

# ML-#2 - Home Work

## 1. Decision Region

$x_1$	$x_2$	class
4	4	pos
4	6	pos
6	2	neg

a) (4,4) (4,6)

$$(x_1 - 4)^2 + (x_2 - 4)^2 \leq (x_1 - 4)^2 + (x_2 - 6)^2$$

$$x_2^2 - 8x_2 + 16 \leq x_2^2 - 12x_2 + 36$$

$$4x_2 - 20 \leq 0$$

$$x_2 - 5 \leq 0$$

b) (4,6) (6,2)

$$(x_1 - 4)^2 + (x_2 - 6)^2 \leq (x_1 - 6)^2 + (x_2 - 2)^2$$

$$x_1^2 - 8x_1 + 16 + x_2^2 - 12x_2 + 36 \leq x_1^2 - 12x_1 + 36 + x_2^2 - 4x_2 + 4$$

$$4x_1 - 8x_2 + 12 \leq 0$$

$$x_1 - 2x_2 + 3 \leq 0$$

c) (6,2) (4,4)

$$(x_1 - 6)^2 + (x_2 - 2)^2 \leq (x_1 - 4)^2 + (x_2 - 4)^2$$

$$x_1^2 - 12x_1 + 36 + x_2^2 - 4x_2 + 4 \leq x_1^2 - 8x_1 + 16 + x_2^2 - 8x_2 + 16$$

