Linear Functions

Consider the following table

x	0	1	2	3	4	5
$f(x)$	-17	-14	-11	-8	-5	-2

+3 +3 +3 +3 +3?

What would you guess if $f(5)$?

x	-2	0	1	2	5	6
$g(x)$	9	7	6	5	2	

−2 −1 −1 −3 ?

This change seems less consistent.

* Here these x values are not changing at the same rate

x	-2	0	1	2	5	6
$g(x)$	9	7	6	5	2	1

-2 $\quad -1$ $\quad -1$ $\quad -3$ $\quad ?$

Let's look at how g(x) changes as the x changes

$$\frac{\text{change in } y}{\text{change in } x} \qquad \frac{7-9}{0-(-2)} = \frac{-2}{2} = -1$$

$$\frac{6-7}{1-0} = \frac{-1}{1} = -1$$

$$\frac{2-5}{5-2} = \frac{-3}{3} = -1$$

That for every change in x g(x) changes by -1

If a function has a constant rate of change, slope, then we call the function a linear function.

A linear function can be written as

$$y = mx + b$$

where m is the rate of change, ie the slope.

ex

Find m and b such that

$g(x) = m \cdot x + b$

x	-2	0	1	2	5	6
$g(x)$	9	7	6	5	2	

$-2 \quad -1 \quad -1 \quad -3 \quad ?$

$$7 = g(0) = m \cdot (0) + b = b$$

$$\boxed{b = 7}$$

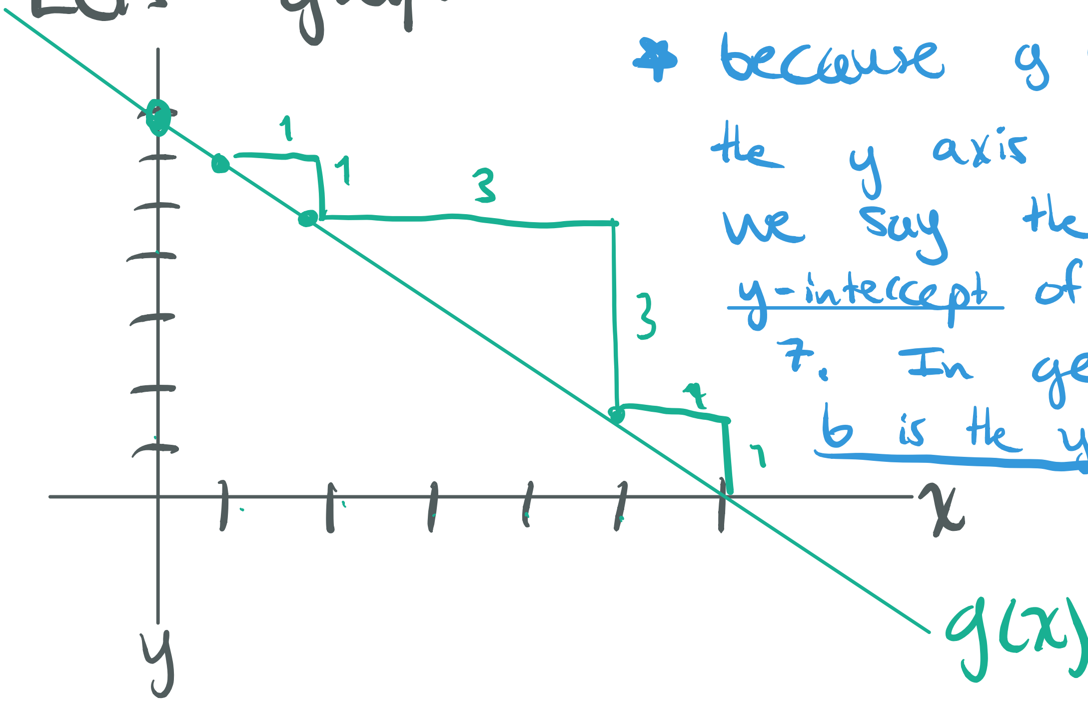
$$6 = g(1) = m \cdot 1 + b = m \cdot 1 + 7$$

