

Lense-Thirring-Effekt

Vortrag im Hauptseminar SoSe 2025

Marvin Henke - 10. Mai 2025

Betreuer: Dr. Nikodem Szpak

Lense-Thirring-Effekt

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- Metrik und Geodäten

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- Einsteinsche Feldgleichungen

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- Gravitoelektromagnetismus

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$$\boldsymbol{\eta} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Lense-Thirring-Effekt

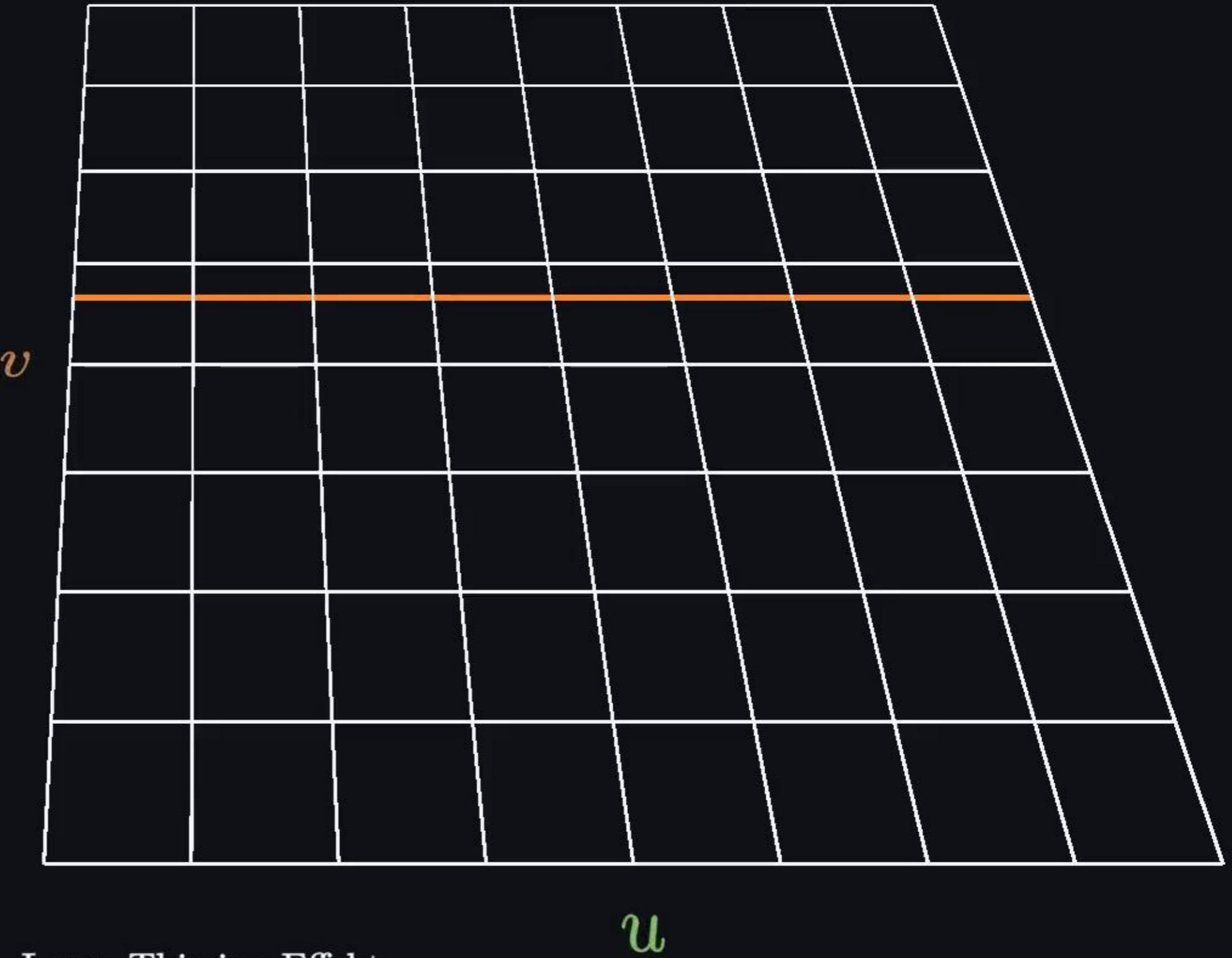
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$$(+,-,-,-)$$

Metrik und Geodäten

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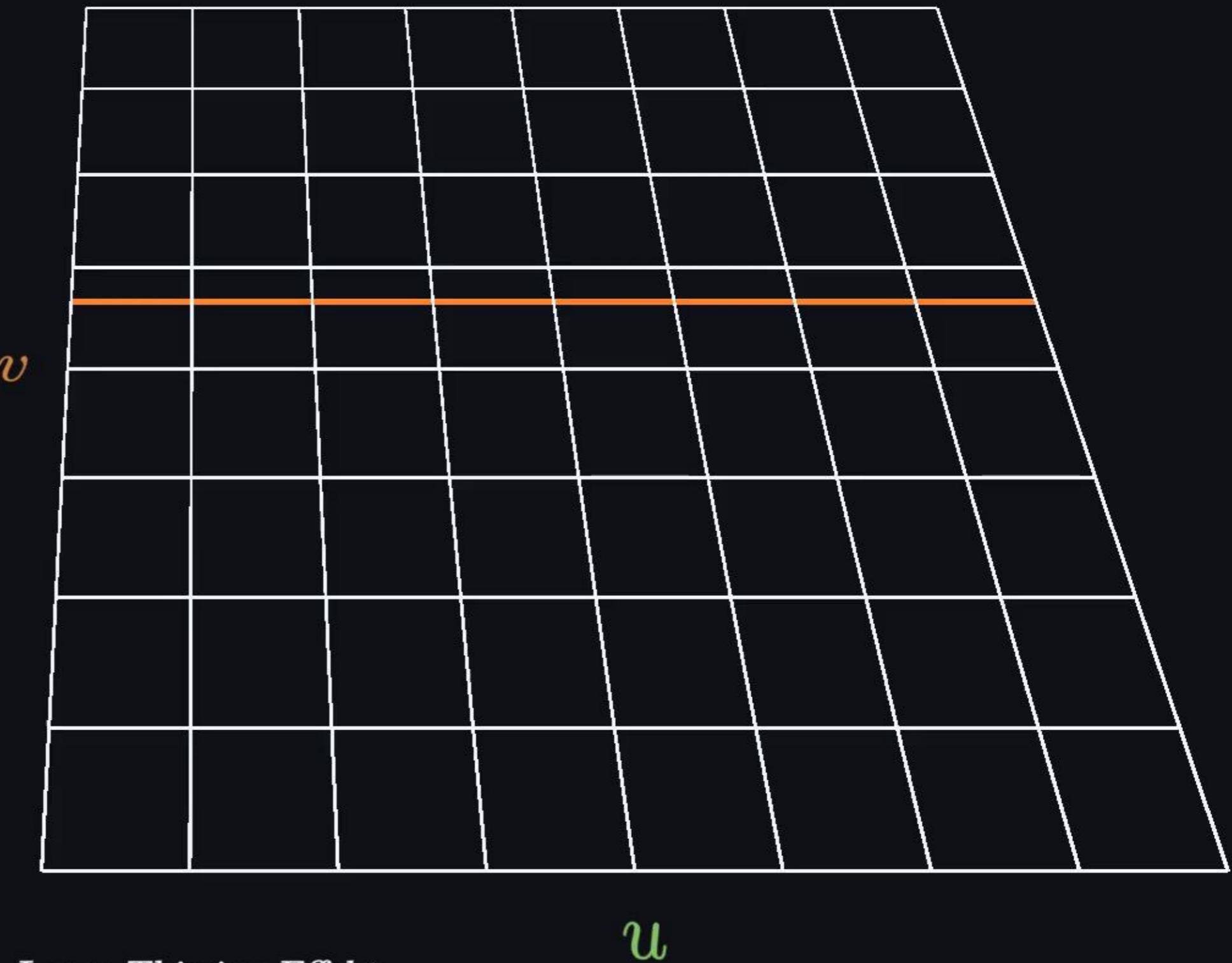


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Metrik und Geodäten

Fläche

$$\vec{x}(u, v) = (u, v, 0)$$



Lense-Thirring-Effekt

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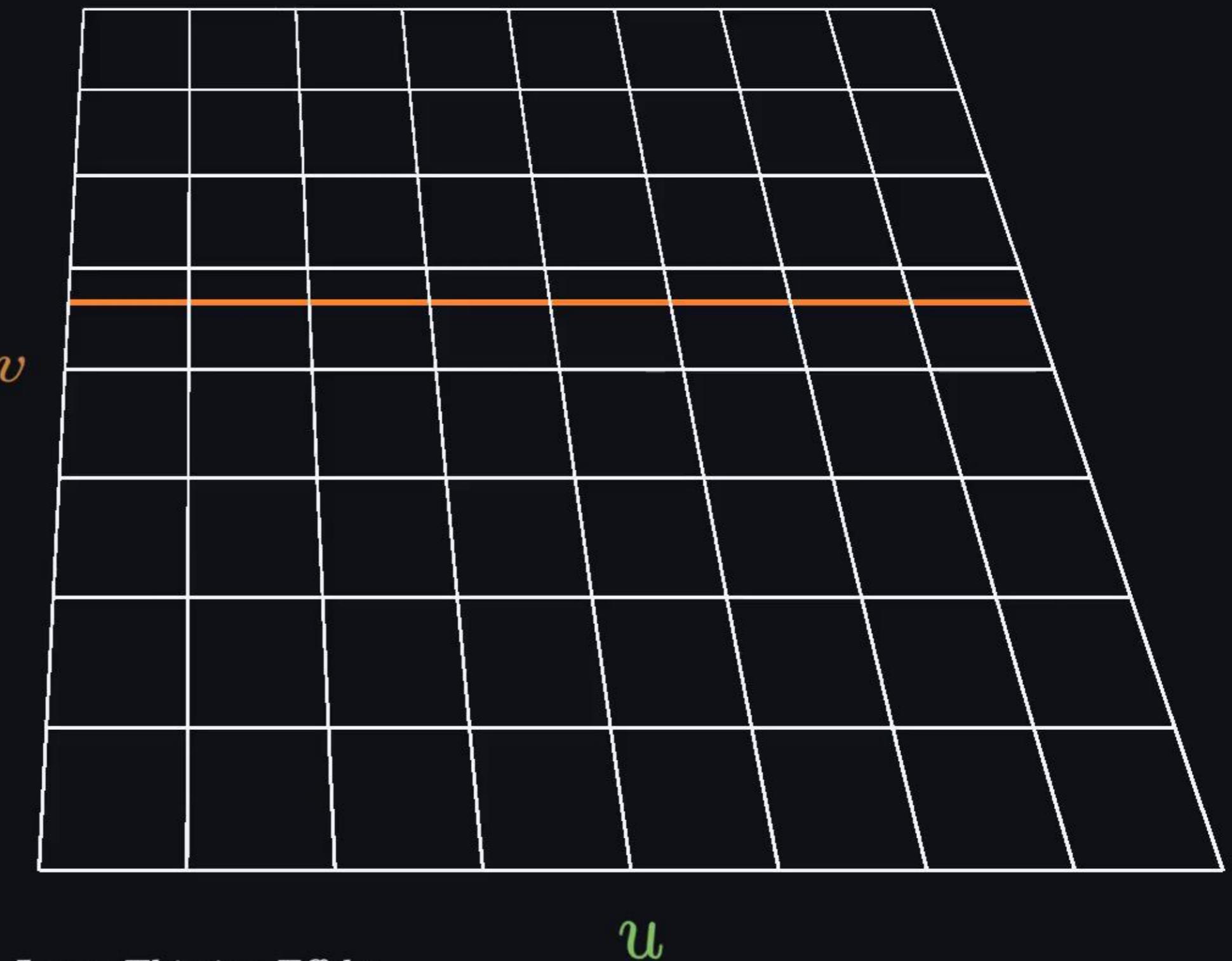
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Metrik

