Analysis Over the efficiency of a multi-threaded implementation of the QuickSort algorithm on multi-core machines, we are studying the time that will take every different methodes individual, and if there is relation between the size of array and the time for sorting, you can find the data we use in folder data.

```r
library(ggplot2)
library(dplyr)
```

```
##
## Attaching package: 'dplyr'

## The following objects are masked from 'package:stats':
##
##     filter, lag

## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```r
library(tidyr)

df <- read.csv("data/measurements_03_47.csv", header = T)
df$Type <- gsub("\\s+", "", df$Type)
head(df)
```

```
##   Size       Type     Time
## 1  100 Sequential 0.000010
## 2  100   Parallel 0.004024
## 3  100   Built-in 0.000013
## 4  100 Sequential 0.000010
## 5  100   Parallel 0.004448
## 6  100   Built-in 0.000014
```

```r
summary(df)
```

```
##       Size              Type                Time
##  Min.   :    100   Length:75          Min.   :0.000009
##  1st Qu.:   1000   Class :character   1st Qu.:0.000210
##  Median :  10000   Mode  :character   Median :0.016149
##  Mean   : 222220                      Mean   :0.051255
##  3rd Qu.: 100000                      3rd Qu.:0.043877
##  Max.   :1000000                      Max.   :0.242869
```

```r
print(df)
```

```
##       Size       Type     Time
## 1      100 Sequential 0.000010
## 2      100   Parallel 0.004024
## 3      100   Built-in 0.000013
## 4      100 Sequential 0.000010
```

```
## 5      100   Parallel 0.004448
## 6      100   Built-in 0.000014
## 7      100 Sequential 0.000009
## 8      100   Parallel 0.003384
## 9      100   Built-in 0.000013
## 10     100 Sequential 0.000010
## 11     100   Parallel 0.003738
## 12     100   Built-in 0.000012
## 13     100 Sequential 0.000010
## 14     100   Parallel 0.003133
## 15     100   Built-in 0.000011
## 16    1000 Sequential 0.000128
## 17    1000   Parallel 0.020407
## 18    1000   Built-in 0.000209
## 19    1000 Sequential 0.000126
## 20    1000   Parallel 0.022003
## 21    1000   Built-in 0.000201
## 22    1000 Sequential 0.000128
## 23    1000   Parallel 0.016149
## 24    1000   Built-in 0.000210
## 25    1000 Sequential 0.000128
## 26    1000   Parallel 0.014594
## 27    1000   Built-in 0.000209
## 28    1000 Sequential 0.000129
## 29    1000   Parallel 0.014905
## 30    1000   Built-in 0.000210
## 31   10000 Sequential 0.001774
## 32   10000   Parallel 0.018943
## 33   10000   Built-in 0.001720
## 34   10000 Sequential 0.001698
## 35   10000   Parallel 0.016226
## 36   10000   Built-in 0.001733
## 37   10000 Sequential 0.001652
## 38   10000   Parallel 0.017348
## 39   10000   Built-in 0.001702
## 40   10000 Sequential 0.001680
## 41   10000   Parallel 0.017302
## 42   10000   Built-in 0.001726
## 43   10000 Sequential 0.001675
## 44   10000   Parallel 0.017386
## 45   10000   Built-in 0.001716
## 46  100000 Sequential 0.020040
## 47  100000   Parallel 0.050548
## 48  100000   Built-in 0.020300
## 49  100000 Sequential 0.020004
## 50  100000   Parallel 0.043119
## 51  100000   Built-in 0.020504
## 52  100000 Sequential 0.019763
## 53  100000   Parallel 0.050735
## 54  100000   Built-in 0.020439
## 55  100000 Sequential 0.019913
## 56  100000   Parallel 0.049806
## 57  100000   Built-in 0.020541
## 58  100000 Sequential 0.019726
```

```
## 59  100000   Parallel 0.044636
## 60  100000   Built-in 0.020252
## 61 1000000 Sequential 0.230648
## 62 1000000   Parallel 0.162221
## 63 1000000   Built-in 0.242869
## 64 1000000 Sequential 0.235778
## 65 1000000   Parallel 0.162137
## 66 1000000   Built-in 0.241607
## 67 1000000 Sequential 0.238383
## 68 1000000   Parallel 0.163279
## 69 1000000   Built-in 0.242786
## 70 1000000 Sequential 0.232921
## 71 1000000   Parallel 0.170237
## 72 1000000   Built-in 0.241583
## 73 1000000 Sequential 0.230096
## 74 1000000   Parallel 0.153896
## 75 1000000   Built-in 0.242492
```

here we calculate the mean and the standard deviation, the confidence interval for time .

```r
dfsum <- group_by(df, Size, Type) %>%
  summarise(num = n(), mean = mean(Time), sd = sd(Time), se = 2*sd/sqrt(num),
            .groups = 'drop')
```

