

Lense-Thirring-Effekt

Vortrag im Hauptseminar SoSe 2025

Marvin Henke - 10. Mai 2025

Betreuer: Dr. Nikodem Szpak

Lense-Thirring-Effekt

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- Metrik und Geodäten

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- Einsteinsche Feldgleichungen

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$$\eta = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Lense-Thirring-Effekt

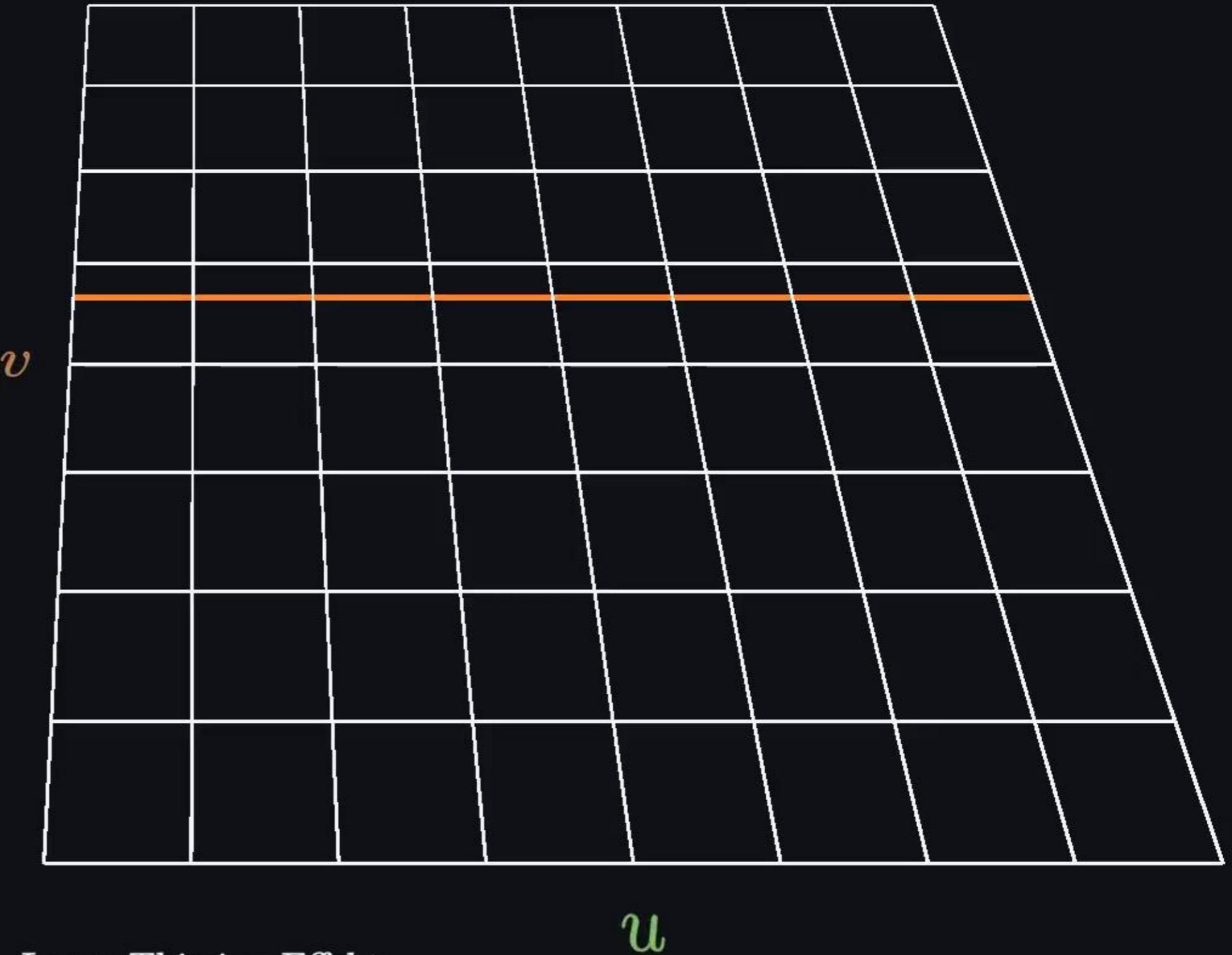
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Metrik und Geodäten

