Graphically Solving Linear Programming Problems

Linear Programming

A LP problem consists of inequality constraints and an objective function.

The goal is to find an optimal solution that satisfies all constraints (ie in the feasible region) and maximizes or minimizes (as specified in the problem) the objective function.

The result of evaluating the obj. function at an optimal solution is called an optimal value.

A contracting firm is increasing its workforce. It wants to hire new surveyors and new project managers. Altogether it needs to hire at least 50 new employees. Surveyors are paid an hourly wage of $22 and managers an hourly wage of $32. The firm does not want to exceed an hourly labor rate of $1260 for the new hires and company protocol says that there should be at least one manager for every 9 surveyors. The firm would like to keep the hourly labor cost as low as possible.

Variables: S #surveyors hired
M #managers hired

Constraints

$S + M \geq 50$

$22S + 32M \leq 1260$

$\frac{S}{M} \leq 9 \Rightarrow S \leq 9M$
$\Rightarrow S - 9M \leq 0$

Objective function

$22S + 32M$, minimize

Fundamental Theorem of Linear Programming

- If an LP has an optimal solution then it will be one of the corners of its feasible region
- If its feasible region is bounded then there will always exist an optimal solution

Solving LP Graphically

0. intepret constraints & obj function from word problem
1. graph constraints
2. find corners

2.5 unbounded feasible region has extra steps

3. Evaluate obj function @ corner points
4. Interpret what is 'optimal' corner

A student field trip is being planned. The two available vans can fit a total of 12 people. To receive permission for the trip, there must be at least two adults. To receive school funding, the number of students cannot be less than half the number of adults. If it costs $10 for adults and $5 for students, what's the cheapest trip? If adults can carry 4 snacks and students can carry 1, what's the largest number of snacks that can be brought?

variables: # adults → x

students → y

constraints

$x + y \leq 12$

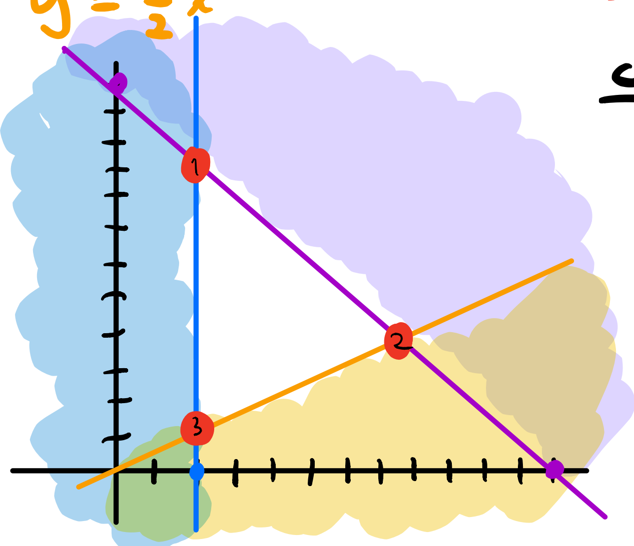
$x \geq 2$

$y \geq \frac{1}{2}x$

Objective functions

$10x + 5y$, minimize

$4x + 1y$, maximize

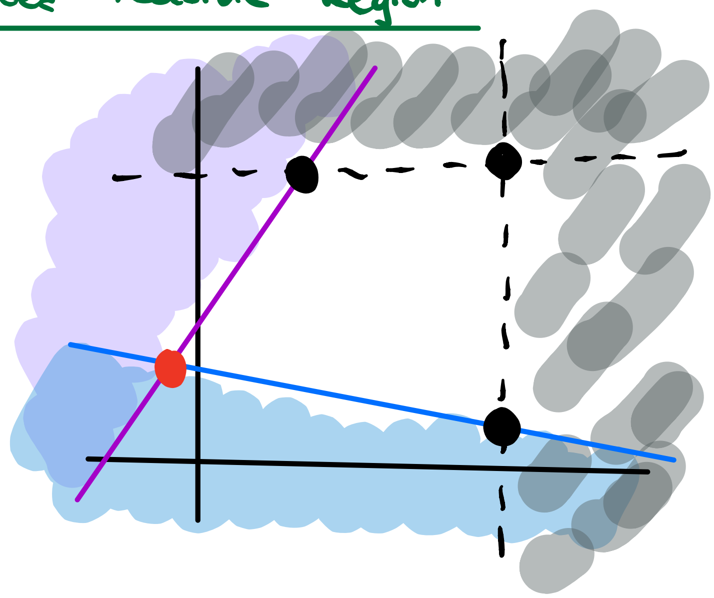


corners	$10x+5y$	$4x+1y$
$(2, 10)$	$10 \cdot 2 + 5 \cdot 10$ 70	18
$(8, 4)$	120	36
$(2, 1)$	25	9

- To minimize cost: 2 adults, 1 student, $25 trip
- To maximize snack: 8 adults, 4 students, 36 snacks

LPs Over Unbounded Feasible Region

The idea for unbounded regions is to add artificial constraints that make the region bounded. This introduces artificial corners.



Evaluate obj for all (original and artificial) corners

- if an original corner is 'optimal' then that corner is the optimal solution
- if an artificial corner is 'optimal' then we say the function is unbounded

Constraints

$x + y \geq 4$

$y \geq 0$

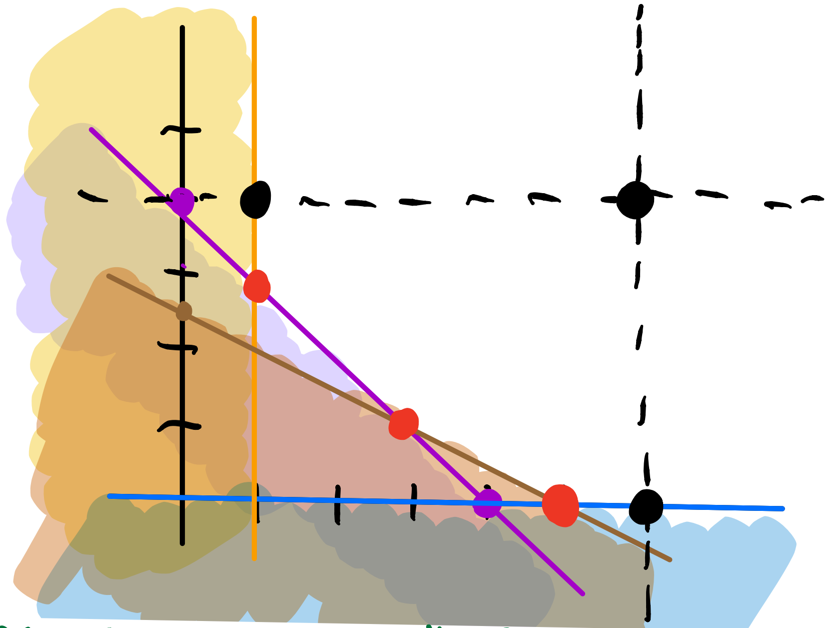
$x \geq 1$

$x + 2y \geq 5$

artificial

$x \leq 6$

$y \leq 4$



corners	min $20x+30y$	max $-4000x+2000y$
(1,3)	110	2000
(3,1)	90	-10000
(5,0)	100	-20000
artificial		
(6,0)	120	-24000
(1,4)	140	4000
(6,4)	240	-16000

- 1st objective $20x+30y$, min, has optimal solution (3,1), and opt value 90
- 2nd objective $-4000x+2000y$, max, is unbounded