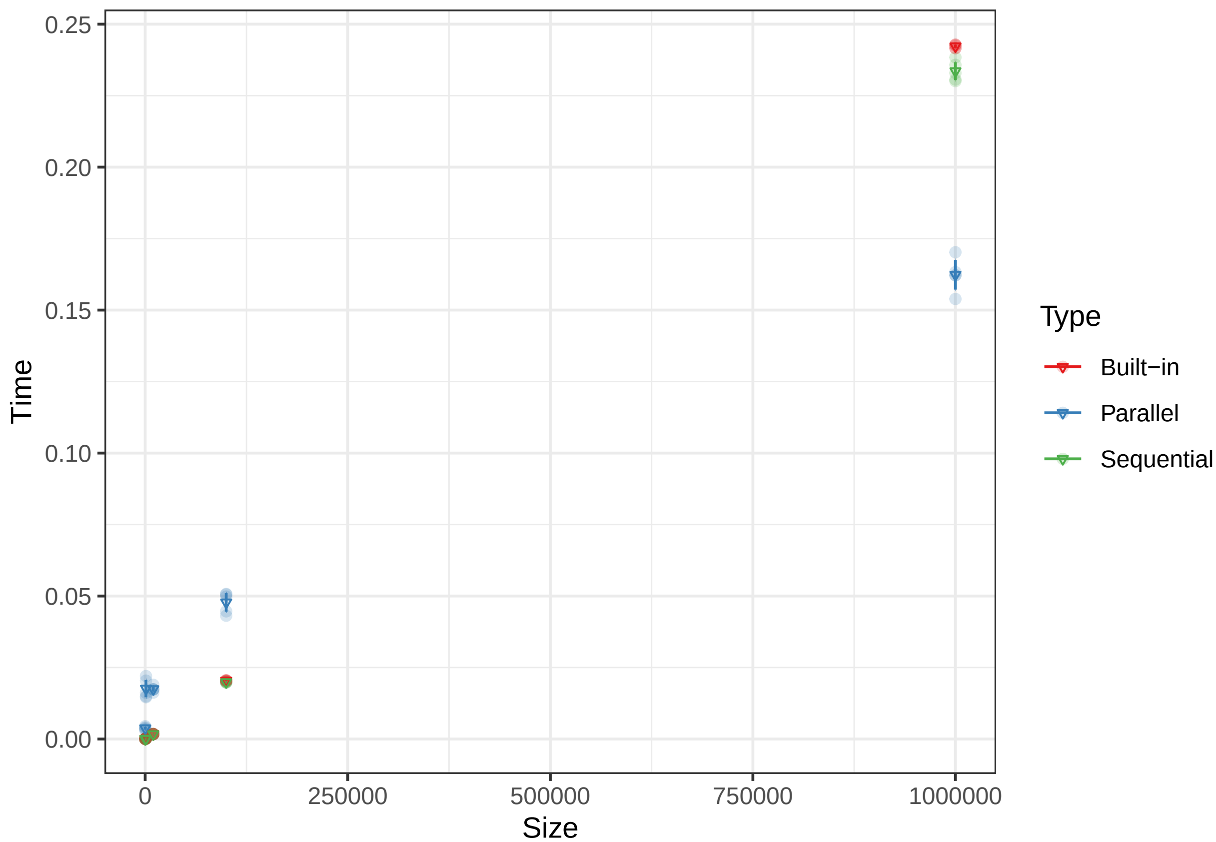
```r
print(dfsum)
```

```
## # A tibble: 15 x 6
##       Size Type         num      mean          sd           se
##      <int> <chr>      <int>     <dbl>       <dbl>        <dbl>
##  1     100 Built-in       5 0.0000126 0.00000114  0.00000102
##  2     100 Parallel       5 0.00375   0.000519    0.000464
##  3     100 Sequential     5 0.0000098 0.000000447 0.000000400
##  4    1000 Built-in       5 0.000208  0.00000383  0.00000343
##  5    1000 Parallel       5 0.0176    0.00338     0.00302
##  6    1000 Sequential     5 0.000128  0.00000110  0.000000980
##  7   10000 Built-in       5 0.00172   0.0000117   0.0000104
##  8   10000 Parallel       5 0.0174    0.000970    0.000868
##  9   10000 Sequential     5 0.00170   0.0000467   0.0000418
## 10  100000 Built-in       5 0.0204    0.000126    0.000113
## 11  100000 Parallel       5 0.0478    0.00361     0.00323
## 12  100000 Sequential     5 0.0199    0.000141    0.000126
## 13 1000000 Built-in       5 0.242     0.000630    0.000563
## 14 1000000 Parallel       5 0.162     0.00580     0.00519
## 15 1000000 Sequential     5 0.234     0.00350     0.00313
```

here we will plot the data with different colors for each unique value in the type column.

first graph size with time without linear regression .

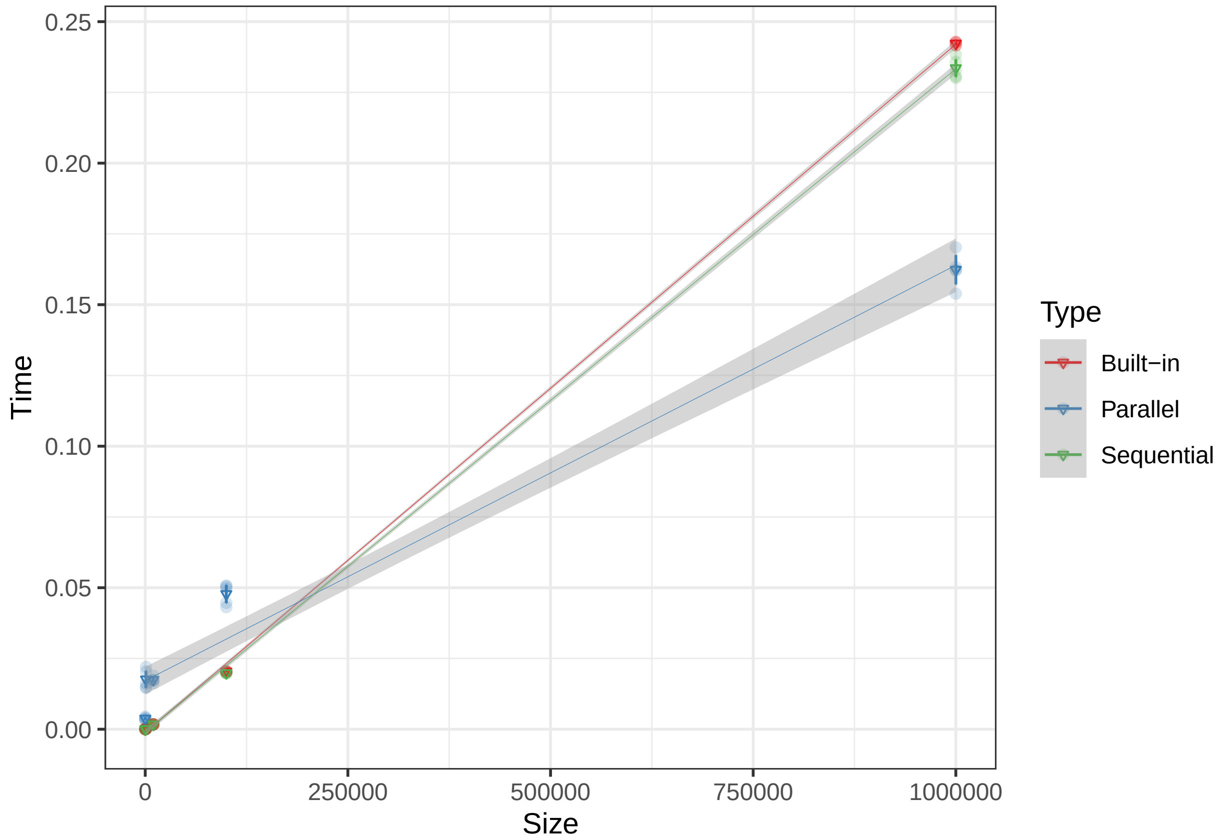
```r
ggplot(df,aes(x=Size,y=Time,color=Type)) +
  scale_color_brewer(palette="Set1") + theme_bw() +
  geom_jitter(alpha=.2,position=position_jitter(width = 0.1)) +
  geom_errorbar(data=dfsum,width=0.1, aes(y=mean,ymin=mean-se,ymax=mean+se)) +
  geom_point(data=dfsum,shape=25, size=1, aes(y=mean,color=Type))
```



second graph size and time with linear regression.

```r
ggplot(df,aes(x=Size,y=Time,color=Type)) +
  scale_color_brewer(palette="Set1") + theme_bw() +
  geom_jitter(alpha=.2,position = position_jitter(width = 0.1)) +
  geom_errorbar(data=dfsum,width=0.1, aes(y=mean,ymin=mean-se,ymax=mean+se)) +
  geom_point(data=dfsum,shape=25, size=1, aes(y=mean,color=Type))+
  geom_smooth(method="lm",linewidth=0.1)
```

```
## `geom_smooth()` using formula = 'y ~ x'
```



Note that the `echo = FALSE` parameter was added to the code chunk to prevent printing of the R code that generated the plot.