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$$c = G = 1$$

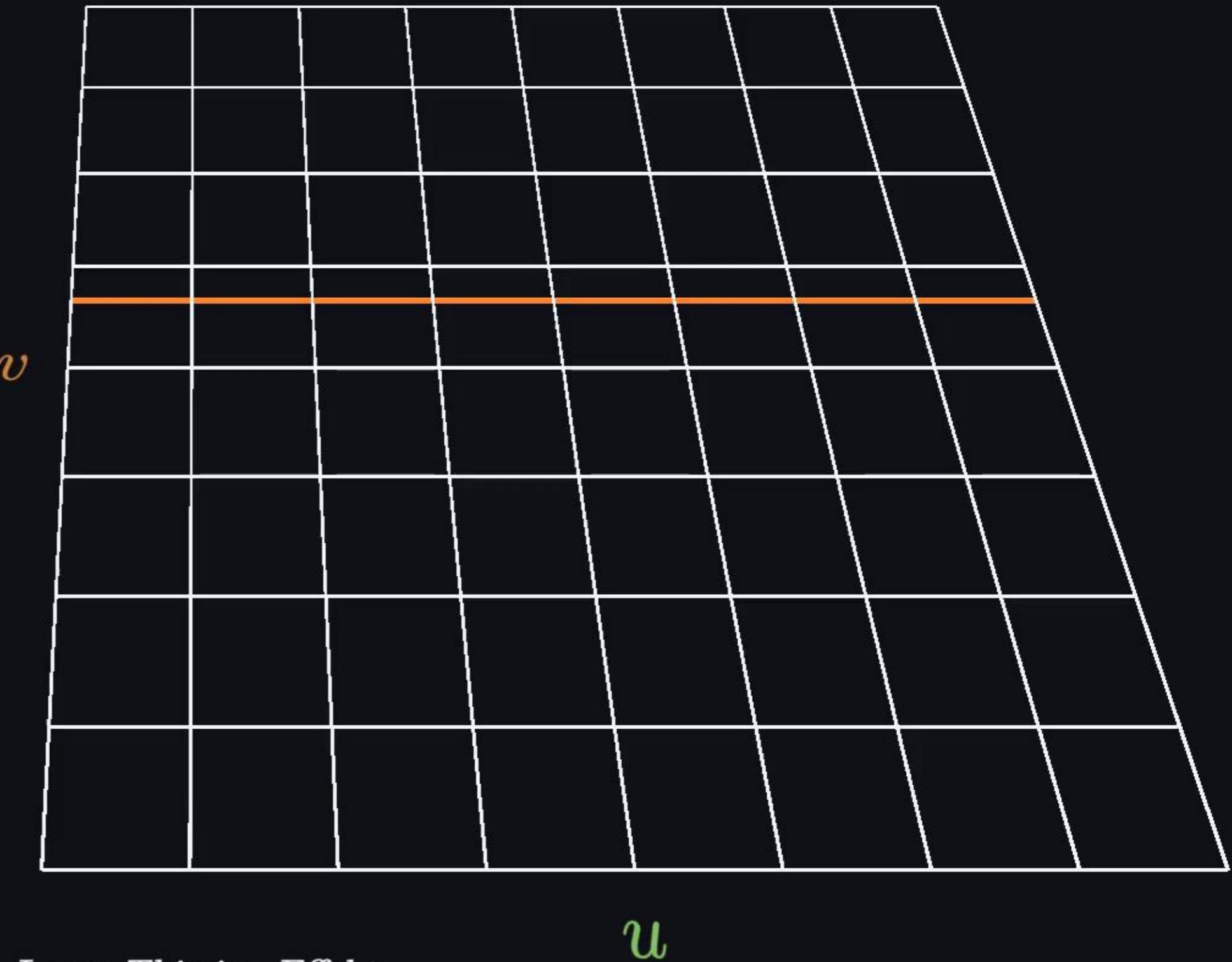
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Metrik und Geodäten

Fläche

$$\vec{x}(u, v) = (u, v, 0)$$



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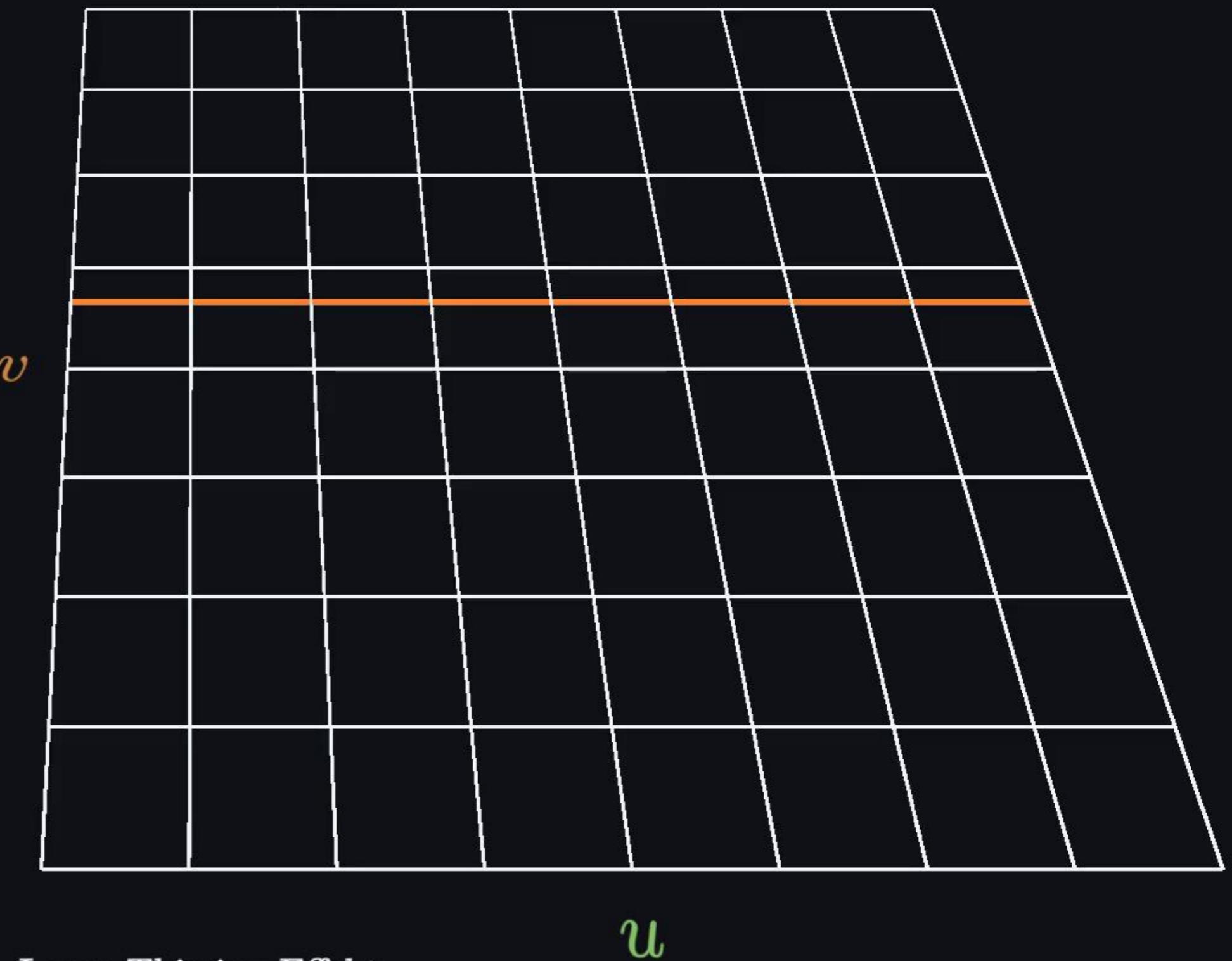
Fläche

