6 Nov Recitation Worksheet for MA141

1. Integration basics
   (a) $\int ax^n \, dx$
   (b) $\int e^x \, dx$
   (c) $\int 1/x + k \, dx$
   (d) $\int \sin x + \cos x \, dx$
   (e) $\int \frac{1}{\sqrt{1-x^2}} \, dx$
2. Note
$$\int g(f(x))f'(x) \, dx = \int g(u) \, du \text{ where } u = f(x)$$
   Rewrite and solve the following integrals given $u(x)$.
   (a) $\int \frac{16x}{\sqrt{8x^2+1}} \, dx$, $u(x) = 8x^2 + 1$
   (b) $\int 3\sqrt{\sin x} \cos x \, dx$, $u(x) = \sin x$
   (c) $\int \frac{-e^{\sqrt{x}}}{2\sqrt{x}} \, dx$, $u(x) = \sqrt{x}$
   (d) $\int \frac{1}{x\sqrt{1-(\ln x)^2}} \, dx$, $u(x) = \ln x$
3. Note
$$\int_a^b g(f(x))f'(x) \, dx = \int_{u(a)}^{u(b)} g(u) \, du \text{ where } u = f(x)$$
   Evaluate the definite integrals
   (a) $\int_4^{16} \frac{1}{2}x^{-1/2} \, dx$
   (b) $\int_0^{\pi/2} \sin(2x) \, dx$
   (c) $\int_0^1 e^x \, dx$
   (d) $\int_0^5 3f(x) + 2 \, dx$ if $\int_0^5 f(x) = 2$
   (e) $\int_{-\pi/2}^{\pi/2} \frac{1}{1+\cos x} \, dx$

Hints:

(i) $\sin 2x = 2 \sin x \cos x$

(ii) $\cos 2x = 2\cos^2 x - 1$

(iii) $\dfrac{d}{dx} \tan x = 1/\cos^2 x$