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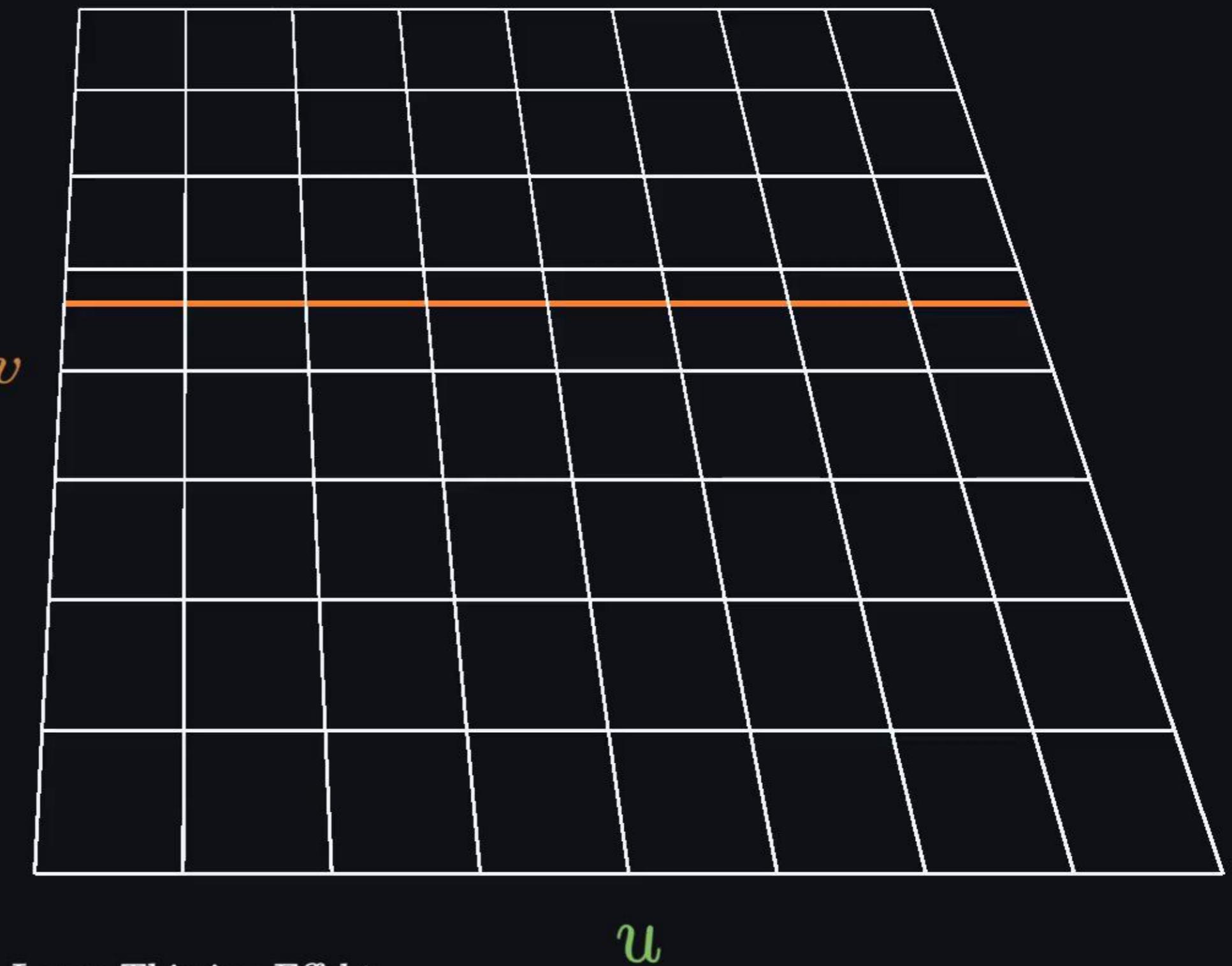
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Geodätengleichung

$$\frac{d^2 \vec{x}^\lambda}{d\tau^2} = 0$$



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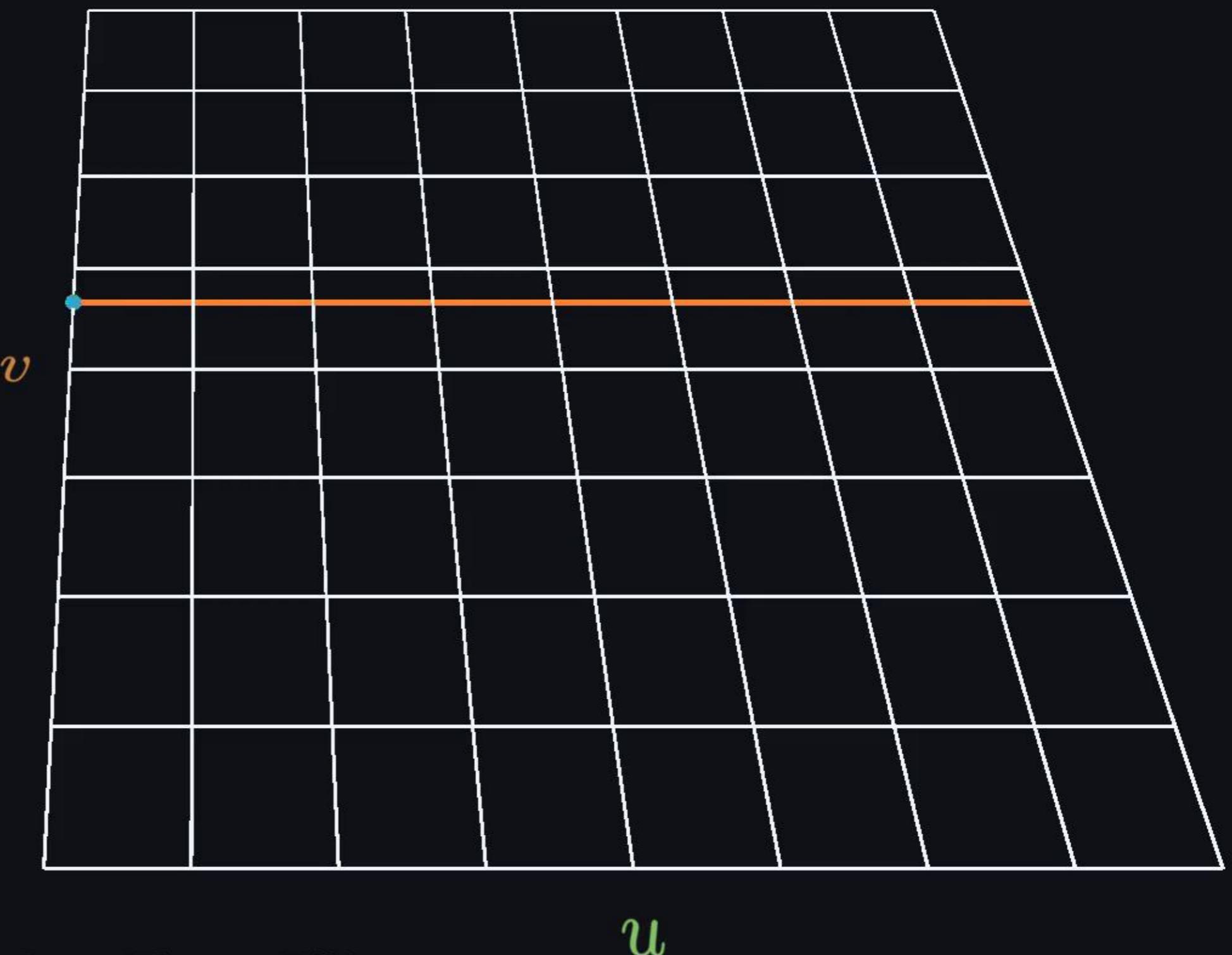
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$$\vec{x} : \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