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Metrik

$$g_{\mu\nu} = \partial_\mu \vec{x} \cdot \partial_\nu \vec{x}$$

$$g = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



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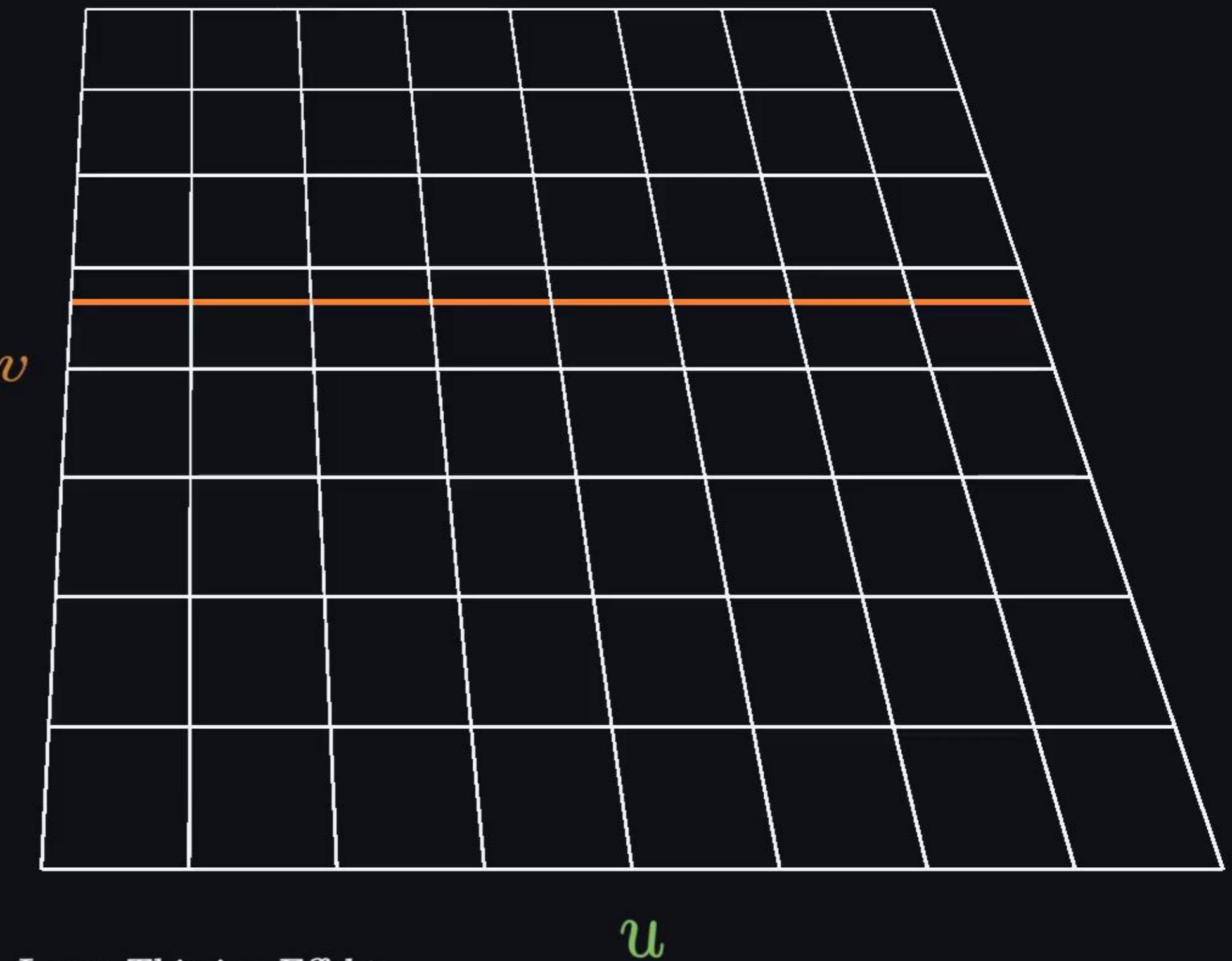
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Geodätengleichung