$$-4x_1 + 4x_2 + 8 \leq 0$$

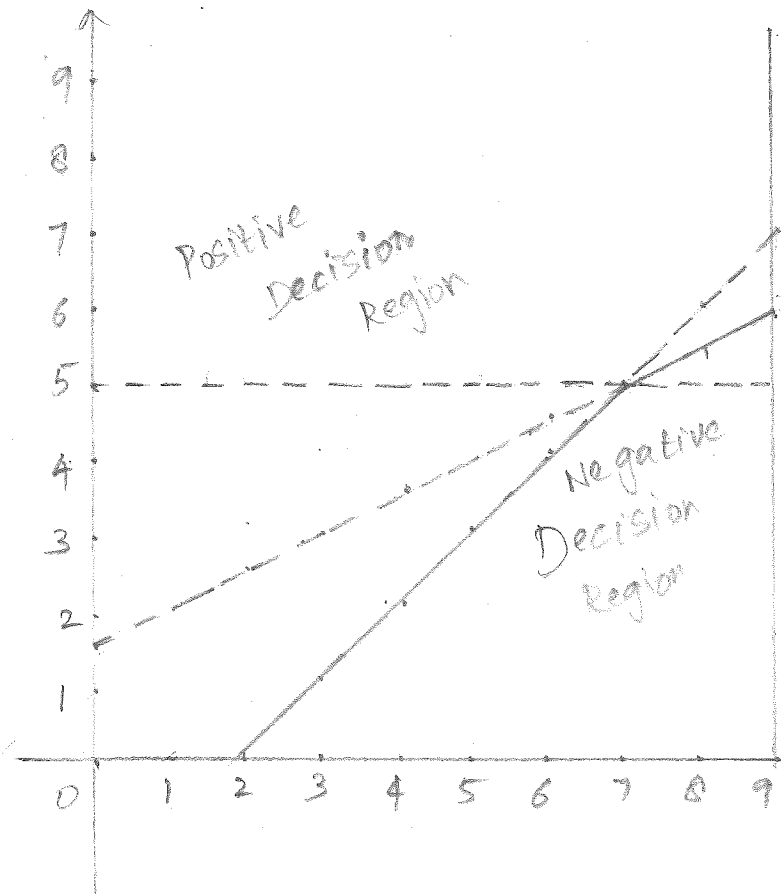
$$-x_1 + x_2 \leq -2$$

Decision boundary determined by the following equations

a)  $x_2 - 5 \leq 0$

b)  $x_1 - 2x_2 + 3 \leq 0$

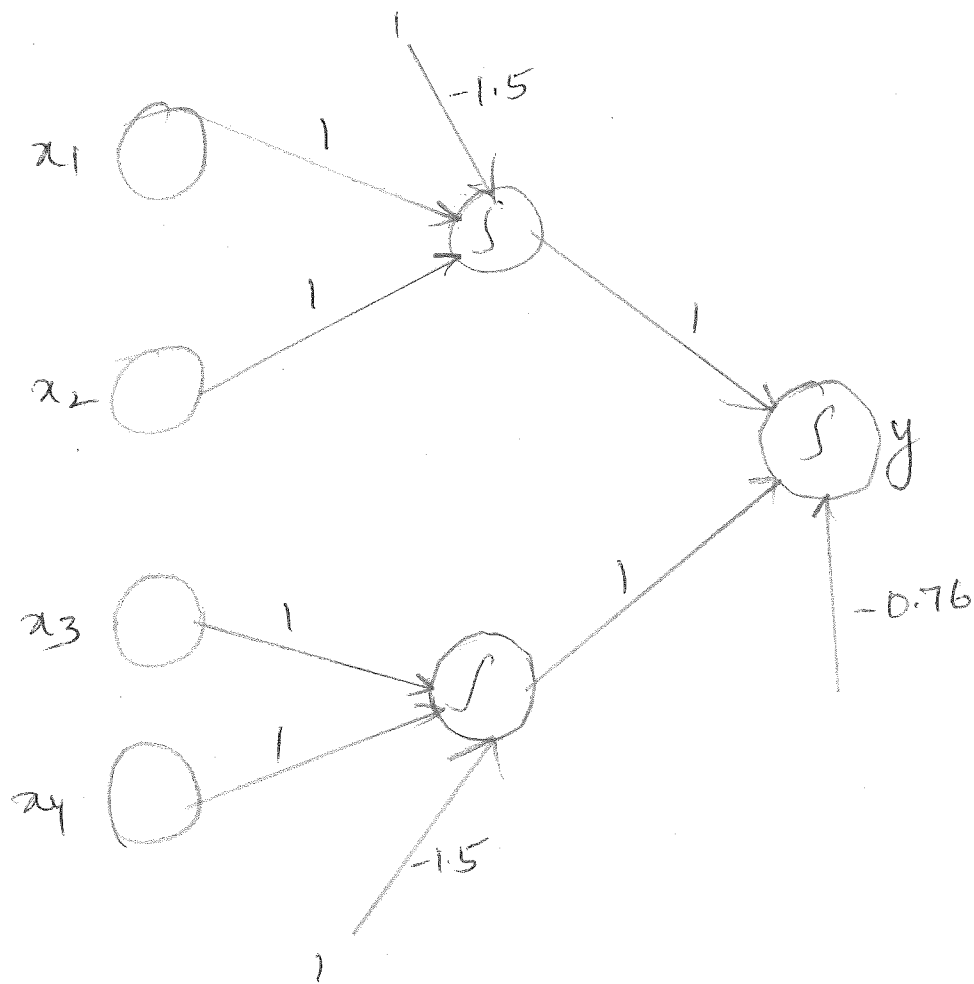
c)  $-x_1 + x_2 + 2 \leq 0$



2. Query Point : (7,10)

Operation	Priority Queue	Distance	Best Distance	Best Node
	(f,10)		∞	
pop f	(h,10)(c,11)	$5\sqrt{2}$	$5\sqrt{2}$	f
pop h	(i,10)(c,11)(g,5)	$5\sqrt{2}$	$5\sqrt{2}$	f
pop i	(c,11)(j,3)(g,5)	3	3	i
pop c	(b,10)(e,10)(j,3)(g,5)	2	2	c
pop b	(e,10)(j,3)(a,4)(g,5)	$2\sqrt{5}$	2	c
pop e	(d,10)(j,3)(a,4)(g,5)	$\sqrt{61}$	2	c
pop d	(j,3)(a,4)(g,5)	$\sqrt{29}$	2	c
pop j, return c				

3. Neural network for  $y = (x_1 \wedge x_2) \vee (x_3 \wedge x_4)$



4.

$$W_{CA} = 1$$

$$W_{CB} = -0.5$$

$$W_{EC} = -2$$

$$W_{FC} = -2$$

$$W_{DA} = -0.5$$

$$W_{DB} = 1$$

$$W_{ED} = 1$$

$$W_{FD} = -2$$

$$O_C = \frac{1}{1 + e^{-z}}$$

$$z = (0 \times 1) + (1 \times -0.5) + (1 \times -0.5) = -1$$

$$\Rightarrow O_C = \frac{1}{1 + e^{+1}} = 0.2689$$

$$O_D = \frac{1}{1 + e^{-(1 \times 1) + (0 \times -0.5) + (1 \times -1)}} = \frac{1}{1 + e^0} = 0.5$$

$$O_E = \frac{1}{1 + e^{-((1 \times 0) + (0.2689 \times -2) + (0.5 \times 1))}} = \frac{1}{1 + e^{-0.379}} = 0.4905$$

$$O_F = \frac{1}{1 + e^{-((1 \times -0.5) + (0.5 \times -2) + (0.2689 \times -2))}} = \frac{1}{1 + e^{-(-2.0378)}} = 0.1153$$

