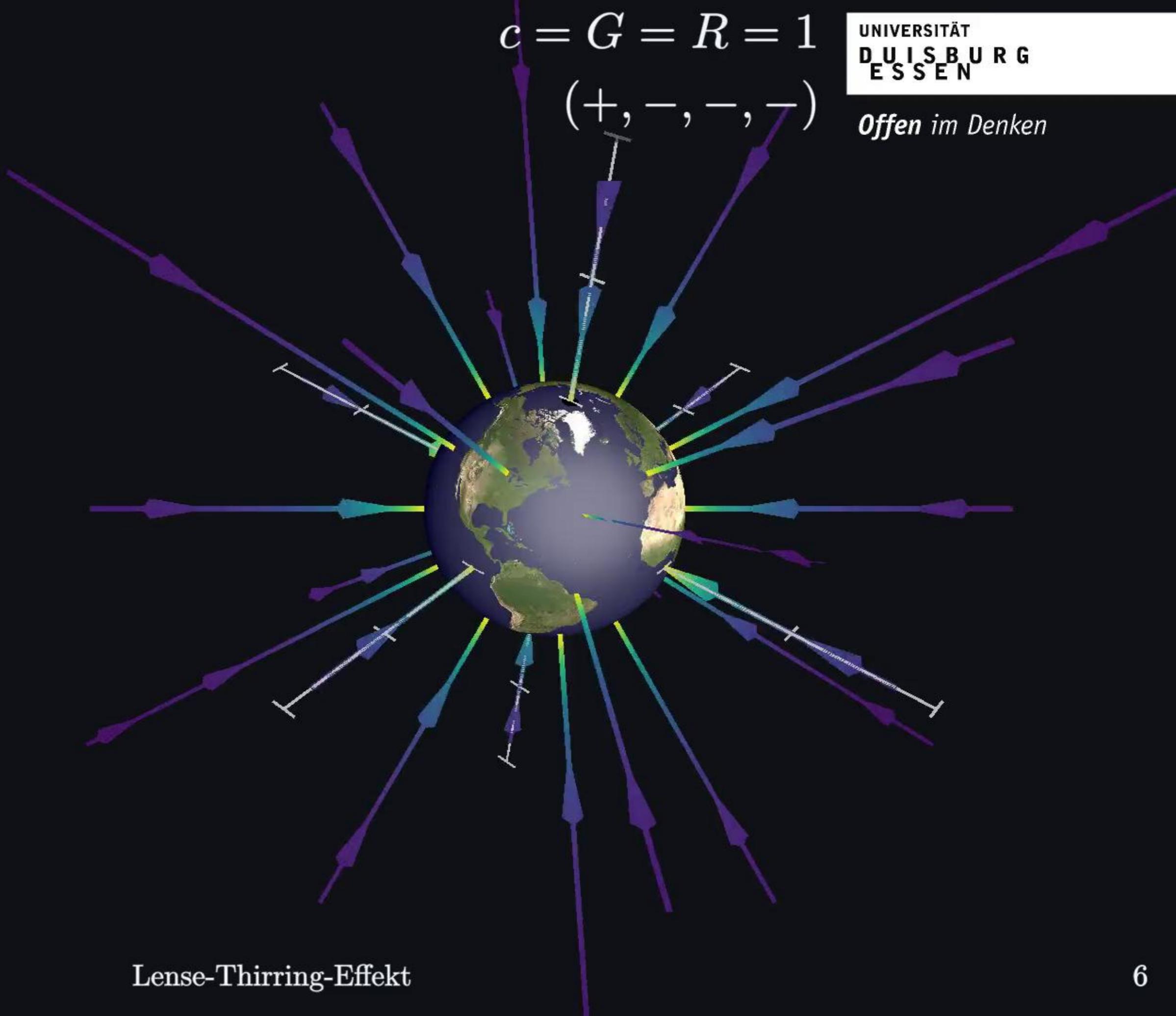
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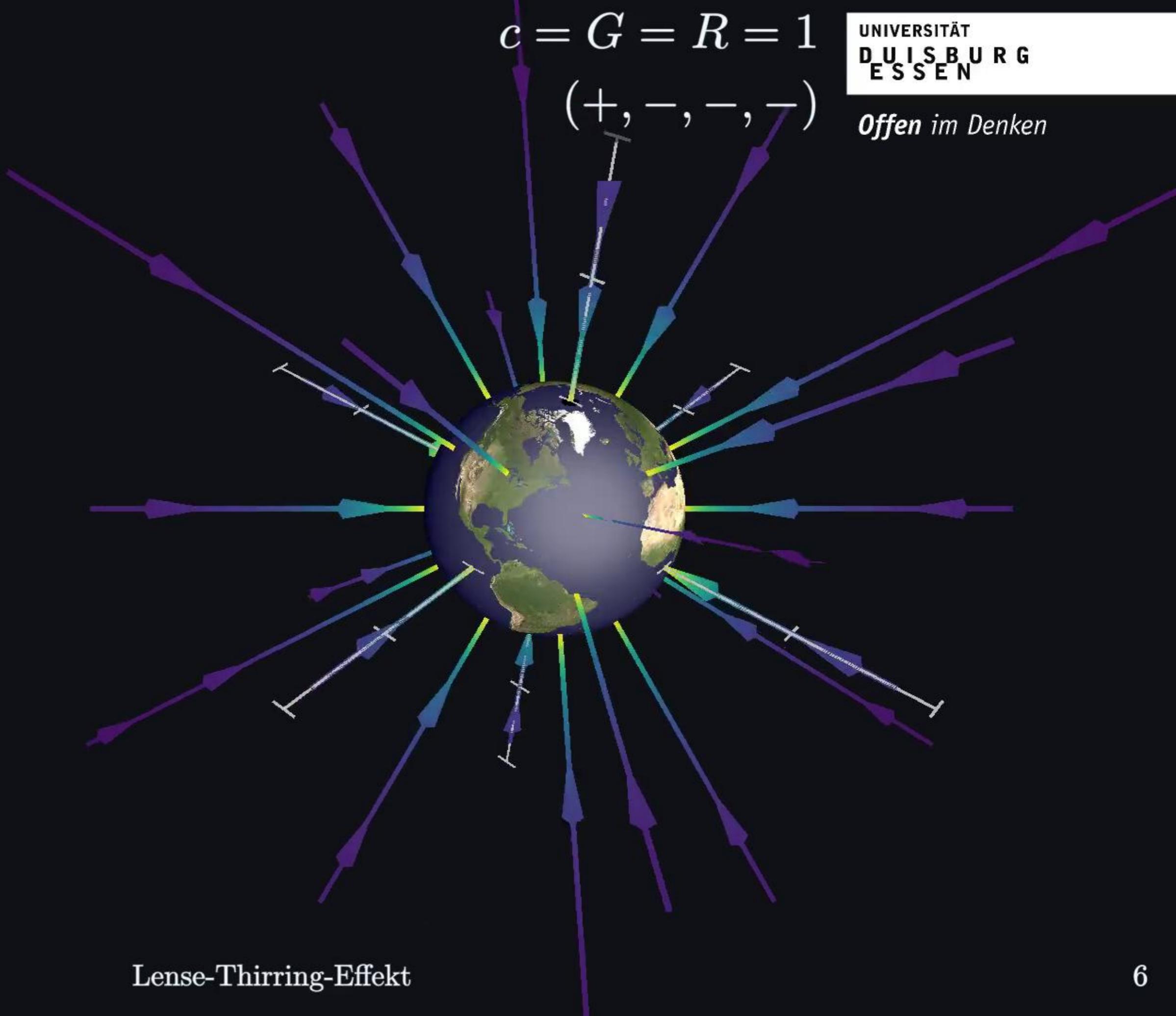
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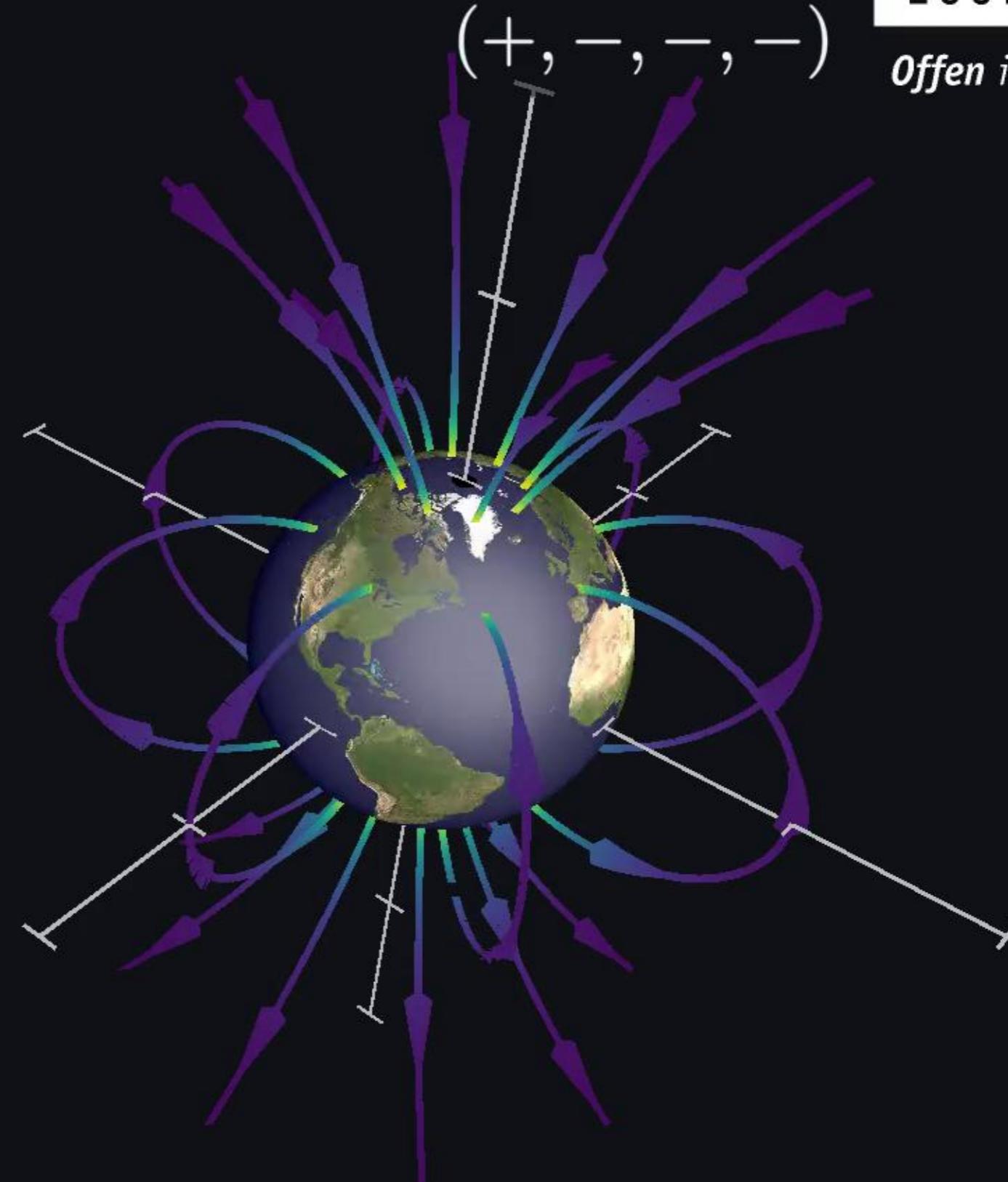