$$\frac{d^2 \vec{x}^\lambda}{d\tau^2} = 0$$



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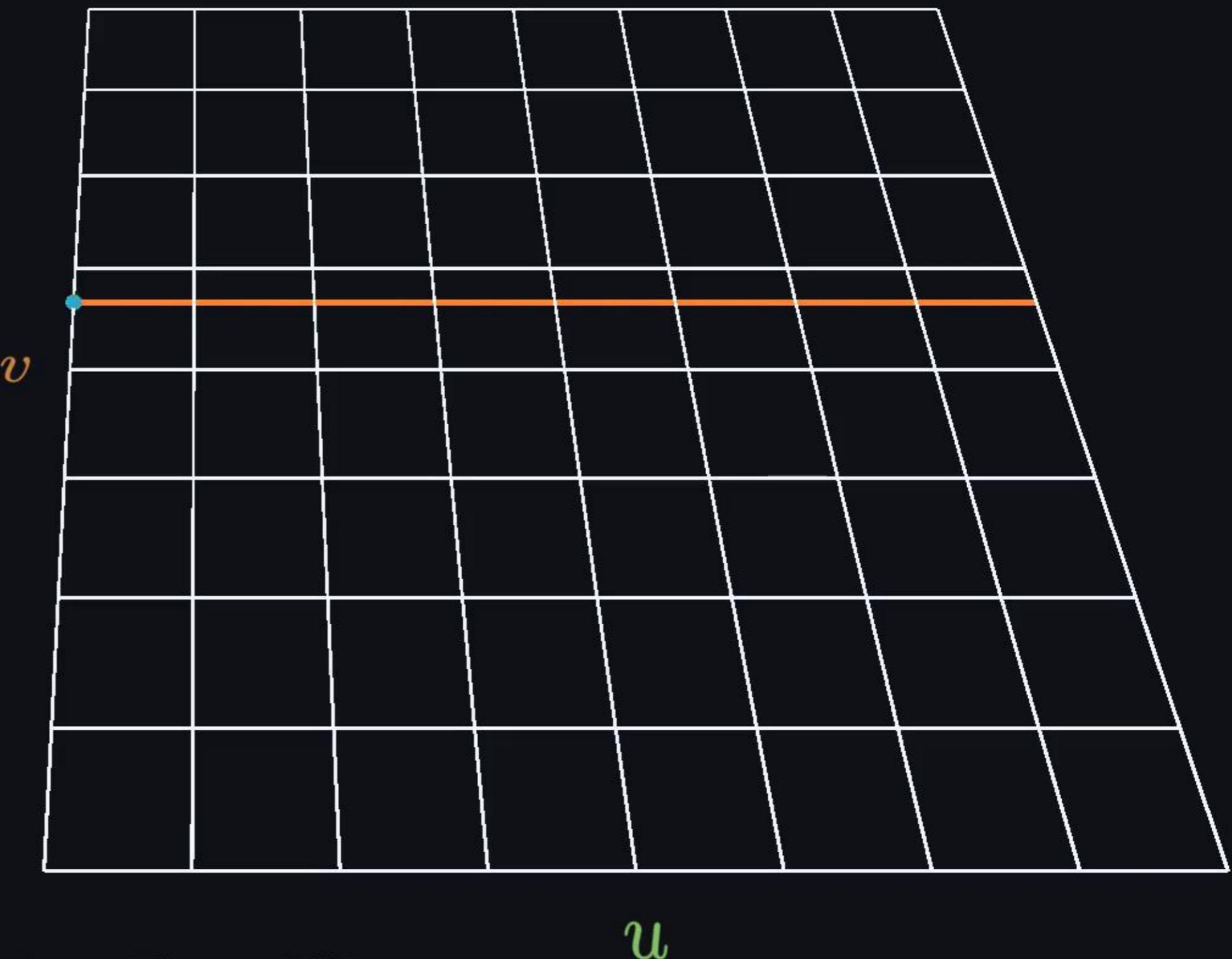
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$$\vec{x} : \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

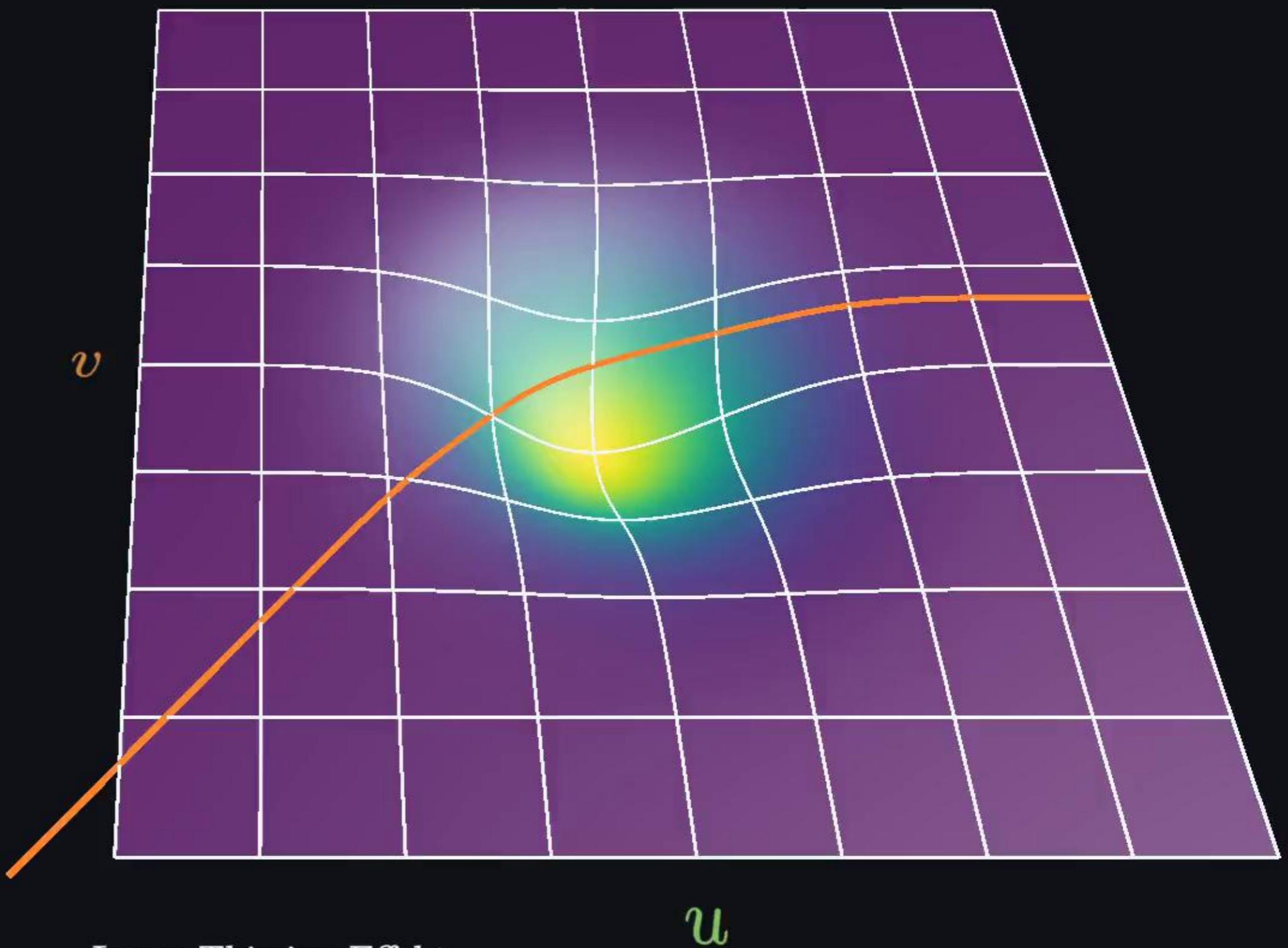
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Geodätengleichung