First we will see the Parallel method for Quicksort algorithm.

We can start with Filter type column .

```r
dfforlm = df %>% filter(Type == "Parallel")
print(dfforlm)
```

```
##       Size     Type     Time
## 1      100 Parallel 0.004024
## 2      100 Parallel 0.004448
## 3      100 Parallel 0.003384
## 4      100 Parallel 0.003738
## 5      100 Parallel 0.003133
## 6     1000 Parallel 0.020407
## 7     1000 Parallel 0.022003
## 8     1000 Parallel 0.016149
## 9     1000 Parallel 0.014594
## 10    1000 Parallel 0.014905
## 11   10000 Parallel 0.018943
## 12   10000 Parallel 0.016226
## 13   10000 Parallel 0.017348
```

```
## 14   10000 Parallel 0.017302
## 15   10000 Parallel 0.017386
## 16  100000 Parallel 0.050548
## 17  100000 Parallel 0.043119
## 18  100000 Parallel 0.050735
## 19  100000 Parallel 0.049806
## 20  100000 Parallel 0.044636
## 21 1000000 Parallel 0.162221
## 22 1000000 Parallel 0.162137
## 23 1000000 Parallel 0.163279
## 24 1000000 Parallel 0.170237
## 25 1000000 Parallel 0.153896
```

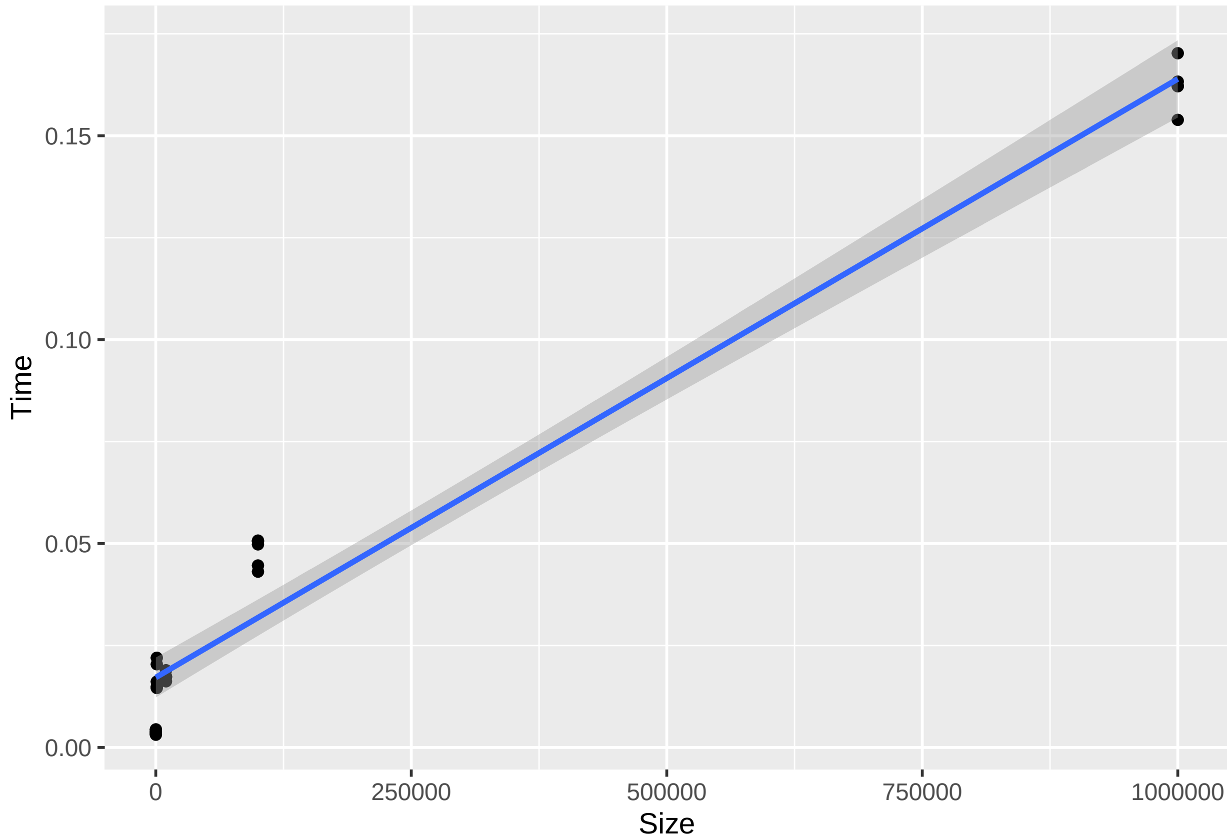
we will try to use linear regression for there type of data wth simple function.

```r
reg <- lm(Time ~ Size,data = dfforlm)
summary(reg)
```