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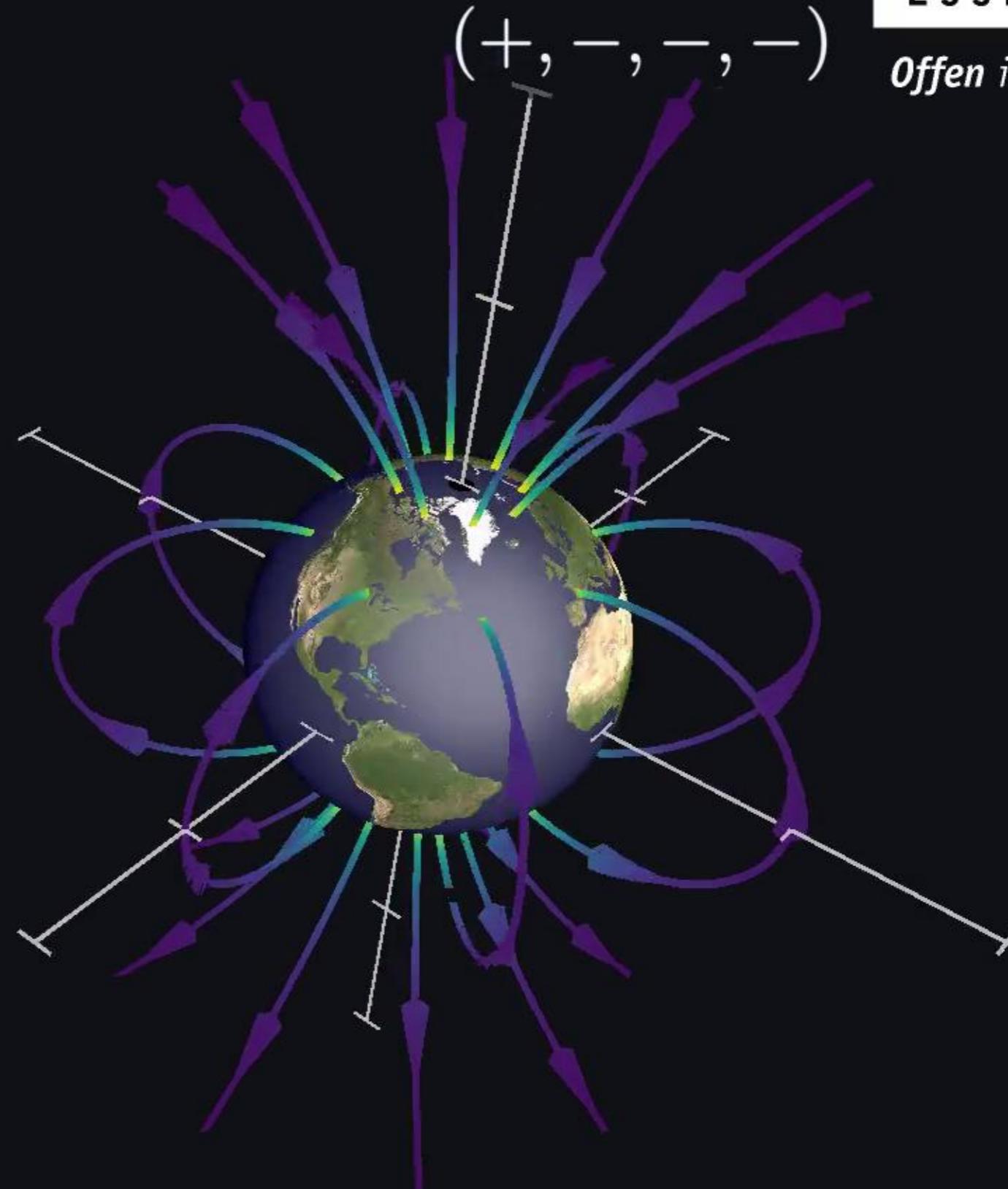
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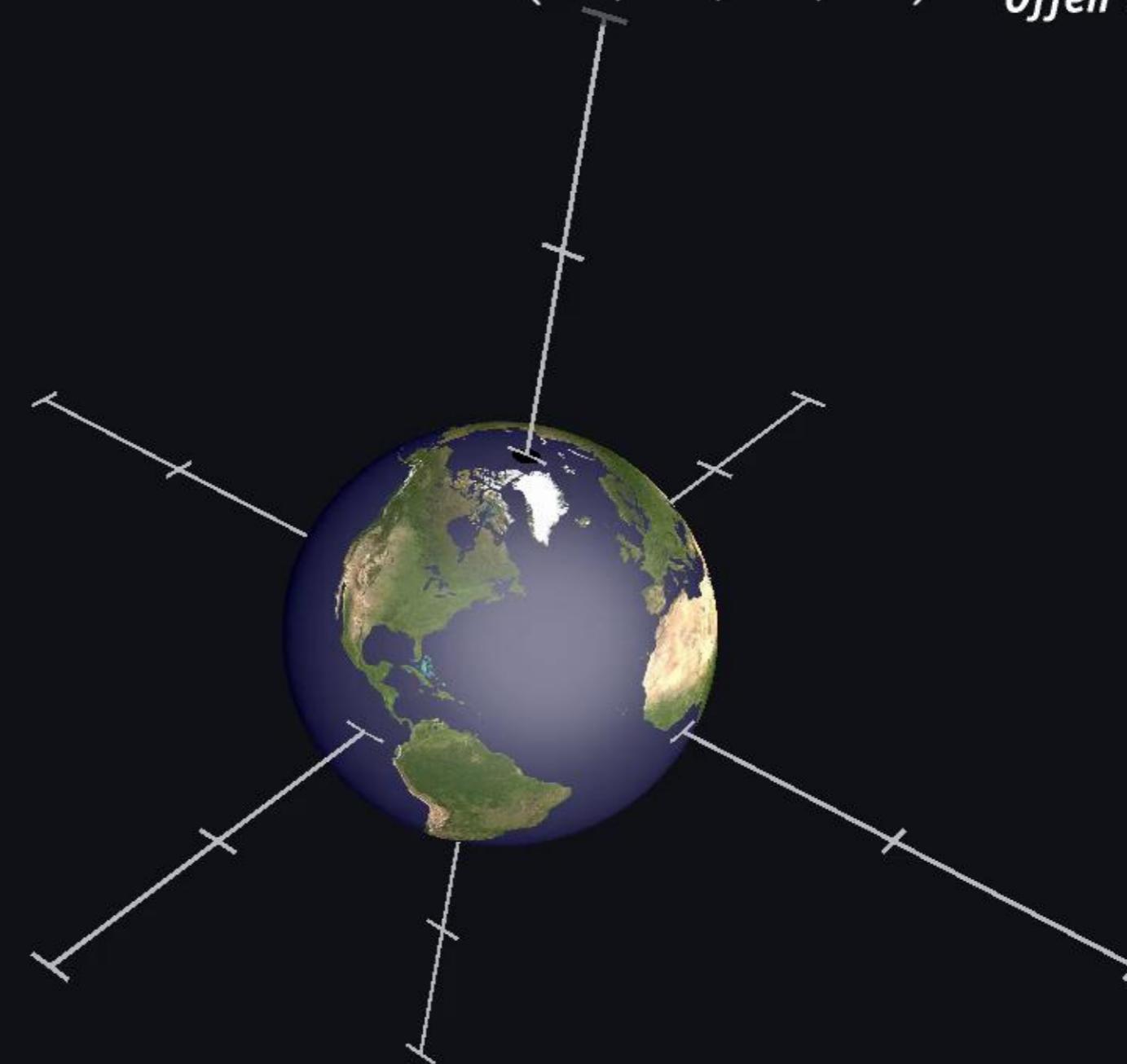
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$$c = G = R = 1$$
$$(+, -, -, -)$$