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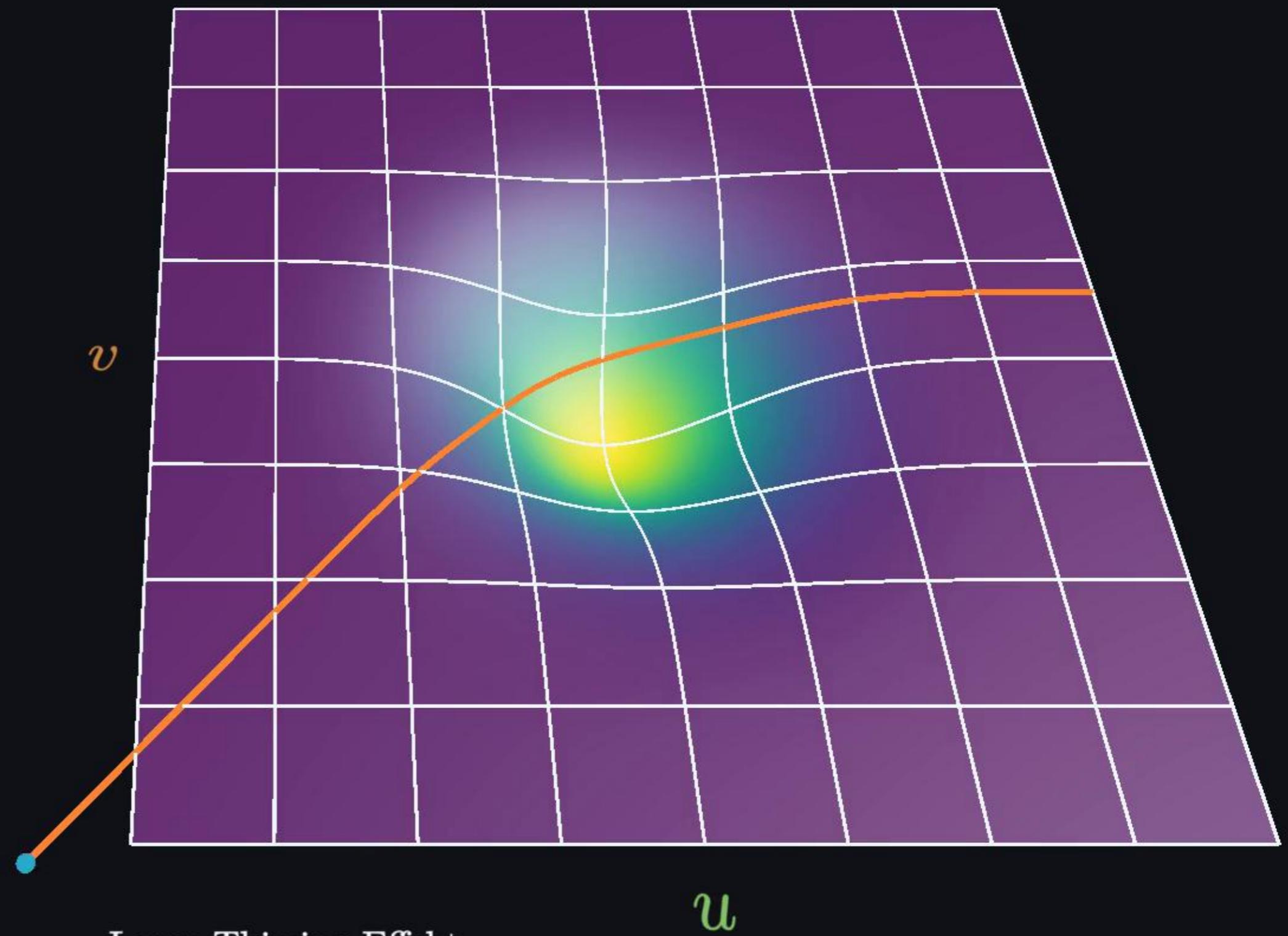
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Einsteinsche Feldgleichungen

Einstein'sche Feldgleichungen

2D Fläche \rightarrow 4D Mannigfaltigkeit

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Energie-Impuls-Tensor: $T_{\mu\nu}$

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Gravitoelektromagnetismus

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Substitutionen: $\vec{E} = \frac{1}{2} \vec{\nabla} h_{00}$, $B_j = -\varepsilon_{jlm} \frac{\partial h_{0m}}{\partial \textcolor{teal}{x}^l}$

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$$\vec{F} = m \left(\vec{E} + \vec{v} \times \vec{B} \right)$$

Rotierende Kugelmasse

$$c = G = 1$$

(+ - -)

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$$\rho(|\vec{x}|) = \rho_0 \Theta(R - |\vec{x}|)$$