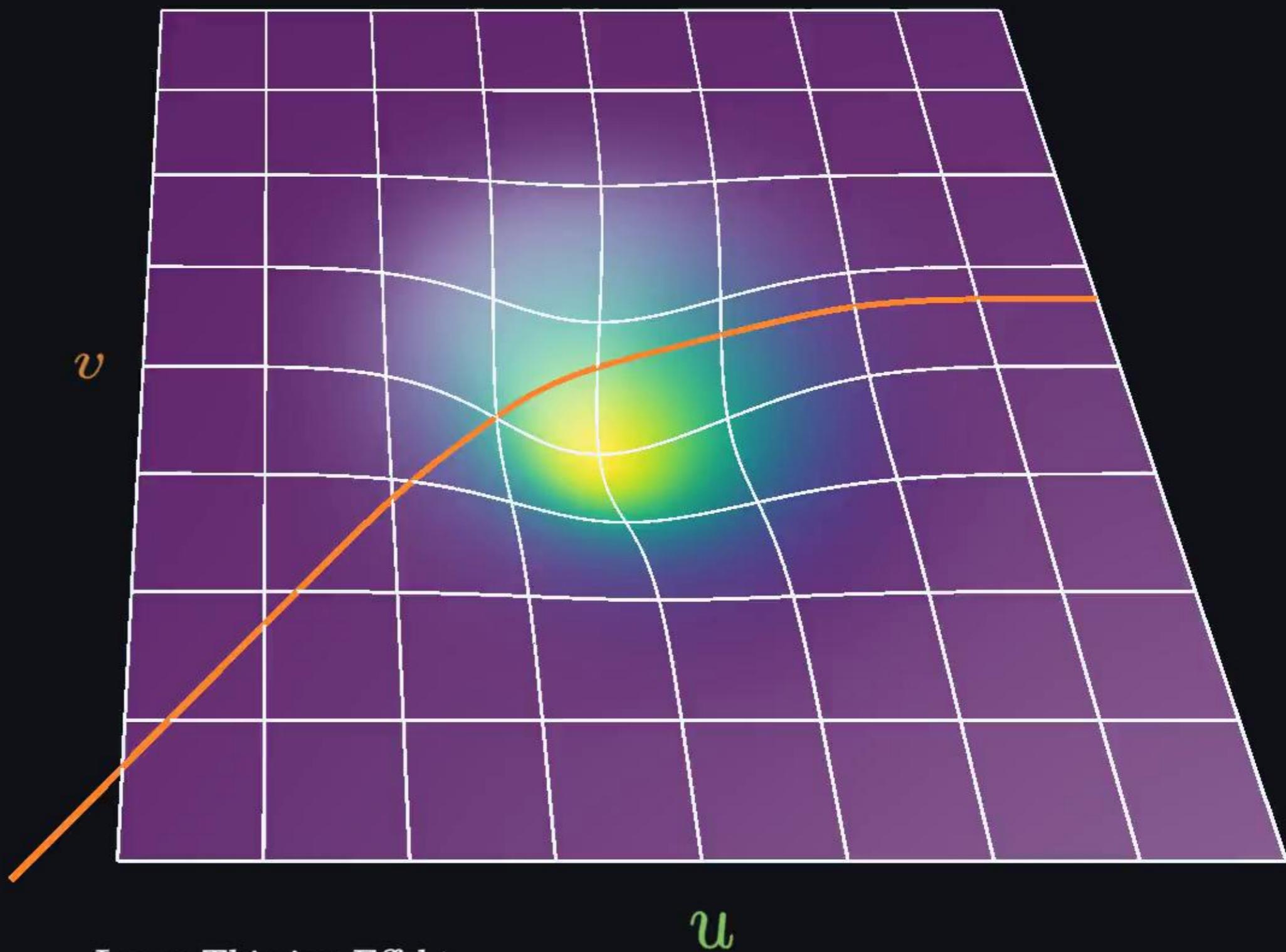
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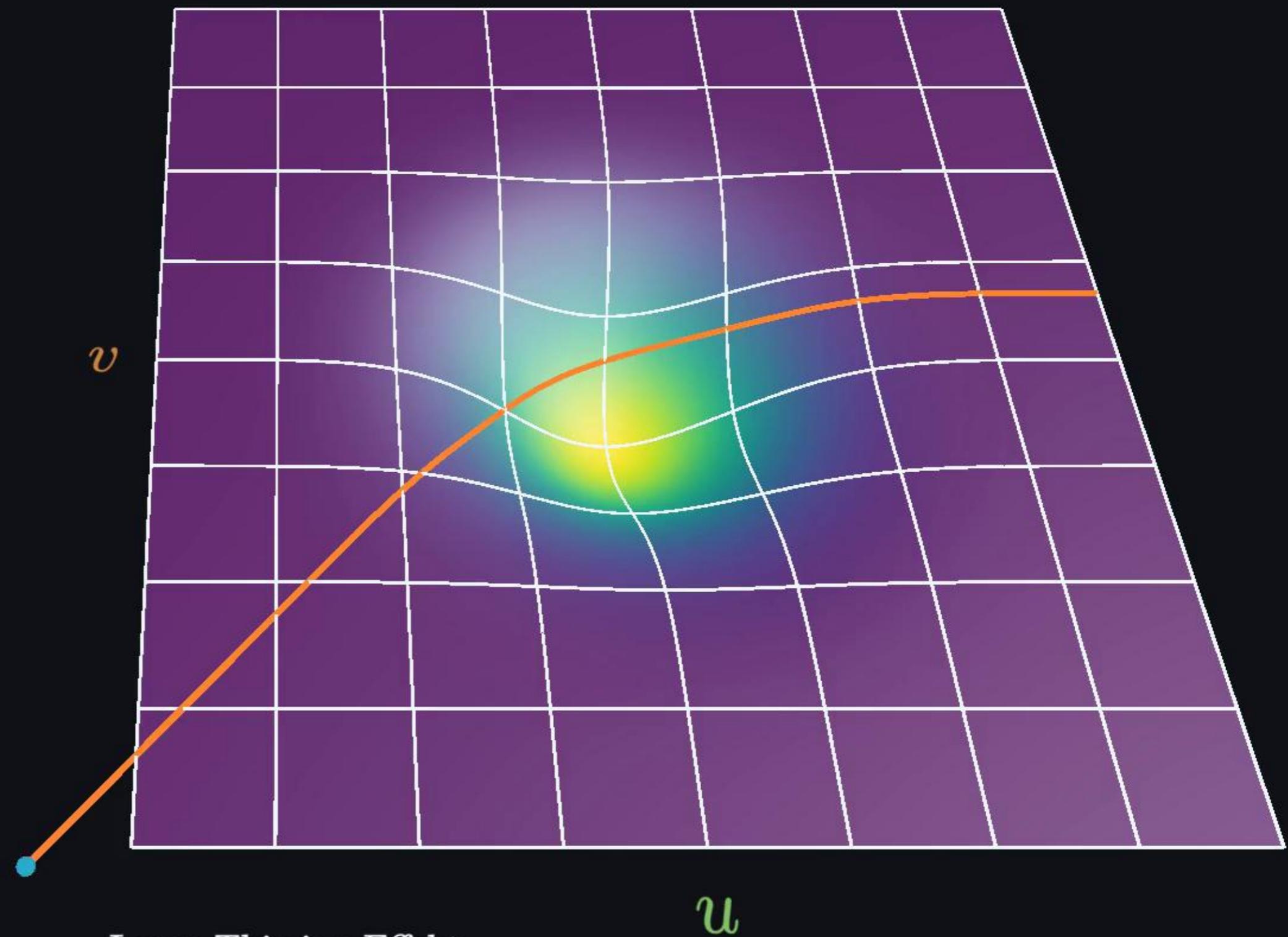
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Einsteinsche Feldgleichungen

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Einstein'sche Feldgleichungen

2D Fläche \rightarrow 4D Mannigfaltigkeit

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Einsteinsche Feldgleichungen

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Einsteinsche Feldgleichungen

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Koordinaten $(ct, x, y, z) \Rightarrow \mathbf{g} \in \mathbb{R}^{4 \times 4}$

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Energie-Impuls-Tensor: $T_{\mu\nu}$

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Linearisierung

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Annahmen: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $h \ll \eta$, $\tau \approx t$

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Gravitoelektromagnetismus

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Substitutionen: $\vec{E} = \frac{1}{2} \vec{\nabla} h_{00}$, $B_j = -\varepsilon_{jlm} \frac{\partial h_{0m}}{\partial \textcolor{teal}{x}^l}$

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Rotierende Kugelmasse

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$$\rho(|\vec{x}|) = \rho_0 \Theta(R - |\vec{x}|)$$



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$$\vec{B} = \frac{2}{|\vec{x}|^3} \left[\vec{S} - \frac{3(\vec{S} \cdot \vec{x}) \vec{x}}{|\vec{x}|^2} \right]$$