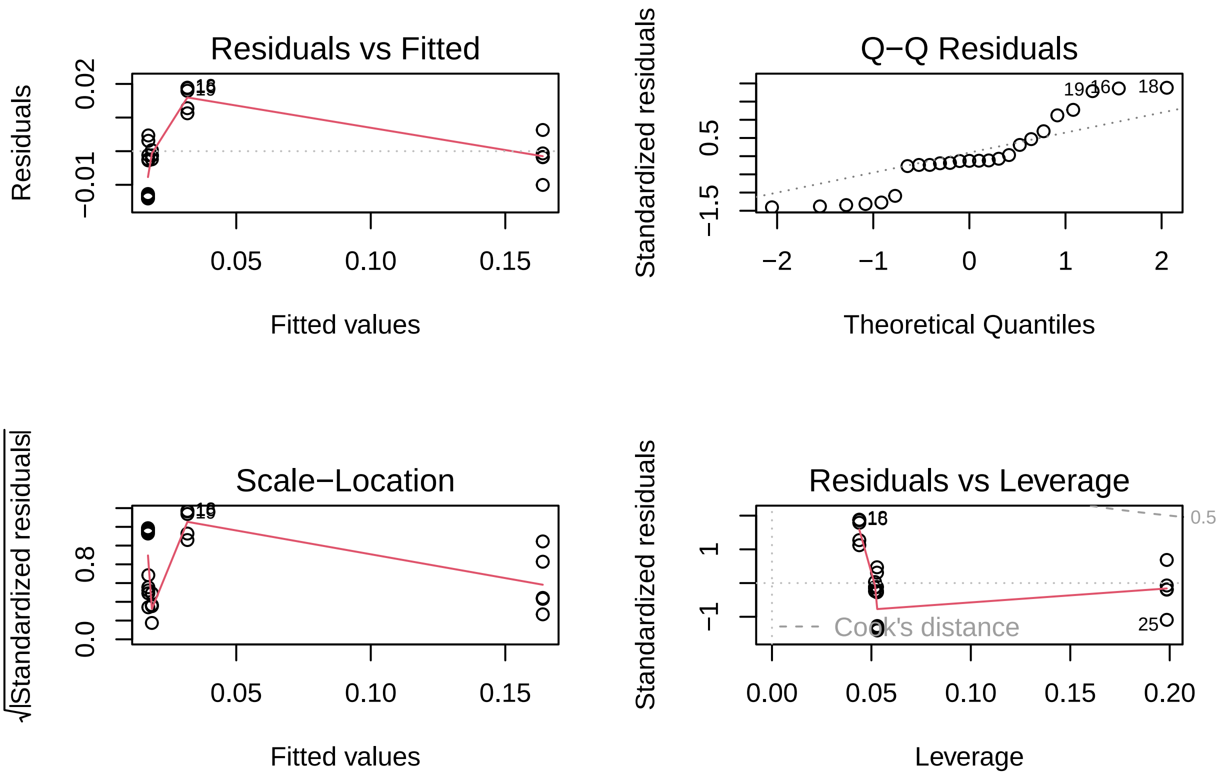
```
##
## Call:
## lm(formula = Time ~ Size, data = dfforlm)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.014052 -0.002723 -0.001290  0.004686  0.018888
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) 1.717e-02  2.364e-03   7.263 2.16e-07 ***
## Size        1.468e-07  5.260e-09  27.904  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.01027 on 23 degrees of freedom
## Multiple R-squared:  0.9713, Adjusted R-squared:  0.9701
## F-statistic: 778.6 on 1 and 23 DF,  p-value: < 2.2e-16
```

so what we knew from the LR model is that we can see the estimated values for slope and the constant the value of time when size is zero are quite small also what is the range for these values +-std error, we can see the test like F test, and the p-value is too small means reject the null hypothesis (consider the coefficients zero)

```r
ggplot(dfforlm,aes(x = Size, y = Time)) + geom_point() + stat_smooth(method = "lm"
                                                                     , formula = y ~ x,geom = "smooth")
```




```r
par(mfrow=c(2,2));plot(reg);par(mfrow=c(1,1))
```



from the above graph fitted value vs residuals I am really not sure here is there pattern or not we need more measurements for this situation in different sizes like n the middel.

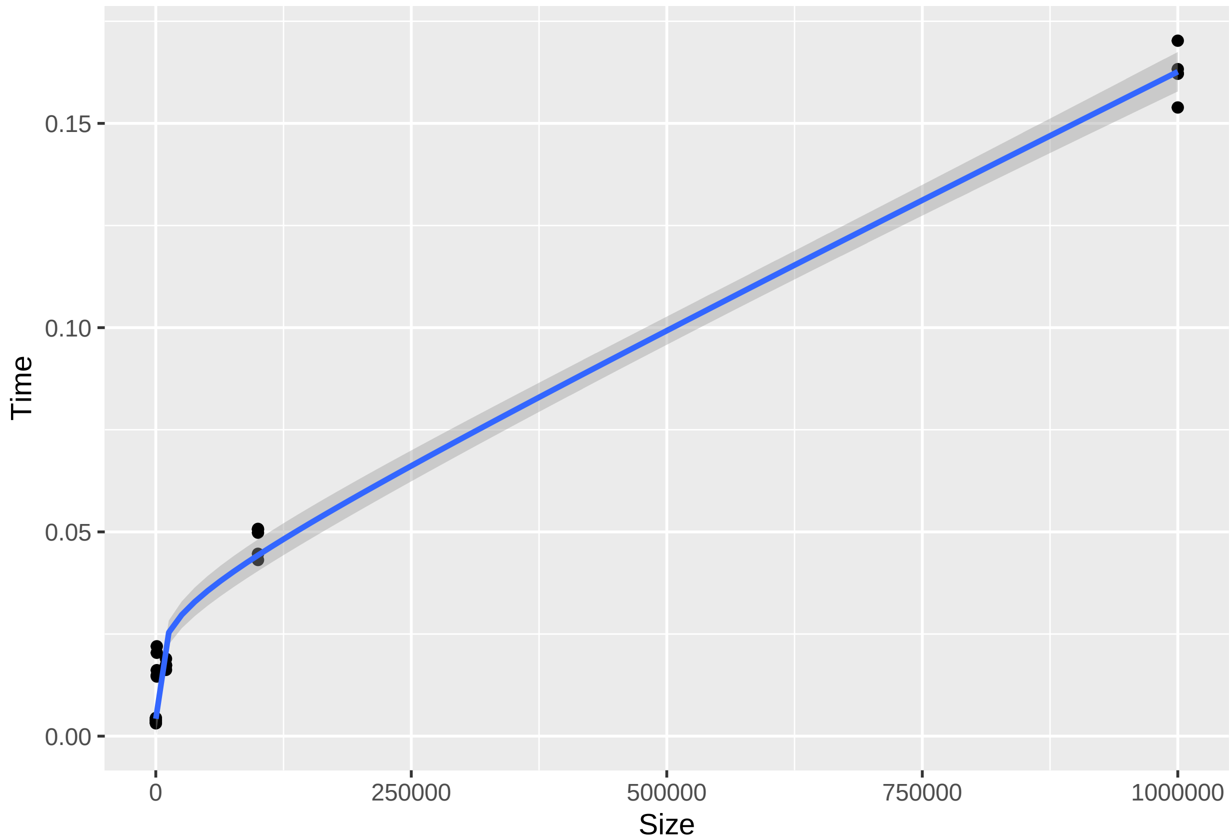
now let's see another function with a linear regression model but now we make it more oriented to our problem since the complexity for the quick sort is n log n approximately.

```
reg2 <- lm(Time ~ log(Size)+Size^2,data = dfforlm)
summary(reg2)
```

```
##
## Call:
## lm(formula = Time ~ log(Size) + Size^2, data = dfforlm)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0087444 -0.0011620 -0.0002334  0.0024763  0.0083303
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.437e-02  4.022e-03  -3.572   0.0017 **
## log(Size)    4.042e-03  4.920e-04   8.214  3.8e-08 ***
## Size         1.212e-07  4.101e-09  29.546  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

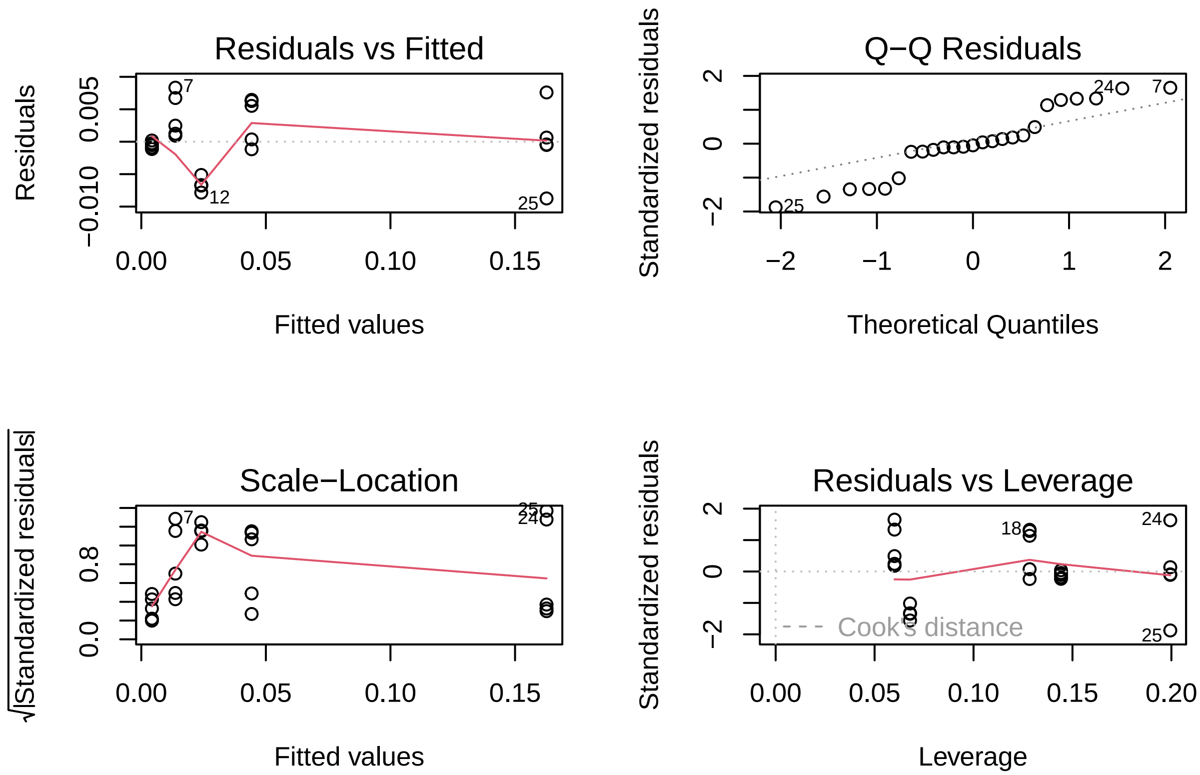
```
##
## Residual standard error: 0.005209 on 22 degrees of freedom
## Multiple R-squared:  0.9929, Adjusted R-squared:  0.9923
## F-statistic:  1548 on 2 and 22 DF,  p-value: < 2.2e-16
```