Trajektorien

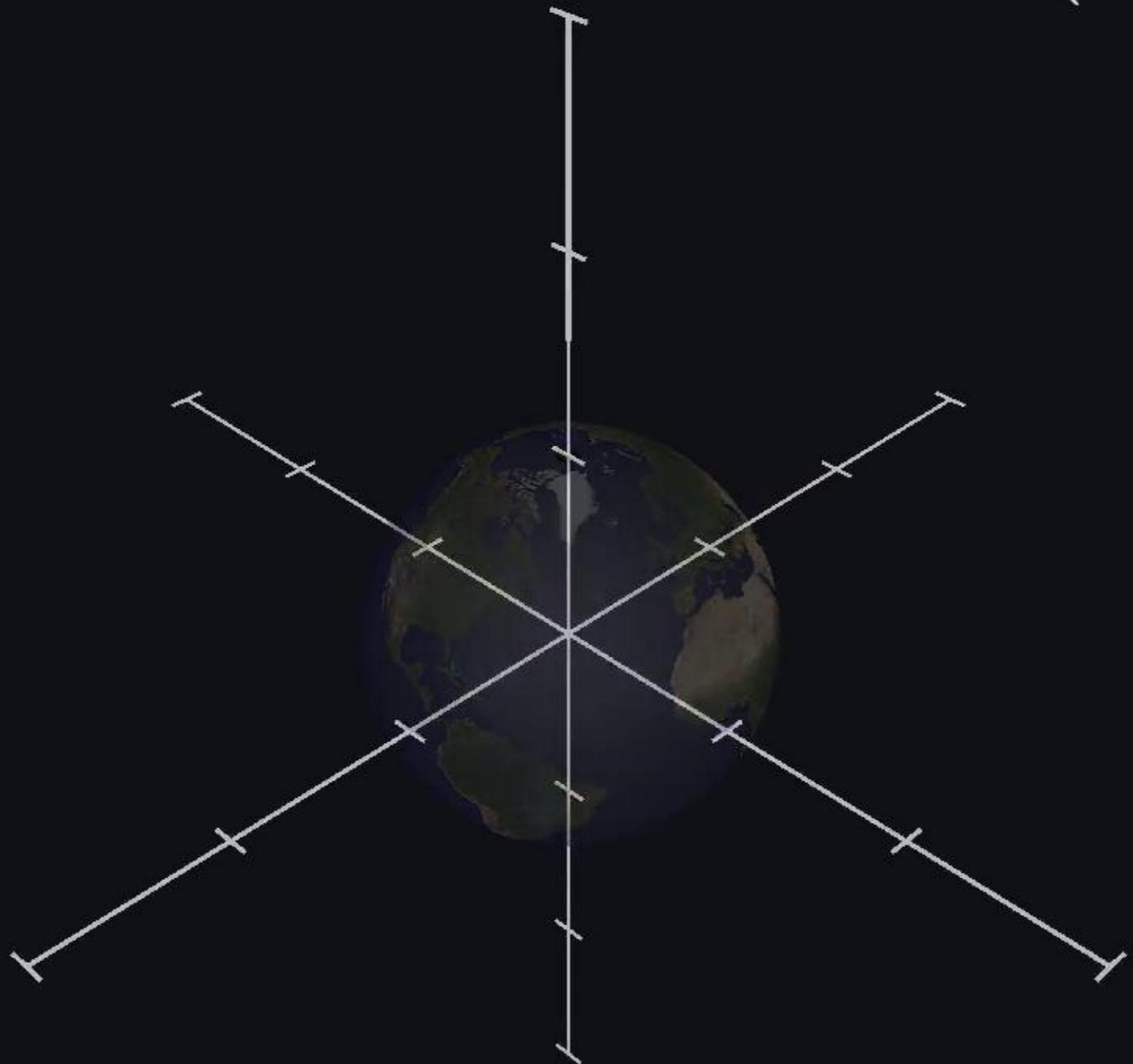


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Trajektorien

$$\vec{F} = m \left(\vec{E} + \vec{v} \times \vec{B} \right)$$

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