$$I = \frac{2}{5} M R^2$$

$$\vec{S} = I \vec{\omega}$$



EM-Felder

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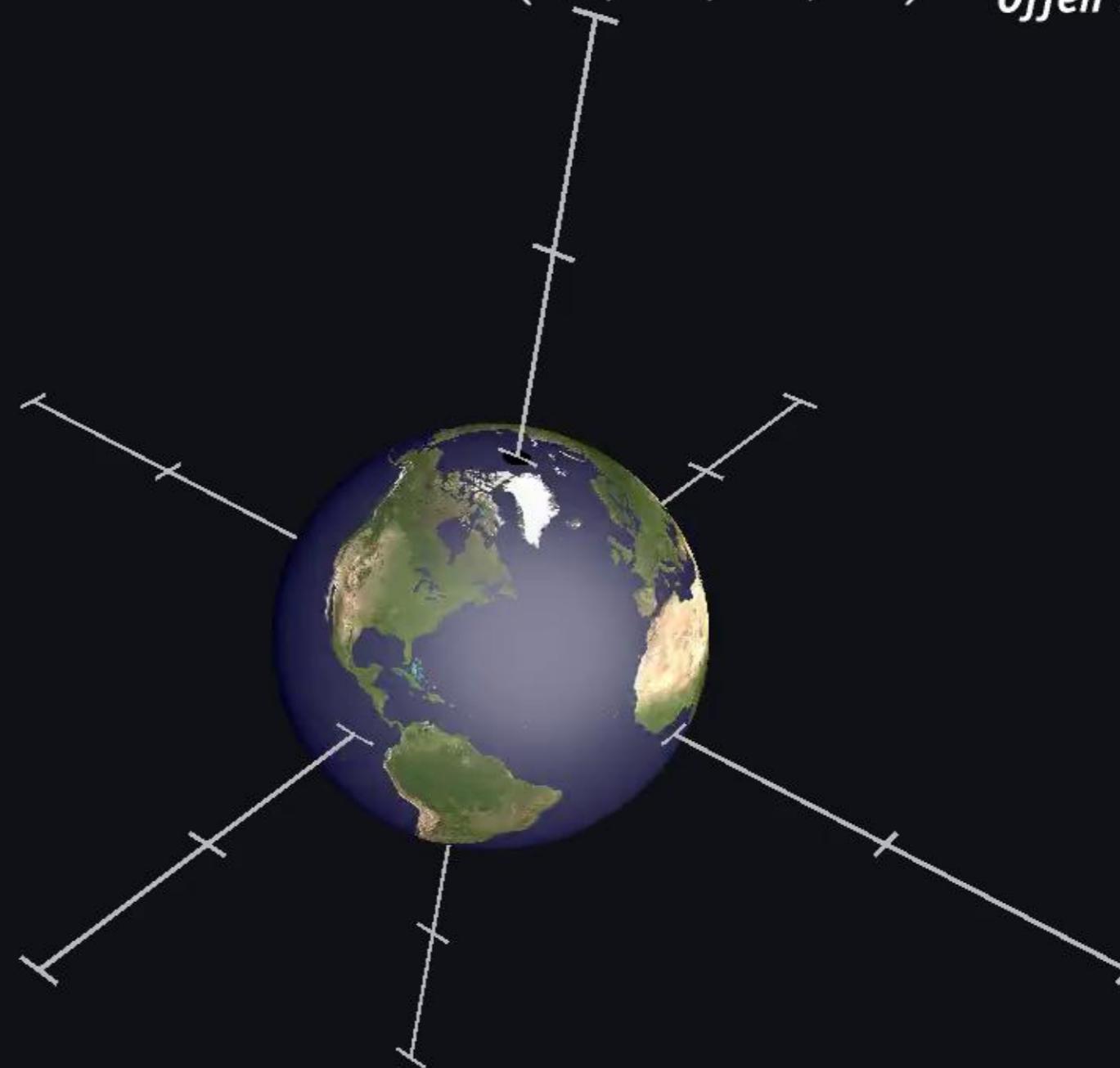
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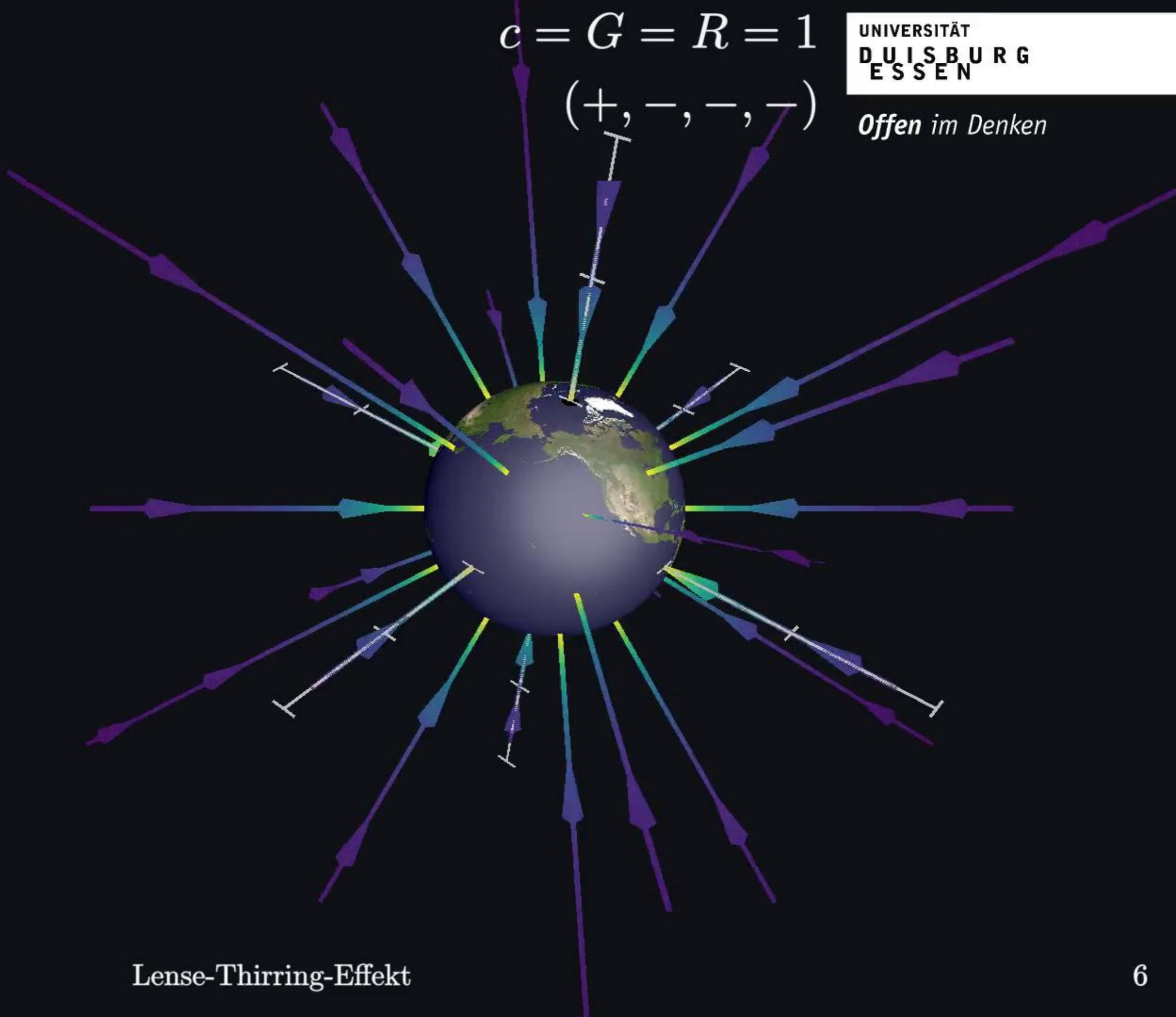
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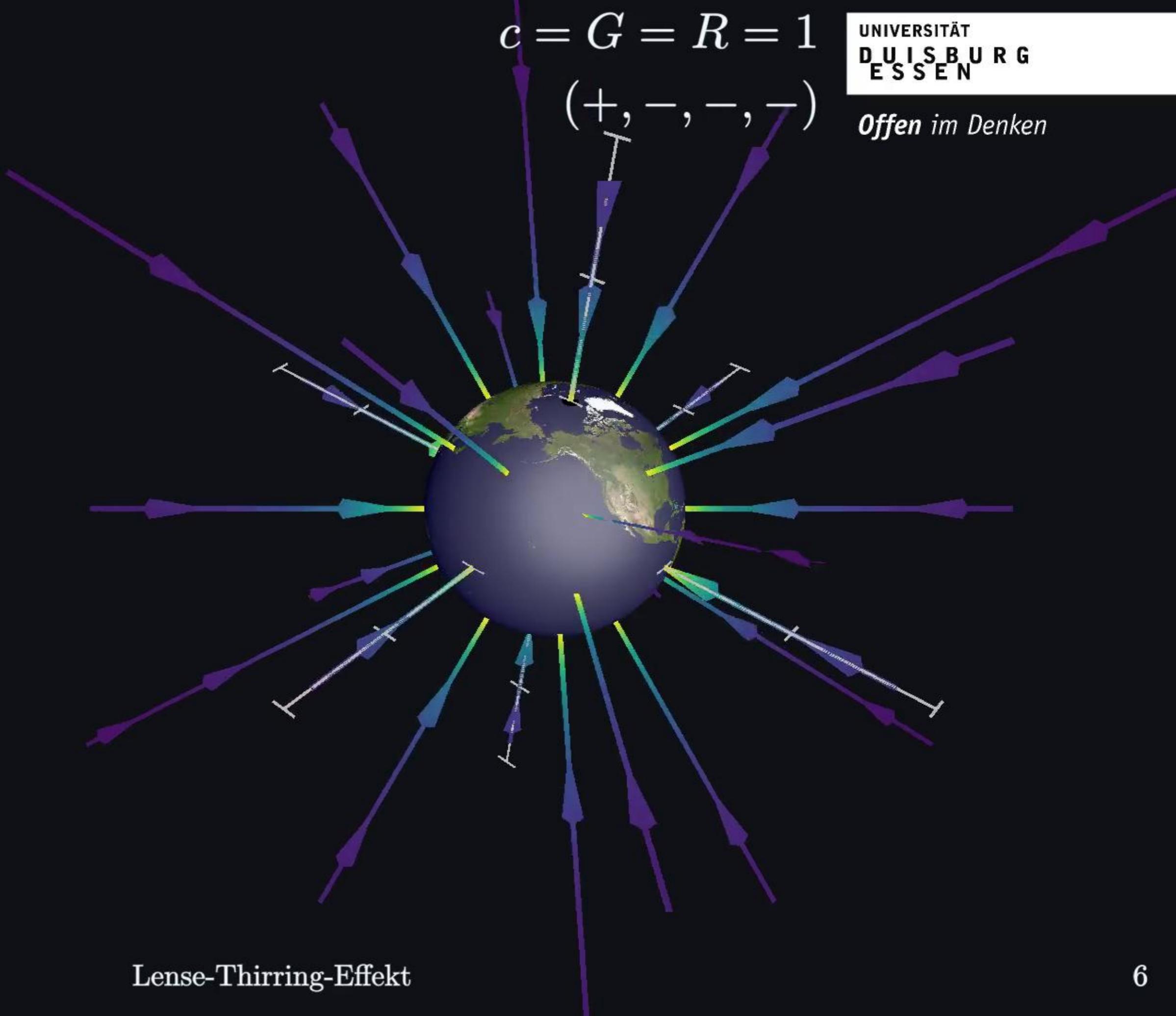
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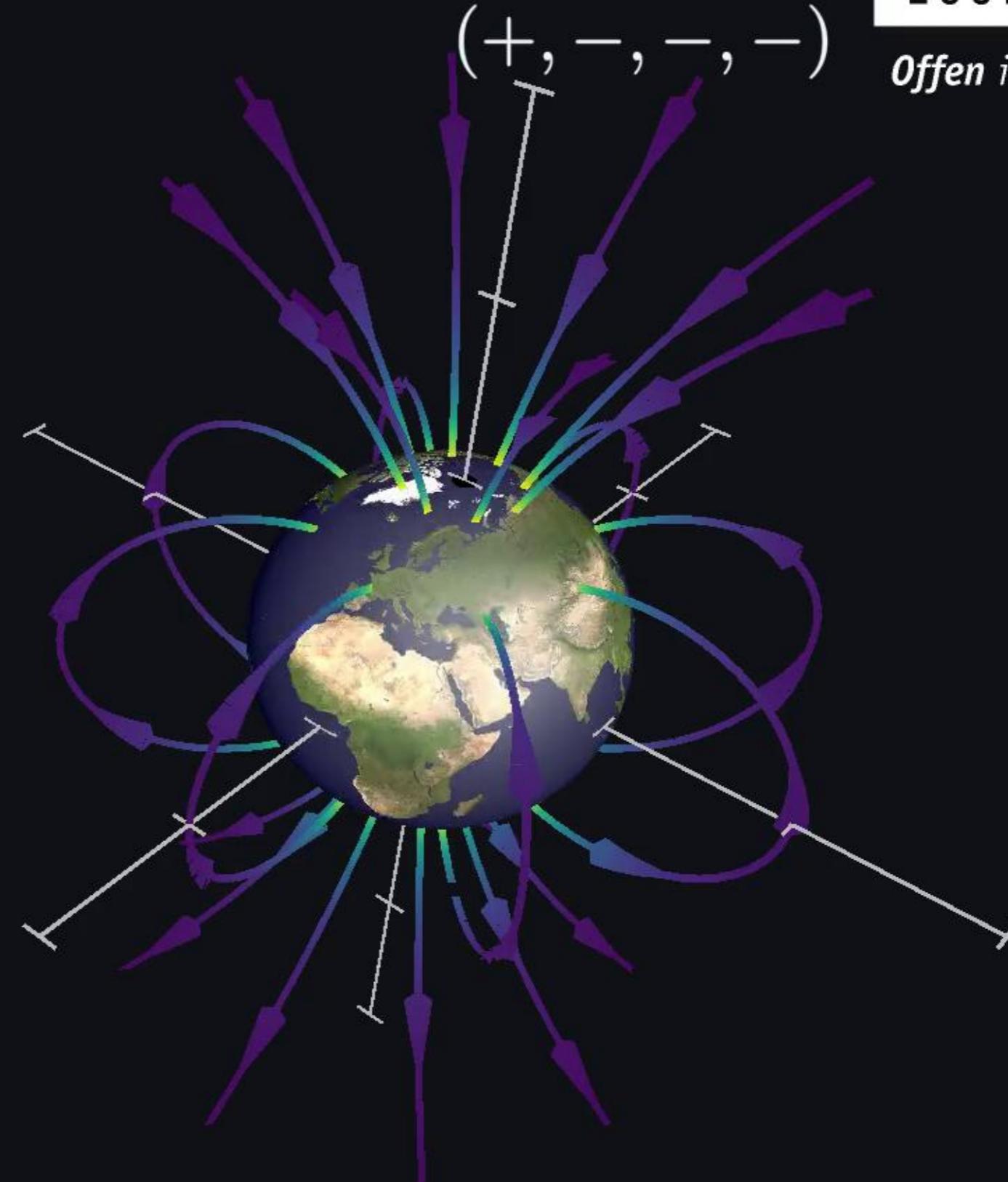
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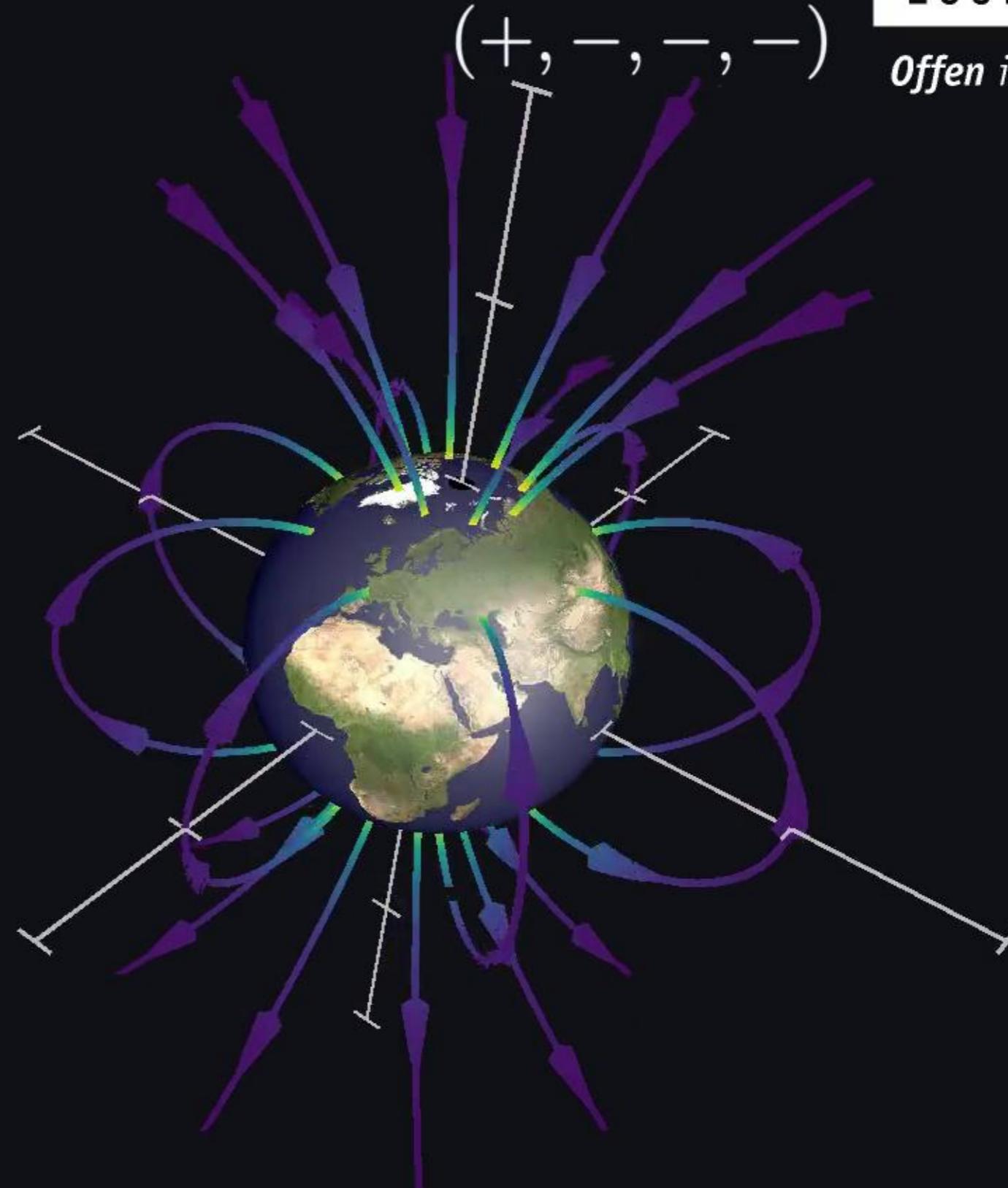
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$$c = G = R = 1$$
$$(+, -, -, -)$$