```
ggplot(dfforlm,aes(x = Size, y = Time)) + geom_point() + stat_smooth(method = "lm", formula = y ~ log(x)
```



we can see how is the model fit the points very well but can we consider the model good ? maybe , but let see the residual points and the summary of the model.

```
par(mfrow=c(2,2));plot(reg2);par(mfrow=c(1,1))
```



From the residuals vs fitted value graph, it is apparent that there is no discernible pattern, resembling more of a random noise distribution. This observation suggests that the model performs well, exhibiting a good fit. However, it is crucial to conduct further experimentation, particularly in the mid-range of values. Drawing a line between two points may seem straightforward, but additional experiments are warranted to confirm the model's reliability, especially as the line begins to exhibit curvature.

here we will use n*log n equation.

```r
reg6 <- lm(Time ~ log(Size)*Size,data = dfforlm)
summary(reg6)
```

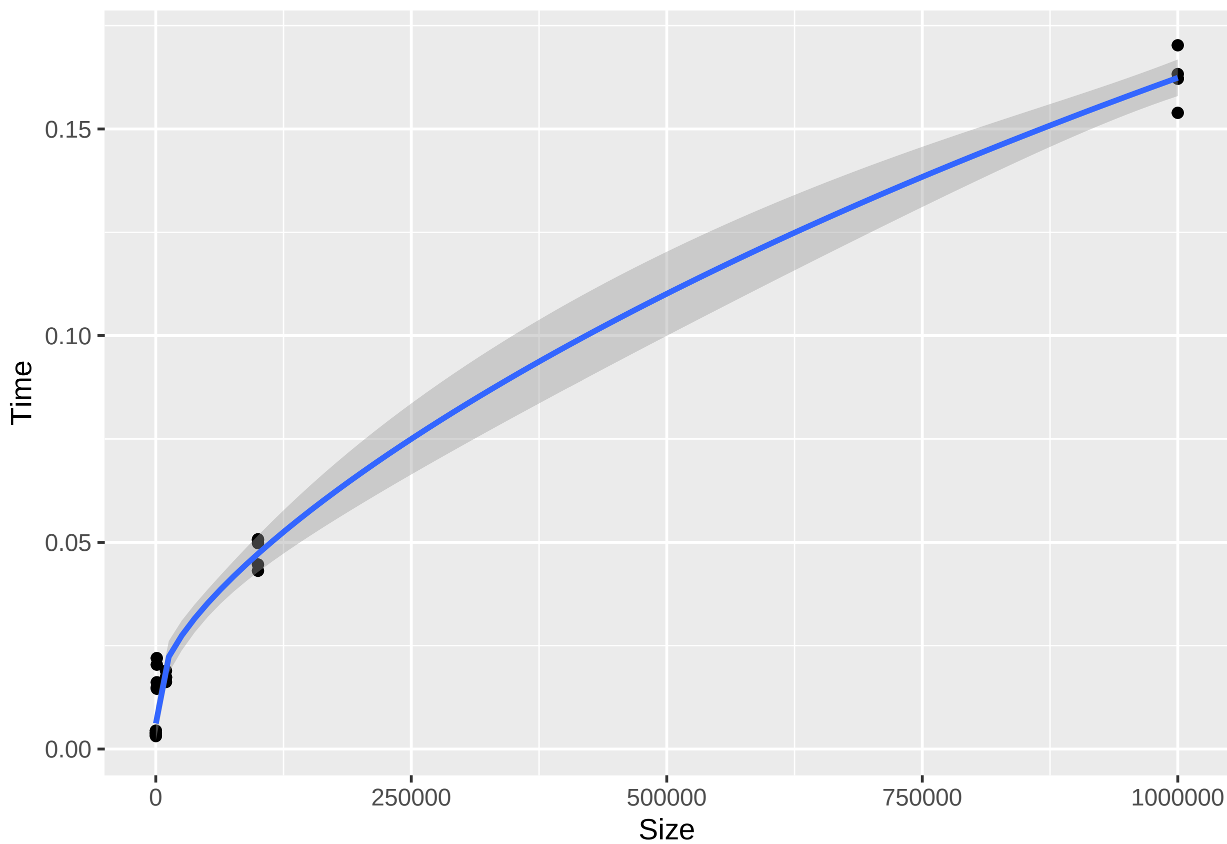
```
##
## Call:
## lm(formula = Time ~ log(Size) * Size, data = dfforlm)
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.008483 -0.003062 -0.001747  0.002682  0.009780
##
## Coefficients:
##                   Estimate Std. Error t value Pr(>|t|)
## (Intercept)     -5.113e-03  5.393e-03  -0.948  0.35386
## log(Size)        2.443e-03  8.171e-04   2.990  0.00698 **
```