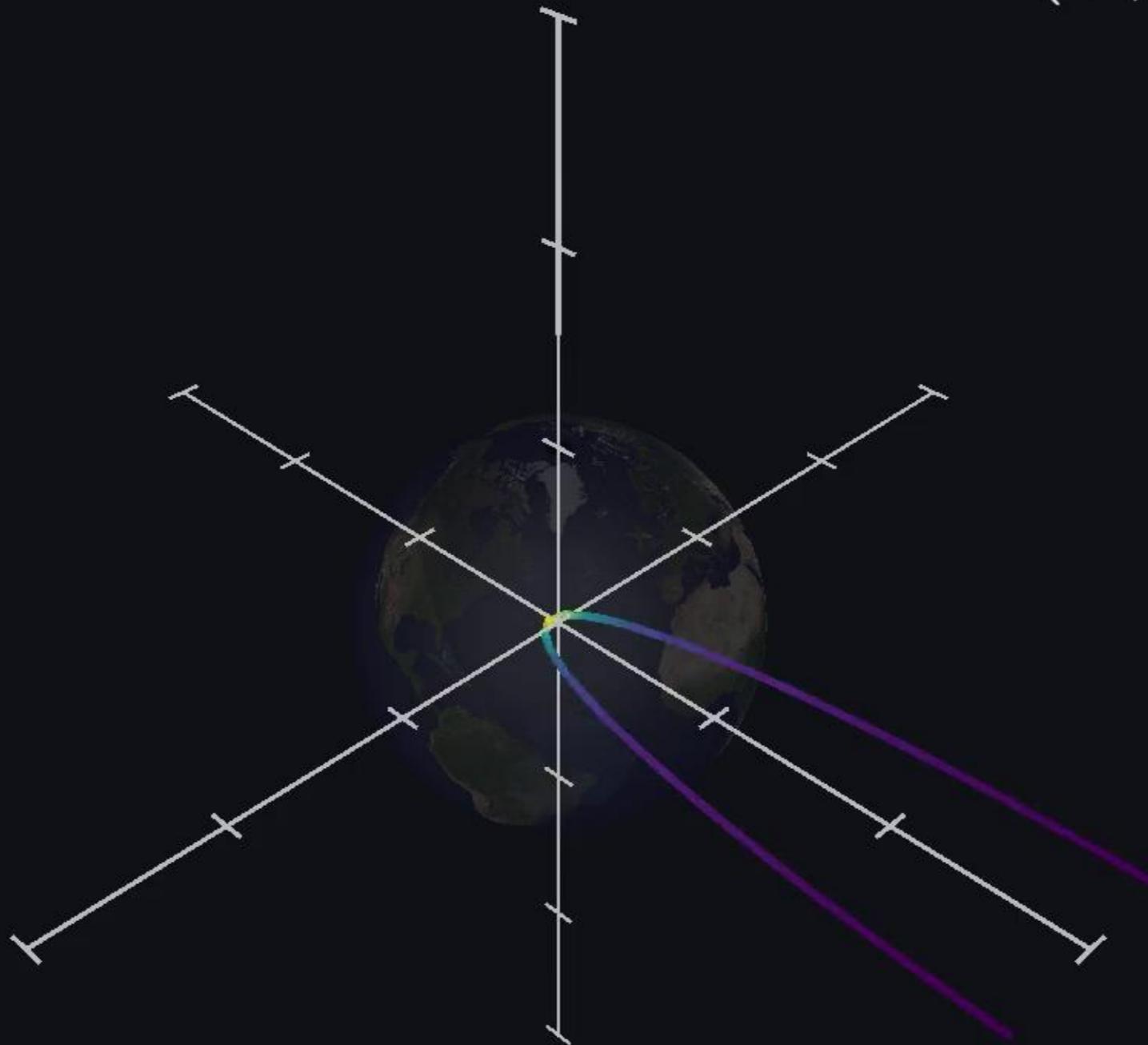


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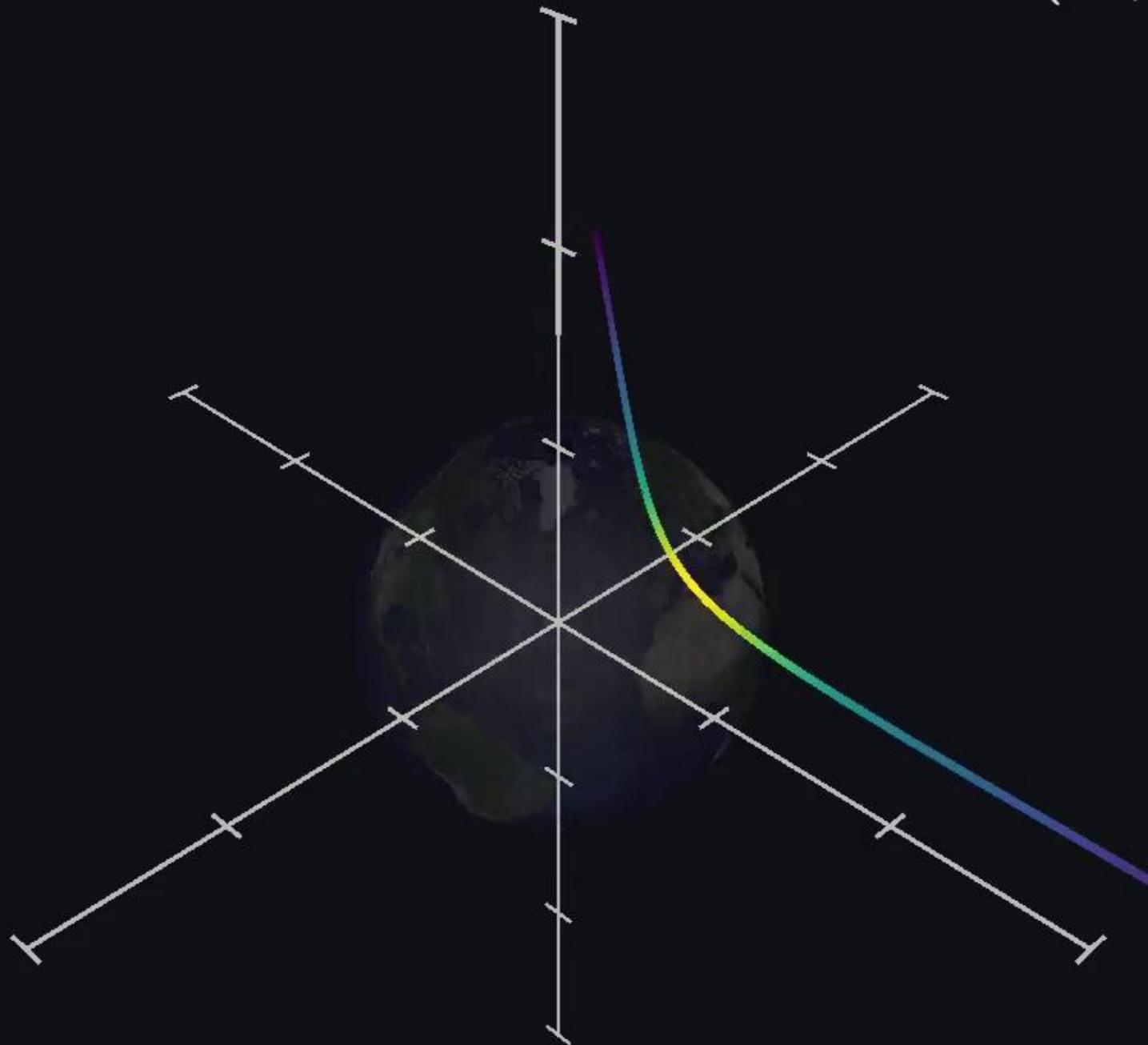


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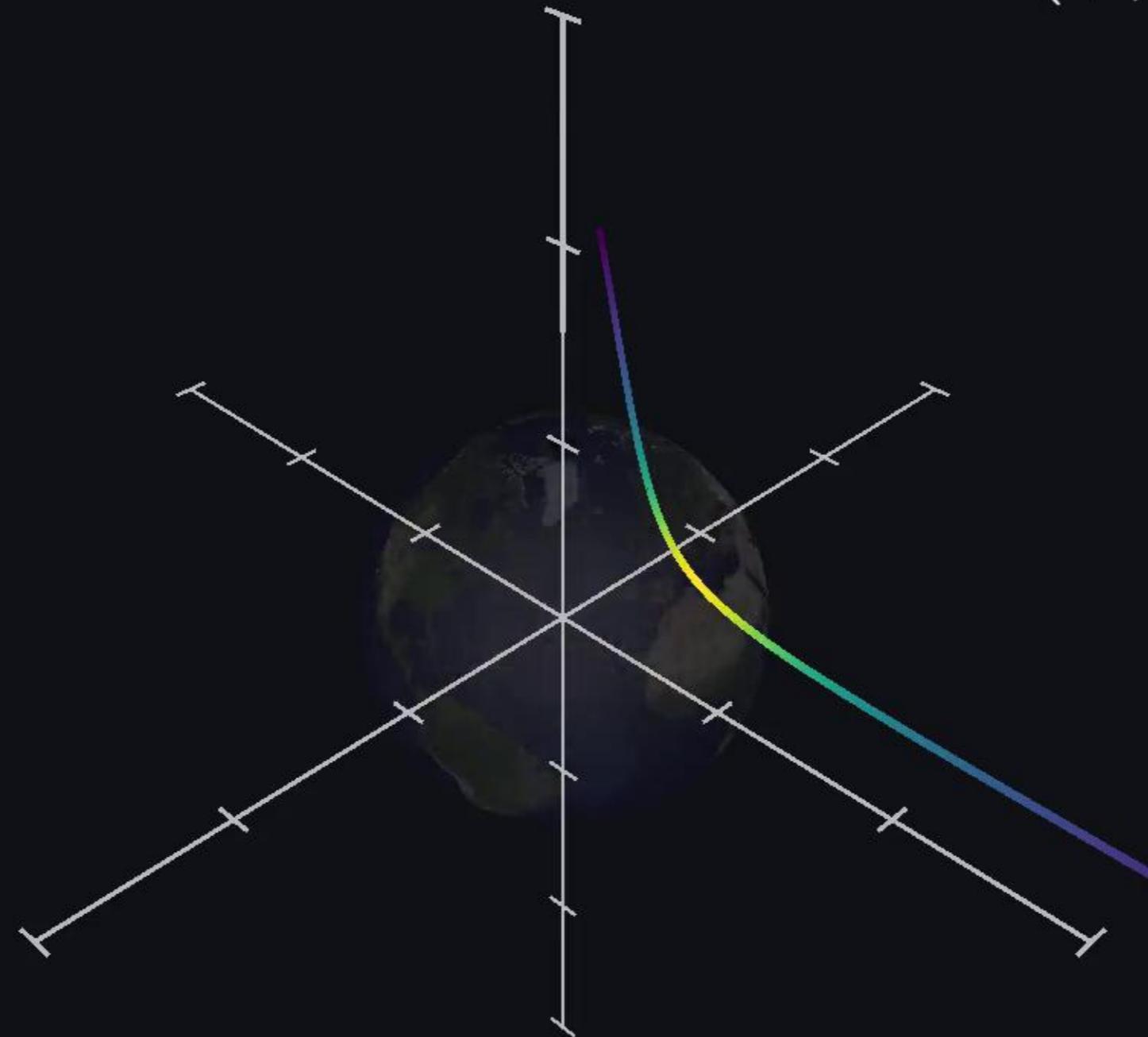