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$$I = \frac{2}{5} M R^2$$

$$\vec{S} = I \vec{\omega}$$



EM-Felder

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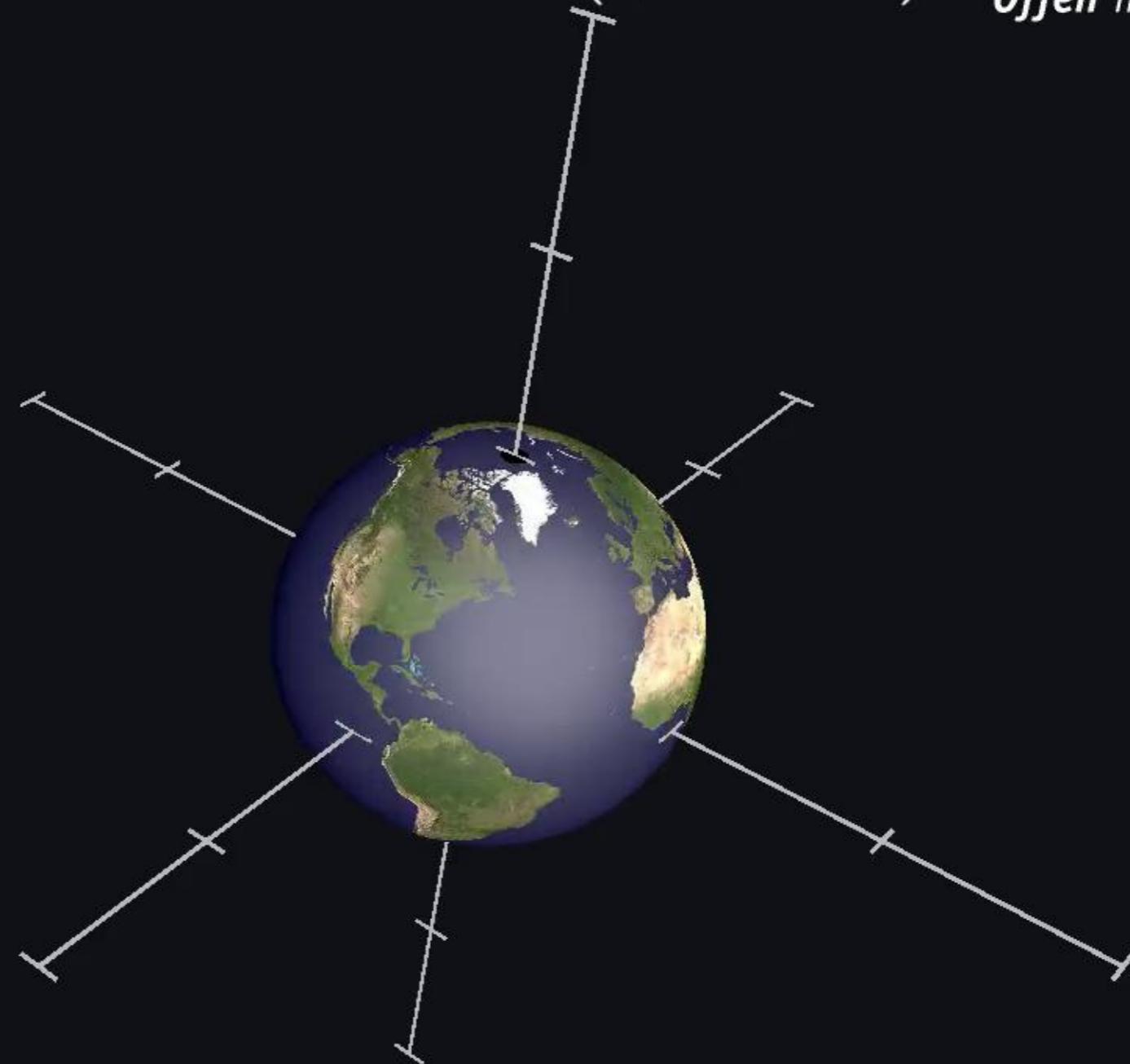