```
## Size             7.842e-07  2.834e-07   2.768  0.01153 *
## log(Size):Size -4.708e-08  2.012e-08  -2.340  0.02922 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.004749 on 21 degrees of freedom
## Multiple R-squared:  0.9944, Adjusted R-squared:  0.9936
## F-statistic:  1244 on 3 and 21 DF,  p-value: < 2.2e-16
```

Question for me:

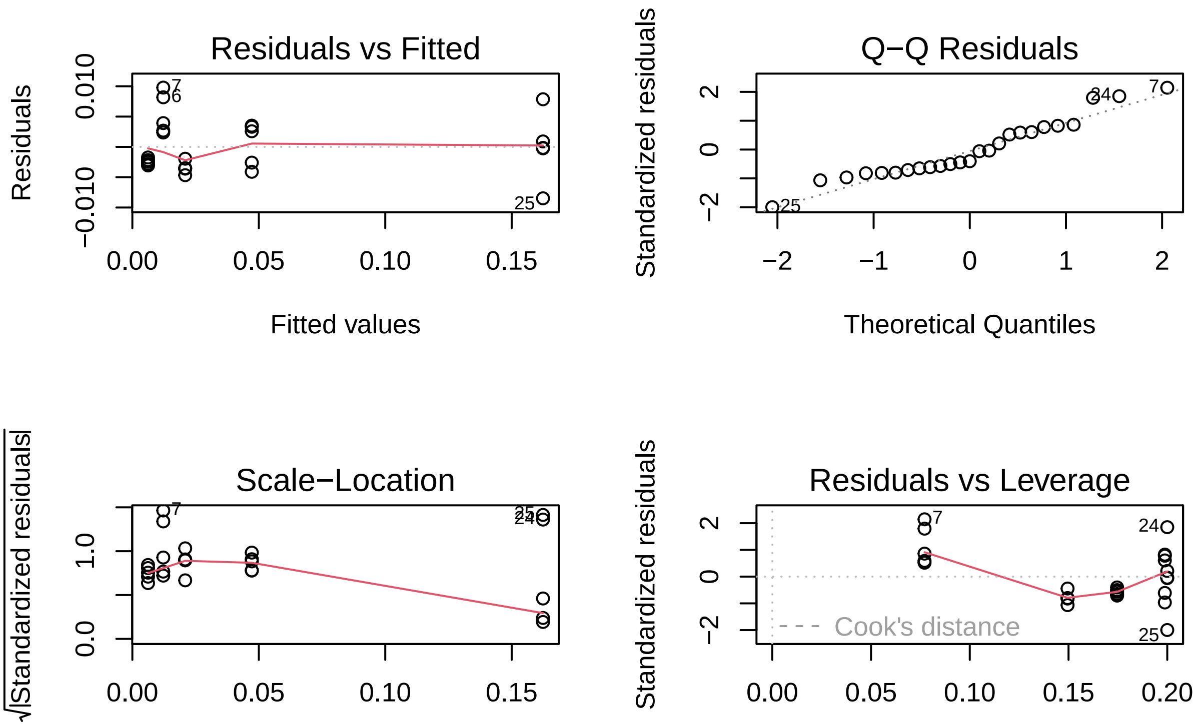
I would like you to give me your opinion about these results and why the pr not too small as other functions.

```
ggplot(dfforlm,aes(x = Size, y = Time)) + geom_point() + stat_smooth(method = "lm",
                                                          formula = y ~ log(x)*x, geom = "smooth")
```



see the graphs to evalute the model.

```
par(mfrow=c(2,2));plot(reg6);par(mfrow=c(1,1))
```



we can see how the residual vs fitted is almost noise and the model is good but still we need more measurements in the middle and around the middle but the shape of the function related to our problem that will help us to consider it good, also from the R-squared: 0.9944 we see it above for it.

we will try with Sequential method for QuickSort Algorithm.

filter first on type column.