Back propagation

$$\Delta W_{FD} = \eta \delta_F \cdot O_D$$

$$\delta_F = 0.1153 \times (1 - 0.1153) \times (0 - 0.1153) = -0.1176$$

$$\Delta W_{FD} = 0.1 \times -0.1176 \times 0.5$$

$$= -0.000588 = -0.0006$$

$$W_{FD} = -2 + (-0.0006)$$

$$\underline{W_{FD} = -2.0006}$$

$$\Delta W_{DA} = \eta S_D O_A$$

$$S_D = O_D \times (1 - O_D) \times (g_E \times w_{ED}) + (g_F \times w_{FD})$$

$$S_E = 0.4905 \times (1 - 0.4905) \times (1 - 0.4905) = 0.1273$$

$$g_F = 0.1153 \times (1 - 0.1153) \times (0 - 0.1153) = -0.01176$$

$$S_D = 0.5 \times (1 - 0.5) \times ((0.1273 \times 1) + (-2 \times -0.01176))$$

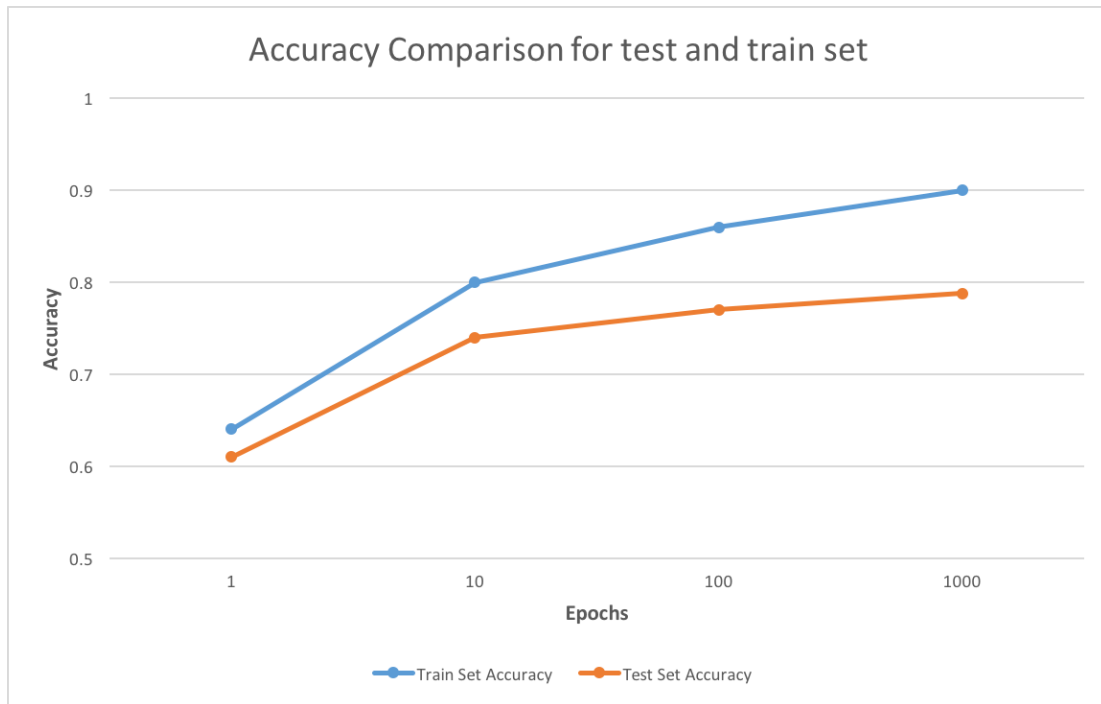
$$= 0.5 \times 0.5 \times (0.1273 + 0.02352)$$

$$= 0.37705$$

$$\Delta W_{DA} = 0.1 \times 0.37705 \times 0 = 0$$

$$\therefore W_{DA} = \underline{\underline{-0.5}}$$

## 5. Comparison of accuracies:



## 6. ROC Curve:

