

CSEN 1002

Task 4: CFG Epsilon & Unit Rules Elimination ¹

¹These slides are based on Lecture 10 of CSEN 502 by Assoc. Prof. Haythem O. Ismail

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- ③ R is the set of **production rules**.
 - Represented in the format $V \longrightarrow (\Sigma \cup V)^*$
- ④ S is the **start variable**

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 - **Note:** This should be done for each occurrence of A on the right-hand side of a rule. That is, it should be repeated for each possible choice of u and v .

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 - **Note:** This should be done for each occurrence of A on the right-hand side of a rule. That is, it should be repeated for each possible choice of u and v .
 - ❸ If $B \rightarrow A \in R$, then unless $B \rightarrow \varepsilon$ has already been removed, let $R = R \cup \{B \rightarrow \varepsilon\}$.

Example

$G_1 :$

$$\begin{array}{lcl} S & \longrightarrow & ASA \mid \mathbf{a}B \\ A & \longrightarrow & B \mid S \\ B & \longrightarrow & \mathbf{b} \mid \varepsilon \end{array}$$

Example

$G_1 :$

$$\begin{aligned} S &\longrightarrow ASA \mid aB \\ A &\longrightarrow B \mid S \\ B &\longrightarrow b \mid \varepsilon \end{aligned}$$

$G_2 :$

$$\begin{aligned} S &\longrightarrow ASA \mid a \mid aB \\ A &\longrightarrow B \mid S \mid \varepsilon \\ B &\longrightarrow b \end{aligned}$$

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$G_2 :$

$$\begin{aligned} S &\longrightarrow \mathbf{AS} \mid ASA \mid \mathbf{S} \mid \mathbf{SA} \mid a \mid aB \\ A &\longrightarrow B \mid S \\ B &\longrightarrow b \end{aligned}$$

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 - ❷ For every rule of the form $B \longrightarrow u \in R$ (where $u \in (V_1 \cup \Sigma)^+$ and $u \notin V$), let $R = R \cup \{A \longrightarrow u\}$.
 - ❸ For every rule of the form $B \longrightarrow C \in R$ (where $C \in V$), then unless $A \longrightarrow C$ has already been removed, let $R = R \cup \{A \longrightarrow C\}$.

Example (Cont'd)

 $G_2 :$

$$\begin{array}{lcl} S & \longrightarrow & AS \mid ASA \mid S \mid SA \mid \mathbf{a} \mid \mathbf{a}B \\ A & \longrightarrow & B \mid S \\ B & \longrightarrow & \mathbf{b} \end{array}$$

Example (Cont'd)

$G_2 :$

$$\begin{aligned} S &\longrightarrow AS \mid ASA \mid S \mid SA \mid \mathbf{a} \mid \mathbf{a}B \\ A &\longrightarrow B \mid S \\ B &\longrightarrow \mathbf{b} \end{aligned}$$

$G_3 :$

$$\begin{aligned} S &\longrightarrow AS \mid ASA \mid SA \mid \mathbf{a} \mid \mathbf{a}B \\ A &\longrightarrow \mathbf{A}S \mid \mathbf{A}SA \mid \mathbf{S}A \mid \mathbf{a} \mid \mathbf{a}B \mid \mathbf{b} \\ B &\longrightarrow \mathbf{b} \end{aligned}$$

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CFG String Representation

G_1 :

$$\begin{aligned} S &\longrightarrow ASA \mid aB \\ A &\longrightarrow B \mid S \\ B &\longrightarrow b \mid \varepsilon \end{aligned}$$

String Representation

S;A;B#a;b#S/ASA,aB;A/B,S;B/b,e

CFG String Representation

G_3 :

$$\begin{aligned} S &\longrightarrow AS \mid ASA \mid SA \mid a \mid aB \\ A &\longrightarrow AS \mid ASA \mid SA \mid a \mid aB \mid b \\ B &\longrightarrow b \end{aligned}$$

String Representation

$S; A; B \# a; b \# S/AS, ASA, SA, a, aB; A/AS, ASA, SA, a, aB, b; B/b$