```
dfforlm2 = df %>% filter(Type == "Sequential")
print(dfforlm2)
```

```
##       Size        Type     Time
## 1      100 Sequential 0.000010
## 2      100 Sequential 0.000010
## 3      100 Sequential 0.000009
## 4      100 Sequential 0.000010
## 5      100 Sequential 0.000010
## 6     1000 Sequential 0.000128
## 7     1000 Sequential 0.000126
## 8     1000 Sequential 0.000128
## 9     1000 Sequential 0.000128
## 10    1000 Sequential 0.000129
```

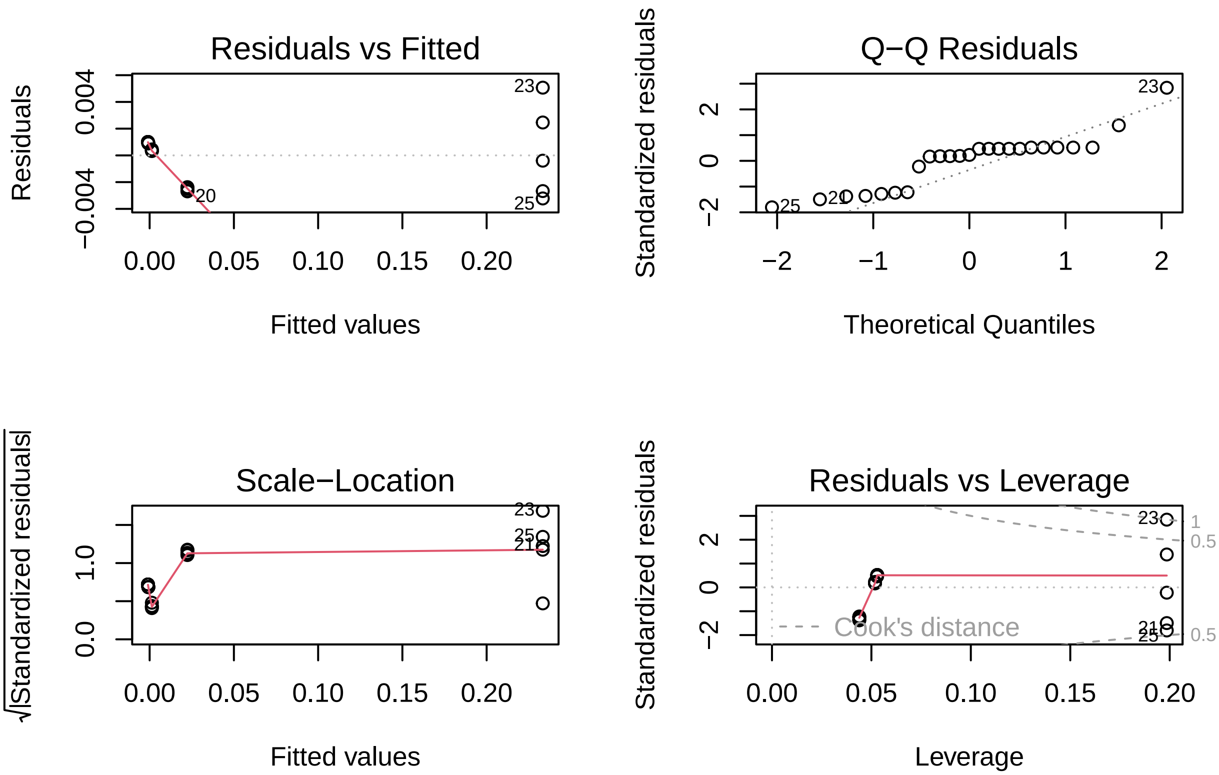
```
## 11   10000 Sequential 0.001774
## 12   10000 Sequential 0.001698
## 13   10000 Sequential 0.001652
## 14   10000 Sequential 0.001680
## 15   10000 Sequential 0.001675
## 16  100000 Sequential 0.020040
## 17  100000 Sequential 0.020004
## 18  100000 Sequential 0.019763
## 19  100000 Sequential 0.019913
## 20  100000 Sequential 0.019726
## 21 1000000 Sequential 0.230648
## 22 1000000 Sequential 0.235778
## 23 1000000 Sequential 0.238383
## 24 1000000 Sequential 0.232921
## 25 1000000 Sequential 0.230096
```

the linear model algo wwith summary and some graphs about how good our model .

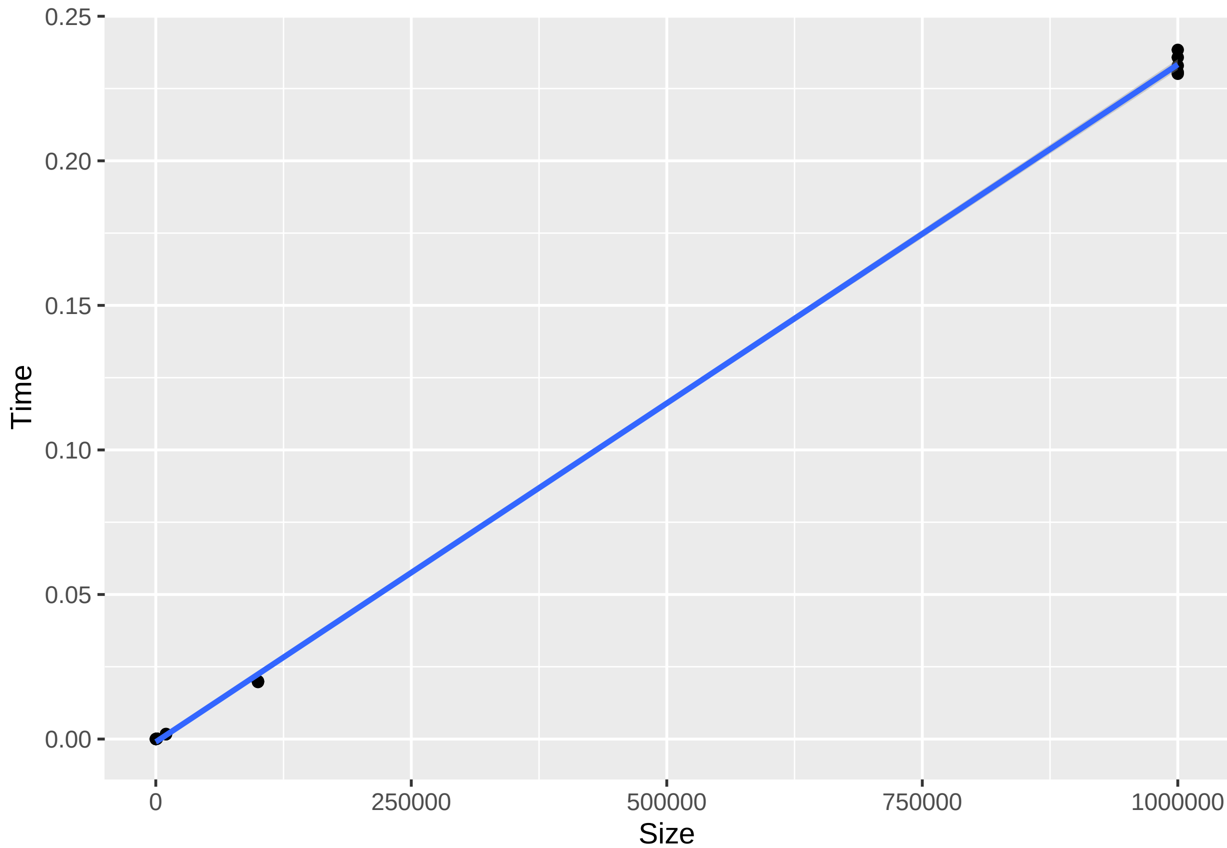
```
reg3 <- lm(Time ~ Size,data = dfforlm2)
summary(reg3)
```

```
##
## Call:
## lm(formula = Time ~ Size, data = dfforlm2)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0032211 -0.0023774  0.0004466  0.0010015  0.0050659
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.016e-03  4.583e-04  -2.217   0.0368 *
## Size         2.343e-07  1.020e-09 229.807   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.001992 on 23 degrees of freedom
## Multiple R-squared:  0.9996, Adjusted R-squared:  0.9995
## F-statistic: 5.281e+04 on 1 and 23 DF,  p-value: < 2.2e-16
```

