



Algorithms: Design
and Analysis, Part II

Greedy Algorithms

A Scheduling Application:
Correctness Proof Part I

Correctness Claim

Claim: Algorithm #2 (order jobs according to decreasing ratios w_i/e_j) is always correct.

Proof: by an Exchange Argument.

Plan: Fix arbitrary input of n jobs.

Will proceed by contradiction.

Let σ = greedy schedule, σ^* = optimal schedule.

Will produce schedule even better than σ^* , contradicting purported optimality of σ^* .

(with σ^*
better than
 σ^*)

Correctness Proof

Assume: all w_j/d_j 's distinct.

Assume: [just by renaming jobs] $\frac{w_1}{d_1} > \frac{w_2}{d_2} > \dots > \frac{w_n}{d_n}$

Thus: greedy schedule σ is just $1, 2, 3, \dots, n$.

Thus: if optimal schedule $\sigma^* \neq \sigma$, then there are consecutive jobs i, j with $i > j$.

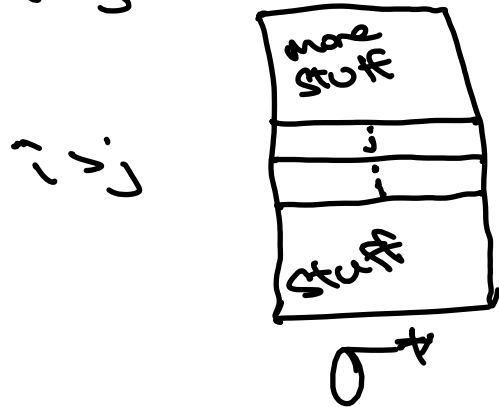
[only schedule where indices always go up is $1, 2, 3, \dots, n$]

Correctness Proof (con'd)

So far: ① $\frac{w_1}{e_1} > \frac{w_2}{e_2} > \dots > \frac{w_n}{e_n}$

② in optimal σ^* , \exists consecutive jobs $i:j$ with $i > j$.

Thought experiment: Suppose we **exchange** order of $i:j$ in σ^* (leaving other jobs unchanged):



exchange
 $i:j$

