hall effect

April 11, 2025

1 Exercise

Consider copper with mass density ρ_m=8.94g/cm^3 and atomic mass 63.546 u.

a) Estimate the value of the atomic mass unit u in grams given that Avogadro's number $N_A = 6.022x10^{23}$

Mass in grams is equal to u/N_A, so we can find this value as:

[1]:
```
copper_u = 63.546
avo_n = 6.022e23
copper_grams = copper_u/avo_n
print(f'The mass of a copper atom is approximately {copper_grams:.5e} g')
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The mass of a copper atom is approximately 1.05523e-22 g
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b) Assume each copper atom contributes one mobile electron. Estimate the carrier density n as the number of copper atoms per unit volume. Find the value of n in carriers per cm^3

As there is one cm^3 of copper, and we know the density of copper and its atomic weight, we can find the number of copper atoms in one cm^3. As 1u $\approx$ 1g/mol:
Moles of copper = $m_{Cu}(g)$/u(g/mol)

[2]:
```
mass_copper = 8.94 # g
moles_copper = mass_copper/copper_u
print(f'There are {moles_copper:5f} moles of copper in one cubic centimeter.')
num_carriers = moles_copper * avo_n
print(f'This means n = {num_carriers:.5e} carriers/cm^3.')
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There are 0.140685 moles of copper in one cubic centimeter.
This means n = 8.47208e+22 carriers/cm^3.
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c) Copper has a resistivity $\rho_e = 1.724x10^{-8}\ \Omega \cdot m$. The carrier charge is the charge of the electron: $q = -1.602x10^{-19}$ Coulomb. Find a value for the mobility μ of the copper. Try to find values online for comparison: how well do they agree?

This can be done using the relationship μ=1/$ne\rho_e$. First, we'll have to convert units as the resistivity is in terms of ohm meters while the number density is in terms of cubic centimeters. As 1 meter = 100 cm, $\rho_e = 1.724x10^{-6}\ \Omega \cdot cm$.

[3]:
```
copper_resistivity = 1.724e-6 # ohm * cm
q_e = -1.602e-19 # C
mu = 1/(num_carriers * q_e * copper_resistivity)
print(f'Mobility of copper = {mu:.5f} cm^2/V s')
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Mobility of copper = -42.73760 cm^2/V s
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Online, values for the mobility of copper electrons is listed as 30-50 cm^2/V·s, at room temperature. My result lands well within that range, so the results agree very well.

d) Suppose a current I=10 A flows across a piece of copper of cross-sectional area A = 1 mm^2 and length l = 1 meter.

e) Given the above resistivity, what is the resistance?

ii) What is the voltage drop?

iii) What is the electric field?

iv) What is the drift velocity in cm/s?

v) What is this drift velocity in furlong/fortnight? 1 furlong/fortnight = 1.66E-2 cm/s = 0.1663 mm/s

vi) What does this tell you about the size of the drift velocity of electrons carrying a fairly large amount of current in copper (2/3 to 1/2 the value that would trip, respectively, 15A or 20A circuit breakers).

[4]:
```
# i
# R=rho * l/A, putting all units into cm & cm^2
area = 0.1 # cm^2
length = 100 # cm
resistance = copper_resistivity * length / area
print(f'i: Resistance = {resistance:.2e} ohm')

# V = Il/qn mu
current = 10 # A
voltage = (current * length)/(q_e * num_carriers * mu)
print(f'ii: Potential difference = {voltage:2e} V')

# E = V/l
e_field = voltage/length
print(f'iii: Electric field = {e_field:.2e} V/cm')

# v_drift = mu * E
v_drift = mu * e_field
print(f'iv: Drift velocity = {v_drift:2e} cm/s')

fur_fort = 1.66e-2 # cm/s
fur_fort_drift = v_drift * fur_fort
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print(f'v: Drift velocity in furlongs per fortnight = {fur_fort_drift:.2e}')

def v_drift(mu, current, resistance, length):
    return (mu * current * resistance)/length

v_drift1 = v_drift(mu, (2/3)*15, resistance, length)
print(f'vi: As 2/3 of 15 = 1/2 of 20, the drift velocity between the two␣
 ↪breakers would be equivalent at {v_drift1:.2e} cm/s')
```

```
i: Resistance = 1.72e-03 ohm
ii: Potential difference = 1.724000e-03 V
iii: Electric field = 1.72e-05 V/cm
iv: Drift velocity = -7.367963e-04 cm/s
v: Drift velocity in furlongs per fortnight = -1.22e-05
vi: As 2/3 of 15 = 1/2 of 20, the drift velocity between the two breakers would
be equivalent at -7.37e-03 cm/s
```