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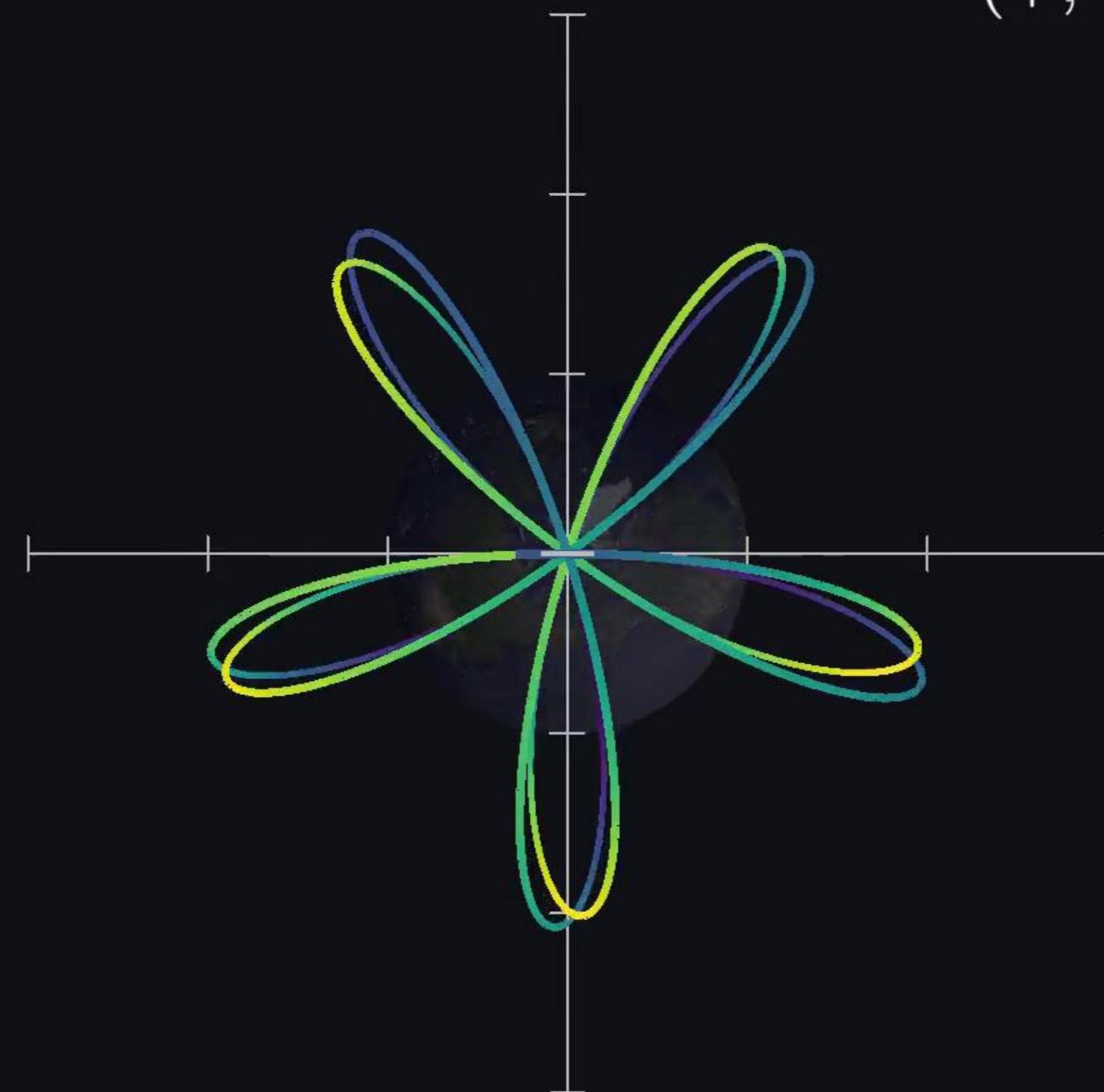


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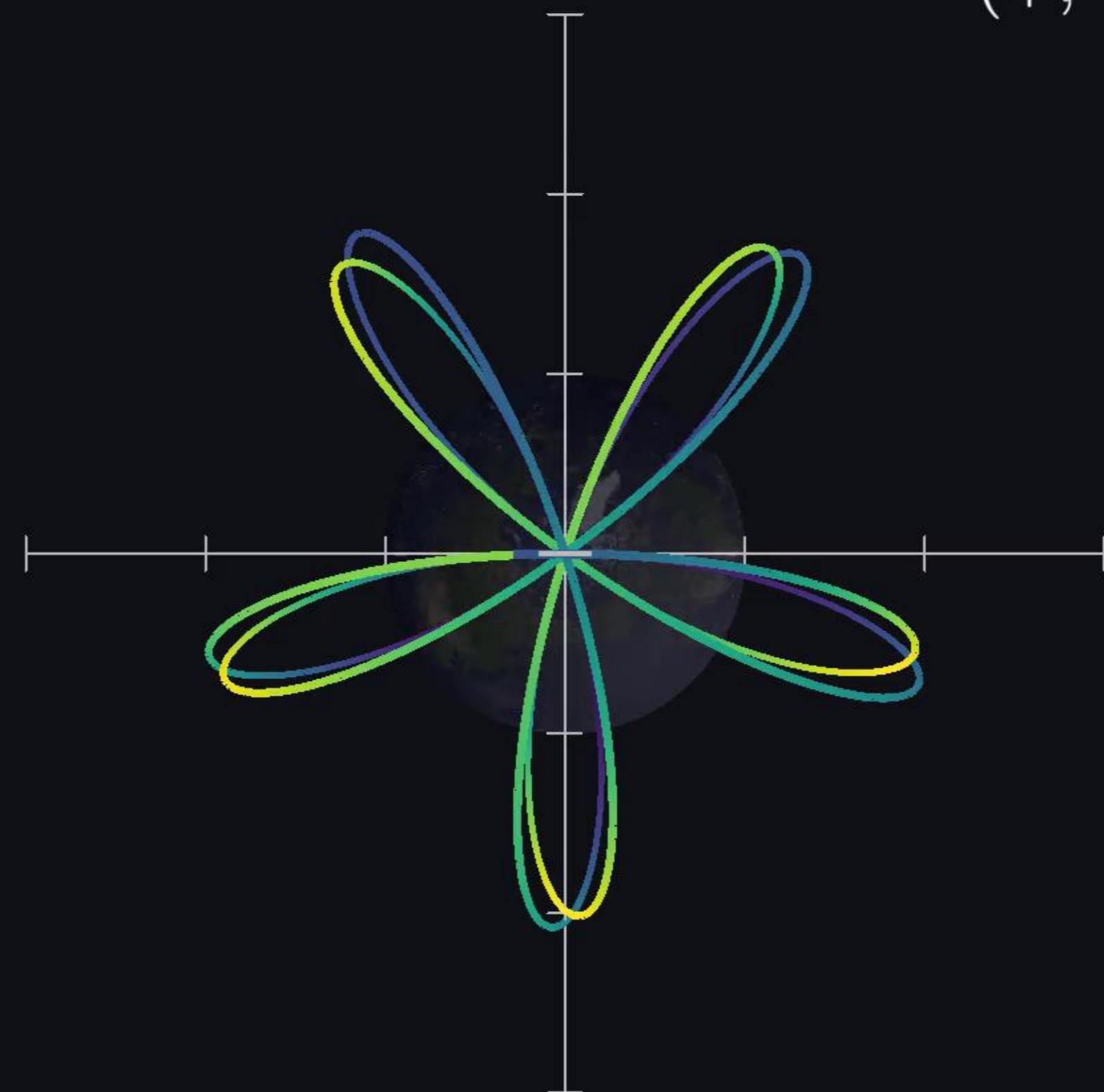