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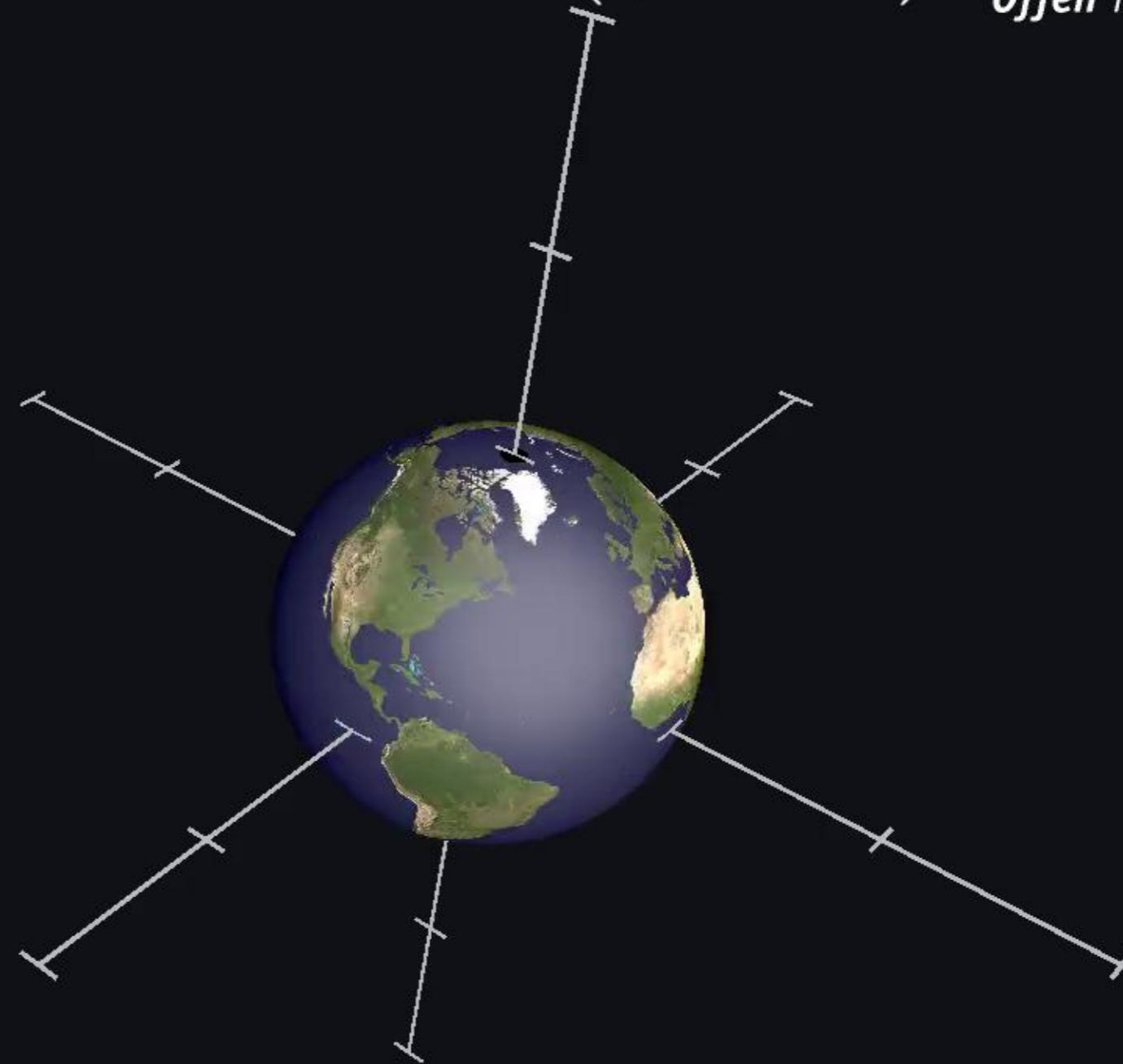
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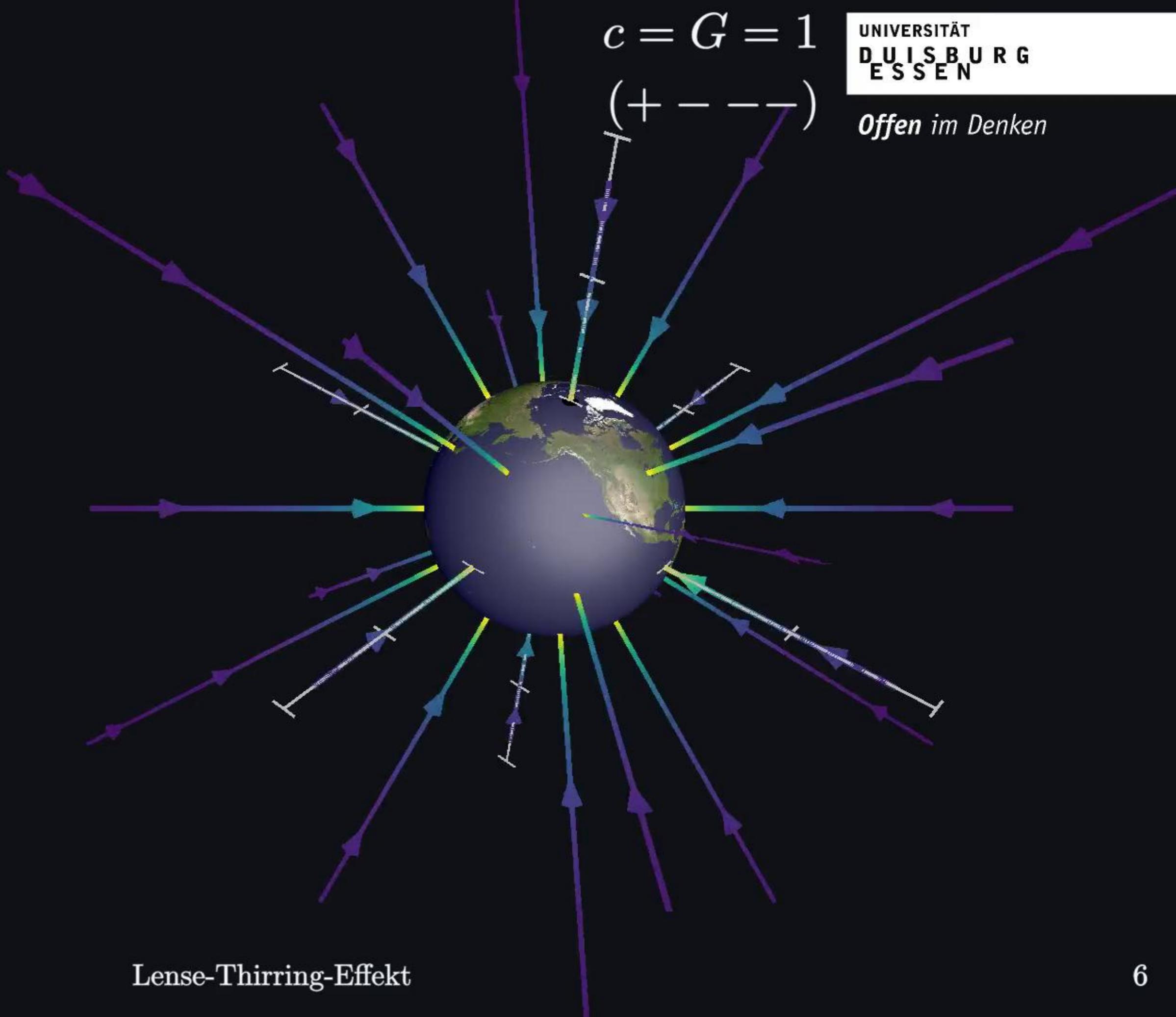