```
par(mfrow=c(2,2));plot(reg3);par(mfrow=c(1,1))
```




```
ggplot(dfforlm2,aes(x = Size, y = Time)) + geom_point() + stat_smooth(method = "lm", formula = y ~ x,ge
```

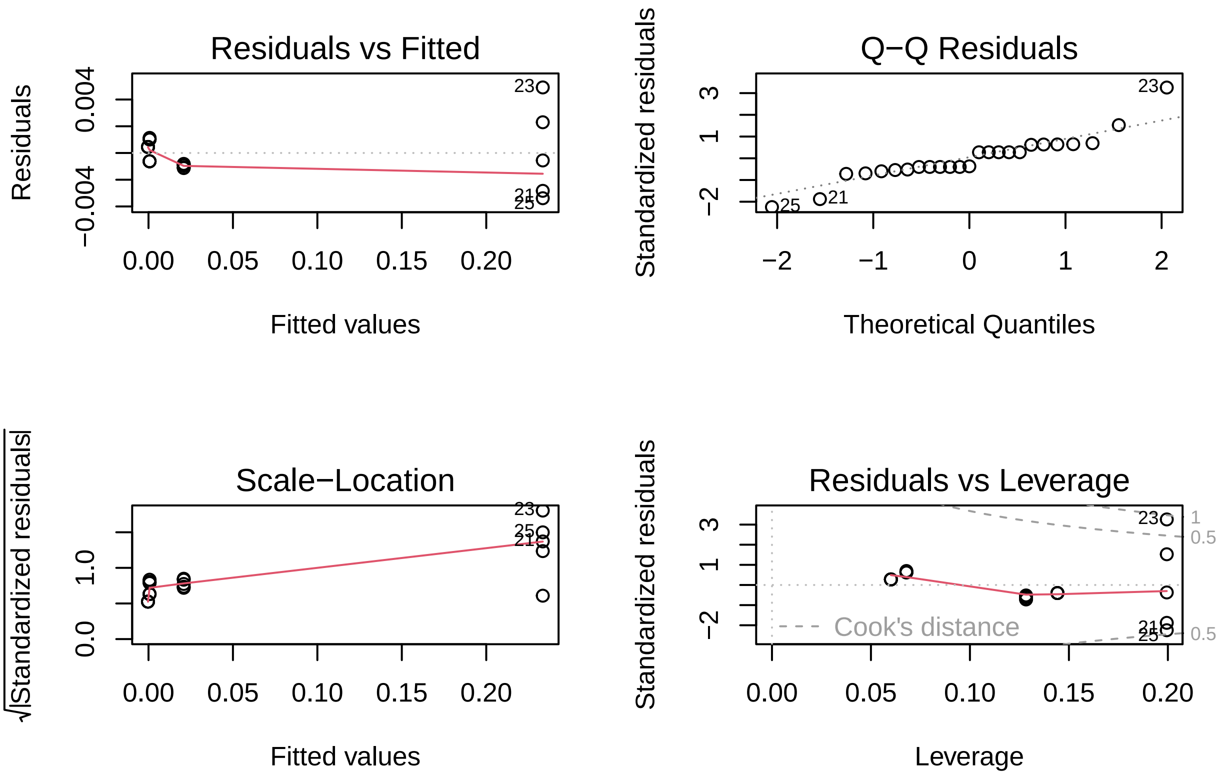


The linear model with different equation.

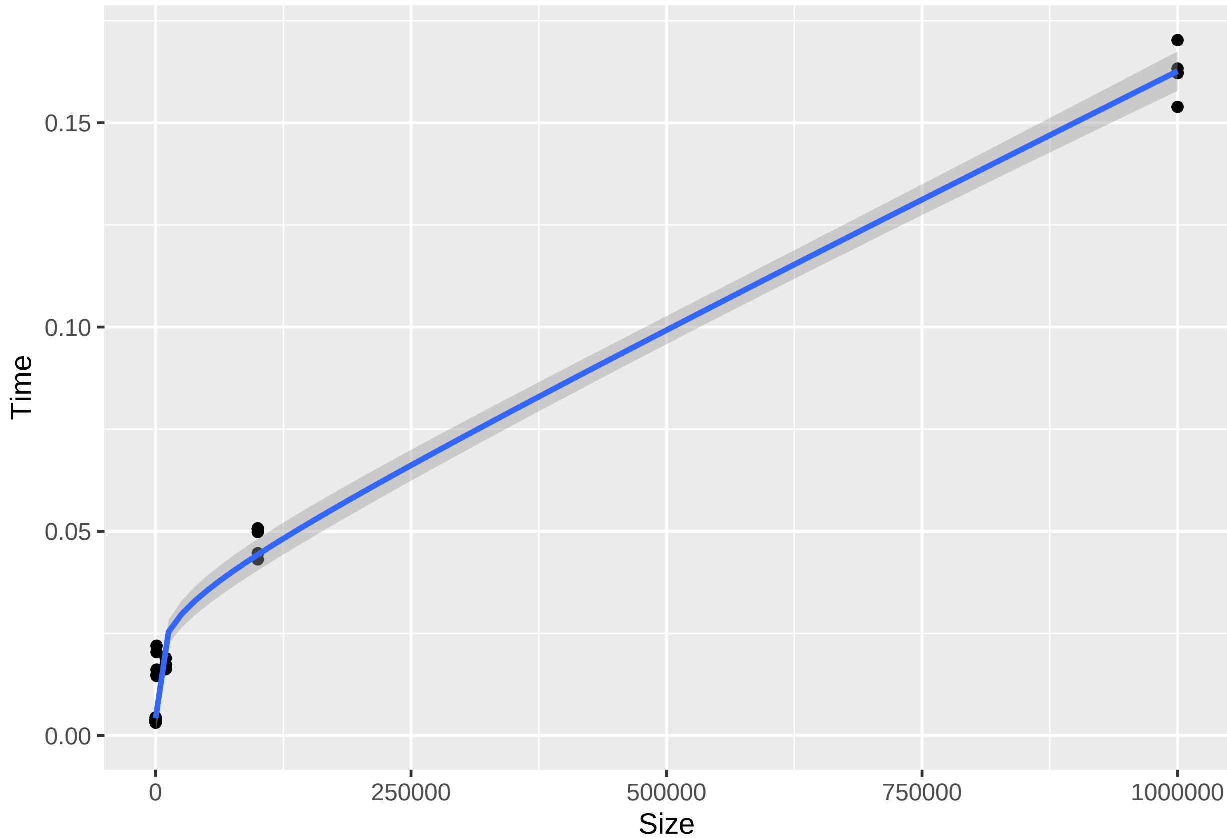
```r
reg4 <- lm(Time ~ log(Size)+Size^2,data = dfforlm2)
summary(reg4)
```

```
##
## Call:
## lm(formula = Time ~ log(Size) + Size^2, data = dfforlm2)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -0.0033836 -0.0008145 -0.0005586  0.0010073  0.0049034
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.948e-03  1.300e-03   2.268  0.03347 *
## log(Size)   -5.080e-04  1.590e-04  -3.195  0.00418 **
## Size         2.376e-07  1.325e-09 179.264  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.001683 on 22 degrees of freedom
## Multiple R-squared:  0.9997, Adjusted R-squared:  0.9997
## F-statistic: 3.698e+04 on 2 and 22 DF,  p-value: < 2.2e-16
```

```r
par(mfrow=c(2,2));plot(reg4);par(mfrow=c(1,1))
```




```r
ggplot(dfforlm,aes(x = Size, y = Time)) + geom_point() + stat_smooth(method = "lm", formula = y ~ log(x)
                                                                     geom = "smooth")
```


0.15
0.10
0.05
0.00
Time
0
250000
500000
750000
1000000
Size