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Raumzeitdarstellung

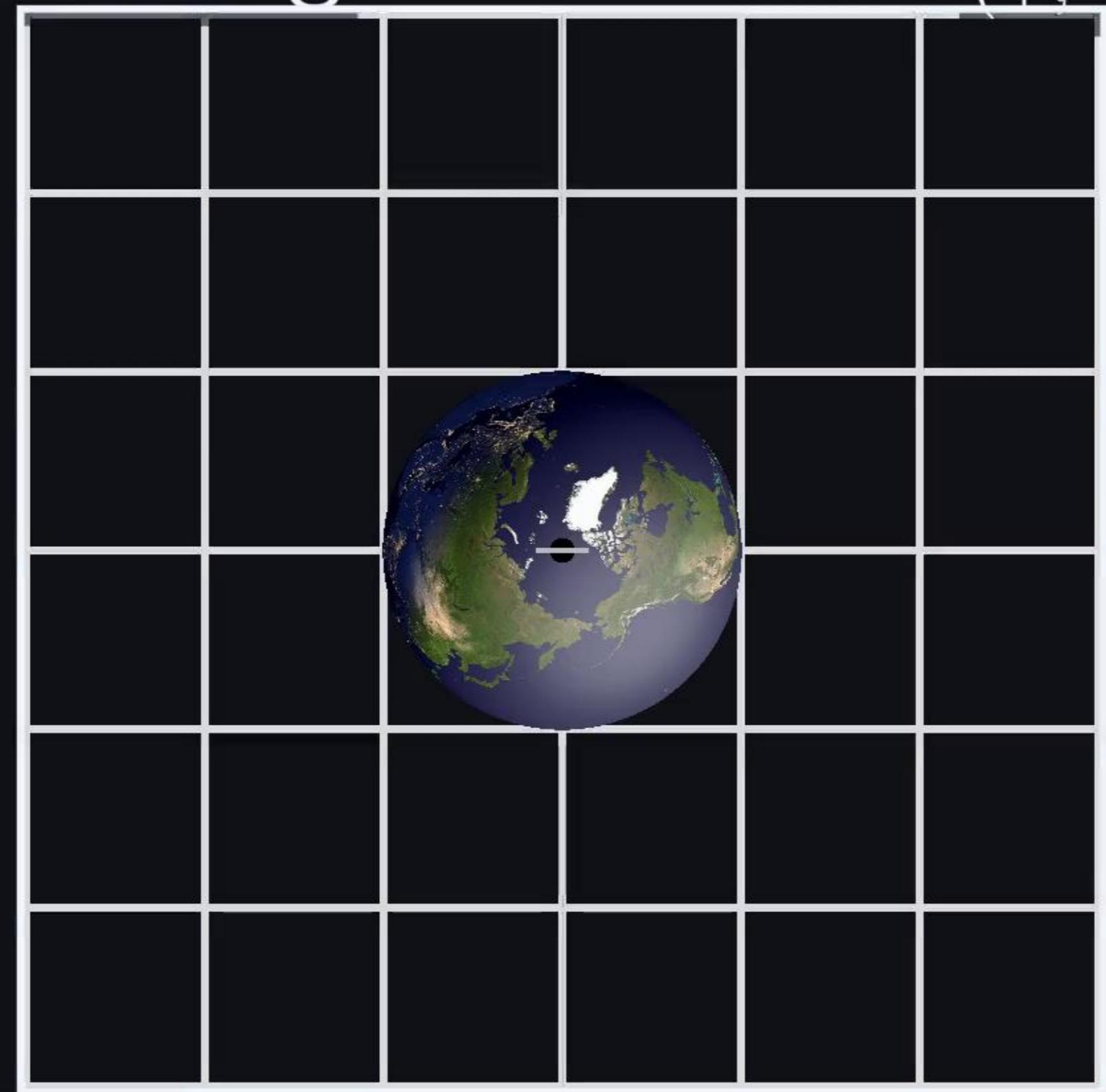


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(+,-,-,-)