$$6 = m + 7$$

$$m = 6 - 7 = -1$$

$$g(x) = -1 \cdot x + 7$$

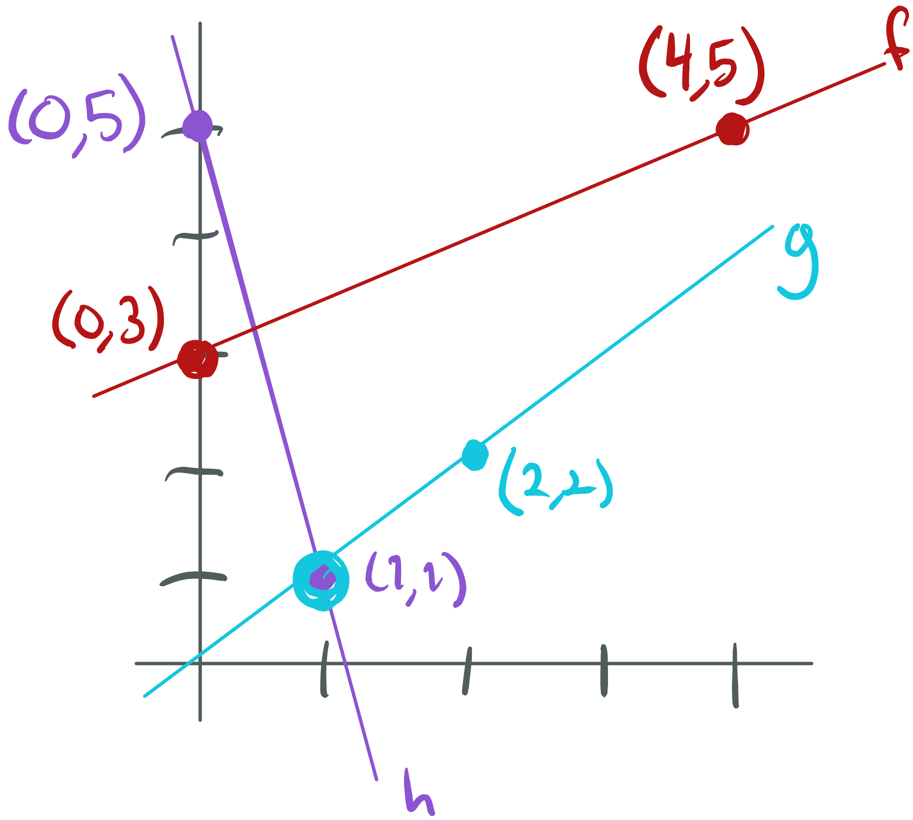
Let's graph this



* because g crosses the y axis at 7 we say the <u>y-intercept</u> of g is 7. In general <u>b is the y-intercept</u>

x	-2	0	1	2	5	6
$g(x)$	9	7	6	5	2	

$-2 \quad -1 \quad -1 \quad -3 \quad ?$



- The slope of f is $\frac{2}{4} = \frac{1}{2}$
- The slope of g is $\frac{1}{1} = 1$
- The slope of h is $\frac{-4}{1} = -4$

So a 'bigger' slope is a steeper line

a negative slope gives a line 'going down'

Two points define a line.
Similarly we can use two points to define a linear function.

ex What is the slope of the line through (1,2) and (6,7)?

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{7-2}{6-1} = \frac{5}{5} = 1$$

What is y-intersect?

$y = mx + b$

- $m = 1$
- at $x = 1$ $y = 2$

$2 = 1 \cdot 1 + b$

$2 = 1 + b$

$b = 1$

The line through (1,2) and (6,7) is

$$\boxed{y = x + 1}$$

Non Example:

x	0	2	4	6
y	0	0.5	1.5	4.5

Is this a linear function?

$$\frac{\Delta y}{\Delta x} = \frac{0.5-0}{2-0} = \frac{0.5}{2} = 4,$$

$$\frac{\Delta y}{\Delta x} = \frac{1.5-0.5}{4-2} = \frac{1}{2}$$

Because $4 \neq \frac{1}{2}$, the rate of change is not constant.

So it is not a linear function.