Trajektorien

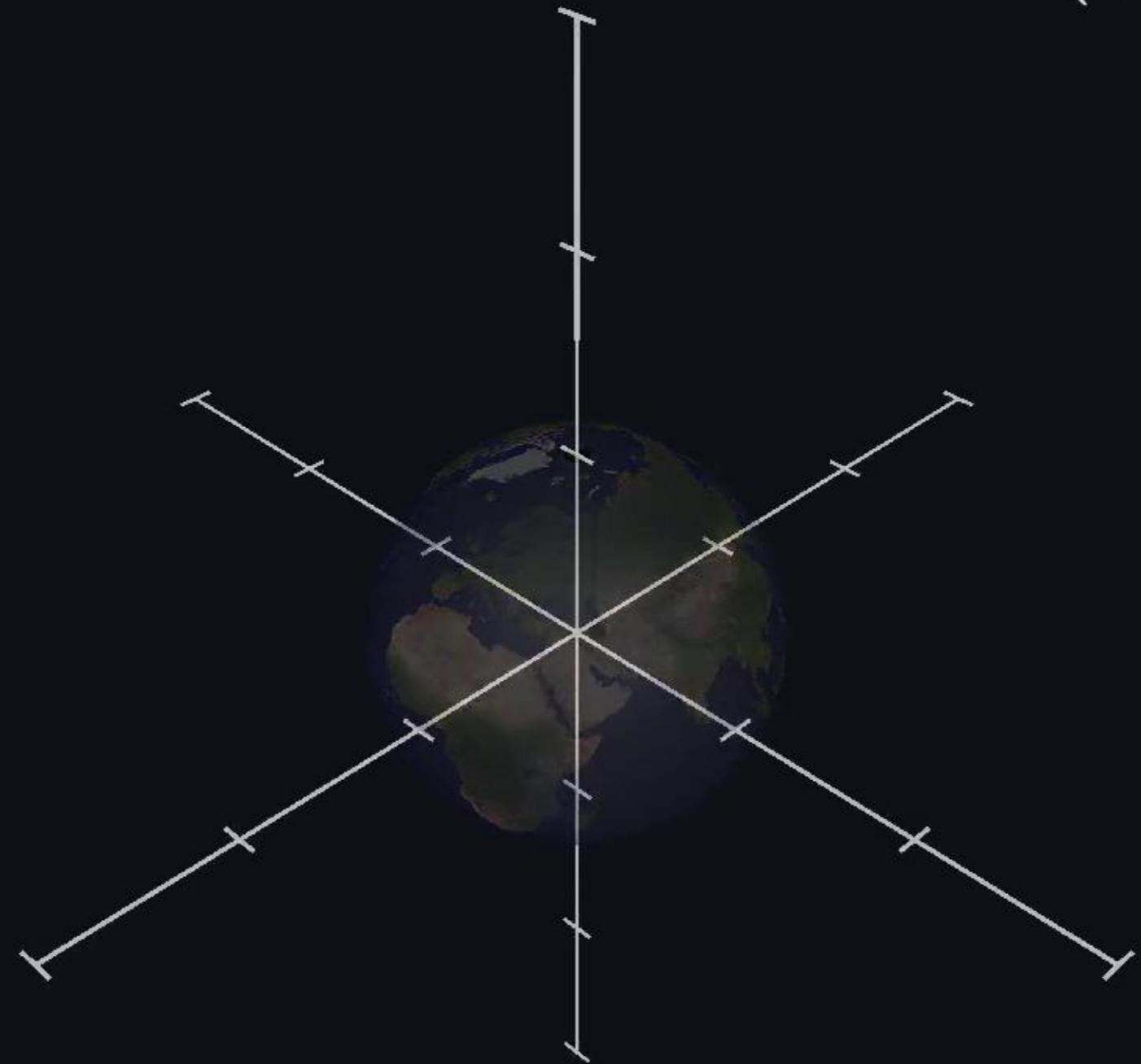


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Trajektorien

$$\vec{F} = m \left(\vec{E} + \vec{v} \times \vec{B} \right)$$

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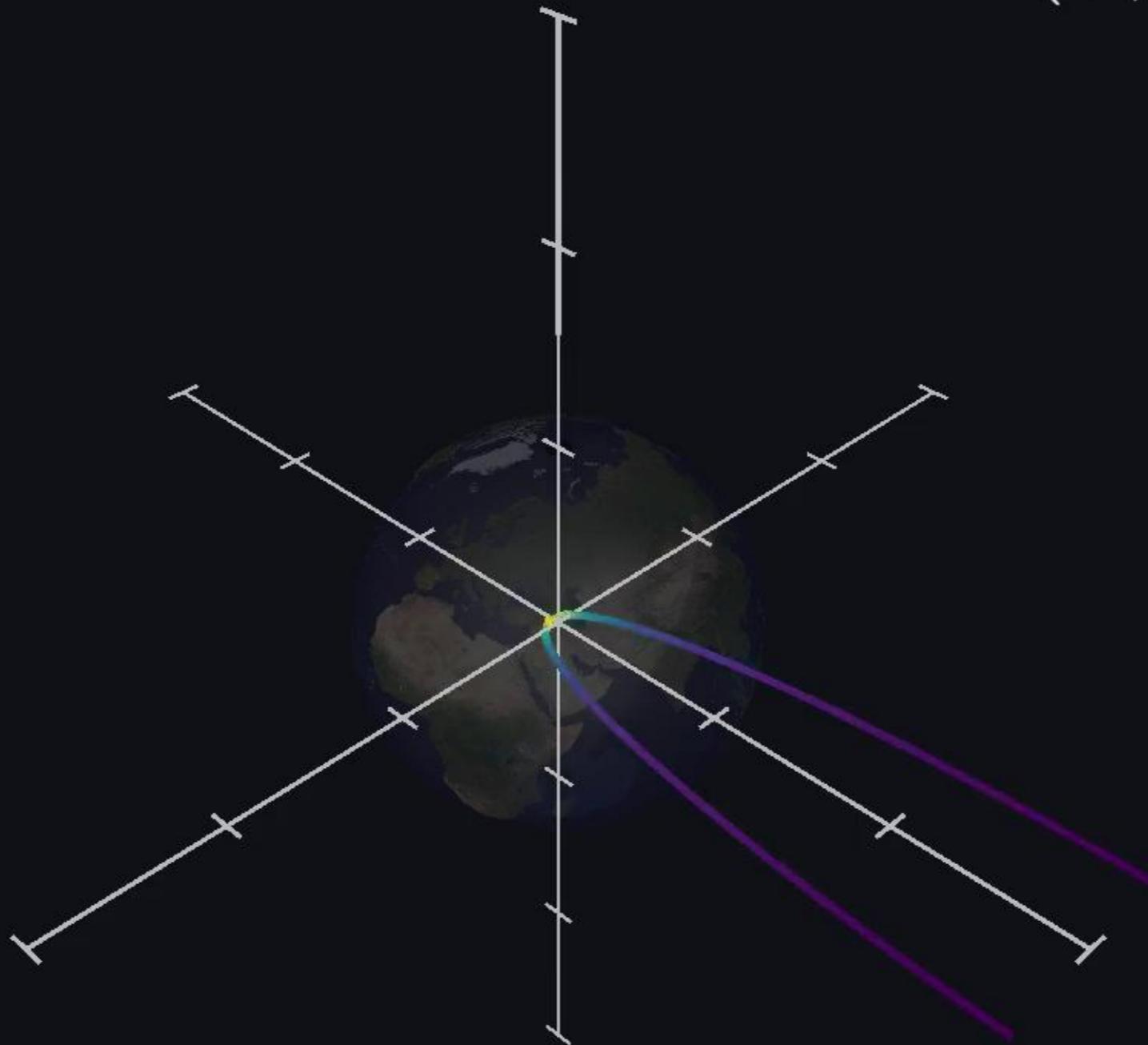