Raumzeitdarstellung

Zeitentwicklung
für jeden
Gitterpunkt
bei $\omega = 0$

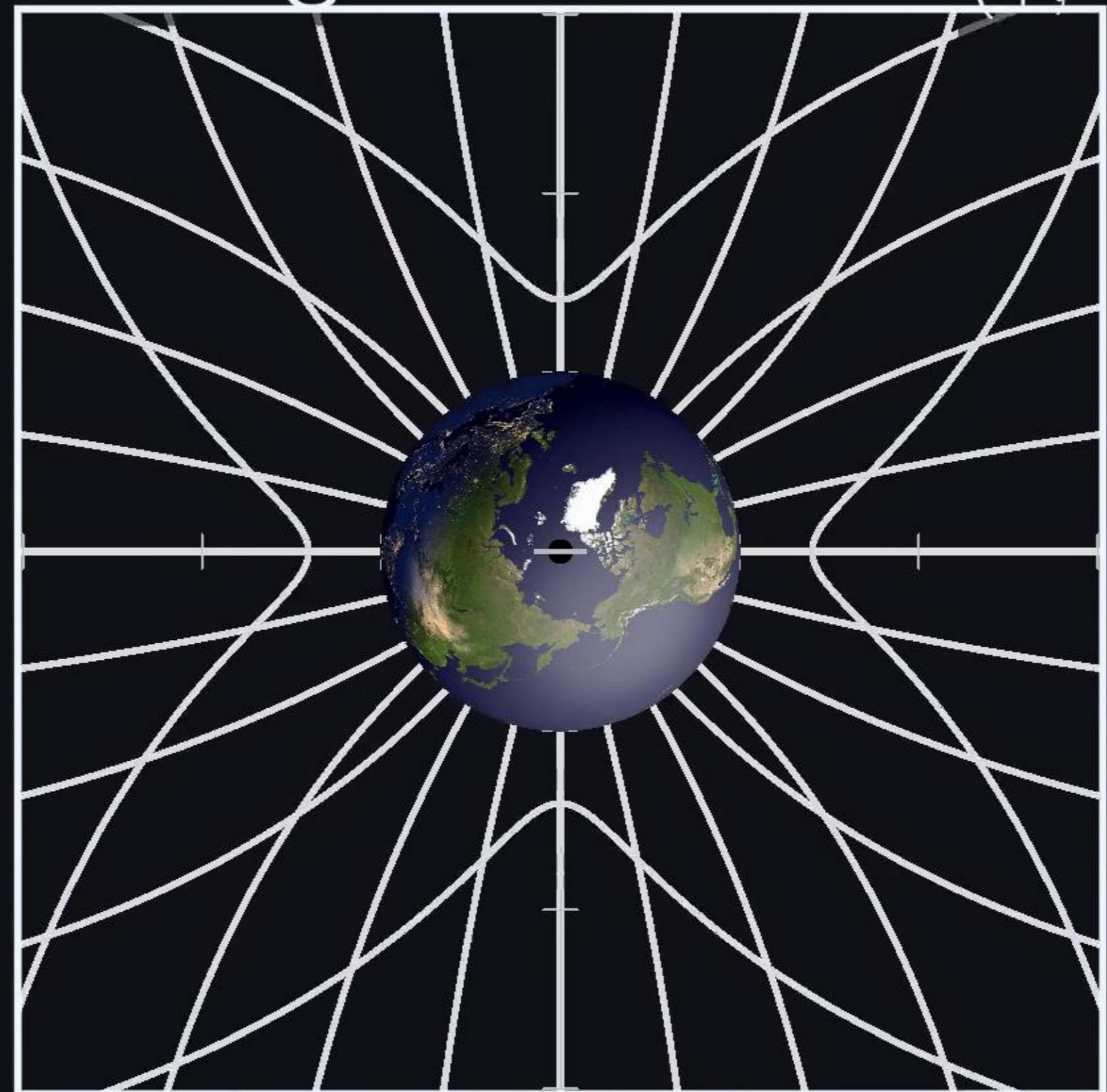


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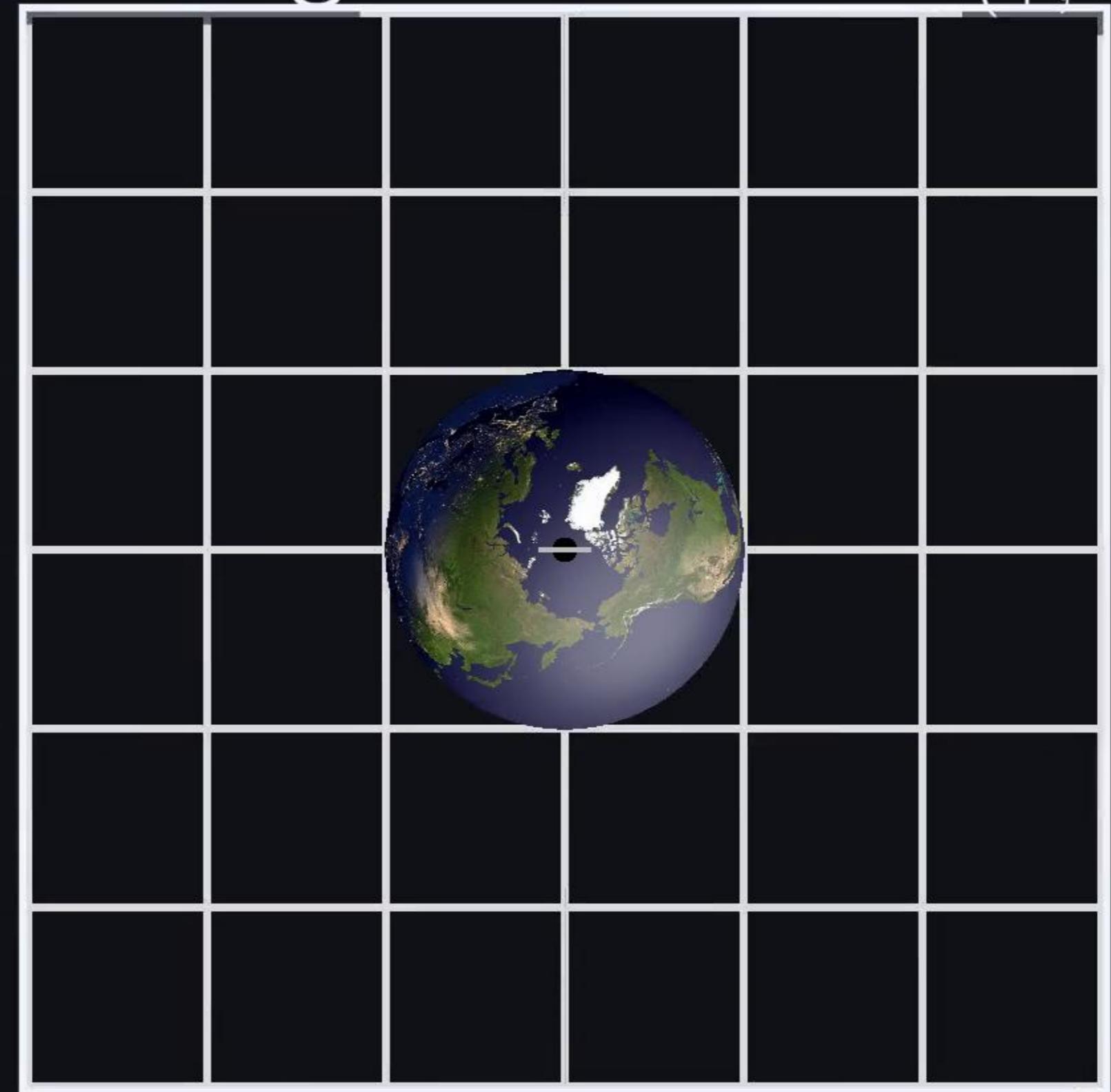
Zeitentwicklung
für jeden
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(+,-,-,-)