There is no m and b

$$y = mx+b$$

ex

Suppose a print shop's costs were linear. If it costs \$1.30 for 10 prints and \$1.72 for 16 prints, how much would 25 prints cost.

- formula will look $y = mx + b$
- 10 prints cost \$1.30 $\rightarrow (10, 1.30)$

$\rightarrow (16, 1.72)$

- Find m:

$$m = \frac{\text{change in cost}}{\text{change in prints}} = \frac{1.72 - 1.3}{16 - 10} = \frac{0.42}{6} =$$

$$= 0.07$$

- find b:

$(10, 1.30) \rightarrow 1.30 = (0.07) \cdot 10 + b$

$$1.30 = 0.7 + b$$

$$b = 1.3 - 0.7$$
$$= 0.60$$

- $y = 0.07x + 0.6$

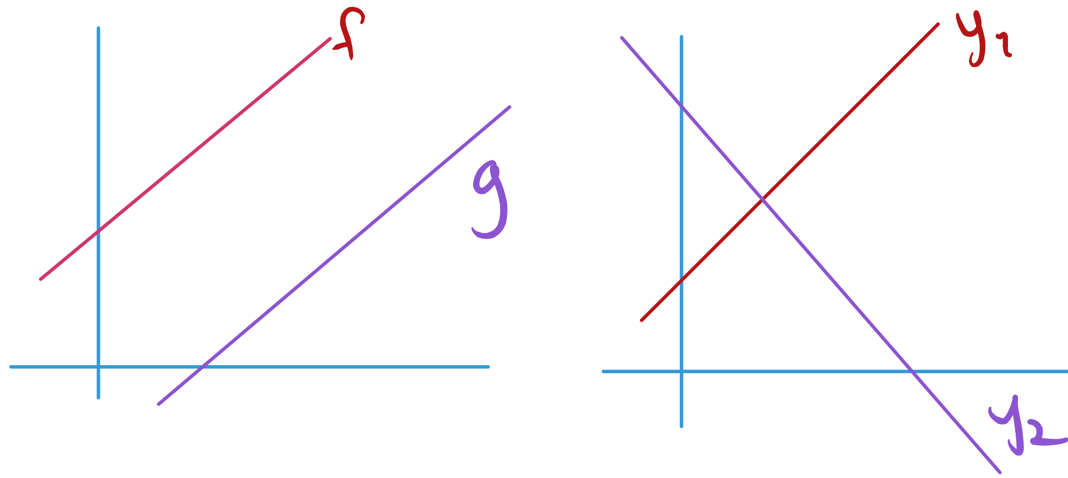
- cost of 25 prints:

$$y = 0.07 \cdot (25) + 0.6$$
$$= 1.75 + 0.6$$
$$= 2.35$$

25 prints will cost \$2.35;

Each print cost 7¢ and there was 60¢ fee to access the printers.

Interpreting parallel and perpendicular



We say 2 lines are parallel if when extended never intersect and perpendicular when they intersect at 90° angles.

<u>parallel</u>: we have <u>equal</u> slope

The <u>slopes</u> of f and g are equal (approximately 1)

<u>perpendicular</u>: have slopes that are negative reciprocals of each other

ex

Find a line parallel to $y=10x+2$ that goes through $(3,7)$

- parallel lines have same slope
- slope of $y=10x+2$ is 10
- $m=10$
- Let's use the point $(3,7)$ to find b

$7 = 10 \cdot 3 + b \quad (y=mx+b)$

$7 = 30 + b$

$b = 7 - 30 = -23$

The line parallel to $y=10x+2$ and through $(3,7)$ is $y=10x-23$

<u>ex</u> Find the line perpendicular to $y = 2x - 4$ that passes through $(3,6)$.

- perpendicular means slope is negative reciprocal
- the slope of $y = 2x - 4$ is 2
- $m = -\frac{1}{2}$
- use $(3,6)$ to find b

$$6 = -\frac{1}{2} \cdot 3 + b$$

$$b = 6 + \frac{3}{2} = \frac{15}{2}$$

$$y = -\frac{1}{2}x + \frac{15}{2}$$