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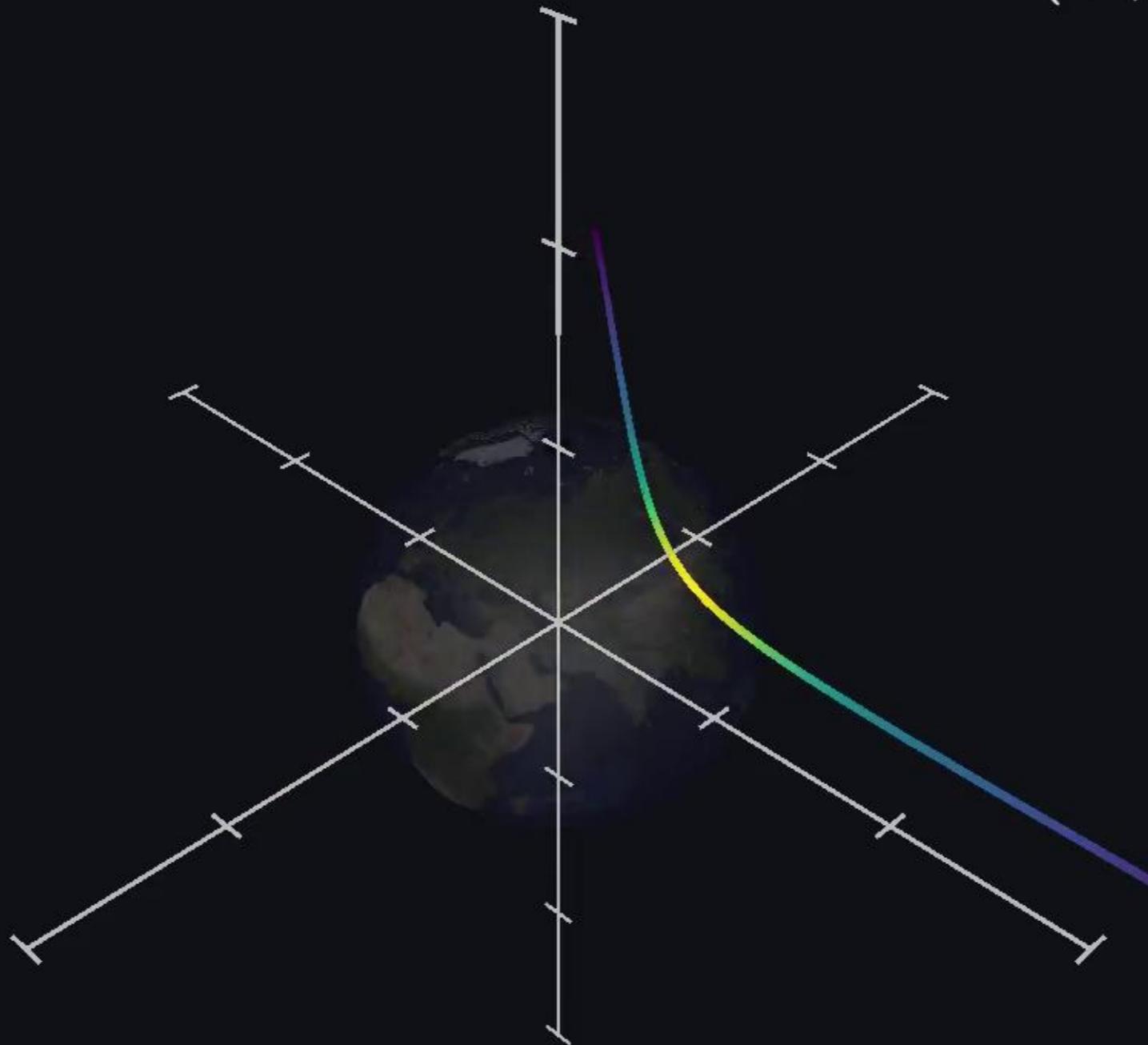


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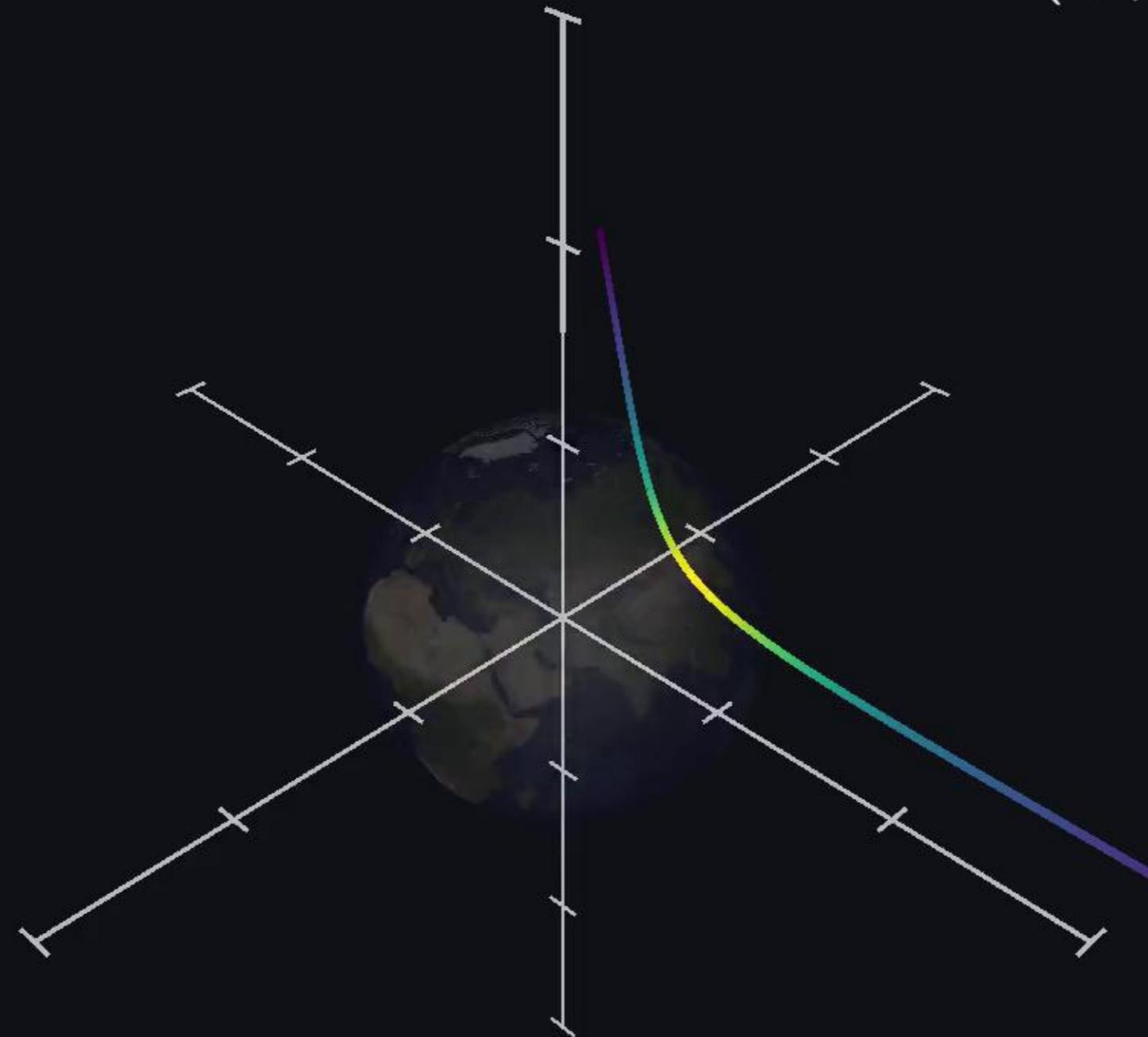


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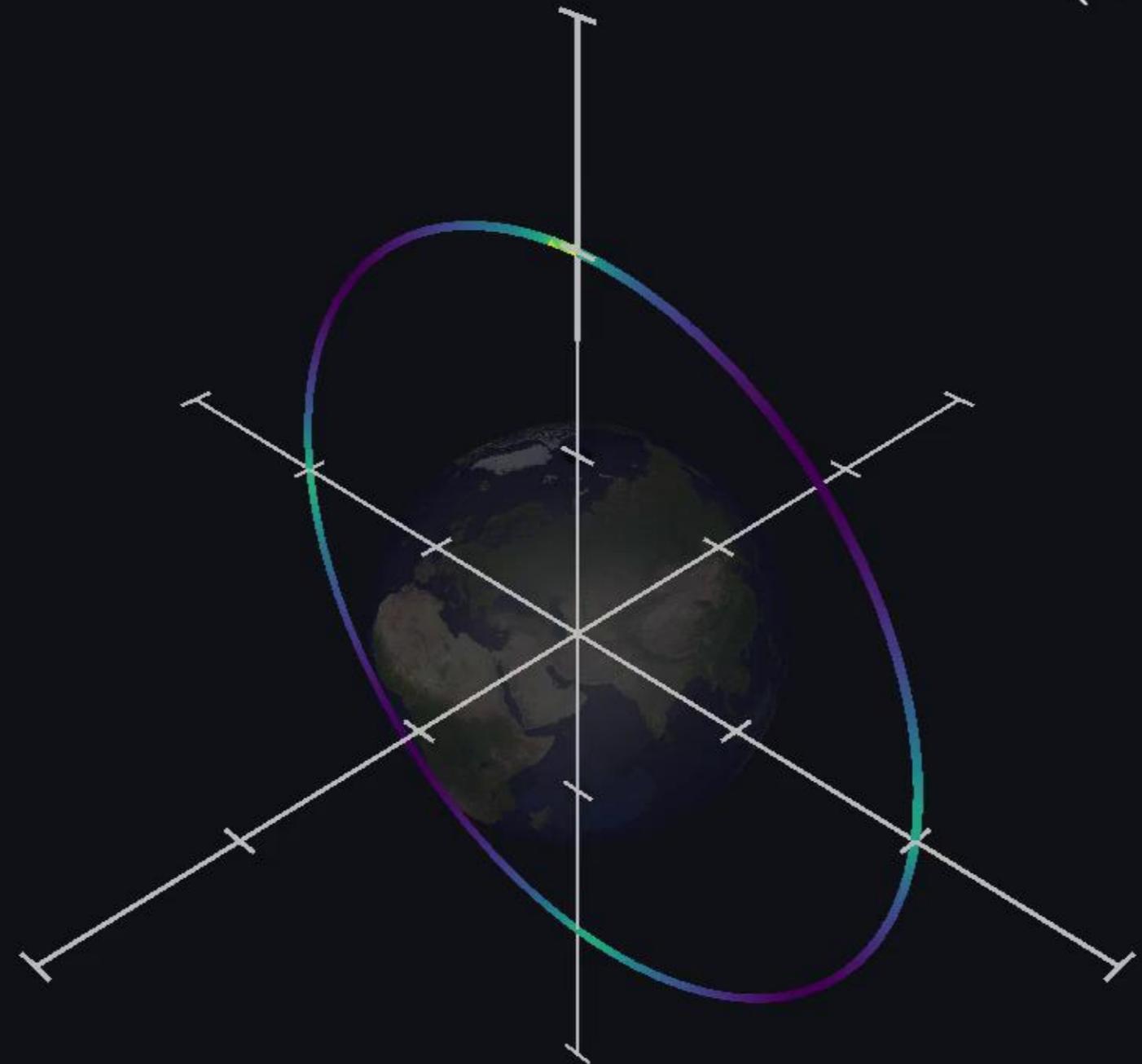