Raumzeitdarstellung

Zeitentwicklung
für jeden
Gitterpunkt
bei $\omega = 1$

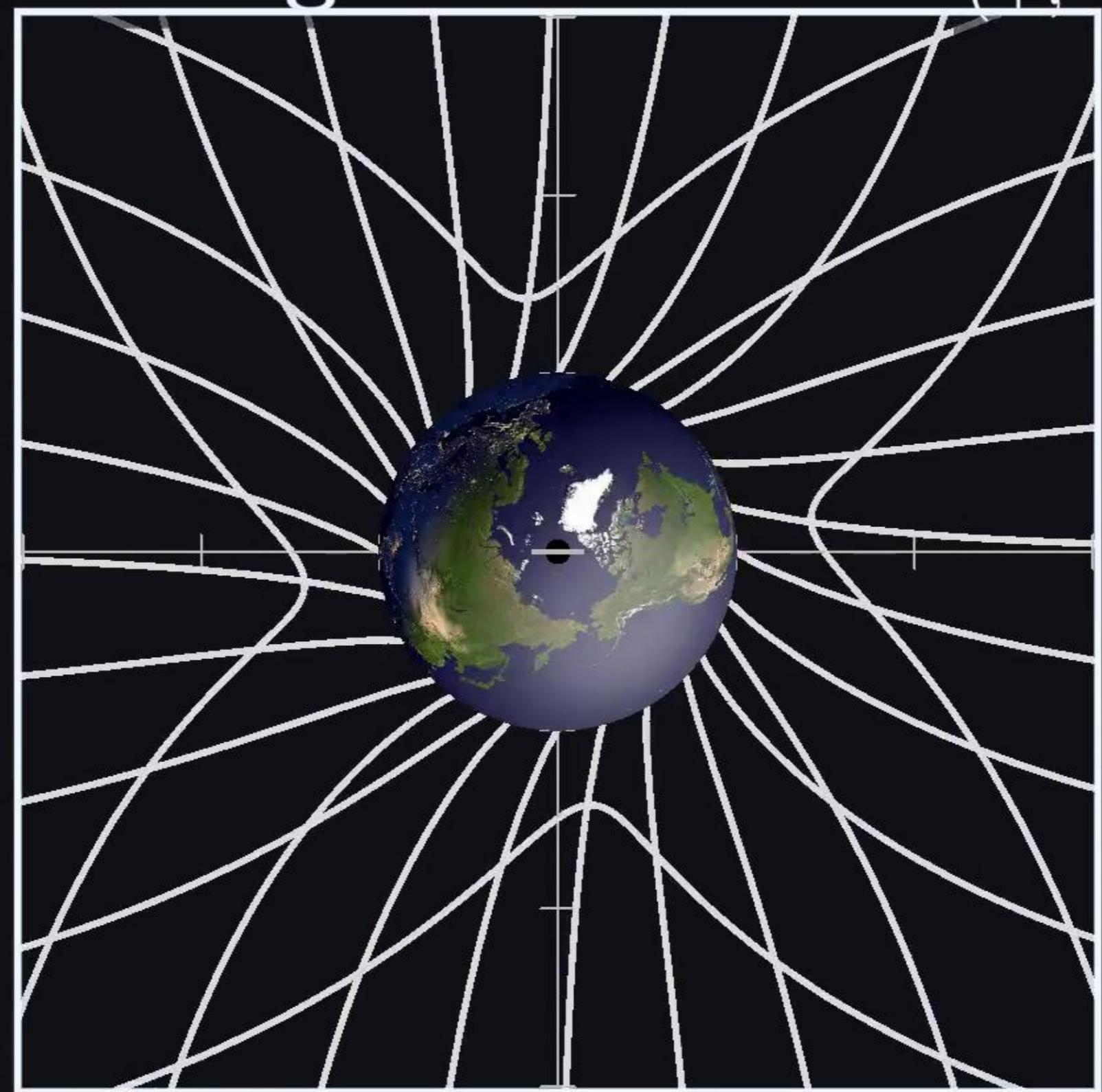


$$c = G = R = 1$$

Raumzeitdarstellung

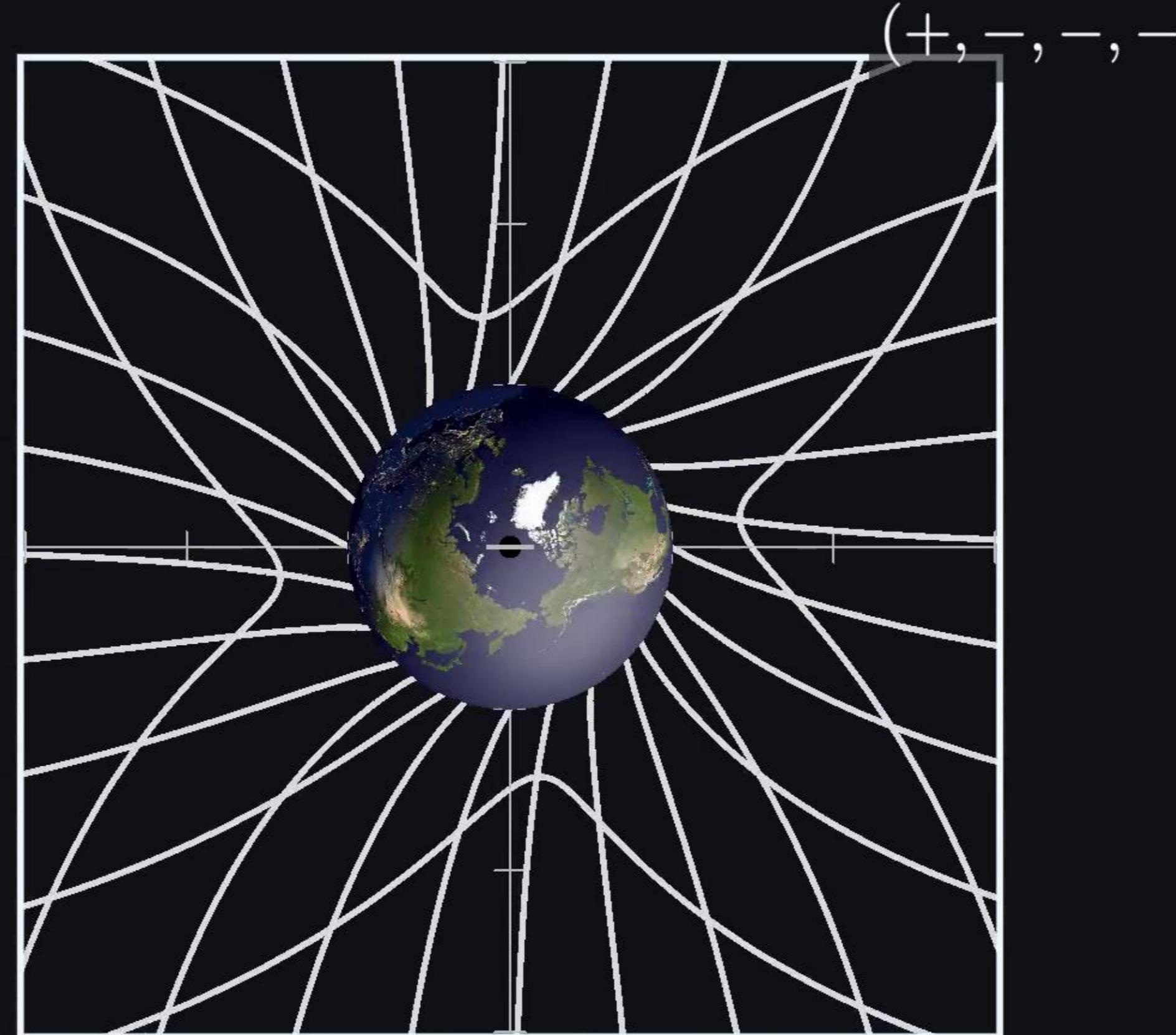
Zeitentwicklung
für jeden
Gitterpunkt
bei $\omega = 1$

(+,-,-,-)



$$c = G = R = 1$$

Präzession



Präzession

$$c = G = R = 1$$

(+, -, -, -)



Lense-Thirring-Effekt

Präzession

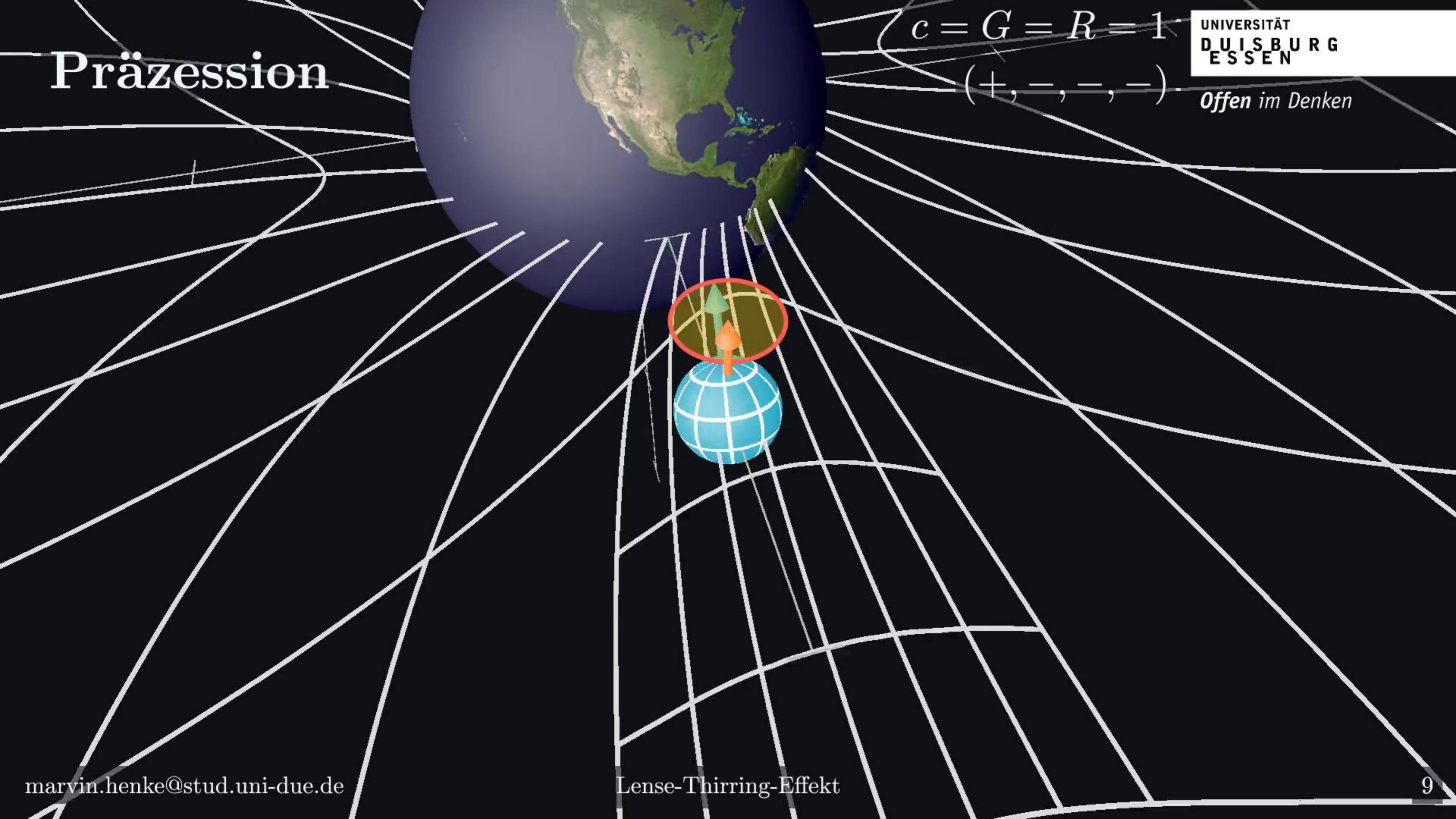
$$c = G = R = 1$$

(+, -, -, -)

Präzession

$$c = G = R = 1$$

$$(+, -, -, -)$$



$$c = G = R = 1$$

(+, -, -, -)

Präzession



Lense-Thirring-Effekt