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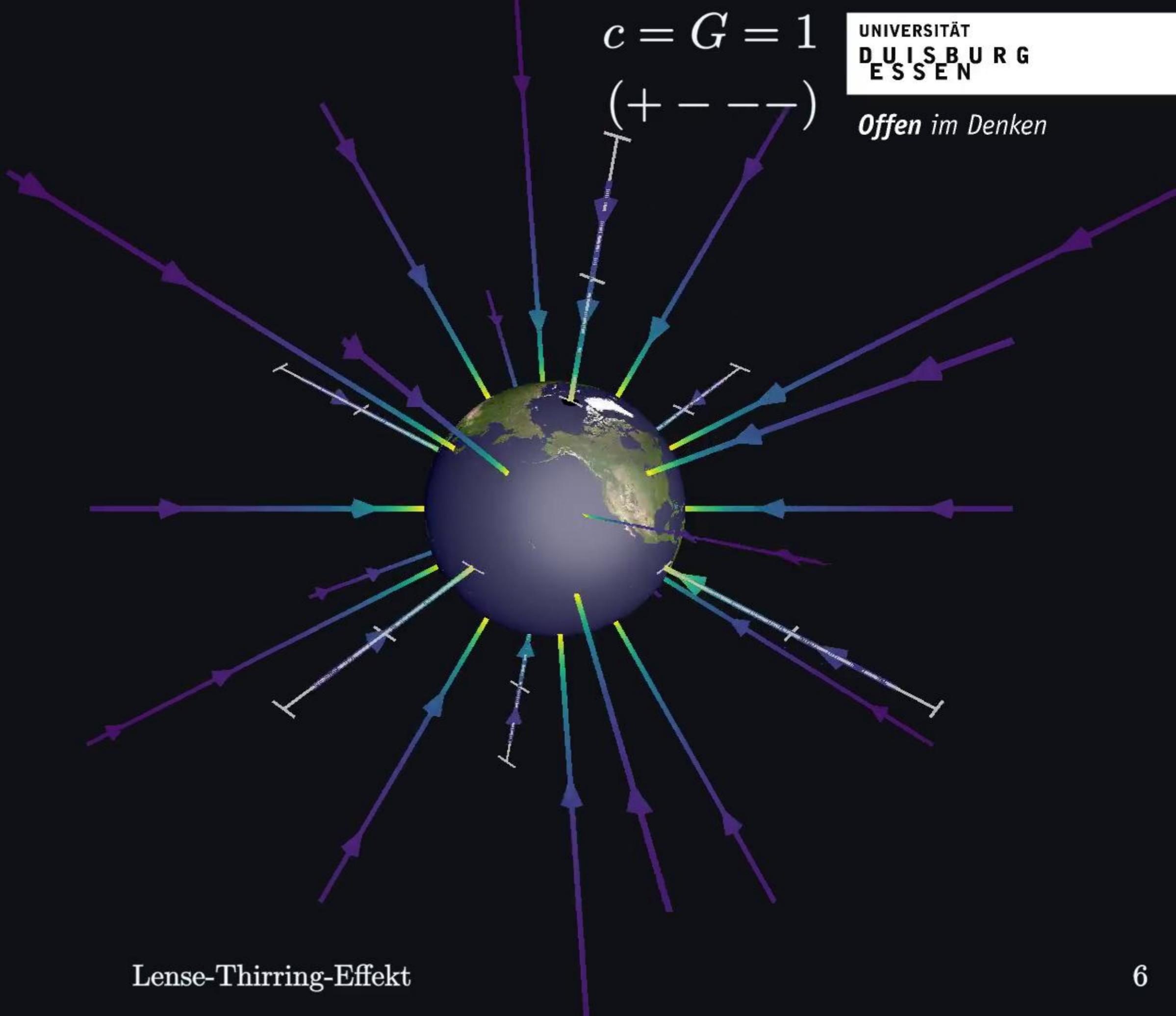
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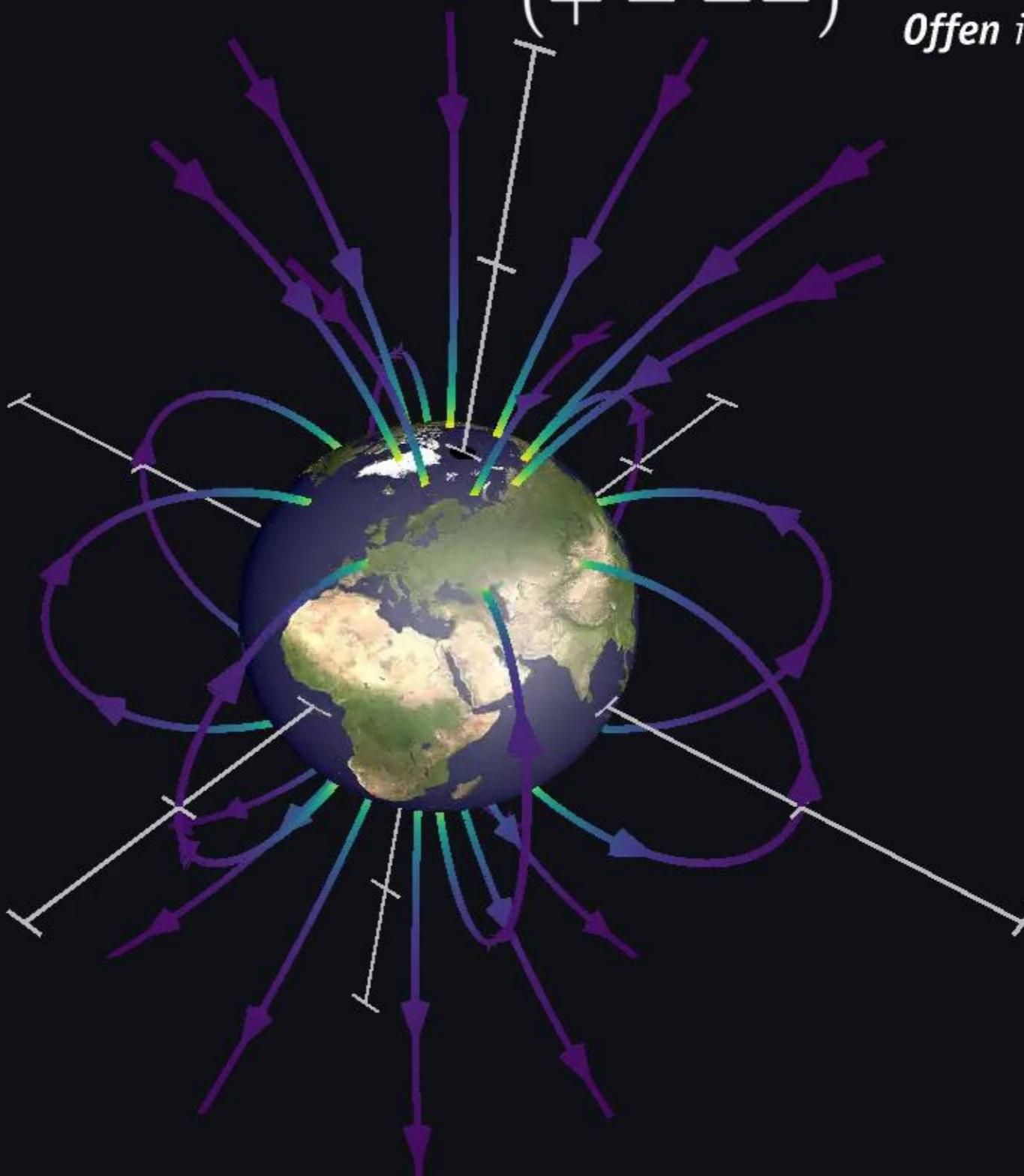
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