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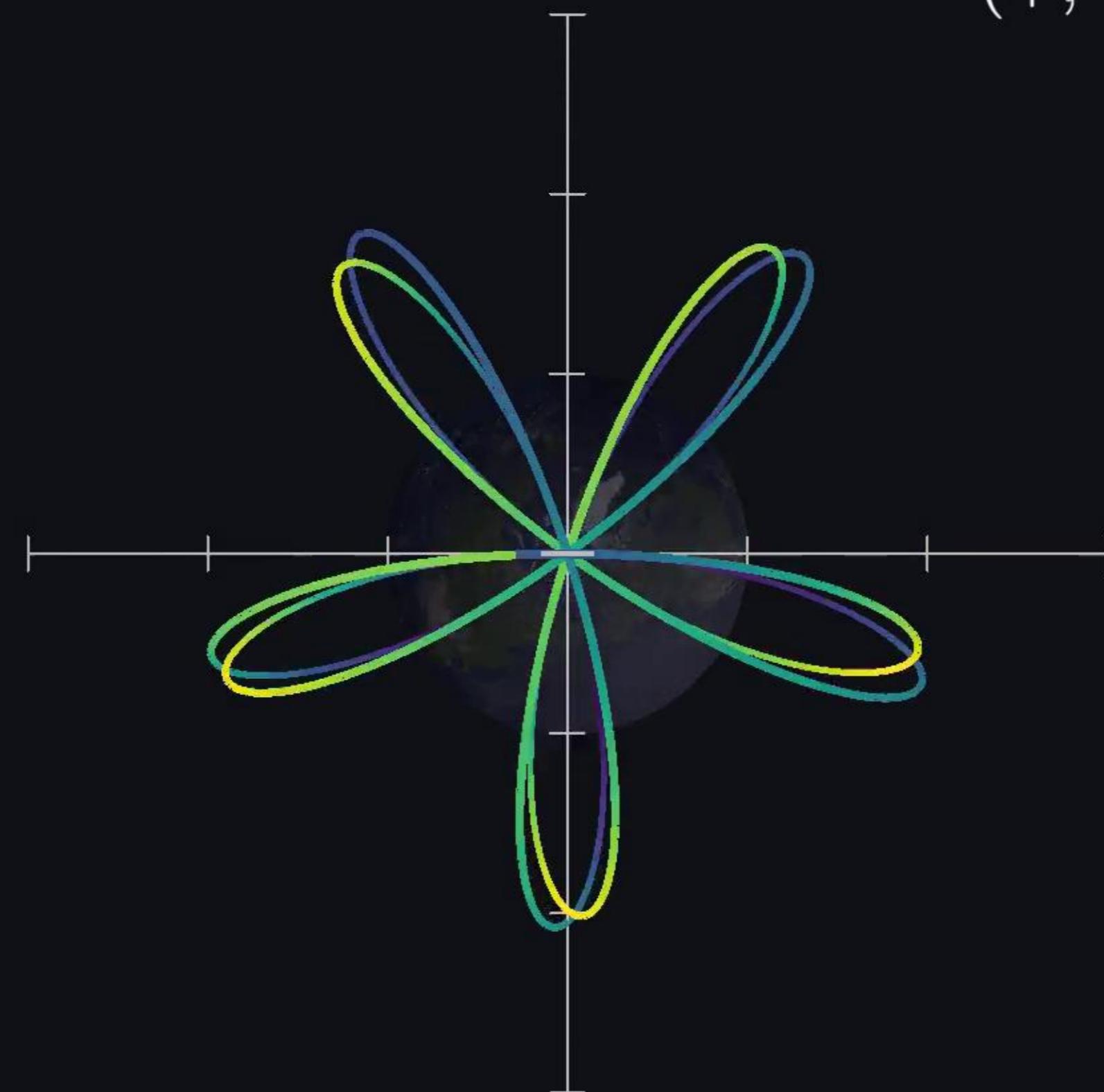


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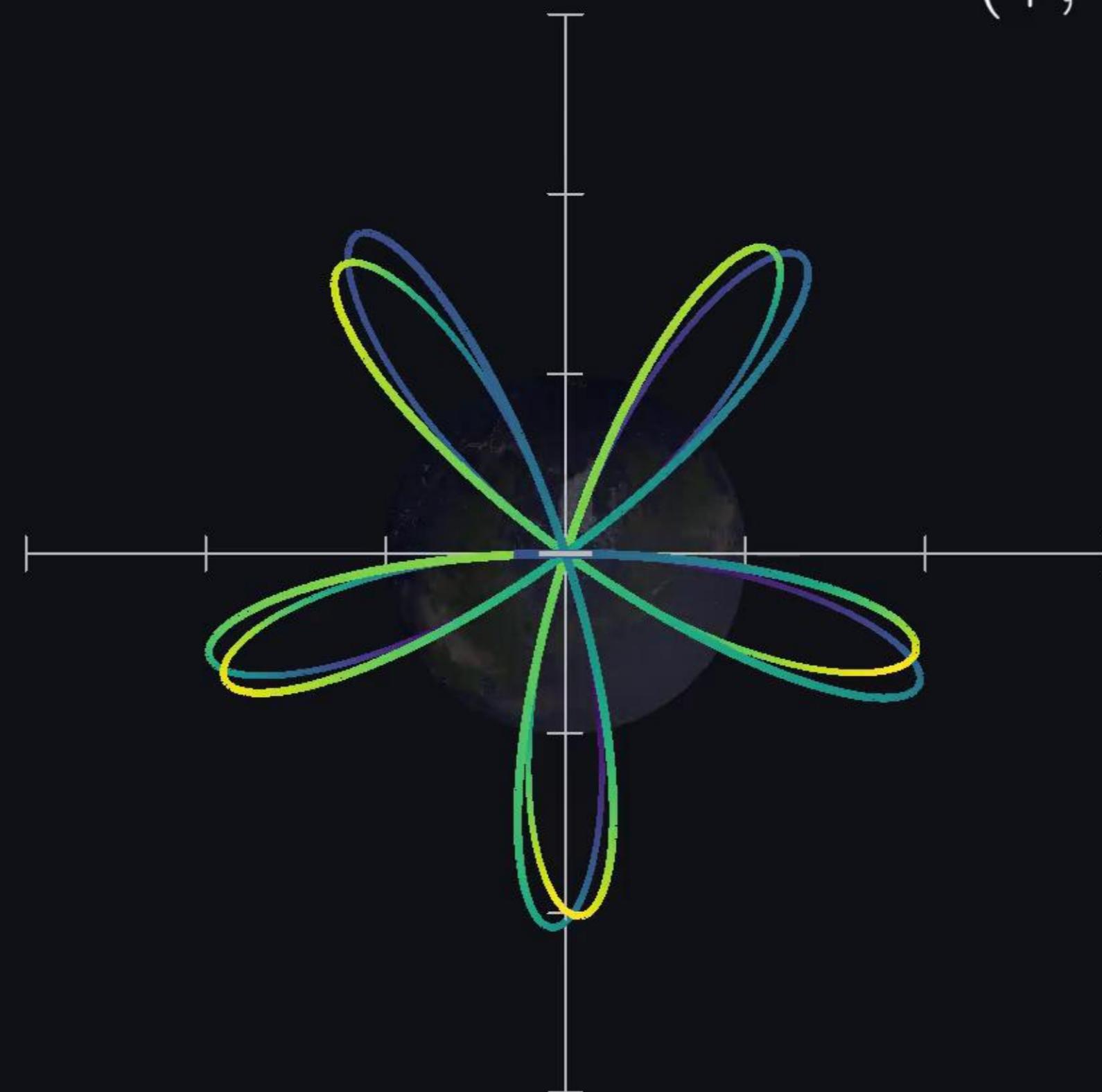


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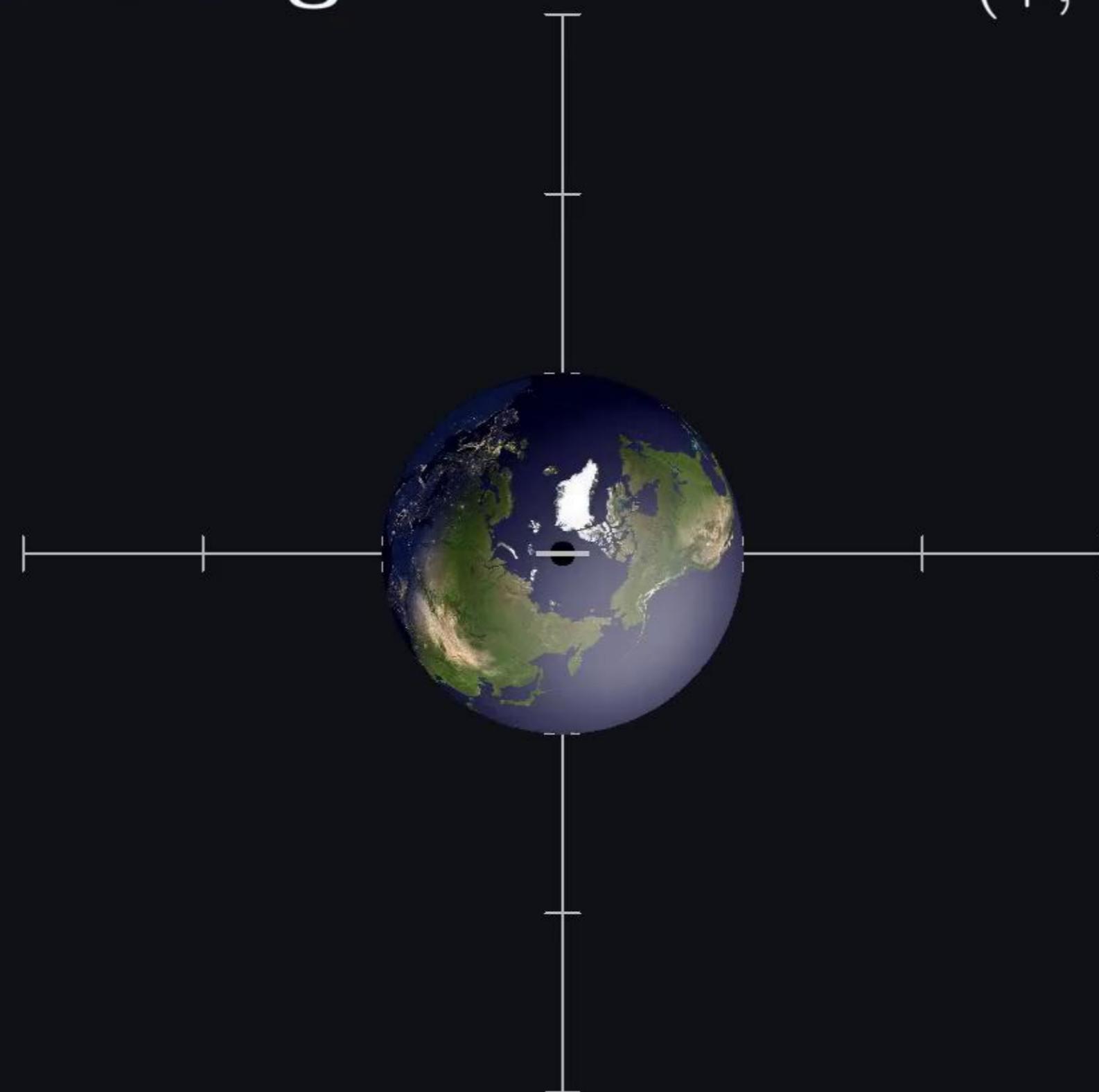
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Raumzeitdarstellung

Zeitentwicklung
für jeden
Gitterpunkt

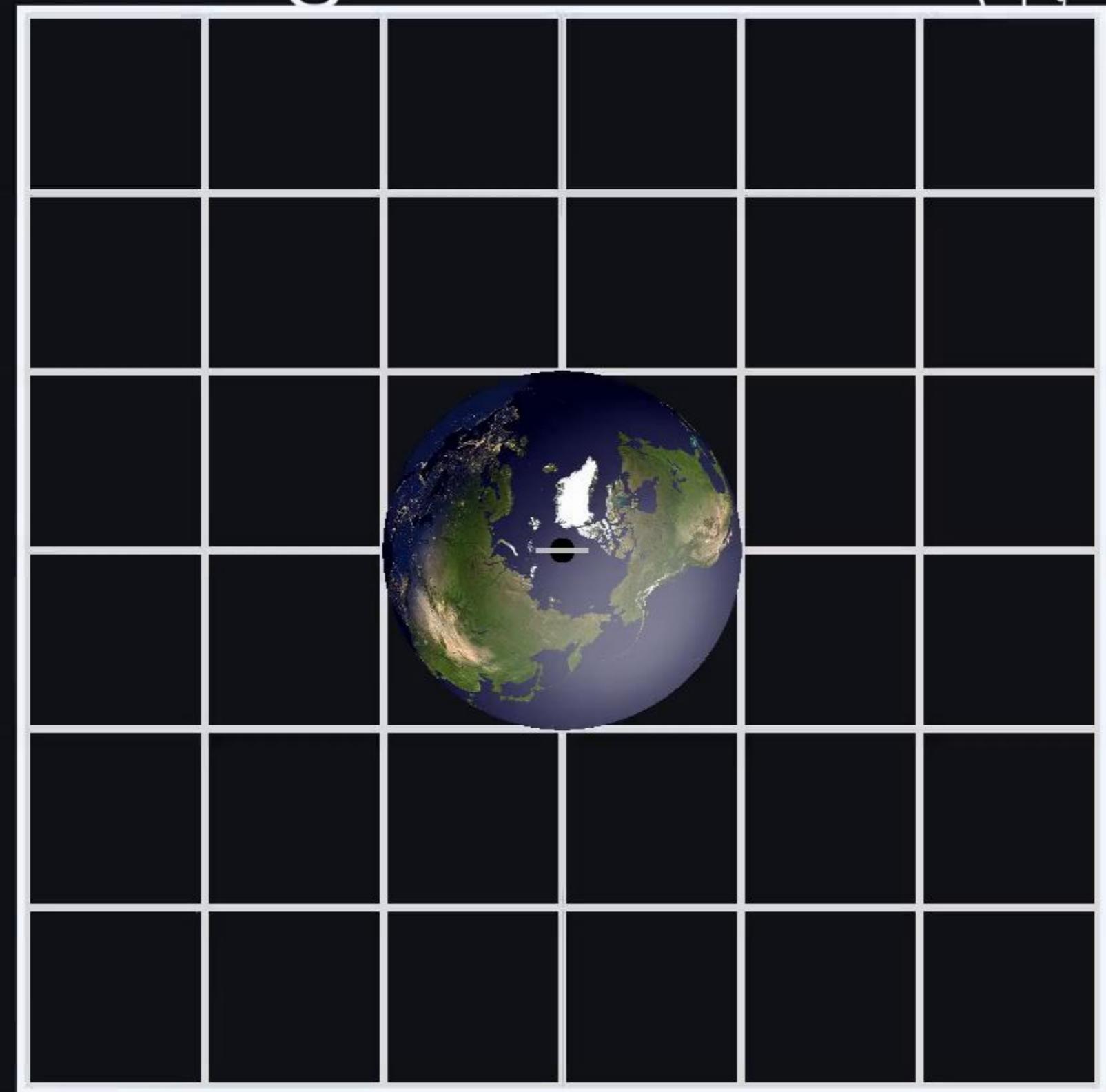


$$c = G = R = 1$$

$$(+, -, -, -)$$

Raumzeitdarstellung

Zeitentwicklung
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Gitterpunkt
bei $\omega = 0$

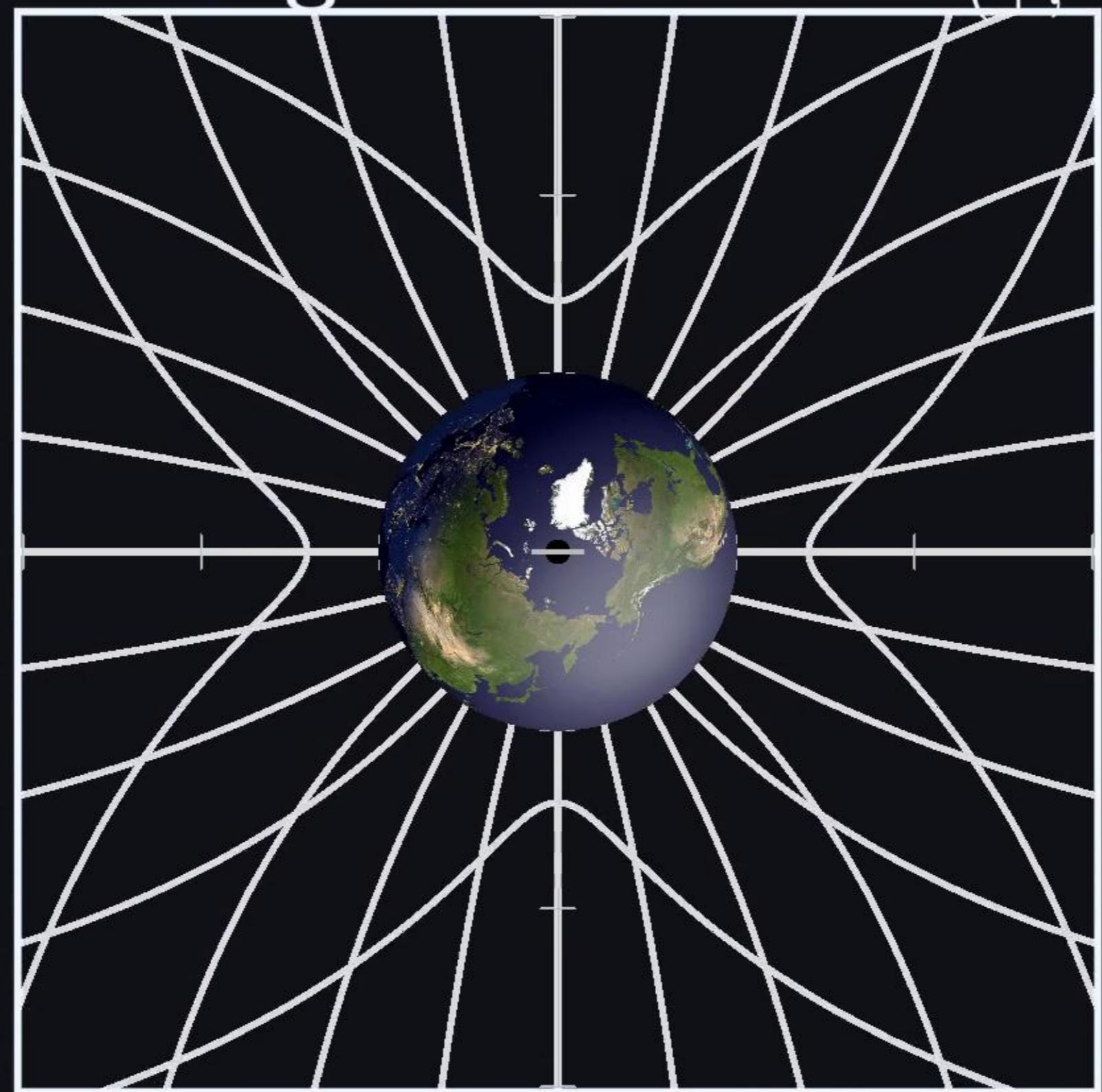


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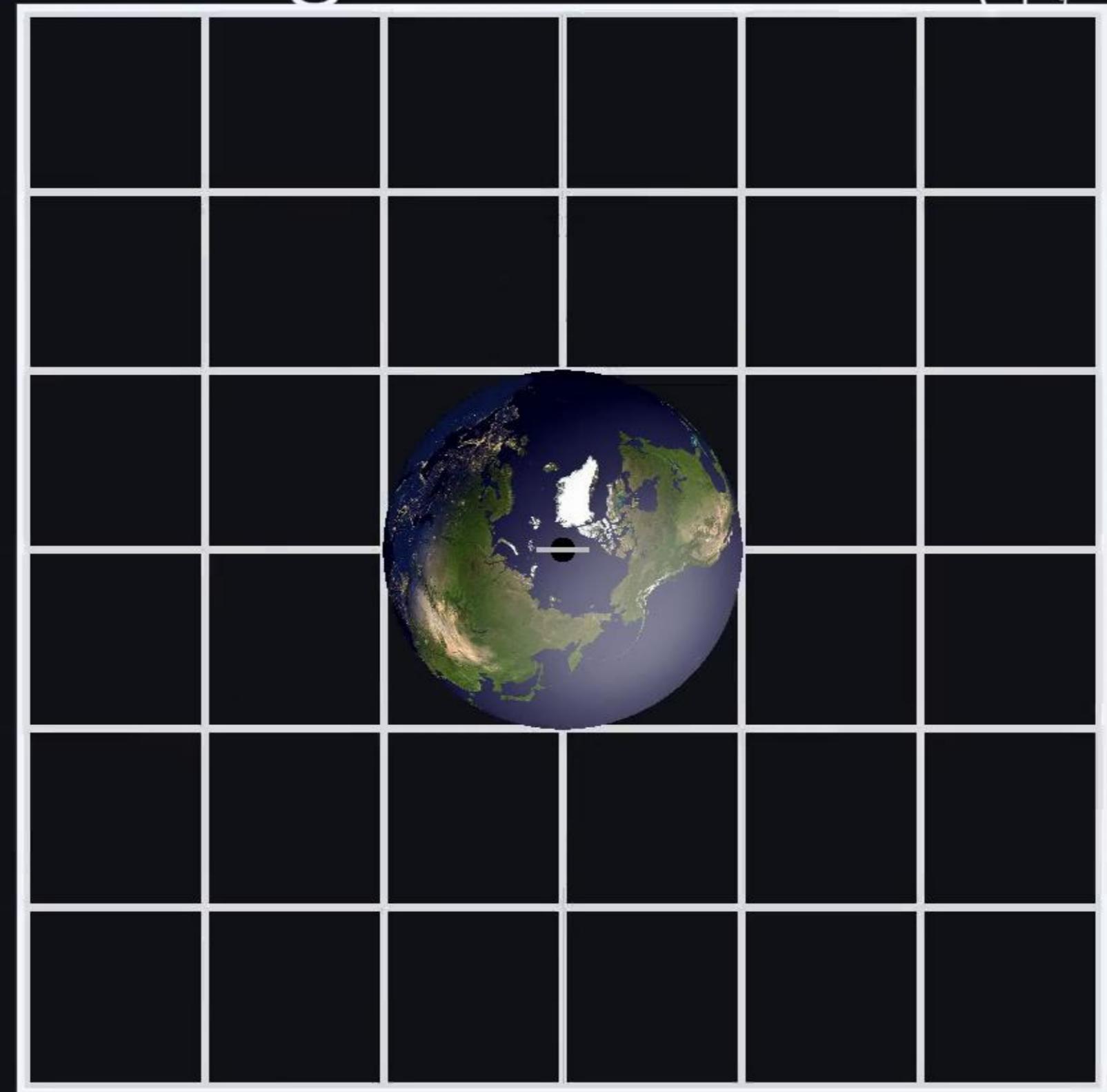
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Raumzeitdarstellung

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